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FOSSIL ENERGY

CHARACTERIZATION OF FLUID FLOW IN NATURALLY FRACTURED RESERVOIRS

Final Report

Work Performed for the Department of Energy
Under Contract No. DE-AC19-80BC10255

Date Published—August 1981

University of Central Florida
Orlanda, Florida

U. S. DEPARTMENT OF ENERGY

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Final Report

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NOMENCLATURE

A .... Cross section area of fracture conduit

ΔA .... Area increment

a .... Aperture of an elliptical shaped fracture conduit

B .... Non-dimensional constant

C_{ij} .... Mass fraction of component phase

c .... Fluid compressibility

D .... Reservoir depth

d .... Half width of fracture aperture

$d\Omega$ .... Differential solid angle

$d\Omega_\omega$ .... differential solid angle in direction of $\vec{\omega}$

e .... Natural logarithm base

$f(\vec{r},\vec{\Omega})$ .... Fracture distribution function

g .... Acceleration of gravity

K .... Permeability

$\overline{\overline{K}}_f$ .... Permeability tensor for fractures

K_{1f}, K_{2f}, K_{3f} .... Principal permeabilities of the fractures

K_{ij} .... Components of fracture permeability tensor

K_f .... Fracture permeability of a parallel plate conduit

$\hat{K}_{ij}$ .... Phase equilibrium constant

k_j .... Relative permeability of phase j

ℓ .... Characteristic half length of a fracture

$N(\vec{r},\vec{\omega})$ .... Hemispherical fracture number density

ΔN .... Numer of fractures passing through area element ΔS

n .... Integer

P	....	Fluid pressure
$\hat{P}$	....	Perimeter of a fracture conduit
$\vec{r}$	....	Position vector
r_H	....	Hydraulic Radius
S_g, S_o, S_w	......	Phase Saturations
S	....	Surface Area
ΔS	....	Surface area increment
T	....	Temperature
t	....	Time
U_o	....	Magnitude of average fluid velocity in fracture
V	....	Volume
V_j	....	Velocity of fluid phase j
V_s	....	Velocity of solid medium
w	....	Height of a parallelipiped fracture
x,y,z	....	Rectangular coordinates
α	....	Fracture orientation angle
β	....	Parameter in fluid interaction term
γ	....	Specific weight
Γ	....	Fluid interaction term between fractures and primary rock matrix
λ	....	Parameter to take into account entrance and exit effects for flow in fractures
ρ	....	Fluid density
ϕ	....	Porosity
μ	....	Fluid viscosity
$\vec{\Omega}$	....	Unit vector in direction of fracture orientation
$\vec{\omega}$	....	Unit vector normal to a plane intersecting S non-orthogonal
Υ	....	Volume element

Subscripts

c .... Capillary pressures

f .... Fracture value

g .... Gas phase

ij .... Hydrocarbon component

j .... Phase

o .... Oil phase

r .... Relative value

w .... Water phase

1 .... Primary rock matrix value

Symbols

$\rightarrow$ .... Vector

$\_$ .... Average value

$=$ .... Tensor

∇ .... Nabla differential operator

$\int$ .... Integral

$\oint$ .... Cyclic integral (integral over a solid angle of 4π)

▯ .... Parallelipiped fracture

Δ .... Triangular fracture

0 .... Elliptical fracture

ABSTRACT

This report summarizes the results of a four month study of the characteristics of multiphase flow in naturally fractured porous media. An assessment and evaluation of the literature was carried out and a comprehensive list of references compiled on the subject. Mathematical models presented in the various references cited were evaluated along with the stated assumptions or those inherent in the equations. Particular attention was focused upon identifying unique approaches which would lead to the formulation of a general mathematical model of multiphase/multi-component flow in fractured porous media. A model is presented which may be used to more accurately predict the movement of multi-phase fluids through such type formations. Equations of motion are derived for a multiphase/multicomponent fluid which is flowing through a double porosity, double permeability medium consisting of isotropic primary rock matrix blocks and an anisotropic fracture matrix system. The fractures are assumed to have a general statistical distribution in space and orientation. A general distribution function, called the fracture matrix function is introduced to represent the statistical nature of the fractures.

