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ANALYTICAL THEORY OF INTERCHANGE AND COMPRESSIONAL ALFVÉN INSTABILITIES IN EBT

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Analytical Theory of Interchange and Compressional Alfvén Instabilities in EBT

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Abstract

The local stability of the EBT plasma is analyzed for the long wavelength perturbations in the frequency regime, $\omega \lesssim \Omega_i$ (Ω_i is ion cyclotron frequency). In addition to the low frequency interchange instability, the plasma can be unstable to the compressional Alfvén wave. Contrary to the previously obtained quadratic dispersion relation in ω for the interchange mode, our dispersion relations for both types of instabilities are cubic in ω. New stability boundaries are found, for the hot electron interchange mode, to relate to the enhanced compressibility of the core plasma in the presence of hot electrons. The compressional Alfvén instability is driven due to the coupling of hot electron magnetic drifts and diamagnetic drift with the compressional Alfvén wave. The stability conditions of these two types of instabilities are opposite to each other.

I. INTRODUCTION

Stability of the plasma in the ELMO Bumpy Torus in the presence of the hot electrons has been studied for the low frequency interchange mode[1-4] driven by the diamagnetic drift and magnetic drifts of the plasma. These earlier works have confirmed that in order to obtain correct stability condition, the hot electrons must be treated by kinetic theory.[2-4] Nelson[2] and Van Dam and Lee[3] have investigated the low frequency interchange mode by retaining only the lowest order hot electron contribution in the electromagnetic part of the dispersion relation, D_{em}. They find that the hot electrons can enhance the compression of the core plasma and influence the stability analysis. The enhancement of the compressibility of the core plasma is due to a near cancellation in D_{em}. Therefore, when the next order hot electron contribution in D_{em} is included, we will expect quite different stability boundary. Since the core plasma compressional Alfvén frequency can be comparable to the hot electron magnetic drift frequency, we also expect the compressional Alfvén wave to be driven unstable by the hot electron magnetic drifts. Therefore, a realistic determination of the stability of EBT plasma should include both the interchange mode and the compressional Alfvén wave.

Because the driving mechanism for these modes is the interaction between the magnetic (∇B and curvature) drifts and diamagnetic drift of the hot electrons and the core plasma, the stability boundary is sensitive to the hot electron distribution function. In this paper we will employ two different models of the hot electron distribution function, δ-function and Maxwellian, to analytically study the stability of EBT plasma. The difference between our theory and the previous works[2,3] will be demonstrated.

II. FORMULATION

Consider a slab model[2,3] with density and magnetic field inhomogenieties in the x direction and an equilibrium magnetic field $\vec{B} = B_z(x)\hat{z} + B_x(x)\hat{x}$ where $B_x << B_z$. We also assume there are no temperature gradients. The plasma equilibrium is composed of three species — warm ions, warm electrons, and hot electrons. Then the equilibrium condition $\nabla(p + B^2/8\pi) = \vec{B}\cdot\nabla\vec{B}/4\pi$ can be written locally as

$$1/L_B = 1/L_c - (\tfrac{1}{2})(\beta_h/L_h + \beta_i/L_i + \beta_e/L_e) \tag{1}$$

where L_B, L_c, L_h, L_i, L_e are the magnetic field gradient, magnetic field curvature, hot electron density, warm ion density and warm electron density scale lengths, respectively. $\beta_s = 8\pi N_s T_s/B^2$ for s = i, e, h, where N denotes the density and T is the temperature. The quasi-neutrality condition, $N_i = N_e + N_h$ relates these equilibrium density scale lengths by

$$1/L_i = (1 - \delta)/L_e + \delta/L_h \tag{2}$$

where $\delta = N_h/N_i = (\beta_h T_i)/(\beta_i T_h)$.

