SANDIA REPORT

SAND87–0043 • UC–37
Unlimited Release
Printed August 1987

A Tutorial on Rogowski Coil Theory and Operation

Charles R. McClenahan

Prepared by
Sandia National Laboratories
Albuquerque, New Mexico 87185 and Livermore, California 94550
for the United States Department of Energy
under Contract DE-AC04-76DP00789

SF2900Q(8-81)

Issued by Sandia National Laboratories, operated for the United States Department of Energy by Sandia Corporation.

Printed in the United States of America
Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road
Springfield, VA 22161

NTIS price codes
Printed copy: A03
Microfiche copy: A01

SAND--87-0043

DE88 000006

A Tutorial on Rogowski Coil Theory and Operation

Charles R. McClenahan

Simulation Applications Division, 1233
Sandia National Laboratories
P. O. Box 5800
Albuquerque, NM 87185

ABSTRACT

New staff members in our division often have little background in pulsed power diagnostics. Nevertheless, they must learn the techniques for measuring the electrical parameters of high voltage, pulsed power generators. This memorandum addresses this need by providing an elementary introduction to the technique of measuring current with Rogowski coils.

Without rigorously deriving all the equations that describe Rogowski coils, this memorandum describes how Rogowski coils work and why a Rogowski coil detects the charge passing through its toroidal opening yet remains insensitive to the details of the charge distribution.

Rogowski coils sense the time derivative of the current (dI/dt), and produce a proportional output. Nevertheless, an experimenter, usually, wants to measure the current. Often, a passive integrator used with a Rogowski coil can provide an output signal proportional to the current passing through the toroidal opening in the Rogowski coil.

In order to illustrate the use of Rogowski coils and integrators, this memorandum derives the equation for calculating the sensitivity of a Rogowski coil, and it compares the calculated and measured sensitivities of an actual Rogowski coil: The calculated sensitivity of 84 ± 13 mV-ms/kA agrees well with the measured sensitivity of 81 ± 2 mV-ms/kA. The integrator constructed for and used with the Rogowski coil described here has a measured time constant of 102 ± 4 ms. Therefore, to determine the current passing through the toroidal opening in the Rogowski coil, the experimenter multiplies the voltage at the output of the integrator by 1.26 ± 0.06 kA/mV.

One can design Rogowski coils by calculating the sensitivity. Nevertheless, the wise experimenter measures the sensitivities of his Rogowski coils whenever possible.

TABLE OF CONTENTS

LIST OF FIGURES

INTRODUCTION

At Sandia, Rogowski coils[1] often measure the large dI/dt's produced by modern pulsed power generators.[2] For example, Rogowski coils on the SPEED and HYDRAMITE accelerators have measured dI/dt's in excess of $1 \cdot 10^{13}$ A/s. Recently, Jim Ramsey of the Naval Support Weapons Center/Crane passed along a request to measure a current that consists of a train of 60 Hz, 10 kA pulses. A welder, used to weld submarine hulls, generates these pulses, and quality control concerns the Navy. The high current and relatively long duration of each pulse make any current viewing resistor (CVR) impractical. Moreover, the large charge transfer on each pulse would saturate the magnetic core of a typical current viewing transformer (CVT), such as a Pearson model 301X. A Rogowski coil, which has no saturable magnetic core might solve this problem. Although I had never before used a Rogowski coil to measure such low frequency currents, I designed and constructed a Rogowski coil to satisfy this request.

While building and documenting the Rogowski coil to satisfy the Navy's request, I concluded that new staff members at Sandia have no convenient reference on measuring current with Rogowski coils; therefore, I wrote this memorandum as an elementary introduction to Rogowski coils. I do not derive all the equations that describe the remarkable properties of a Rogowski coil; nevertheless, I do describe how a Rogowski coil works and why it senses the total current passing through it without sensing the details of the current. These details include current distribution and location. After discussing Rogowski coil theory, I calculate and measure the sensitivity of the Rogowski coil that I built to satisfy the Navy's request. An experimenter often passively integrates the output from a Rogowski coil. I derive the equations describing a passive, resistor-capacitor (RC) integrator and measure the time constant of the integrator that I constructed to accompany the Rogowski coil for the Navy.

THEORY

In this section I discuss the theory of Rogowski coils. Nevertheless, I do not rigorously derive the theory. I take what might be called a "hand waving" approach. The interested reader may refer to the original paper by Rogowski and Steinhaus[1] or to Huddlestone and Leonard[3] for a more rigorous derivation of Rogowski coil theory.

A Rogowski coil consists of a conductor wrapped around the surface of a torus. Furthermore, one end of the conductor connects to a conductor which runs along the circumference of the circle defined by the major diameter of the torus. The free ends of the outer conductor and the central conductor comprise the output terminals. Typically, the center conductor of a coaxial cable forms the central conductor and a wire wound on the dielectric core of the cable after the outer jacket and shield have been removed from the outer conductor. Figure 1 depicts a Rogowski coil.

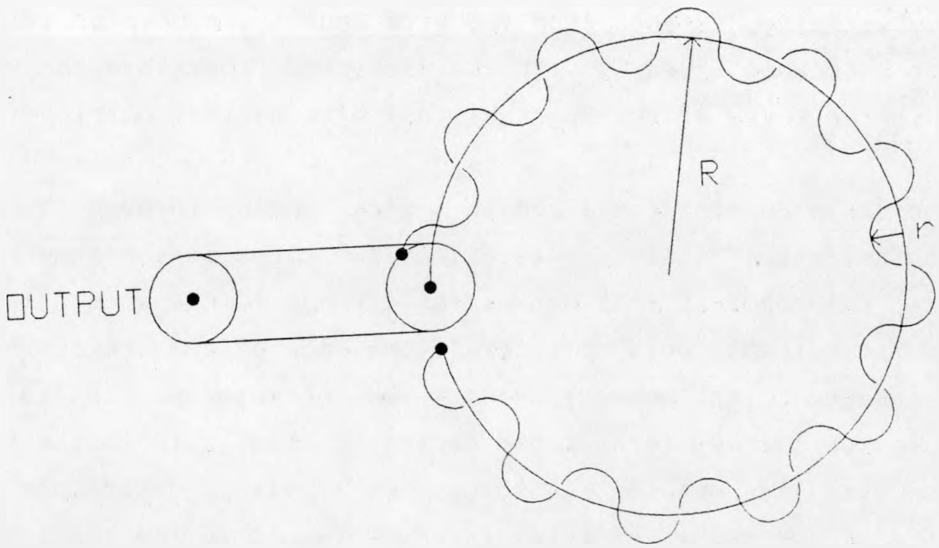

Figure 1. Schematic diagram of a Rogowski coil.

Any magnetic flux cutting the plane of the major diameter of the torus will induce voltages in both the central conductor and the toroidal turns. However, the two induced voltages will oppose each other, and if the torus has a small minor diameter compared to the major diameter, the induced voltages will have very nearly the same magnitude. Therefore, any axial flux will generate comparatively small voltages in the Rogowski coil.