Simplifying assumptions are made concerning flow in the individual fractures and a hemispherical volume integration of the microscopic fracture flow equation is performed to arrive at a generalized Darcy type of flow equation with a symmetric permeability tensor evolving for the fractures. The equations are generalized for multiphase/multicomponent flow through a fractured medium with variable rock and fluid properties.

The report concludes with a brief discussion concerning methods for solving the resulting flow equations. Conclusions and recommendations for further study are also presented.

CHAPTER I

LITERATURE SURVEY AND DISCUSSION OF PREVIOUS RESEARCH

1.1 INTRODUCTION

The flow of fluids in fractured porous media was first developed in the petroleum industry in the 1940's. This work stemmed from the observation that well productivity could be significantly increased in many cases by artificially fracturing the medium near the wellbore. Since that time many researchers have added to the volume of literature that exists on fractured media. A comprehensive list of references concerning flow in fractured porous media is given by Wilson and Witherspoon [1] and Kessler and Greenkorn [2]. In particular the former lists 261 citations and the later 301 citations concerning the flow in porous media and/or fractured media. Of course time did not permit the assessment of all of the references cited in these two documents. In addition to these two treatises additional work reported in the literature is listed in the general set of references at the end of this report.

The reference materials cited were obtained by conducting an extensive literature search within the time allotted, to identify those articles pertinent to the subject matter. Sources used in the literature survey consisted of the Engineering Index, Energy Abstracts, Petroleum Abstracts, Physical and Chemical Abstracts, Dissertation Abstracts, U.S. Government documents, State and University research documents, private industry reports, and the open literature. In order to cross reference as many of the data banks as possible, key words were selected and used to locate research articles and abstracts which pertained to the subject matter of interest. The library research staff at the University of

Central Florida provided support to maximize the effort in identifying those research papers and documents which appeared to address the problem of flow in fractured porous media. As a final cross check an independent computerized literature search was performed by the State Technological Application Center (STAC) which is located at the University of Central Florida. This organization has direct computer access to any literature data bank in the United States. By assigning key words and then searching by various combinations, research papers of interest were identified. From this process abstracts of papers and reports which appeared to be relevant to the subject matter in question could be immediately printed out by the computer. After reviewing the abstract, the decision was then made as to whether the work was significant enough to warrant further investigation. Complete copies of those papers which passed this screening process were then secured and studied in detail. As these documents were received the methodologies and techniques employed by the researchers were assessed as to their applicability to the flow of multiphase fluids in naturally fractured media.

1.2 MODELING OF FRACTURED POROUS MEDIA

When one attempts to model the flow of fluids through any type of medium, the researcher must decide which kinds of fluids and the type of flow he desires to model. In this context there is always the questions as to whether the flow in the medium is laminar or turbulent. This is of particular importance when one considers the flow in fractured media since flow in the fractures is normally considered to be continuum flow. It is generally agreed that turbulent flow may occur when abnormally large fractures are found in the vicinity of abnormally large pressures gradients such as in the early phases of well drawdown [3].

In general the term turbulent flow in porous media implies that Darcy's law is not adequate to describe the flow phenomena. In cases where deviation from Darcy's law occurs, the Forchheimer equation is normally used to describe the flow phenomena. In the case of fractured porous media where a major part of the flow takes place via the fractures, it is possible that the flow can become truly turbulent. However, as has been demonstrated for a majority of the encounters with fracture flow, the laminar flow regime probably prevails [1].

The development of fracture flow models has proceeded along two different approaches. These are the statistical approach and the enumerative approach. In the statistical approach a fractured rock mass is considered to be a statistically homogeneous medium consisting of a combination of fractures and porous rock matrix. In this modeling approach the fractures are considered ubiquitous and the system is called statistically homogeneous because the probability of finding a fracture at any given point in the system is considered to be the same as finding one at any other point. This idealized fracture system is then considered to behave as a type of porous medium. In the enumerative approach a fractured rock media is studied by attempting to model the actual geometry of the fractures and porous rock matrix. In this approach, the location, orientation, and aperture variations for each individual fracture must be considered.