We are interested in electromagnetic perturbations with zero parallel wavenumber, $k_\parallel = 0$, and long perpendicular wavelength $k_\perp\rho_i << 1$, where ρ_i is the ion gyroradius. We also restrict ourselves to local analysis and set $k_x = 0$. Then the perturbations can be specified by the electrostatic potential ϕ and the x-component of the vector potential A_x. The dispersion relation can be derived from the Vlasov equations and Maxwell's equations. In the limits $\omega_{pi}/\Omega_i >> 1$, $\omega/ck << 1$ and $\omega \lesssim \Omega_i$, where ω_{pi}, Ω_i are the ion plasma and cyclotron frequency respectively, c is the velocity of light, the dispersion relation can be simplified to the form[5]

$$D_{es}D_{em} + \left(\frac{\beta_i}{2}\right)D_{ct}^2 = 0 , \qquad (3)$$

where

$$D_{es} = \frac{\omega_{di}(\omega_{*i} - \omega_{di})(1 + \tau_e)}{(\omega - \omega_{di})(\omega - \omega_{de})} - \left(\frac{\omega - \omega_{*i}}{\omega - \omega_{di}}\right)\frac{\Omega_i^2 b_i}{(\omega - \omega_{di})^2 - \Omega_i^2}$$

$$- \left(\frac{\omega_{*e} - \omega_{de}}{\omega - \omega_{de}}\right)(\delta/\tau_e) + (C_1 - 1)(\delta/\tau_h) ,$$

$$D_{em} = 1 + \beta_i \left(\frac{\omega - \omega_{*i}}{\omega - \omega_{di}}\right)\left(1 + \frac{(\omega - \omega_{di})^2}{2b_i[(\omega - \omega_{di})^2 - \Omega_i^2]}\right)$$

$$+ \beta_e \left(\frac{\omega - \omega_{*e}}{\omega - \omega_{de}}\right) + \beta_h C_2 ,$$

$$D_{ct} = \left(\frac{\omega - \omega_{*i}}{\omega - \omega_{di}}\right)\frac{\Omega_i^2}{(\omega - \omega_{di})^2 - \Omega_i^2} + \left(\frac{\omega - \omega_{*e}}{\omega - \omega_{de}}\right)(1 - \delta) + C_3\delta ,$$

$$\omega_{*s} = \lambda_s b_i^{1/2}\tau_s(\rho_i/L_s)\Omega_i,\ b_i = (k_y\rho_i)^2,\ \rho_i^2 = T_i/m_i\Omega_i^2 ,$$

$$\omega_{ds} = \omega_{Bs} + \omega_{cs},\ \omega_{Bs} = \lambda_s b_i^{1/2}\tau_s(\rho_i/L_B)\Omega_i ,$$

$$\omega_{cs} = \lambda_s b_i^{1/2}\tau_s(\rho_i/L_c)\Omega_i,\ \tau_s = T_s/T_i,\ s = h, e, i,$$

and $\lambda_s = 1$ for ion and -1 for electron. For Maxwellian hot electrons, the constants C_1, C_2 and C_3 are given by

$$C_1 = (2/\pi^{1/2})(\omega - \omega_{*h}) \int_0^\infty dx \int_{-\infty}^\infty dy\ x \exp(-x^2 - y^2)/\omega' , \qquad (4(a))$$

$$C_2 = (1/\pi^{1/2})(\omega - \omega_{*h}) \int dxdy\, x^5 \exp(-x^2 - y^2)/\omega' \quad , \qquad (4(b))$$

and

$$C_3 = (2/\pi^{1/2})(\omega - \omega_{*h}) \int dxdy\, x^3 \exp(-x^2 - y^2)/\omega' \quad , \qquad (4(c))$$

where

$$\omega' = \omega - (\omega_{Bh} x^2/2 + \omega_{ch} y^2) \ .$$

If we further make the approximation with $\omega' \approx \omega - \omega_{dh}$ in Eqs. 4(a) - 4(c) and perform the integrations analytically, then this modified Maxwellian hot electron model gives

$$C_1 = C_2 = C_3 = (\omega - \omega_{*h})/(\omega - \omega_{dh}) \quad . \qquad (5)$$

This turns out to be a good approximation when we compare the numerical solutions from both Eqs. (4) and (5).[5] In the following we will employ Eq. (5) for the analytical investigation. If a delta function is used for hot electrons, then

$$C_1 = 2C_2 = C_3 = (\omega - \omega_{*h})/(\omega - \omega_{dh}) \quad . \qquad (6)$$

Note that the only difference between Eqs. (5) and (6) is in C_2 and we will write $C_2 = \hat{C}_2\ (\omega - \omega_{*h})/(\omega - \omega_{dh})$ with $\hat{C}_2 = 1$ for the Maxwellian hot electrons and $\hat{C}_2 = 1/2$ for the δ-function hot electrons.