A radial flux will not induce any voltage in the central conductor. In principle, the toroidal windings present a very small area for radial flux to cut; nevertheless, spiral windings with a quite large pitch frequently comprise the toroidal turns, and radial flux can induce a significant voltage in a helical turn. However, any radial flux must cut these turns in both the positive and negative sense, and the induced voltages very nearly cancel. Therefore, any radial flux will generate comparatively small voltages in a Rogowski coil.

Only azimuthal flux induces significant voltages in a Rogowski coil. Consider the flux generated by the current in a wire parallel to the major axis of the torus. Furthermore, suppose this wire lies entirely outside the torus. The flux cuts the turns nearest the wire in one sense, and those on the opposite side of the Rogowski coil in the opposite sense. Moreover, the field generated by the current decreases inversely with the distance from the wire, but the number of turns the flux cuts increases linearly with the distance. Therefore the voltages induced in the turns of the Rogowski coil by a current outside the torus cancel.

Consider a current contained in a wire running through the torus. The azimuthal magnetic field cuts all of the turns in the same sense. Therefore, the Rogowski coil senses the current in the wire. If the wire and its current would move toward one side of the torus, the voltage induced in the nearest turns would increase, but the voltage induced in the further turns would decrease. Similarly to the situation where the wire lies outside the torus, the $1/r$ field dependence and r dependence of the number of turns involved result in the total induced voltage remaining nearly constant.

Therefore, a Rogowski coil senses primarily those currents flowing through the opening in the torus. Moreover, a Rogowski coil does not sense the location of the current within the torus.

OPERATIONAL CONSIDERATIONS

As with all diagnostic probes, a Rogowski coil must satisfy requirements peculiar to the application. A Rogowski coil that works well in one application may not work at all in another. For example, an insensitive Rogowski coil designed for high energy pulsed power applications may produce a few microvolts in another application. In addition to the sensitivity, an experimenter must consider the frequency response when he chooses a Rogowski coil.

Too large or too small an output signal greatly magnifies the difficulty in recording the experimental data. Moreover, an inappropriate signal level can make the measurement impossible. For example in a pulsed power device, electromagnetic noise can swamp a too-small output. Furthermore, with too large a signal, transmission line components such as cables, connectors, and attenuators can fail. Such failures result in distortion or loss of the signal. In order to choose the appropriate Rogowski coil, the experimenter must determine two things: the anticipated driver (dI/dt) and the coil sensitivity. In the next section, I discuss how to calculate the sensitivity of a Rogowski coil. The experimenter, of course, must determine the anticipated dI/dt.

A Rogowski coil's frequency response can distort the signal. Suppose an experimenter wants to measure a 100 MHz current. Further suppose that the Rogowski coil has an intrinsic inductance of 1 μH and must drive a 50 Ω load. Therefore, the Rogowski coil has an inherent L/R rise time of 20 ns, but the signal has a period of only 10 ns. The inductance of the Rogowski coil distorts the signal beyond recognizability and makes the Rogowski coil unsuitable for this application.

Having cautioned the reader on the danger of a high inductance Rogowski coil, I will make two further points about the inductance. First, a high inductance rarely becomes a problem: typically the signal level becomes too high long before the inductance does. Second, a large inductance and a built-in shunt resistor can make the Rogowski coil self-integrating.[4, 5] Some experimenters use self-integrating

Rogowski coils, and I shall briefly discuss them in the section on integrating the Rogowski coil output.

While the inductance of a Rogowski coil rarely creates frequency response problems, the physical length of a Rogowski coil can. Suppose a Rogowski coil that encircles a current has a major diameter of 1 m. Then the length of the coil exceeds 3 m, and the 10 ns required for a signal to travel around the circumference twice (once in the coils and once in the center conductor) determines the lower limit for the observable rise time. Any signal with a shorter rise time becomes seriously distorted. Large, fast pulsed power accelerators, such as SPEED,[6] often present this difficulty, and an experimenter must design his Rogowski coils accordingly.

Figure 2 depicts an insidious trap that can befall the unwary experimenter. The figure displays the outputs from two Rogowski coils observing the same current. I made the Rogowski coil that produced the lower wave form by winding copper wire on the core of a length of RG-58C/U coaxial cable, but I wound the Rogowski coil that produced the upper wave form on a length of small diameter Microdot cable. Small diameter coaxial cables typically have copper-coated-steel center conductors. The peculiar wave form results from magnetization of the steel center conductor. Clearly, an experimenter would not want to use such a Rogowski coil, and he must take care when selecting the cable for a small diameter Rogowski coil.

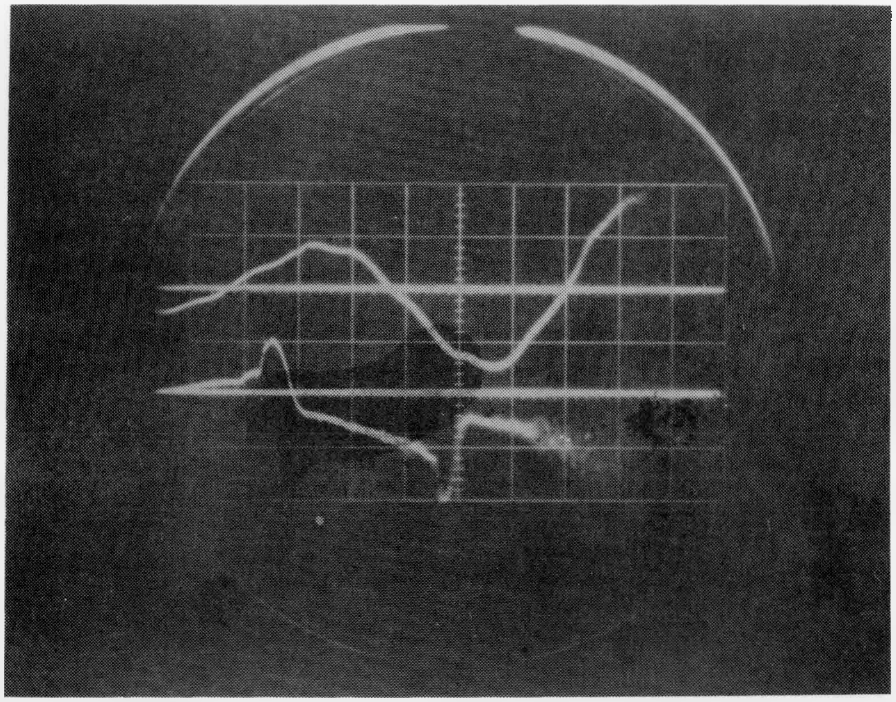

Figure 2. Two Rogowski coils observing the same current. A Rogowski coil wound on a small diameter coaxial cable with a steel center conductor produced the upper trace. A Rogowski coil wound on RG-58C/U cable produced the lower trace.

CALCULATION OF THE SENSITIVITY

In order to illustrate the technique for calculating the sensitivity of a Rogowski coil, in this section I calculate the sensitivity for a coil that I made. First, let me define the sensitivity: The ratio of the voltage output from the Rogowski coil to the time rate of change of the current (dI/dt) flowing through the opening of the Rogowski coil defines the sensitivity, s.

