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Potential Hydraulic Modelling Errors Associated with Rheological Data Extrapolation in Laminar Flow (U)

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Potential Hydraulic Modelling Errors Associated with Rheological Data Extrapolation in Laminar Flow

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Abstract: *The potential errors associated with the modelling of flows of non-Newtonian slurries through pipes, due to inadequate rheological models and extrapolation outside of the ranges of data bases, are demonstrated. The behaviors of both dilatant and pseudoplastic fluids with yield stresses, and the errors associated with treating them as Bingham plastics, are investigated.*

The melter feed in the Savannah River Site Defense Waste Processing Facility, DWPF, process is a non-Newtonian slurry. It is pumped over a wide range of flow rates, through pipes of various diameters, and both laminar and turbulent flows are encountered. The slurries exhibit yield stresses, and they are modelled as Bingham plastics, even though the stress/strain rate behavior is generally non-linear. This paper describes the potential hydraulic modelling errors associated with: assuming Bingham plastic behavior where a more complicated rheology model is required, and extrapolating stress/strain rate behavior beyond the data base. This study is confined to laminar flow.

DISCUSSION

The DWPF melter feed slurries consist of mixtures of glass frit, waste sludge, and water. The particle size distributions are bimodal. The glass frit consists of irregular shaped particles, resembling broken glass, with a mean diameter of 150 μm, and the sludge consists of much smaller particles. The slurries are modelled as Bingham plastics, and the data to support these models are obtained with a concentric cylinder viscometer. Figure 1 is a typical plot of viscometer data for five different slurries. The slurries differ in the percent weight of total solids. The vertical ordinate of the plot is the shear stress, and the horizontal ordinate is the strain rate. The viscometer data show that the fluids exhibit yield stress type behavior, and they also clearly show non-linear behavior for non-zero strain rates. The bottom three rheograms in figure 1 exhibit shear thinning or pseudoplastic behavior, and the top two rheograms exhibit shear thickening or dilatant behavior.

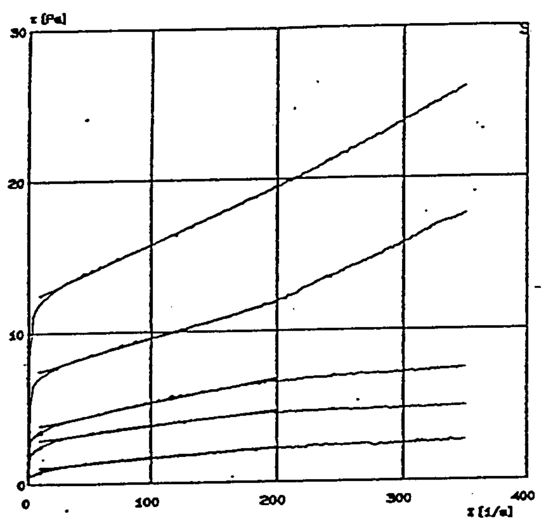

Fig. 1 Concentric cylinder viscometer data, shear stress vs strain rate, for simulated DWPF slurries.

The non-linear behavior in the rheograms cannot be represented by a Bingham plastic model, in which the stress is assumed to be a linear function of the strain rate once the yield stress is exceeded. Equation 1 is the functional form of a Bingham plastic, (BP). This is a two parameter model. In order to model the non-linear behavior, a model with more degrees of freedom is required. A yield/power law, (YPL), or Herschel-Bulkley model is a simple three parameter model that will accommodate the non-linear behavior. Equation 2 is the functional form of a yield/power law model.

$$\tau_{rz} = \tau_o + \eta_o \dot{\gamma} \quad (1)$$

$$\tau_{rz} = \tau_o + \eta_o (\dot{\gamma})^n \quad (2)$$

Rheological models are necessary for predicting flow versus pressure drop behavior of non-Newtonian fluids. Equation 3 is the Buckingham-Reiner equation for flow rate as a function of pressure drop for a Bingham plastic, and equation 4 is the analogous equation for a yield/power law fluid, [1]. Dimensional analysis of pipe flow with a yield/power law fluid shows that the non-dimensional pressure drop is a function of the length to diameter ratio, the Reynolds number, and the Hedstrom number, where the Reynolds and Hedstrom numbers are defined in equation 5. Setting "n" equal to one results in the analogous relations for a Bingham plastic.