III. INTERCHANGE MODE

Let us first consider the low frequency interchange instability with $\omega_{*i}<\omega<\omega_{dh}, \omega_{*h}$. In this frequency regime, the three terms in the disper relation, Eq. (3), can be simplified to yield

$$D_{es} \simeq b_i + (\omega_{*i} - \omega_{di})[(1 + \tau_e)\omega_{di}/\omega^2 + \delta/\omega] ,$$

$$D_{em} \simeq \bar{D}_{em} + \hat{c}_2\beta_h\left(\frac{\omega_{*h} - \omega_{dh}/2}{\omega_{dh}}\right)\omega ,$$

$$\bar{D}_{em} = 1 + \beta_i + \beta_e + \hat{C}_2\beta_h\omega_{*h}/\omega_{dh} ,$$

and

$$D_{ct} \simeq \int \omega_{*h}/\omega_{dh} + (1 + \tau_e)(\omega_{*i} - \omega_{di})/\omega .$$

We note that Eq. (7) will give rise to the usual quadratic dispersion rela if we neglect the second term in D_{em} which is of $O(\omega/\omega_{dh})$. However, the electron contribution will give a near cancellation in $\bar{D}_{em}$ and result i enhanced compression of the core plasma.[2,3] Therefore, we must treat $\bar{D}_{e}$ be the same order as the second term in D_{em} at marginal stability. From (7) the dispersion relation can be written as a cubic in ω:

$$\omega^3 + A_2\omega^2 + A_1\omega + A_o = 0 ,$$

where

$$A_2 = [b_i D_{em} + \frac{\beta_i}{2} (\delta \frac{\omega_{*h}}{\omega_{dh}})^2 + \hat{C}_2 \beta_h (\frac{\omega_{*i} - \omega_{di}}{\omega_{di}})^2 \delta]/S \quad ,$$

$$A_1 = [\hat{C}_2 \beta_h (\frac{\omega_{*h} - \frac{\omega_{dh}}{2}}{\omega_{dh}}) \omega_{di} (1 + \tau_e) + \bar{D}_{em} \delta + \beta_i \delta (\frac{\omega_{*h}}{\omega_{dh}})(1 + \tau_e)] (\frac{\omega_{*i} - \omega_{di}}{S})$$

$$A_o = [\bar{D}_{em} \omega_{di} + (\frac{\beta_i}{2})(1 + \tau_e)(\omega_{*i} - \omega_{di})] \frac{(1 + \tau_e)(\omega_{*i} - \omega_{di})}{S} \quad ,$$

$$S = \hat{C}_2 \beta_h (\frac{\omega_{*h} - \frac{\omega_{dh}}{2}}{\omega_{dh}}) b_i \quad .$$

Without hot electrons ($\delta = \beta_h = 0$), Eq. (8) reduces to a quadratic in ω and describes core plasma interchange instability. With hot electrons the condition for stable interchange mode is that

$$Q^3 + R^2 \leq 0 \quad , \tag{9}$$

where

$Q = (A_1 - A_2^2/3)/3$, $R = (A_1 A_2 - 3A_o)/6 - A_2{}^3/27$, and the frequency at marginal stability is $\omega = -(R^{1/3} + A_2/3)$. In Fig. 1 we plot the stability boundary from Eq. (9) in the $\beta_i - \beta_h$ space for the δ-function hot electrons with the fixed parameters: $\rho_i/L_h = \rho_i/L_i = -0.04$, $L_c/L_i = 40$, $\tau_h = 10^3$, $\tau_e = 1$, $\omega_{pi}/\Omega_i = 25$, $m_i/m_e = 1837$, and $k_y \rho_i = 0.1$. Within the closed stability boundary, the interchange mode is stable. The solution from Eq. (3) (denoted by exact) is also shown for comparison, and our cubic dispersion relation gives amazingly good results.