I made a Rogowski coil consisting of 22 gauge, enameled, solid copper wire wound tightly and closely on the dielectric core of a length of RG-214/U coaxial cable. I stripped approximately 20 cm of the shield conductor from one end of the cable and soldered the 22 gauge wire to the end of the shield. I wound the wire onto the cable until I reached the end of the stripped section of the coaxial cable. Then I soldered the other end of the wire to the center conductor of the cable. Finally I covered the turns with heat shrinkable, plastic tubing and bent the coil into an approximate circle. Using the 0.80 ± 0.04 cm average of the inner and outer diameters for the diameter of the turns and the measured, average turn density of $13.3 \pm 0.7\ \text{cm}^{-1}$, I calculated the sensitivity of the Rogowski coil. What follows describes that calculation.

According to Ampere's Law,[7] a current, I, carried by a long, straight wire generates an azimuthal magnetic field at a distance R from the wire:

$$B = \frac{\mu_o I}{2 \pi R} \qquad ,$$

where $\mu_o = 4 \pi \cdot 10^{-7}$. Consider a single, small, circular loop of wire with radius r that lies in a plane that also contains the long, straight wire. The center of this loop lies at a distance R from the straight wire. I define "small" to mean $r \ll R$. The magnetic field lines generated by the current in the straight wire intersect the area bounded by the circular loop normally. Since $r \ll R$, the magnetic field changes negligibly over the area of the loop, and the the current generates a magnetic flux within the circular loop:

$$\Phi = \frac{\mu_0 \, I \, r^2}{2 \, R} \quad .$$

Therefore, neglecting the sign, a time varying current, $\dot{I}$, generates a time varying magnetic flux, $\dot{\Phi}$, which induces a voltage in the circular wire:

$$V = \dot{\Phi} = \frac{\mu_0 \, \dot{I} \, r^2}{2 \, R} \quad .$$

Now, consider a circuit consisting of N circular loops, all with the same radius r, all at the same distance R from the straight wire, and all coplanar with the wire. Furthermore, the centers of these turns all lie in a plane perpendicular to the straight wire. Therefore, these turns lie on the surface of a torus, which encircles the long, straight wire. By connecting these loops in series, the induced voltages add to give a net voltage of,

$$V = \frac{\mu_0 \, N \, \dot{I} \, r^2}{2 \, R} = \mu_0 \, \pi \, n \, \dot{I} \, r^2 \quad .$$

In this equation, I replaced the linear turn density, $N \, / \, (2 \, \pi \, R)$, by n. Using the definition of the sensitivity,

$$s = \frac{V}{\dot{I}} = \mu_0 \, \pi \, n \, r^2 \quad .$$

Since I performed this derivation using MKS units, substituting $r = 4.0 \cdot 10^{-3}$ m and $n = 1.33 \cdot 10^{3}$ m^{-1} results in

$$s = 8.4 \cdot 10^{-8} \text{ V-s/A.}$$

In order to assess the significance of this calculation, I now estimate the uncertainty of the sensitivity calculation. Using the definition of the turn density, I estimate the uncertainty in the sensitivity:[8]

$$\delta s = s \left[\left(\frac{\delta n}{n} \right)^2 + 4 \left(\frac{\delta r}{r} \right)^2 \right]^{1/2} .$$

Moreover, using the definition of the turn density, I can calculate the uncertainty of the sensitivity:

$$\delta s = s \left[\left(\frac{\delta N}{N} \right) + \left(\frac{\delta R}{R} \right)^2 + 4 \left(\frac{\delta r}{r} \right)^2 \right]^{1/2} .$$

If R varies around the circumference of the coil, its effects on the sensitivity will cancel to some extent, and the above equation overstates the uncertainty somewhat. Nevertheless, in order to provide some indication of the uncertainty, I use the above equation.

I estimate these three uncertainties:

$$\frac{\delta N}{N} = .05 \quad ,$$

$$\frac{\delta R}{R} = 0.1 \quad , \text{ and}$$

$$\frac{\delta r}{r} = 0.05 \quad .$$

Therefore,

$$s = 8.4 \pm 1.3 \cdot 10^{-8} \text{ V-s/A.}$$

ROGOWSKI COIL CALIBRATION

An experimenter can use a calculation similar to the one performed above to estimate the magnitude of the output he can expect from a Rogowski coil. Nevertheless, several factors contribute to make this calculation somewhat uncertain. Four examples come readily to mind: First, a real Rogowski coil, especially a small diameter one often deviates significantly from a circle. Second, precisely and uniformly winding the turns poses difficulties. Third, a small Rogowski coil may violate the condition $r \ll R$, and the magnetic field may vary significantly with position in each loop. Fourth, for very fast signals, skin depth effects can change the effective area of the individual turns. For these reasons, the above calculation can only guide the user. The wise experimenter will directly measure the sensitivity of the Rogowski coil. Moreover, he will measure the sensitivity with a signal whose frequency closely matches the frequency at which he intends to use the Rogowski coil. In this section I describe the measurement of the Rogowski coil's sensitivity that I calculated above.

As the first step in measuring the sensitivity, I used the Tektronix model R7844, dual-beam oscilloscope's internal 1 kHz square wave output to determine the vertical sensitivity and the sweep speed for both channel A and channel B. (See Figures 3 and 4.)

For both beams, I found the sweep speed accurate to within an uncertainty of 0.5 percent. For channel A, I corrected the vertical deflection by multiplying the result by 0.988 ± 0.007. For channel B, I calculated a vertical correction factor of 1.00 ± 0.03. The high gain of channel B, necessary to make the measurement, introduced more noise relative to the signal level and resulted in a larger uncertainty for channel B. As an alternative to calculating the correction factors, I could have adjusted the gain on the vertical amplifiers. Either approach satisfies the need for accuracy; although, the latter alternative would have simplified the computations, slightly. In either case, the uncertainties remain the same.

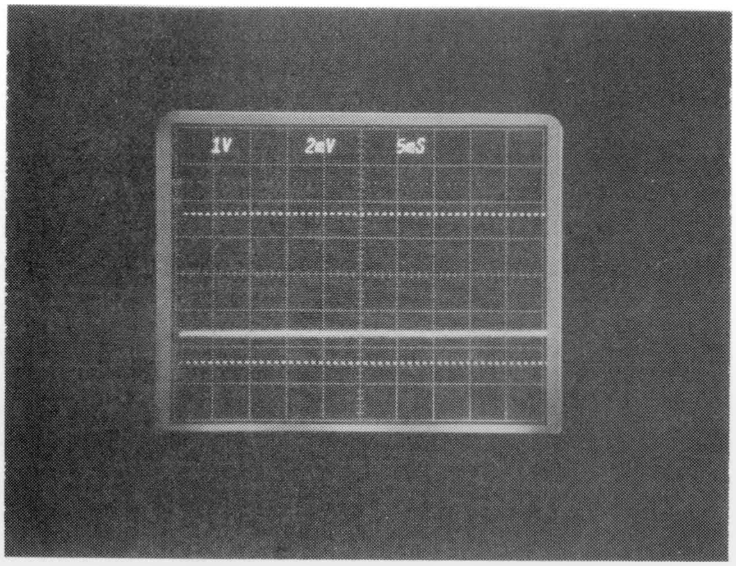

Figure 3. The calibration of channel A resulted in a vertical deflection correction factor of 0.988 ± 0.007 and a sweep speed correction factor of 1.000 ± 0.005.