1.3 THE STATISTICAL APPROACH

Models developed utilizing the statistical approach will be discussed first. Elkins and Skov [4,5] have applied this approach to study the anisotropic fracture permeability associated with the Spraberry Field in Texas. The rock matrix in this field consists of tight sandstones whose permeability is about 1 millidarcy. These researchers were concerned with the displacement of oil by water in a water-wet fractured reservoir [5]. Considering the extensive system of orthogonal vertical joints as an anisotropic medium, from a number of drawdown tests they were able to construct permeability ellipsoids whose axes were aligned reasonably well with the observed fractured system. This is referred to as a one medium statistical model since flow in the porous rock matrix was not considered. Other researchers have developed similar one-medium statistical models and they are discussed in reference 1.

A two-medium statistical model for transient flow in a fractured rock media was developed by Barenblatt, Zheltov, and Kochina [5]. Intact uniform blocks of porous rock are considered to coexist with a second system, consisting of the fractures. The rock matrix system and the fracture system are considered to behave as a homogeneous and isotropic porous medium. Since the rock matrix system is considered to be isolated blocks surrounded by the higher permeability fracture system, flow is considered to proceed from the rock matrix which acts as a reservoir, into the fractures which then carry the fluid to the wellbore. Utilizing this approach gives rise to a doubly porous medium, one porosity associated with the rock matrix and a second porosity associated with the fractures. The equations governing the flow of a fluid through a doubly porous medium are coupled by a fluid interaction term. The interaction term describes the mass flux of fluid from the primary rock matrix (pores) into the fractures in terms of the pressure difference between pores and fractures. However, this approach requires the introduction of two pressures at each point in the system. One value represents the average pressure of the fluid in the pores of the rock matrix, and the other value represents the average pressure in the fractures near that point. Since these values represent average pressures in the neighborhood of a point, it is necessary to assume that any infinitely small volume contains not only a large number of pores but also a larger number of fractures. In this original paper of Barenblatt, Zheltov, and Kochina the development was limited to single phase flow of a liquid. However, in succeeding papers Barenblatt extended his solution to include two-phase liquid-gas flow [7].

Warren and Root [8] further developed the two-medium statistical model by extending it to study transient flow phenomena in fractured porous systems. Their model consisted of homogeneous isotropic porous rock matrix blocks coexisting with a homogeneous anisotropic fracture system which also behaves as a porous media. Conceptially their model is the same as that of Barenblatt, et.al. except the fractures must occur in two parallel orthogonal sets oriented along the directions of principal permeability. The fractures normal to the principal axes are uniformly spaced and are of constant width. Flow can occur between the rock matrix (called primary porosity) and the fractures (called secondary porosity), but flow through the primary porosity elements can not occur. They do publish some field data to establish that some fractured reservoirs do behave as predicted by their theory. Again, Warren and Root limit their model to single phase flow of a slightly compressible liquid. In addition, the primary and secondary porosities were treated as a constant in their analysis. Odeh [9] closely followed Warren and Root's paper and though quite similar to the latter researchers work, the conclusions were quite different (see discussion at end of reference 9). The fundamental difference between the two papers is that Odeh treated the fractures as being uniform throughout the reservoir as opposed to the homogeneous anisotropic fractures of Warren and Root. However, Odeh did present some field data which supported his approach but cautioned that fractured reservoirs in general may not obey his theory.

The method of Warren and Root [8] was extended by Kazemi [10, 11] to study the pressure transient analysis of a well centrally located in a finite circular reservoir. In an accompanying paper by Kazemi, Seth and Thomas [12] the significance of well interference tests in fractured media was investigated. Their results are compared with those obtained from a one-medium model and they conclude that while the one-medium model is insufficient at small values of time, the results for the two models converge for large times. As in the previous papers discussed, consideration was only given to single phase, constant property flow. Saad [13] considers a water bearing porous medium with one set of parallel fractures. Flow in a direction perpendicular to the fracture set is considered to occur primarily within the porous medium. From the results of the drawdown tests Saad calculates the anisotropic permeabilities both parallel and normal to the fractures, and assigns the first permeability to the fracture system and the second one to the porous media.