We have also plotted in Fig. 1 the stability boundary from the quadratic dispersion relation[2,3] by neglecting the 2nd term in D_{em} in Eq. (7). The lower stability boundary (core plasma interchange mode) is a good approximation because it is mainly determined by $D_{es} = 0$. But the upper stability boundary (hot electron interchange mode; unstable roughly when $\tilde{D}_{em} \gtrsim 0$) does not turn around at small β_i which is due to the absence of the $O(\omega/\omega_{dh})$ term in D_{em}. In general, the quadratic dispersion relation predicts more optimistic results than our cubic dispersion relation. Below $\delta = 1$ line is the forbidden region with $N_h > N_i$.

For the Maxwellian hot electrons, the stability boundary in $\beta_i - \beta_h$ space is shown in Fig. 2 for the same set of parameters as in Fig. 1. Our results are very good in comparison with the solution from Eq. (3). Again stability boundary from the quadratic dispersion relation is also shown for comparison. The lower stability is good, but the upper stability boundary is again over optimistic.

Figure 3 shows that the finite Larmor radius stabilization of the low frequency interchange mode. With the same set of parameters as in Fig. 1, the stability boundaries for two different values of $k_y\rho_i$ ($k_y\rho_i$ = 0.1, 0.05) are plotted in the $\beta_i - \beta_h$ space. As $k_y\rho_i$ is reduced the stability boundary moves toward larger β_h and does not intersect with the β_i and β_h axes. Therefore, if β_i is small, no matter how large β_h is there is no stability.

IV. COMPRESSIONAL ALFVÉN WAVE

Now we consider the compressional Alfvén wave with $\omega \sim k_yV_A \sim \omega_{dh}$, but $\omega > \omega_*$, ω_d for the warm species. Since $(k_yV_A)^2 = (2b_i/\beta_i)\,\Omega_i^{\,2}$, k_yV_A can be of the same order as Ω_i. Therefore, one might expect the compressional Alfvén wave to couple not only with the hot electron magnetic drifts but also with the ion cyclotron waves. In this frequency regime the three terms in the dispersion relation, Eq. (3), can be simplified to yield:

$$D_{es} = b_i \frac{\Omega_i^2}{\Omega_i^2 - \omega^2} ,$$

$$D_{em} = 1 + \beta_i + \beta_e + \frac{\beta_i}{2b_i} \frac{\omega^2}{(\omega^2 - \Omega_i^2)} + \hat{C}_2\beta_h \left(\frac{\omega - \omega_{*h}}{\omega - \omega_{dh}}\right) , \tag{10}$$

$$D_{ct} = \frac{\Omega_i^2}{\omega^2 - \Omega_i^2} + (1 - \delta) + \delta \left(\frac{\omega - \omega_{*h}}{\omega - \omega_{dh}}\right) .$$

And the dispersion relation becomes

$$\left\{ \frac{\omega^2}{\Omega_i^2} \left[1 + \delta(\omega_{dh} - \omega_{*h})/(\omega - \omega_{dh})\right] - \delta^2 \left(\frac{\omega_{dh} - \omega_{*h}}{\omega - \omega_{dh}}\right)^2 \right.$$

$$\left. - \frac{2b_i}{\beta_i} \left[1 + \beta_i + \beta_e + \hat{C}_2\beta_h\left(\frac{\omega - \omega_{*h}}{\omega - \omega_{dh}}\right)\right]\right\}/(\omega^2 - \Omega_i^2) = 0. \tag{11}$$