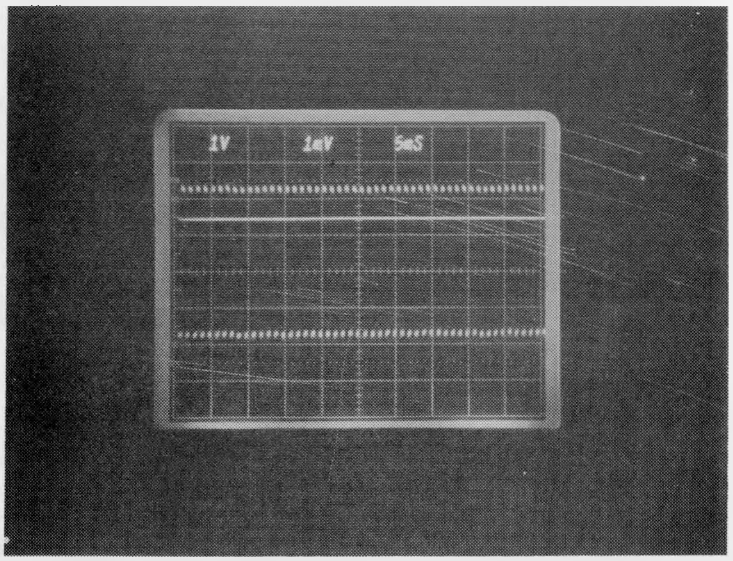

Figure 4. The calibration of channel B resulted in a vertical deflection correction factor of 1.00 ± 0.03 and a sweep speed correction factor of 1.000 ± 0.005.

Since I designed this Rogowski coil to detect 60 Hz signals, I needed a high-current, 60 Hz source in order to measure the sensitivity. I replaced the tip of a soldering gun with a 2-foot-length, insulated, heavy wire--the center conductor from RG-214/U coaxial cable. Thus modified, the soldering gun would generate an approximately sinusoidal current with a peak-to-peak difference of more than 150 amperes. I routed the length of center conductor through the Rogowski coil and a Pearson model 301X current transformer. I displayed the outputs from the current transformer and Rogowski coil on the oscilloscope calibrated above and recorded the display on film. Figure 5 depicts the resulting oscillogram. The upper trace displays the current transformer output, and the lower trace displays the Rogowski coil output.

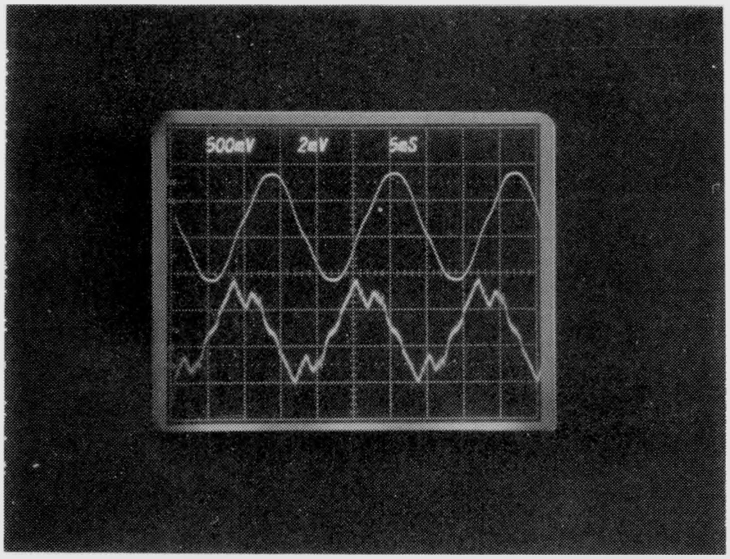

Figure 5. The Pearson model 301X current transformer output and the Rogowski coil output. With a gauge factor of 100 A/V, the vertical deflection corresponds to about 50 A/Div for the current transformer output. The vertical deflection of the Rogowski coil output corresponds to 2 mV/Div.

Using a Tektronix 4956 graphics tablet and a Tektronix 4052 graphics computer, I digitized and analyzed the oscilloscope traces.[9]

Figures 6 and 7 depict the digitized output from the computer. Figure 6 depicts the current waveform from the Pearson current transformer, and Figure 7 depicts the output from the Rogowski coil. The symbols each represent one digitized point, and the straight line connecting the

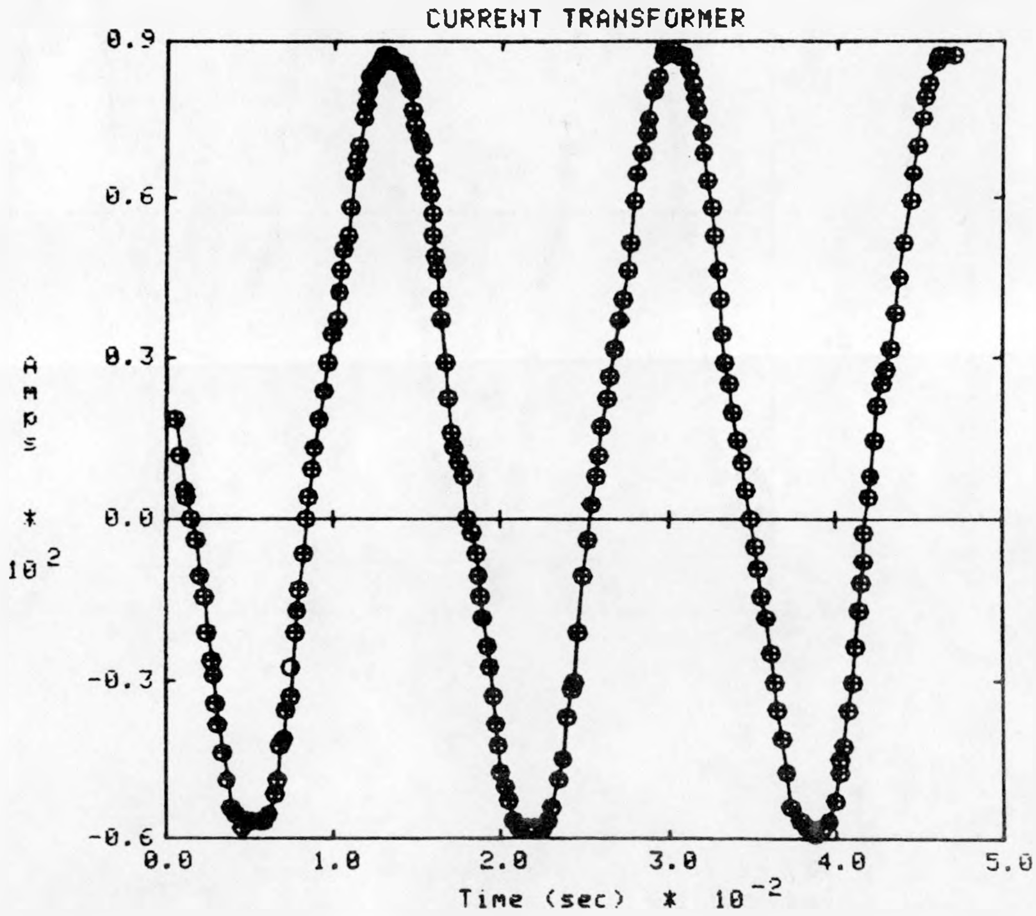

Figure 6. Digitized output waveform from the Pearson model 301X current transformer.