In their study of blood flow in capillaries, Lew and Fund [14] have taken into account the anisotrophy of the non-rigid porous medium by utilizing a statistical space averaging approach. A "pore matrix function" is introduced to describe the structure of the network of pores in the anisotropic non-rigid media. By assuming the fluid is Newtonian, the flow in an individual tubule of the pore network can be described by the Navier-Stokes equations. The equations governing the macroscopic motion of the viscous fluid through the porous media are derived by averaging the motion of the fluid through individual elements of the pores over a small volume of the porous medium. A generalized Darcy's law is obtained, in which the permeability tensor is explicitly expressed in terms of the pore matrix function. The permeability tensor which evolves is a second rank symmetric tensor and may be inferred from experimental data.

Duguid and Lee [15] adapted Lew and Fung's [14] technique to the flow of a single phase fluid (water) through fractured porous media. In their development the fractured porous medium is treated as an elastic, incompressible solid containing two different porosities. The primary porosity is considered to be isotropic and the secondary porosity associated with the fractures which separate the primary blocks is anisotropic depending upon the spatial distribution of the fractures. The fluid is considered to be slightly compressible, and the fluid velocity in both the primary pores and the fractures is assumed to be small. This latter assumption allows the nonlinear portion of the acceleration term in the equation of motion to be neglected. Two sets of governing equations are required to describe the flow in the fractured porous media, one for each type or porosity. The resulting sets of equations are coupled by the interaction of the fluid in the primary pores with the fluid in the fractures.

Yamamoto, Padgett, Ford, and Bougeguira [18] developed a very specialized gas-oil multiple component (compositional) model for flow within fractured reservoirs. Their model is used to simulate matrix block behavior for a single block, with a fracture located along the mid-depth of the block. The fracture represents the boundary conditions around the block and flow along the fracture is not considered.

In an earlier paper by Duguid and Abel [16] the researchers present a system of equations similar to those derived by Duguid and Lee[15].

The governing flow equations were solved by the former authors by employing a finite element Galerkin method. Results are presented for transient incompressible flow in a confined leaky aquifer. However, the two problems the authors present solutions for are artificial in nature but do provide insight into the physical character of flow through fractured porous media. Specific details of the deriviations and the fracture model are contained in another report by Duduid and Lee [17].

Kleppe and Morse [19] and Kazemi, Merrill, Porterfield, and Zeman [20] extended Warren and Root's [8] single phase flow equations to two-phase, two and three dimensional models, respectively. However, the validity of Kleppe and Morse's model was verified against data obtained from a laboratory experiment involving a single matrix block contained in a circular tube surrounded by an annulus to represent the fracture. The results of their laboratory experiments agreed well with their simulator results. The equations presented in the paper are the ones normally found in the literature which describe the flow in a single porosity system. It is not clear from their work as to how the fractures affect the flow. However, they were attempting to match inbibition experiments as opposed to describing the total flow problem in the rock matrix - fracture system. Kazemi, et.al. [20] on the other hand, were concerned with the total flow problem and their model is quite general for oil-water flow in fractured reservoirs. Their model accounts for relative fluid mobilities, gravity forces, imbibition, and variations in reservoir properties although the porosities in the fracture and the primary rock matrix are considered to be constant. However, their model is limited in that the fractures must occur in two parallel orthogonal sets oriented along the directions of principal permeability as is the case of Warren and Root's [8] model. Fractures which are randomly oriented cannot be handled by Kazemi's et.al. model.

In a similar treatment Rossen [21] considered the rock matrix blocks as source and sink terms in a conventional reservoir simulator which models only the fracture system. The source/sink terms are functions of rock matrix and fluid properties with fracture saturation and pressure defining the boundary conditions. These functions are derived by history matching simulations or independently by laboratory experiment or single matrix block simulation, i.e. it is assumed that these functions are known.