We see that the compressional Alfvén wave decouples from the ion cyclotron waves even in the presence of hot electrons. We further note that in the limits $\delta = 0$ and $\beta_h = 0$ (i.e., there are no hot electrons), Eq. (11) recovers the well known compressional Alfvén wave with $\omega^2 = k_y^2 v_A^2(1 + \beta_i + \beta_e)$. The compressional Alfvén wave mainly couples with and, hence, is destabilized by the hot electron magnetic drifts and diagmagnetic drift. If we also assume that

$$\delta^2 \left(\frac{\omega_{dh} - \omega_{*h}}{\omega - \omega_{dh}}\right)^2 < M_{in}\left[1, \left(\frac{\omega}{\Omega_i}\right)^2\right] \tag{12}$$

then Eq. (11) can be cast into a cubic form in ω:

$$Z^3 + A_2 Z^2 + A_1 Z + A_o = 0, \tag{13}$$

$$Z = (\omega/k_y\rho_i\Omega_i) \ ,$$

$$A_2 = -\left(\omega_{dh}/k_y\rho_i\Omega_i\right) \ ,$$

$$A_1 = -\frac{2}{\beta_i}\left(1 + \beta_i + \beta_e + \hat{C}_2\beta_h\right) < 0 \ ,$$

$$A_o = 2\left[(1 + \beta_i + \beta_e)\omega_{dh} + \hat{C}_2\beta_h\omega_{*h}\right]/(k_y\rho_i\Omega_i) \ .$$

Since A_o, A_1, and A_2 are independent of k_y, the stability boundary is also independent of k_y but the frequency ω is linear in k_y. From Eq. (9), the condition for stable solution is given by

$$\frac{2}{3}(C-A)B^2 + (4C^2 - 3A^2)B + 8C^3 \geq 0 \ , \tag{14}$$

where

$$A = C_2\beta_h(1 - 3\frac{\omega_{*h}}{\omega_{dh}}) - 2(1 + \beta_i + \beta_e) \ ,$$

$$B = \left(\frac{\omega_{dh}}{k_y\rho_i\Omega_i}\right)^2\beta_i > 0 \ ,$$

$$C = 1 + \beta_i + \beta_e + \hat{C}_2\beta_h > 0.$$

In general $\omega_{*h}/\omega_{dh} < 0$ and it is possible to obtain unstable solution only when $(C-A) = 3\left(1 + \beta_i + \beta_e + \hat{C}_2\ \beta_h \frac{\omega_{*h}}{\omega_{dh}}\right) = 3\ \bar{D}_{em} < 0$. We note that this is opposite to the instability condition for hot electron interchange mode. On the other hand, $\bar{D}_{em}$ > is sufficient for stable solution. Considering $L_c/L_B >> 1$ and δ-function hot electrons with $\hat{C}_2 = 1/2$, we find

$$(C - A) = 3\left[1 + \beta_i + \beta_e - \left(1 + \frac{\beta_i}{\beta_h}\frac{L_h}{L_i} + \frac{\beta_e}{\beta_h}\frac{L_h}{L_e}\right)^{-1}\right] \tag{16}$$

and the compressional Alfvén wave is always stable for δ-function hot electrons when $L_h/L_i > 0$ and $L_h/L_e > 0$. This has been confirmed by numerical solutions. If $\tilde{D}_{em} < 0$, then in the limit β_i, $\beta_e \ll \beta_h$, the stability condition, Eq. (14), can be approximately expressed as

$$\left(\frac{\tau_h \rho_i}{2L_h}\right)^2 \beta_i \beta_h^{\,2} \lesssim \left(\frac{-x + (x^2 - 4y)^{1/2}}{2}\right) , \tag{17}$$

where

$$x = \frac{2(1 + \hat{C}_2\beta_h)^2 - \frac{3}{2}\left(\hat{C}_2(\beta_h - 6) - 2\right)^2}{\left[1 + \beta_i + \beta_e - 2\hat{C}_2\left(1 + \frac{\beta_i}{\beta_h}\frac{L_h}{L_i} + \frac{\beta_e}{\beta_h}\frac{L_h}{L_e}\right)^{-1}\right]} > 0 ,$$

and

$$y = 4\,(1 + \hat{C}_2\beta_h)^3 / \left[1 + \beta_i + \beta_e - 2\hat{C}_2\left(1 + \frac{\beta_i}{\beta_h}\frac{L_h}{L_i} + \frac{\beta_e}{\beta_h}\frac{L_h}{L_e}\right)^{-1}\right] < 0.$$

Note that for the Maxwellian hot electron model, $\hat{C}_2 \approx 1$ and x and y are weak functions of β_i and β_e for β_i, $\beta_e < \beta_h \lesssim 1$. However, for the δ-function hot electron model, $\hat{C}_2 = 1/2$ and x and y become inversely proportional to β_i and β_e.