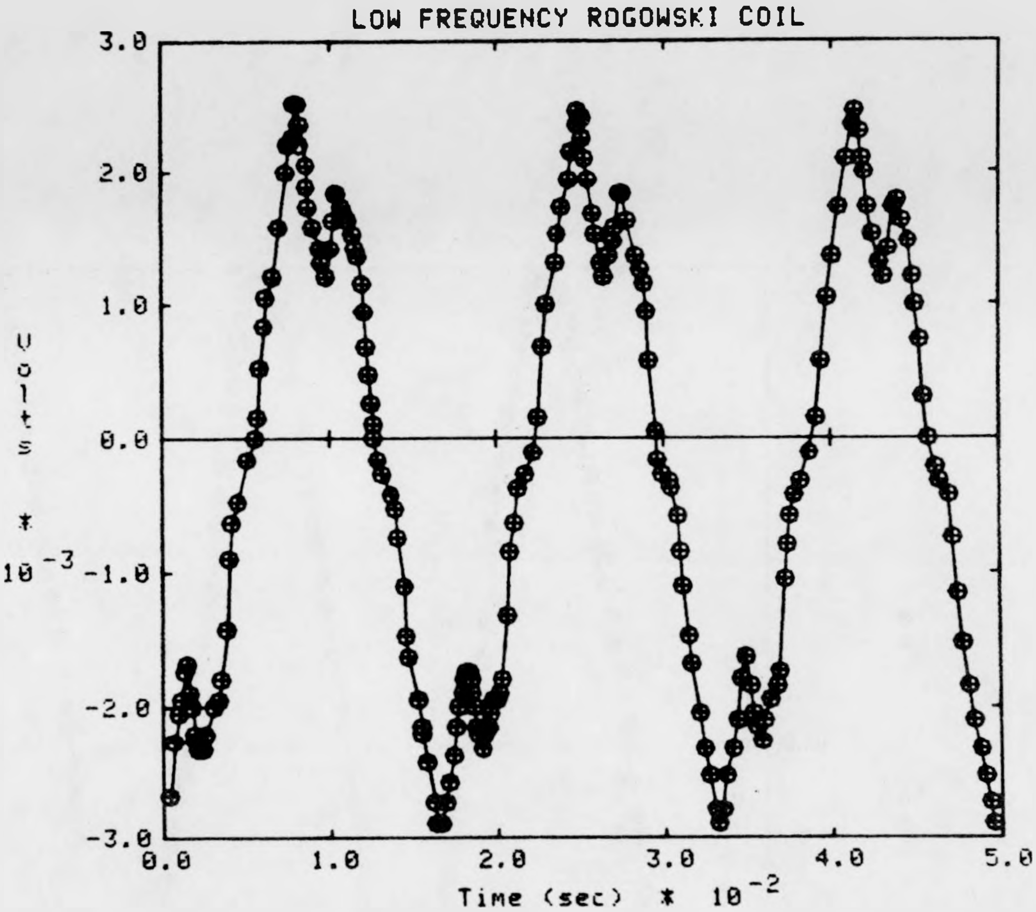

Figure 7. Digitized output waveform from the Rogowski coil.

symbols represents a linear interpolation between the points. Since the oscillograms have no baseline, the digitizing process arbitrarily determines the zero level. However, the fitting routine determines the correct baseline, and the arbitrary baselines cause no difficulty.

Suppose the functions R(t) and I(t) respectively describe the output voltage of the Rogowski coil and the current flowing through the opening in the coil at the time t. Then, the gauge factor, $g = 1/s$, relates the Rogowski coil output and the current through the following equation:

$$I(t) = g \int_0^t R(\zeta)\, d\zeta \quad .$$

In principle, one can simply perform a least squares fit to determine g. Nevertheless, when one digitizes the two data waveforms, several complications occur: (1) shifting of the reference baselines used for digitizing relative to the true baselines, (2) rotation of the reference baselines relative to the true baselines, and (3) inconsistent choice of zero times. Therefore, the resultant, integrated Rogowski coil output probably contains an additive, quadratic error and incorrect limits on the integral. Consequently, in order to determine the Rogowski coil sensitivity, I numerically integrated the digitized Rogowski coil output and assumed a model of the following form:

$$\eta(t) = a_0 + a_1(t-t_0) + a_2(t-t_0)^2 + g \int_0^{t-t_0} R(\zeta)\, d\zeta \quad .$$

Finally, the fitting procedure consists of finding that combination of the five parameters (a_0, a_1, a_2, g, and t_0) that minimizes the least squares difference between the model ($\eta(t)$) and the current (I(t)) as determined from the current transformer. With a nonlinear model, such as this one, one commonly uses Gauss linearization to estimate the parameters.[10] In addition to using the Tektronix 4052 graphics computer to digitize the data, I used it to perform the iterative process and find the best estimates of the parameters.

Figure 8 depicts the fit. Of all the parameters only the gauge factor, g, has any physical significance. Arbitrary choices made while digitizing the oscillograms determine the other parameters. I digitized the two traces several times and determined the best value for the gauge factor: $g = 1.23 \pm 0.03 \cdot 10^{7}$ A/V-s. This uncertainty results from several uncertainties: the uncertainty in the fit, the uncertainty in the current transformer calibration, and the uncertainties in the oscilloscope calibrations. Therefore, I determined the best estimate for the Rogowski coil sensitivity:

$$s = \frac{1}{g} = 8.1 \pm 0.2 \cdot 10^{-8} \text{ V-s/A} \quad .$$

The prediction and the measurement agree remarkably well. Nevertheless, the construction process may lead to large uncertainties in the sensitivity calculation. If an experimenter uses a Rogowski coil for precise current measurements, he cannot rely on a calculation of the sensitivity. He must measure the sensitivity. Nevertheless, a sensitivity calculation can provide a useful design guide.