$$Q = \frac{\pi \Delta P R^4}{8 L \eta_o}\left[1 - \frac{8}{3}\frac{L\tau_o}{\Delta P R} + \frac{16}{3}\left(\frac{L\tau_o}{\Delta P R}\right)^4\right] \quad (3)$$

$$Q = \frac{4\pi L n}{(n+1)\Delta P \eta_o^{1/n}}\left\{\frac{R^2}{2}\left(\frac{\Delta P R}{2L} - \tau_o\right)^{\frac{n+1}{n}} - \frac{4nL^2}{\Delta P^2}\left[\frac{\left(\frac{\Delta P R}{2L} - \tau_o\right)^{\frac{3n+1}{n}}}{3n+1} + \frac{\tau_o\left(\frac{\Delta P R}{2L} - \tau_o\right)^{\frac{2n+1}{n}}}{2n+1}\right]\right\} \quad (4)$$

$$\mathrm{Re} = \frac{\rho v^{2-n} D^n}{\eta_o} \qquad He = \frac{\rho \tau_o D^{2n} v^{2(1-n)}}{\eta_o^2} \quad (5)$$

The transitions from laminar to turbulent flow of a Bingham plastic and a yield/power law fluid are also defined in terms of the Reynolds and Hedstrom numbers. Hanks, [2], developed a general method for determining the laminar stability threshold of any time-independent viscous fluid for which a stress/strain rate constitutive relation is known. He used the method to determine the critical Reynolds numbers for Bingham plastics and Powell-Eyring fluids, and demonstrated good agreement between theory and data. Hanks method was used to derive the appropriate relations for predicting the onset of transition to turbulence of a yield/power law fluid. First equation 6 is used to solve for α_c. This parameter is then substituted into equation 7, which is solved for the critical or transition Reynolds number. The analogous relations for a Bingham plastic are equations 8 and 9, [1].

$$\frac{\alpha_c}{(1-\alpha_c)^{\frac{n+2}{2-n}}} = \frac{\rho^{\frac{n}{2-n}} \tau_o R^{\frac{2n}{2-n}}}{\eta_o^{\frac{2}{2-n}}}\left[\frac{n}{808(n+2)^{\frac{n+2}{n+1}}}\right]^{\frac{n}{2-n}} \quad (6)$$

$$\mathrm{Re}_c = \frac{3232 n^{1-n}(n+2)^{\frac{n+2}{n+1}}}{(n+1)^{2-n}(1-\alpha_c)^{\frac{n+2}{n}}}\left[\frac{1}{2}(1-\alpha_c)^{\frac{n+1}{n}} - \frac{n\alpha_c(1-\alpha_c)^{\frac{2n+1}{n}}}{2n+1} - \frac{n(1-\alpha_c)^{\frac{3n+1}{n}}}{3n+1}\right]^{2-n} \quad (7)$$

$$\frac{\alpha_c}{(1-\alpha_c)^3} = \frac{He}{16800} \quad (8)$$

$$\mathrm{Re}_c = \frac{He}{8\alpha_c}\left(1 - \frac{4}{3}\alpha_c + \frac{1}{3}\alpha_c^4\right) \quad (9)$$

Bingham plastic and yield/power law models are completely empirical, so it is imprudent to extrapolate outside of the range of the data base. The system modelling requirements determine need for rheological data. The data shown in figure 1 is sufficient for modelling pipe flows in which the absolute value of the wall velocity gradient (strain rate) is less than 350 1/s. This will be shown to be quite limiting.