The stability boundary in $\beta_i - \beta_h$ space for the compressional Alfvén wave from Eq. (14) is shown in Fig. 3 for the Maxwellian hot electron model with the same parameters as in Fig. 1. The approximate solutions are very good in comparison with the exact numerical solutions of the dispersion relation Eq. (3). The behavior of the stability boundary at small β_i can be very well explained by Eq. (17). Now the stability window is enclosed by both the interchange and the compressional Alfvén stability boundaries. As $k_y\rho_i$

decreases the stable region shrinks mainly due to the shift of the interchange stability boundary. The compressional Alfvén stability boundary is rather insensitive to $k_y\rho_i$ for $k_y\rho_i \ll 1$.

For δ-function hot electron model, the compressional Alfvén wave may become unstable only when $L_h/L_i < 0$ and $L_h/L_e < 0$ and at somewhat higher β_i and β_h than the Maxwellian hot electron model case. This has been confirmed by the numerical solutions of the full dispersion relation.

V. CONCLUSION

In this paper we have correctly analyzed the local solutions of the low frequency interchange and the compressional Alfvén instabilities of the EBT plasma in the frequency regime $\omega \lesssim \Omega_i$. The analytical solutions are then compared to the numerical solutions of the full dispersion relation with good agreement. These instabilities are mainly determined by the magnetic drifts and diamagnetic drift of the hot electrons. Therefore the stability boundary is very sensitive to the hot electron distribution function. Two different models of hot electron distribution function, δ-function and isotropic Maxwellian, are employed in our analysis and yield very different results. Unlike the previously obtained quadratic dispersion relation in ω,[2,3] our simplified dispersion relations are cubic in ω for both types of the instabilities. For the low frequency interchange mode, our cubic dispersion relation is due to an extra hot electron term of $O(\omega/\omega_{dh})$ in the electromagnetic part of the dispersion relation, D_{em}. This term is ignored in the previous quadratic dispersion relation, but is important because at marginal stability the O(1) terms in D_{em} nearly cancel with each other and becomes the same order as the $O(\omega/\omega_{dh})$ term. The stability turns out to be more pessimistic than predicted in the previous theories.

For the compressional Alfvén wave, our cubic dispersion relation is due to the compressional term in D_{em} and is obtained in the limit $\delta^2(\omega_{dh} - \omega_{*i})^2/(\omega - \omega_{dh})^2 < M_{in}[1, (\omega/\Omega_i)^2]$. The stability boundary is independent of $k_y\rho_i$ and the frequency is linear in $k_y\rho_i$. With $L_h/L_i > 0$ and $L_h/L_e > 0$, the compressional Alfvén wave is shown to be stable for δ-function hot electrons and can be unstable for Maxwellian hot electrons. With $L_h/L_i < 0$ and $L_h/L_e < 0$ the compressional Alfvén wave can be unstable for both models of hot electron distribution function.

Then the stability window of the EBT plasma is determined by both the interchange stability boundary and the compressional Alfvén stability boundary. This somewhat pessimistic result for EBT stability may be improved by a nonlocal calculation in a realistic geometry and with a proper equilibrium including anisotropic hot electron distribution and temperature gradients.

Acknowledgments

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Figure Captions

Fig. 1 Marginal interchange stability boundaries in $\beta_i - \beta_h$ space for the δ-function hot electron model. The fixed parameters are $\rho_i/L_i = \rho_i/L_h = -0.04$, $L_c/L_i = 40$, $k_y\rho_i = 0.1$, $\omega_{pi}/\Omega_i = 25$, $T_h/T_i = 10^3$, $T_e/T_i = 1$, and $m_i/m_e = 1837$. Solutions of the quadratic, cubic, and full dispersion relations are shown for comparison. The compressional Alfvén wave is stable for the set of parameters.