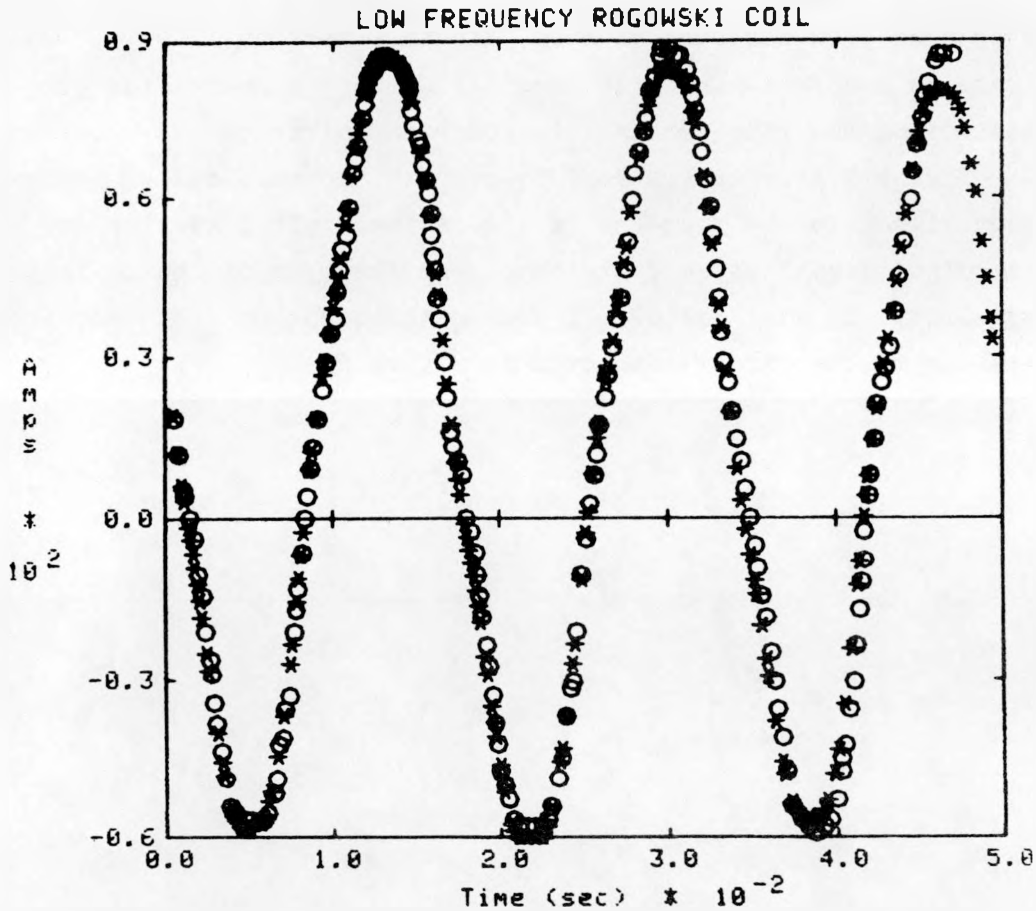

Figure 8. Numerically integrated Rogowski coil output fit to the current transformer output. The fit results in an estimated gauge factor of $g = 12.3 \pm 0.3$ kA/V-ms.

INTEGRATING THE ROGOWSKI COIL OUTPUT

While a Rogowski coil generates a signal proportional to dI/dt, an experimenter usually wants to measure the current; therefore, he must integrate the Rogowski coil output. For many uses, particularly when the signal falls entirely within a limited time interval, numerical integration of the Rogowski coil output satisfies the experimental requirements. However, sometimes the experimenter must record the integrated signal directly. Often, a passive, resistor-capacitor (RC) integrator provides the solution. In this section I derive the equations for an RC integrator and illustrate a method for precisely determining the time constant for such an integrator.

Figure 9 illustrates an RC integrator schematically. Applying the input signal to the free end of the resistor, the experimenter records the output signal present at the common junction of the resistor and the capacitor. In what follows, I derive the equation that describes the behavior of the circuit depicted in Figure 9.

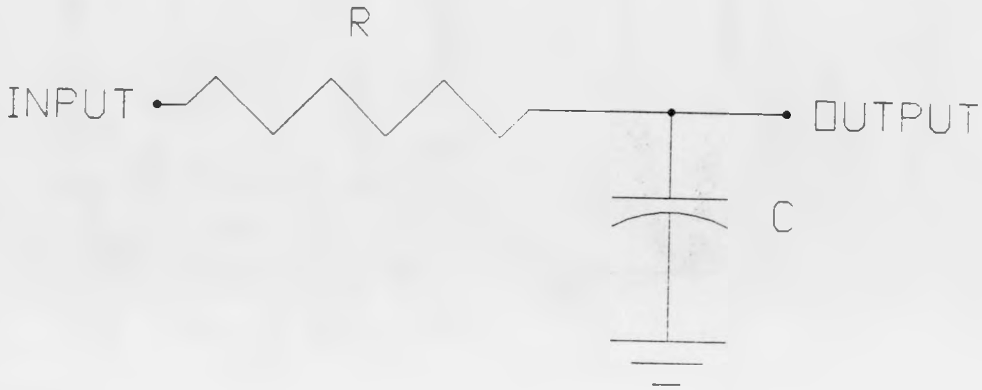

Figure 9. Schematic diagram of a passive, RC integrator.

With the capacitor uncharged at some time, $t = 0$, I apply a sinusoidal voltage to the input: $V(t) = V_s \sin(\omega t)$. With a charge, Q, on the capacitor, C, and a current, $\dot{Q}$, in the resistor, R, the following inhomogeneous differential equation describes the behavior of this circuit:

$$R\,\dot{Q} + \frac{1}{C}\,Q = V_s \sin(\omega t) \quad .$$

I will use the standard technique to solve this differential equation[11].

Substituting $\tau = RC$, I solve the homogeneous equation:

$$Q_h = A\,e^{-t/\tau} \quad .$$

This derivation will later yield the value of the presently undetermined constant A. Because of the sinusoidal character of the inhomogeneous part of the equation, I try a particular solution with the undetermined coefficients B and Γ:

$$Q_p = B\,\sin(\omega t) + \Gamma\,\cos(\omega t) \quad .$$

I find B and Γ by first differentiating Q_p and substituting into the inhomogeneous equation. After collecting the terms,

$$\left(\frac{B}{C} - R\,\omega\,\Gamma\right)\sin(\omega t) + \left(R\,B\,\omega + \frac{\Gamma}{C}\right)\cos(\omega t) = V_s\,\sin(\omega t).$$

We note that

$$\left(\frac{B}{C} - R\,\omega\,\Gamma\right) = V_s \quad , \text{ and}$$

$$\left(R\,B\,\omega + \frac{\Gamma}{C} \right) = 0 \quad .$$

Solving for B and Γ, and adding Q_p and Q_h gives,

$$Q = Q_p + Q_h = \frac{V_s\,C}{1 + (\tau\,\omega)^2} \left[\sin(\omega t) - \tau\,\omega\cos(\omega t) \right] + A\,e^{-t/\tau} \; .$$

Finally using the condition Q(0) = 0, I have the solution to the differential equation:

$$Q = \frac{V_s\,C}{1 + (\tau\,\omega)^2} \left[\sin(\omega t) + \tau\,\omega \left(e^{-t/\tau} - \cos(\omega t) \right) \right] \; .$$

I define the time period of interest:

$$0 \leq t \lesssim \frac{2\,\pi}{\omega} \quad .$$

Assuming that $t << \tau$, then

$$\tau\,\omega >> 2\,\pi \quad \text{, and}$$

$$Q \simeq \frac{V_s\,C}{\tau\,\omega} \left(1 - \cos(\omega t) \right) \quad .$$

Therefore the following equation approximates the output voltage:

$$V_o(t) = \frac{Q}{C} \simeq \frac{V_s}{\tau} \int_0^t \sin(\omega\zeta)\, d\zeta \quad .$$

If the input voltage, $V_i(t)$, consists of more than one frequency component, all the frequency components integrate independently. Therefore, so long as $\tau >> t$,

$$V_o(t) = \frac{Q}{C} \simeq \frac{1}{\tau} \int_0^t V_i(\zeta)\, d\zeta \quad ,$$

for any arbitrary waveform.