ANALYSIS

Predicted pipe flows through the DWPF melter feed line for Bingham plastic and yield/power law models of the fluids with the top three rheograms in figure 1 are compared. The three rheograms are for slurries with 54.17%, 49.36%, and 47.34% total weight solids respectively from the top. Ten points were picked off each of the rheograms and a yield/power law model, equation 2, was fit to the data. Figure 2 shows the ten data points, the yield/power law model, and a Bingham plastic model based on data up to a strain rate of 200 1/s, the procedure used at DWPF, for each of the three rheograms. The rheological model parameters for figure 2 are shown in table 1.

The melter feed line is a 3/8" line with an equivalent length of 16.217 m. Figures 3 through 5 show the predicted flow rates of the three slurries through the melter feed line, as a function of the frictional pressure drop. These flow rates are calculated with equation 3 for the Bingham plastic fluids and equation 4 for the yield/power law fluids. The plots span the range of strain rates from 50 to 350 1/s. There is good agreement between the Bingham plastic and the yield/power law fluid flow rates for the 54.17% total weight solids slurry. The two rheological models for this slurry are also in good agreement over the entire range of strain rates. The other two slurries show significant differences between

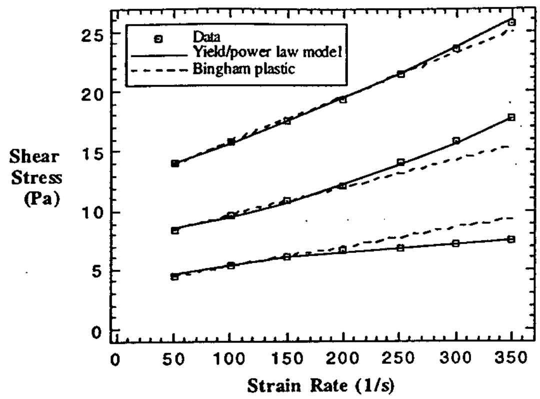

Fig. 2 YPL and BP models of the top three slurry reograms in fig. 1: 54.17%, 49.36%, and 47.34% total wt. solids respectively from the top.

% Total Wt. Solids	*Yield Stress*	*Consistency*	*Exponent*
54.17	12.8	0.00978	1.23
	12.08	0.03726	1
49.36	8.01	0.0014	1.509
	7.26	0.02329	1
47.34	1.582	0.785	0.3427
	3.668	0.01623	1

Table 1: Rheological model parameters for the three slurries shown in figure 2.

the predicted flow rates of the two rheological models at higher strain rates. The Bingham plastic model predicts higher flow rates than the yield/power law model, at the upper end of the range of pressure drops, for the 49.36% total weight solids slurry, and the opposite behavior is predicted for the 47.34 % total weight solids slurry. In both cases the yield/power law models will be better since they are in better agreement with the rheological data. For the dilatant fluid, the Bingham plastic model over predicts the flow rate, and for the pseudoplastic fluid, the Bingham plastic model under predicts the flow rate. The danger of extrapolating beyond a strain rate of 350 1/s is evident in figures 4 and 5. The differences between the predicted flow rates for the two rheological models increase rapidly with increasing flow rate.

Whether a flow is laminar or turbulent is a function of the Reynolds number. The transition Reynolds number for flow of a yield/power law fluid is calculated with equations 8 and 9, and for a Bingham plastic with equations 6 and 7. The predicted transition Reynolds numbers for the three slurries are shown in table 2, along with the Reynolds numbers at a strain rate of 350 1/s. The rheological data in figure 1 falls well short of covering the laminar flow range. The transition Reynolds numbers for the various rheological models differ significantly. This suggests that the Reynolds numbers of the various rheology models cannot be compared, but the Reynolds number has significance only in the context of a specific rheological model.