Fig. 2 Marginal stability boundaries in $\beta_i - \beta_h$ space for the Maxwellian hot electron model. The parameters are the same as in Fig. 1. Solutions of the quadratic and cubic dispersion relations are shown for comparison. The stability window is enclosed by the interchange and the compressional Alfvén stability boundaries.

Fig. 3 Finite Larmor radius effects ($k_y\rho_i = 0.05, 0.1$) on the marginal stability boundaries in $\beta_i - \beta_h$ space for the Maxwellian hot electron model. The compressional Alfvén stability boundary is independent of $k_y\rho_i$. The other parameters are the same as in Fig. 1.

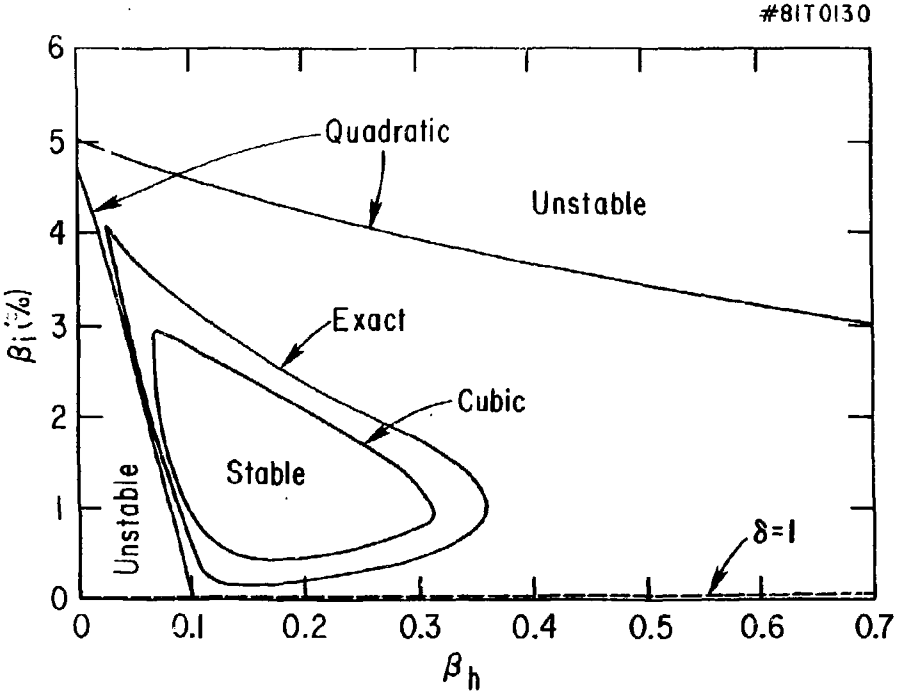

#81T0130
Quadratic
Unstable
Exact
Cubic
Stable
Unstable
δ=1
β_i (%)
β_h
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
1
2
3
4
5
6