This derivation assumes two conditions: $\tau = RC >> t$ and $Q(t{=}0) = 0$. Typically, by choosing $RC \geq 10 \cdot t$, an experimenter satisfies the former condition. However, he often overlooks the second. If the voltage at the integrator's input remains zero for much longer than the RC time constant, and the Rogowski coil output arrives during a short time interval, then the experiment usually satisfies the second condition. Most pulsed power applications generate this type of signal, and therefore, satisfy the second condition. On the other hand, a continuous output signal from the Rogowski coil often violates the second condition, and the integrator will give erroneous results. The user must take care to satisfy the condition that $Q(t{=}0) = 0$.

I have so far failed to mention one more, implicit condition on the integrator: The recording instrument, typically an oscilloscope, must have a much larger input impedance than the resistor in the integrator. Otherwise, the resistor and the instrument input impedance will act as a voltage divider. A typical oscilloscope has an input impedance of 1 MΩ, and this condition requires the integrator to have a resistor of less than about 100 kΩ. Obviously, one should avoid using this type of integrator with oscilloscopes having 50 Ω input impedances.

At this point I shall digress briefly to mention an alternate integrator design. This alternate design uses three components connected in a "T" network: two identical resistors and a capacitor, with one lead from each of the three components connected to a common point. (See Figure 10.) The free end of either resistor forms the input, and the free end of the other resistor forms the output. Furthermore, the resistance of each resistor equals the impedance of the signal cable, normally 50 Ω. When an experimenter uses this type of integrator, he must terminate the signal cable at the recording instrument input. If the recording instrument has a high input impedance compared to the cable impedance, the experimenter can use a terminator to provide the impedance match.

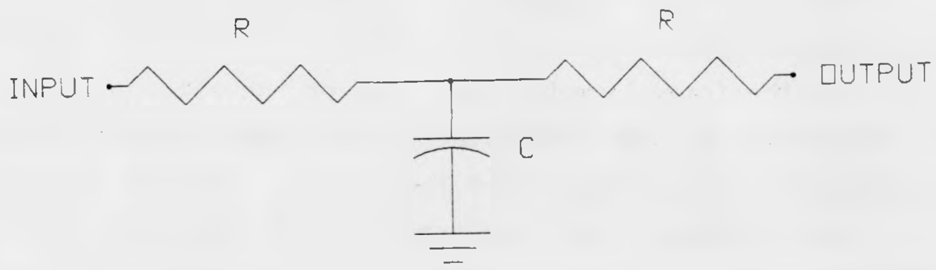

Figure 10. Schematic diagram of a "T"-network, passive, RC integrator.

This type of integrator has three major advantages: First, one can use a high frequency oscilloscope, which typically has a 50 Ω input impedance. Second, because this type of integrator looks like a terminator for the times of interest, one may place it anywhere in the cable. Third, this symmetric integrator performs equally well when connected in either orientation. On the other hand, this type of integrator has two major disadvantages: First, the output resistor reduces the output signal by half. Second, requiring the resistance to equal the cable impedance limits the choice of capacitance. An experimenter would most often uses this type of integrator with very fast events. I simply mention this design for the sake of completeness, and I shall not discuss it further.

The integrator I built consists of a 24 kΩ input resistor and four 1 μF capacitors in parallel comprising the integrating capacitor. Therefore, I expect a time constant of $\tau \approx 0.1$ s. I placed these components inside a small aluminum box measuring approximately one inch square by two inches long and having two BNC connectors for the input and output signals. For convenience, I installed a 50 Ω resistor between the input and ground. This resistor terminates the input cable, but it does not alter the integrator's operation.

To measure the integrator's time constant, I pulsed it with a single, square pulse from a Tektronix model PG 502 pulse generator. Using the integrator equation, and substituting,

$$V_i(t) = V_i = \text{constant} \quad ,$$

then

$$\tau = \frac{V_i}{\frac{V_o(t-t_0)}{(t-t_0)}} \quad .$$

Since integrating a constant voltage results in a ramp voltage, dividing the input voltage by the output slope results in the time constant:

$$\tau = \frac{V_i}{\frac{dV_o}{dt}} \quad .$$

The oscillogram depicted in Figure 11 shows the input and output pulses. Using the calibration factors I measured for the oscilloscope, I calculate the time constant:

$$\tau = 0.102 \pm 0.004 \text{ s} \quad .$$

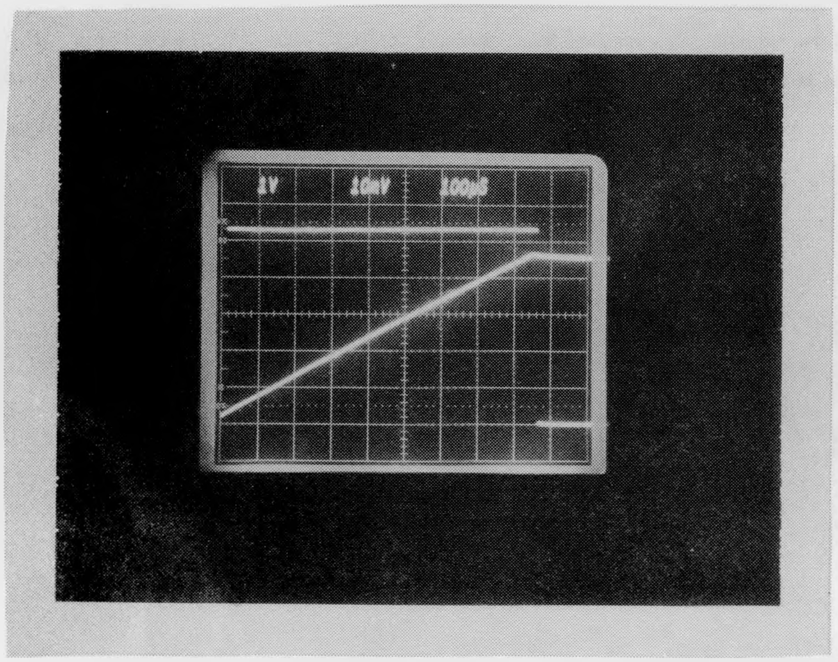

Figure 11. Calibrating an RC integrator using a square pulse input to the integrator and the ramp output.

Finally, I know everything I need to know in order to use the Rogowski coil and integrator to measure currents. The following equation relates the current flowing through the opening in the Rogowski coil to the integrator output signal:

$$I = \frac{\tau}{s} V_o = 1.26 \pm 0.06 \cdot 10^6 \; V_o \quad .$$

In an earlier section, I mentioned the existence of self-integrating Rogowski coils. A self-integrating Rogowski coil differs from a normal Rogowski coil in that a small-value resistor shunts the output signal to ground. Figure 12 depicts a self-integrating Rogowski coil, schematically. Compare Figure 12 with Figure 9. By replacing R with L, C with 1/R, and Q with I in the equation which follows Figure 9,