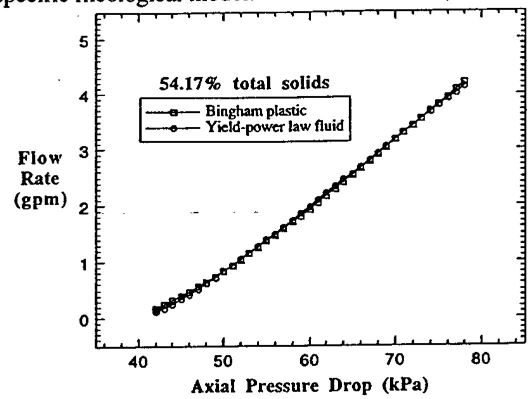

Fig. 3 Flow rate through the melter feed line as a function of the frictional pressure drop.

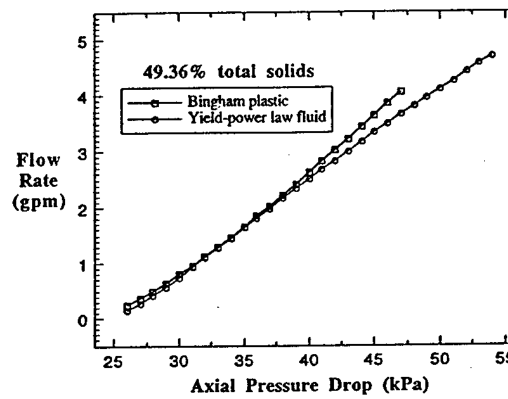

Fig. 4 Flow rate through the melter feed line as a function of the frictional pressure drop.

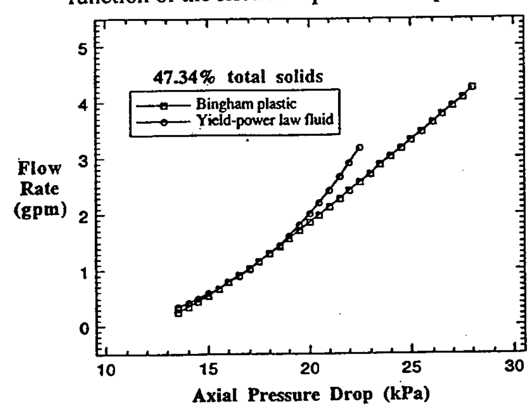

Fig. 5 Flow rate through the melter feed line as a function of the frictional pressure drop.

% Total Wt. Solids	Rheological Model	Transition Re	Re @ 350 1/s
54.17	YPL	3138.6	1028.2
	BP	2366.1	569.8
49.36	YPL	4692.5	2719.4
	BP	2475.4	890.1
47.34	YPL	795.7	171.9
	BP	2483.2	1368.4

Table 2: Calculated laminar to turbulent flow transition Reynolds numbers for the rheological models of the three slurries shown in figure 2.

The DWPF sampling system has a 1/2" schedule 40 pipe with an equivalent length of 52.4 m. The range of expected flow rates is between 4 and 8 gpm. The expected wall strain rates will exceed 350 1/s, and therefore the rheological data shown in figure 1 is inadequate to form a basis for modelling this flow. To show the potential errors introduced by using a Bingham plastic model based on low strain rate data to describe the flow behavior of a fluid that is more appropriately modelled as a yield/power law fluid, the flow of a postulated dilatant yield/power law fluid will be considered. Figure 6 shows the assumed rheological behavior of the postulated fluid over the entire laminar flow range. The rheograms cover the strain rate range from 50 to 2000 1/s. Also shown are two Bingham plastic models of the fluid, one is based on the entire range of strain rates, and the other is based on the strain rates up to 300 1/s. The second Bingham plastic model is the one that would be derived from concentric cylinder viscometer data. Flow rates in the sample line, of fluids with the three constituitive relations shown in figure 6, are calculated as functions of the frictional pressure drop. The predicted flow rates as functions of the pressure drop are shown in figure 7. The plots of flow rates go up to the point that transition from laminar to turbulent flow is predicted to occur. As you would expect, flow rates of the Bingham plastic model based on strain rates below 300 1/s agree well with the lower flow rates of the yield/power law fluid, and the Bingham plastic model based on the entire strain rate range agrees better with the yield/power law model at higher flow rates. For this assumed fluid, a single Bingham plastic model is inadequate to model flows in both the melter feed lines and the sample lines. A constituitive relation that is in good agreement with the rheological data over the entire laminar range is the best alternative

Figure 8 shows the Reynolds numbers as functions of the wall strain rate for the three fluid models. While the laminar/turbulent transition Reynolds numbers differ significantly for the three fluid models, transition is predicted to occur at a wall strain rate between 1400 and 1500 1/s.