Closmann [22] developed an aquifer model for fissured reservoirs by adapting the theory of Warren and Root [8] and Odeh [9]. In his approach it was assumed that the reservoir had constant properties and flow only occurred from the rock matrix to the fissures and through the fissures to the inner aquifer boundary.

Swann [23, 24] and Williams [25] have also studied reservoir performance in fractured media utilizing models developed along the theories put forth by Barenblatt et.al. [6] and Warren and Root [8]. In Swann's first paper, effort was directed toward relating the fractured reservoir properties to the well test plots as opposed to describing the flow phenomena occurring in the rock matrix blocks and the fractures. In his second paper an analytical theory is presented which describes water flooding in fractured reservoirs. However, the conclusion is drawn that since the model matches the inbibition experiments of Kleppe and Morse [19] and Mattax and Kyte [26] for a single matrix block it can be used to predict cumulative oil production, water-oil ratio and the saturation profiles in the fractures for a water flood project for a fractured reservoir. Although the approach is different from the other approaches discussed, it does not lend itself to describing the flow phenomena taking place within the primary rock matrix/fracture system.

A recent doctoral dissertation by Menouar [27] presents a numerical study of the displacement of oil by water in heterogeneous reservoirs. Discussion and consideration is given to flow in fractured porous media although the governing equations used in the numerical simulator developed are based upon a single porosity system without any consideration given to the flow in a rock matrix/fractured system. The results presented are primarily concerned with the imbibition process occuring at the boundary of a single matrix block. Although other statistical models are referenced in the various papers cited they are quite similar to those already discussed and do not warrant additional comments.

1.4 THE ENUMERATIVE APPROACH

Models appearing in the literature which rely upon the identification of the geometrical properties of the specific fractures, such as orientation and aperture, will now be considered. In general, these models rely upon a knowledge of the flow characteristics that are taking place in the individual fracture segments. Generally the assumption is made that the flow in the fractures can be modeled as infinite parallel plate flow. This assumption is made because of the simple mathematical expressions which evolve to describe the fluid flow in such conduits, and because it closely approximates the planar tendencies of many fractured surfaces.

General studies of the parallel plate model have been conducted by a multitude of researchers. These are listed in the report of Wilson and Witherspoon [1] and will not be reproduced here. In addition an excellent treatise on general fluid flow in porous media is given by Bear [28] wherein the treatment of parallel plate flow is discussed in detail. Wilson and Witherspoon [1] utilize an enumerative approach to model steady state laminar flow of fluids through a two-dimensional system of idealized fractures in a matrix of permeable blocks. Given a certain fracture geometry, matrix permeabilities, and boundary conditions, equations are derived which gives the steady state pressure head at any point in the medium. From the pressure head the volume flow rates are then found via Darcy's law. The fractures are treated as infinite parallel plate conduits with impermeable smooth walls. Directional permeabilities are derived assuming fracture orientation, and directional principal permeabilities can be experimentally determined from a core sample. The resulting directional permeabilities are therefore, a function of fracture aperture, permeability orientation, and fluid properties. The directional permeabilities are functions of the permeability of the fracture K_f, which in the case of a parallel plate model is

$$K_f = \frac{b^2\gamma}{12\mu} \tag{1}$$

where b = aperture of the fracture

γ = specific weight of fluid

μ = viscosity of the fluid

Using such an approach the authors derive a set of governing equations which are then solved using a finite element program using triangular elements for the pressure head and hence the volume flow rates at any point in the medium. Again, this approach assumes the individual fracture geometrics are known throughout the flow field. Several other enumerative models are discussed by Wilson and Witherspoon [1] and references cited concerning the researchers involved.

Gale, et.al. [29] utilized an enumerative approach to analyze the effect of fluid movement in rock systems where flow is influenced by the behavior of deformable fractures. They conclude that the simple fracture geometry considered may not represent most natural fractured reservoirs.