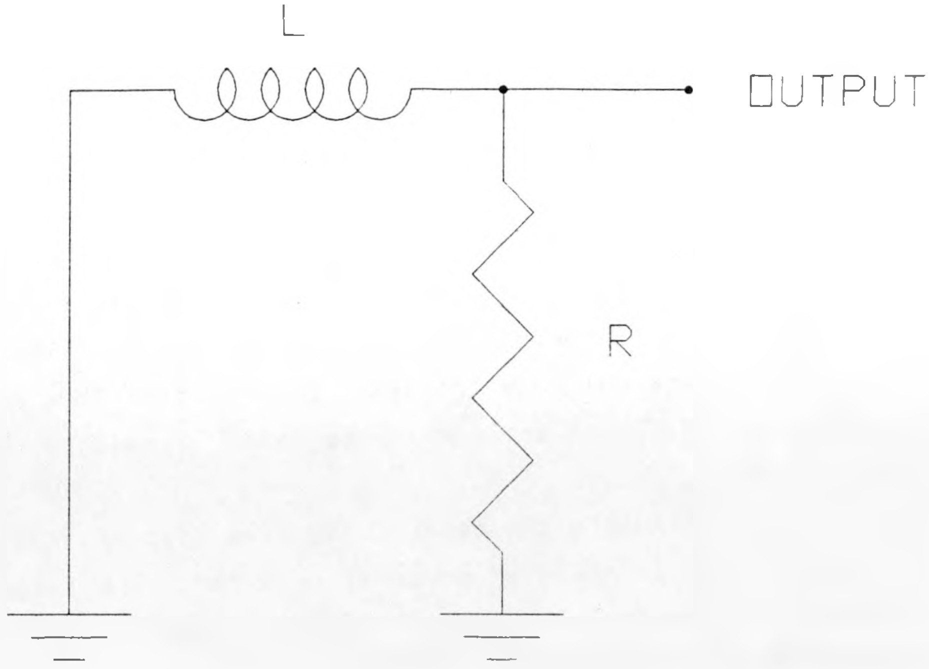

Figure 12. Schematic diagram of a self-integrating Rogowski coil.

I have the equation which describes the circuit depicted in Figure 12:

$$L \dot{I} + R I = V_s \sin(\omega t) \qquad .$$

I solve this equation in the same way as before:

$$V_o(t) = R I \approx \frac{R}{L} \int_0^t V_i(\zeta) \, d\zeta \qquad .$$

The corresopnding conditions become, L/R >> t and I(t=0) = 0. Moreover, the signal cable, or recording instrument, must have a much larger input impedance than the shunt resistance.

CONCLUSION

I have used Rogowski coils to measure dI/dt's approaching $1 \cdot 10^{14}$ A/s. In this paper, I describe a Rogowski coil that can measure dI/dt's as low as $3 \cdot 10^{4}$ A/s. Clearly, one can use Rogowski coils to measure a wide range of dI/dt's. The design of Rogowski coils makes them insensitive to the exact current distribution. Therefore, when an experimenter does not know the exact current distribution, Rogowski coils offer a distinct advantage over other magnetic probes, such as $\dot{B}$ loops.

One can calculate the sensitivity of a Rogowski coil, but it should be directly measured whenever possible. The following equation relates the output of the Rogowski coil to the current flowing through the opening in the Rogowski coil:

$$\frac{dI}{dt} = \frac{1}{s} V_o \quad .$$

Moreover, the following equation defines the sensitivity, s, for an ideal Rogowski coil in terms of the turn density (the number of turns per meter around the circumference of the Rogowski coil), n, and the radius of the turns, r:

$$s = \frac{V}{\dot{I}} = \mu_0 \pi n r^2 \quad ,$$

where $\mu_0 = 4 \pi \cdot 10^{-7}$.

By using a passive RC integrator, an experimenter can directly record a current waveform, rather than a dI/dt waveform. In order to use a passive, RC integrator, the experimenter must ensure that the circuit satisfies three conditions: a long time constant relative to the times of interest ($RC = \tau >> t$), no charge on the capacitor in the integrator at the beginning of the time interval under observation

($Q(0) = 0$), and a small resistance in the integrator compared to the input impedance of the recording instrument ($R << Z_i$).

One can calibrate passive, RC integrators by using a single square pulse and recording the input voltage and the output slope. The ratio of the input voltage to the output slope yields the time constant. The Rogowski coil sensitivity, s, and the integrator time constant, $\tau = RC$, relate the output signal from the integrator to the current flowing through the toroidal opening in the Rogowski coil:

$$I = \frac{\tau}{s} V_o \quad .$$

REFERENCES

1 W. Rogowski and W. Steinhaus, Arch. Elektrotech. 1, 141 (1912).

2 Charles R. McClenahan, Thomas P. Wright, Jeffrey P. Quintenz, Robert C. Backstrom, and Phillip W. Spence, Triaxial Diode on HYDRAMITE, SAND85-0740, (Albuquerque, NM: Sandia National Laboratories, 1985).

3 S. L. Leonard in Plasma Diagnostic Techniques, R. H. Huddlestone and S. L. Leonard, ed., Academic Press, New York, pp. 8ff (1965).

4 D. G. Pellinen and P. W. Spence, Rev. Sci. Instrum. 42, 1699 (1971).

5 D. G. Pellinen, M. S. Di Capua, S. E. Sampayan, H. Gerbracht, and M. Wang, Rev. Sci. Instrum. 51, 1535 (1980).

6 G. A. Zawadzkas, P. A. Kuenstler, and L. M. Choate, Sandia National Laboratories Radiation Facilities, SAND83-0598, (Albuquerque, NM: Sandia National Laboratories, 1983).

7 J. R. Reitz and F. J. Milford, Foundations of Electormagnetic Theory, Addison-Wesley Publishing Company, Reading, Mass., pp. 148ff (1967).

8 Stuart L. Meyer, Data Analysis for Scientists and Engineers, John Wiley & Sons, New York, pp. 39ff (1975).

9 I describe a similar program in, Charles R. McClenahan, Digitizing Program for the Tektronix 4050 Series Computers With the Tektronix 4956 Graphics Tablet, SAND85-2651, (Albuquerque, NM: Sandia National Laboratories, 1986).

10 J. V. Beck and K. J. Arnold, Parameter Estimation in Engineering and Science, John Wiley & Sons, New York, pp. 340ff (1977).

11 Shepley L. Ross, Differential Equations, Blaisdell Publishing Company, Waltham, MA, pp. 389ff (1964).

Distribution:

1000	V. Narayanamurti
1200	J. P. VanDevender
1220	M. W. Cowan
1230	J. E. Powell
1231	J. R. Lee
1232	W. Beezhold
1232	W. P. Ballard
1232	D. E. Beutler
1232	N. E. Counts
1233	L. M. Choate
1233	G. A. Carlson
1233	S. R. Crawford
1233	M. A. Hedemann
1233	W. H. McAtee
1233	C. R. McClenahan (25)
1233	A. W. Sharpe (25)
1233	T. F. Thornhill
1234	R. J. Leeper
1234	G. T. Baldwin
1235	J. M. Hoffman
1235	M. C. Clark
1236	G. A. Zawadzkas
1236	P. Micono
1236	K. A. Mikkelson
1236	J. P. Reilly
1236	R. L. Westfall
1240	K. R. Prestwich
1250	T. H. Martin
1260	D. L. Cook
1270	R. B. Miller
1542	M. A. Stark
3141	S. A. Landenberger (5)
3151	D. S. Rix
3151	C. G. Shirley
3151	M. L. Garner (3)
3154-1	L. H. Dalin (28) for DOE/DSTI
8024	P. W. Dean