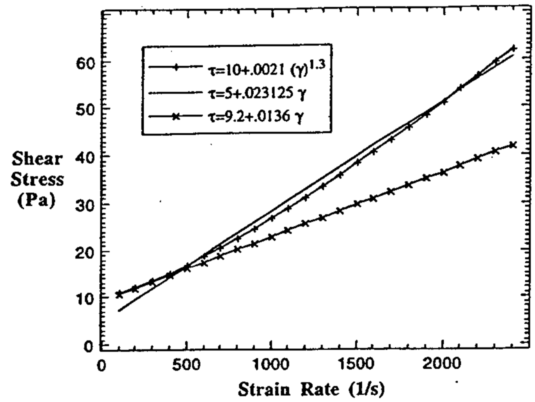

Fig. 6 The assumed dilatant YPL fluid and two BP models of the fluid.

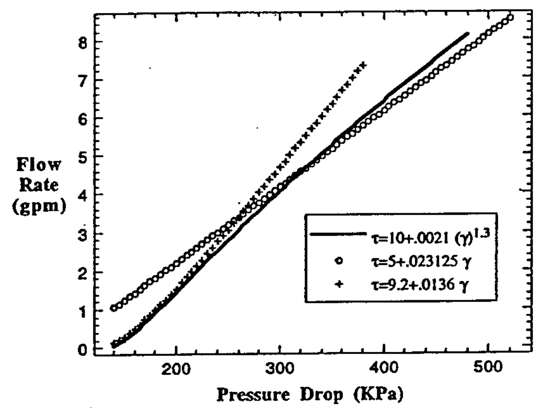

Fig. 7 Flow rate through the DWPF sample line of the fluid shown in figure 6.

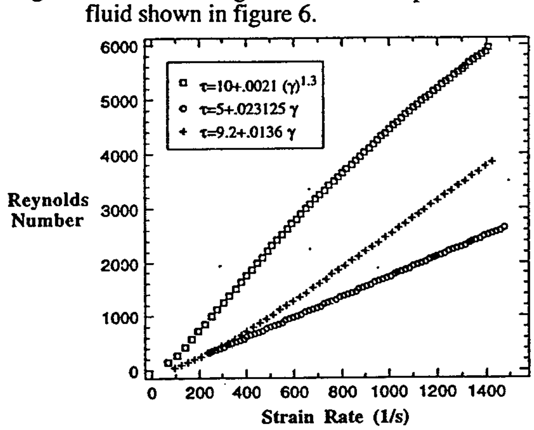

Fig. 8 Reynolds numbers in the DWPF sample line as functions of the wall strain rates.

Figure 9 shows the percent error in the predicted flow rates of the two Bingham plastic models of the assumed fluid, relative to the predicted flow rate of the yield/power law model. At very low flow rates, the relative errors of both Bingham plastic models are large, but the absolute flow rates are small so the absolute errors are small. The relative error of the model based on strain rate data below 300 1/s increases linearly for flow rates above 2.0 gpm, and the relative error is 25% at the predicted transition to turbulence. This plot clearly shows the danger of extrapolating the constituitive relations beyond the range of the data on which it is based.