Snow [30] developed a general anisotropic permeability tensor for fractured porous media by considering the manner in which a fixed oriented hydraulic gradient effects flow in a rock volume with a number of differently oriented fractures. The fractures are assumed to be infinite in extent, have constant average spacing, and identical apertures and orientations. In a subsequent paper Snow [31] extended this approach to allow for randomly distributed orientations and apertures. However, if flow in the system of porous rock blocks is to be considered for this latter approach, it must be treated as acting independent of the fracture system. In a follow-on paper Snow [32] presented results from pressure-test data which permitted an estimate of the average spacing and the average aperture of water-conducting fractures in undisturbed rock masses.

Asfari and Witherspoon [33] used an enumerative approach to investigate mathematical models of idealized systems of vertical fractures consisting of parallel plate conduits. Regular systems of fractures were chosen in such a manner that the length of the fracture could be characterized in terms of the dimension of a representative elemental area associated with the fracture. By changing the permeability of the fracture relative to the matrix, steady state solutions were obtained which demonstrated that the effective permeability of the fractured system depend upon both fracture conductivity and dimensionless parameters representing fracture density. Finite element algorithms were used to solve the mathematical models discussed. Excellent agreement between their analytical results and those of other researchers were obtained.

Peaceman [34,35] investigated the convective mixing taking place within the oil-filled portion of a fissured system. He utilized a single vertical parallelipiped shaped figure of length L, width W and aperture D_f tilted at an arbitrary angle with the local vertical in order to include gravity effects. The fracture permeability was assumed to be that of a parallel plate conduit similar to that given by equation 1. Since the work was focused upon evaluating matrix-fissure transfer effects and density inversion in the fracture, consideration was not given to the total flow problem through a reservoir made up of these types of fissures surrounded by primary rock matrix blocks. The numerical results obtained from the study while preliminary in nature, indicate that convection in fractures may play a significant role in the performance of fractured reservoirs undergoing pressure depletion.

Other mathematical models based upon the enumerative approach are cited in the various papers, discussed and referenced in this report. However, none of these models are general and applicable enough to warrant detailed discussion of the approaches involved. They are primarily concerned with special cases rather than attempts to model general fracture networks.

1.5 STATISTICAL vs ENUMERATIVE APPROACH

When one assesses the literature on modeling of fluid flow through fractured porous media, it is obvious that the statistical approach dominates in terms of the quantity of work which has been published on the subject in question. However, this is primarily due to the fact that the enumerative approach requires an accurate description of the complex geometry of natural fractured systems. To accomplish this, a number of techniques have been proposed (see Chapter I of reference 1) for obtaining fracture spacing, orientation apertures and other fracture parameters required to carry out an accurate enumerative model of natural fractured reservoirs.

In general the statistical approach is preferred over the enumerative approach whenever the size of the rock matrix block becomes small in relation to the size of the boundaries [1]. However, use of the statistical approach, involving treatment of the fracture system as an equivalent porous medium, can under certain circumstances lead to invalid results when fracture spacing is large. In general, according to Wilson and Witherspoon [1], when the smallest dimension of the system boundary is more than 50 times as long as the longest dimension of the larger rock matrix block the statistical approach is preferred over the enumerative approach.

1.6 SUMMARY OF LITERATURE SURVEY

The main thrust of the literature survey was to identify and assess the potential of the physical/mathematical models which currently exist in the published media to adequately describe multiphase/multicomponent flow through natural fractured porous media. From the results obtained it is clear that no specific paper, report or other published document adequately treats this very important flow problem. Since a multitude of data bases were researched to uncover a model of this nature, it is believed that a general formulation of this problem has yet to be published. This is as one would suspect since the necessity of a model of this type for fractured reservoirs has only become evident in the past several years with the advent of enhanced recovery processes looking more and more attractive from an economic point of view. For primary and secondary water flood recoveries, the models contained in the literature were apparently adequate. However, the models which do appear in the published literature are either too restrictive for general application or were developed for very specialized cases of particular interest to an organization, group or individual. Furthermore, the various approaches given in the literature do provide insight into developing a more general model. In this regard the literature survey has provided a real service in an attempt to characterize the flow of multi-phase,multi-component fluids through fractured porous media.