Fig. 1

81T0131

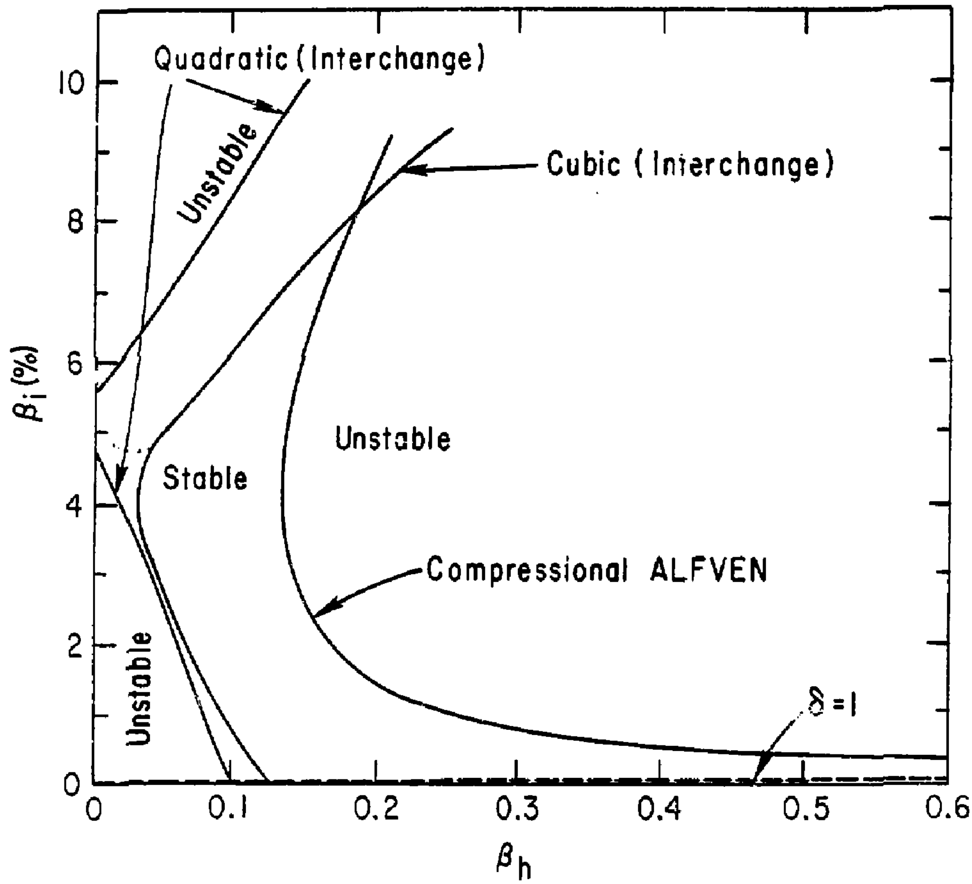
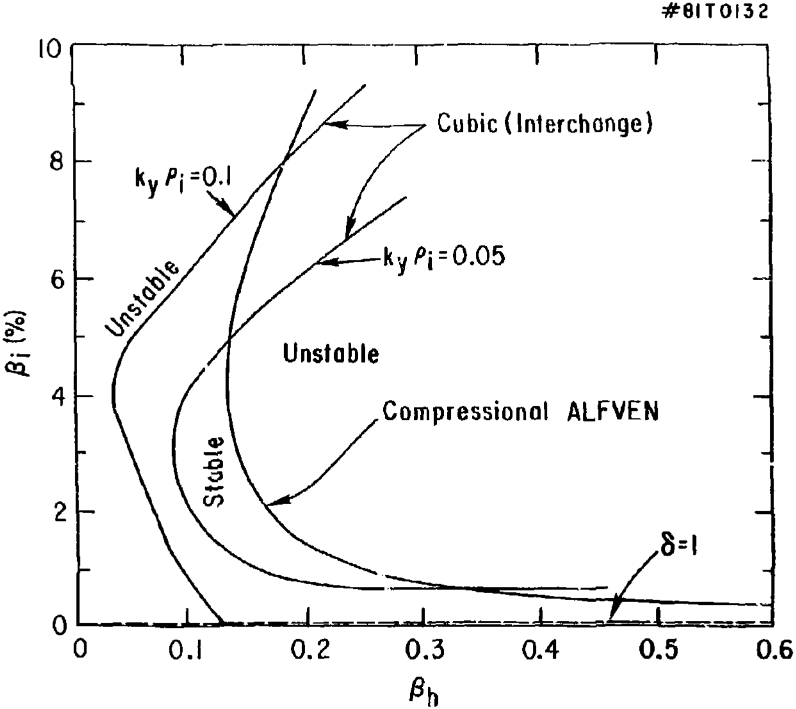


Fig. 2



#81T0132
Cubic (Interchange)
$k_y \rho_i = 0.1$
$k_y \rho_i = 0.05$
Unstable
Unstable
Stable
Compressional ALFVEN
$\delta = 1$
β_i (%)
β_h
0
2
4
6
8
10
0
0.1
0.2
0.3
0.4
0.5
0.6

Fig. 3