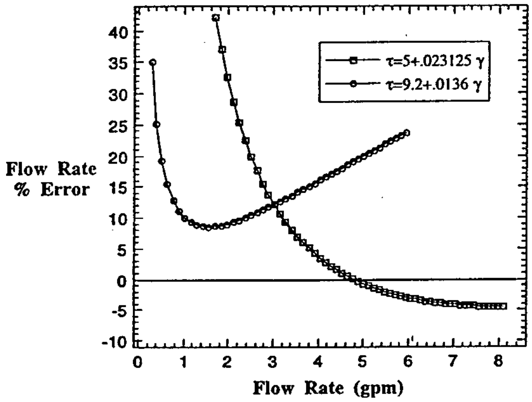

Fig. 9 Percent error in the predicted flow rates in the DWPF sample line for the two BP models, relative to the YPL model.

The results of similar calculations with an assumed pseudoplastic fluid, shown in figure 10, are presented in figures 11 through 13. The results are similar to those for the dilatant fluid, and the same conclusions apply. Transition to turbulence occurs at a slightly higher flow rate, between 8.0 and 9.0 gpm, than with the dilitant fluid, and the wall strain rate at transition is also higher, between 1500 and 1800 1/s. Figures 11 through 13 correspond respectively with figures 7 through 9, and they are presented without further discussion.

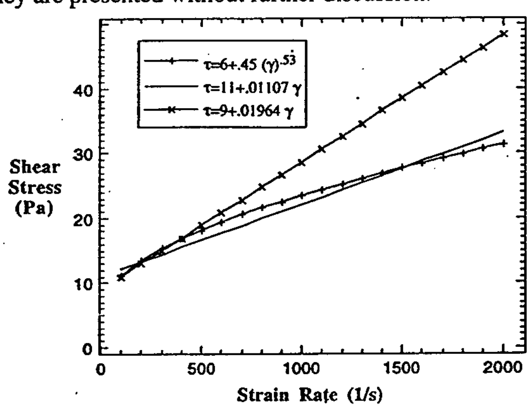

Fig. 10 The assumed pseudoplastic YPL fluid and two BP models of the fluid.

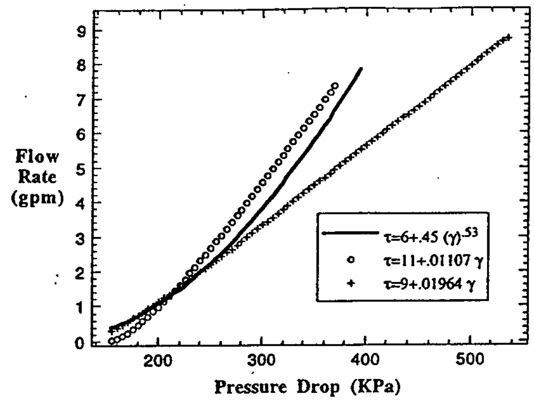

Fig. 11 Flow rate through the DWPF sample line of the fluid shown in fig. 10.

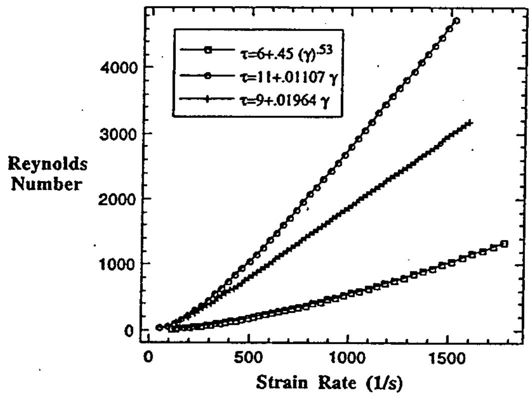

Fig. 12 Reynolds numbers in the DWPF sample line as functions of the wall strain rates, for the fluids shown in fig. 10.