In petroleum reservoir modeling it is more appropriate to derive governing-flow equations based upon a statistical approach as opposed to the enumerative approach. This is particularly true since there is a greater concern with overall fluid movement throughout a reservoir as opposed to the movement through the individual pores or fractures. Certainly one must model on a microscopic scale to describe the flow in a small region. However, the macroscopic flow equations are derived by taking statistical averages of the microscopic equation over a much larger region but yet still small enough to apply the theories of the infinitesmal calculus. Although one can employ the enumerative approach to the problem,the characterization of the individual fractures would require considerable more time and experimental data to solve the governing-flow equations with any degree of confidence. As has been pointed out in the section on enumerative modeling there are situations where adequate fracture geometry exists and the enumerative approach,can yield accurate and useful results. However, in general there is a lack of sufficient geometric fracture data to warrant an enumerative approach and the formulation of the model which is given in the next chapter is based upon the statistical approach.

CHAPTER II

FORMULATION OF A GENERAL MODEL FOR FLOW THROUGH FRACTURED POROUS MEDIA

2.1 BASIC ASSUMPTIONS

In the formulation of a mathematical model for naturally fractured reservoirs the following basic assumptions are made:

1. The fractured porous media is treated as an elastic incompressible solid which contains two kinds of porosity. One porosity is associated with the primary rock matrix and the second porosity is associated with the fractures.
2. The primary porosity in the primary rock matrix is considered to be isotropic, whereas, the porosity in the fracture is considered anisotropic.
3. Fluid velocities in both the primary pores and the fractures are assumed to be small.
4. Primary rock matrix pore volume is independent of fracture pressure and conversely for the fracture volume.

Other assumptions are introduced as necessary.

2.2 PRIMARY AND FRACTURE POROSITY EQUATIONS

As fluids are removed from a confined reservoir or are replaced by other fluids having different compressibilities, the fluid pressure in the medium is decreased and the reservoir is compressed because of the increased overburden load carried by the granular skeleton. If the solid grains are assumed incompressible, consolidation can only occur through re-arrangement of the granular skeleton. This re-arrangement alters the size of the flow conduits in both the primary pores and the fractures thus changing the flow characteristics of the media. Hence, this phenomena results in a change of porosity with time. To model this effect, we consider the total volume of the medium to be:

$$V = V_s + V_f + V_1 \tag{2}$$

In equation 2, V_s is the volume of the solids, V_f is the volume of the fractures and V_1 is the volume of the pores in the primary rock matrix. In terms of porosity for each component of the medium

$$\phi_1 = \frac{V_1}{V}, \quad \phi_f = \frac{V_f}{V}, \quad \phi_s = \frac{V_s}{V} \tag{3}$$

Taking the derivative of ϕ_1, with respect to time we obtain upon re-arranging and using the definition of ϕ_1

$$\frac{d\phi_1}{dt} = \frac{1}{V}\left[\frac{dV_1}{dt} - \phi_1 \frac{dV}{dt}\right] \tag{4}$$

Differentiating V with respect to time and recalling we have assumed V_s is constant then,

$$\frac{dV}{dt} = \frac{dV_1}{dt} + \frac{dV_f}{dt} \tag{5}$$

Substituting equation (5) into equation (4) and simplifying we obtain

$$\frac{d\phi_1}{dt} = \frac{1}{V}\left[(1-\phi_1)\frac{dV_1}{dt} - \phi_1 \frac{dV_f}{dt}\right] \tag{6}$$

Similarly, for the fracture porosity, we obtain

$$\frac{d\phi_f}{dt} = \frac{1}{V}\left[(1-\phi_f)\frac{dV_f}{dt} - \phi_f \frac{dV_1}{dt}\right] \tag{7}$$