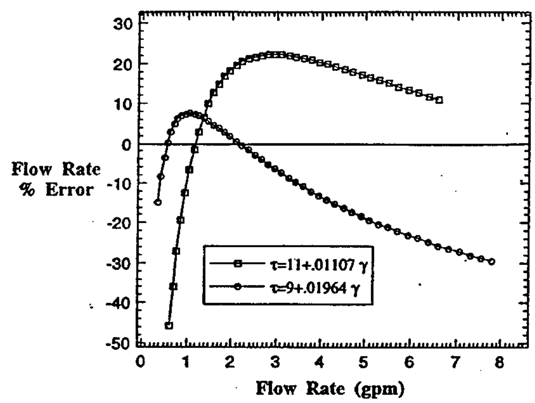

Fig. 13 Percent errors in the predicted flow rates in the DWPF sample line for the two BP models, relative to the YPL model.

CONCLUSIONS

It is difficult to obtain high strain rate data with a concentric cylinder viscometer, because of centrifugal effects and wall slip. Good low strain rate data, that allow determination of the yield stress, can be obtained with this type of viscometer. The interpretation of data collected with a concentric cylinder visometer is also straightforward. A capillary viscometer is a device in which the axial pressure drop of fully-developed flow through a straight tube is measured. The laminar flow range up to transition can easily be covered with this type of viscometer, though it is not ideal for measuring the behavior of fluids at very low flow rates, or wall strain rates. The two types of viscometer are therefore complementary, and they can be used to cover the rheometry of slurries from close to the yield stress to the transition to turbulence.

A capillary viscometry is simple and inexpensive. All one needs to do is measure the axial pressure drop in a straight pipe in which the flow is fully-developed. The data to be collected is the pressure drop and the flow rate. This data can be converted to shear stress versus strain rate data with the Rabinowitsch equation, equation 10, [3].

$$-\left(\frac{dv_z}{dr}\right)_{r=R} = \frac{1}{\pi R^3 \tau_R^2}\frac{d}{d\tau_R}\left(\tau_R^3 Q\right) \qquad (10)$$

The left side of equation 10 is the absolute value of the wall strain rate. By substituting an expression for the wall shear stress, as a function of the length and the pressure drop and expanding the differential term on the right side of equation 10, the wall strain rate can be expressed as a function of the flow rate and the pressure drop, in a form convenient for calculating the wall strain rate, equation 11.

$$\dot{\gamma} = \frac{Q}{\pi R^2}\left(\frac{d(\ln Q)}{d(\ln \Delta P)} + 3\right) \qquad (11)$$

The strain rate can be obtained directly from a plot of the natural log of the flow rate versus the natural log of the pressure drop. If pressure drop versus flow rate data is obtained up to the point of transition to turbulent flow, and low strain rate data is obtained with a concentric cylinder viscometer, a plot of shear stress versus strain rate for the entire laminar range can be easily made.

With rheological data that spans the entire laminar range, good rheological models can be developed. The yield/power law model, with one more degree of freedom than the Bingham plastic model, is better able to fit data over a wide range of strain rates. Equations for predicting the flow rate versus pressure drop relationship and the point of transition to turbulent flow have been derived for a yield/power law fluid, so all of the calculative tools for analyzing flows in hydraulic networks are available. Whether a single model can adequately fit the data over the entire laminar range or several models have to be tacked together in a piecemeal fashion, can be determined only after the rheology data is obtained and plotted.

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NOTATION

D	Tube diameter (m)
He	Hedstrom number
L	Length (m)
n	Yield/power fluid exponent
Q	Flow rate (m^3/s)
r	Radial coordinate (m)
R	Tube radius (m)
Re	Reynolds number
Re_c	Critical Reynolds number for laminar/turbulent transition
v	Velocity (m/s)
v_z	Axial velocity component (m/s)
α_c	Critical value of the non-dimensional yield radius
ΔP	Axial pressure drop (Pa)
$\dot{\gamma}$	Strain rate (1/s)
η_o	Consistancy for a Bingham plastic or a yield/power law fluid (Pa-s) or ($kg/m\text{-}s^{2-n}$)
ρ	Density (kg/m^3)
τ_{rz}	Shear stress (N/m^2)
τ_o	Yield stress for a Bingham plastic or a yield/power law fluid (N/m^2)
τ_R	Wall shear stress (N/m^2)

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