The compressibility of a fluid at constant temperature is defined according to

$$c = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T \tag{8}$$

Introducing equation 8 into equations 6 and 7 and further assuming that the change of pore volume in the rock matrix is independent of the pressure in the fracture and conversely for the change in fracture volume we obtain the following equations for the rock matrix porosity and the fracture porosity respectively

$$\frac{d\phi_1}{dt} = -(1-\phi_1)\ c_1 \frac{dP_1}{dt} + \phi_1 c_f \frac{dP_f}{dt} \tag{9}$$

$$\frac{d\phi_f}{dt} = -(1-\phi_f)\ c_f \frac{dP_f}{dt} + \phi_f c_1 \frac{dP_1}{dt} \tag{10}$$

c_1 is the compressibility of the fluid in the primary rock matrix and c_f is the compressibility of the fluid in the fractures. P_1 and P_f are the fluid pressures in the primary pores and fractures respectively. Since for multi-phase flow the composition of the fluid in the rock matrix and the composition of the fluid in the fractures can be different, the fluid compressibilities in the respective void spaces will be different. If it is assumed that at a point in the reservoir the fluid compositions are the same in both fractures and matrix pores (or single phase flow), then for a slightly compressible fluid Duguid and Lee [15] show that

$$\frac{d\phi_1}{dt} = \phi_1 \phi_f c \frac{dP_f}{dt} - (1-\phi_1)\ \phi_1 c \frac{dP_1}{dt} \tag{11}$$

$$\frac{d\phi_f}{dt} = \phi_1\ \phi_f\ c \frac{dP_1}{dt} - (1-\phi_f)\ \phi_f c \frac{dP_f}{dt} \tag{12}$$

In general the total derivative in equations 9-12 is expressed as

$$\frac{d}{dt}(\ ) = \frac{\partial}{\partial t}(\ ) + \vec{V}_s \circ \nabla \tag{13}$$

In equation 13, $\vec{V}_s$ represents the velocity at which the solid medium is moving and ∇ is the nabla differential operator. For the problem under consideration $\vec{V}_s \equiv 0$ and the total derivative becomes a partial derivative, hence equations 9 and 10 are rewritten as,

$$\frac{\partial \phi_1}{\partial t} = \phi_1 c_f \frac{\partial P_f}{\partial t} - (1-\phi_1)\, c_1 \frac{\partial P_1}{\partial t} \qquad (14)$$

$$\frac{\partial \phi_f}{\partial t} = \phi_f c_1 \frac{\partial P_1}{\partial t} - (1-\phi_f)\, c_f \frac{\partial P_f}{\partial t} \qquad (15)$$

The compressibilities c_1 and c_f are given by

$$c_1 = c_g S_{g1} + c_o S_{o1} + c_w S_{w1} \qquad (16)$$

$$c_f = c_g S_{gf} + c_o S_{of} + c_w S_{wf} \qquad (17)$$

S_g, S_o, S_w and c_g, c_o, c_w are the respective saturations and compressibilities of the various phases in the rock matrix pores and the fractures.

2.3 CONSERVATION EQUATIONS

The conservation of mass in the primary rock matrix and the fractures are obtained by doing a mass balance on an elemental volume of the medium containing fractures and pores. Since it is desired to be general in the formulation the following definitions are introduced

C_{ig}, C_{io}, C_{iw} = mass fractions of the ith component in the gas phase, oil phase,and water phase, respectively.

$\rho_g \vec{V}_g$, $\rho_o \vec{V}_o$, $\rho_w \vec{V}_w$ = mass flux densities of the gas phase, oil phase,and water phase, respectively

Γ_j = mass of fluid phase j flowing from primary rock matrix pores into the fractures per unit time per unit volume of medium

ρ_g, ρ_o, ρ_w and $\vec{V}_g$, $\vec{V}_o$, $\vec{V}_w$ are densities and velocities of the different phases. Γ_j is a fluid interaction term taking place between primary pores and fractures.

The mass balance in the primary pores yields the following continuity equation for the fluid in the primary rock matrix.

$$\nabla \cdot [\Sigma C_{ij1} \rho_j \vec{V}_{1j}] + \Sigma \Gamma_j + \frac{\partial}{\partial t} [\phi_1 \Sigma C_{ij1} \rho_j S_{j1}] = 0 \tag{18}$$

For the fractures the continuity equation is

$$\nabla \cdot [\Sigma C_{ijf} \rho_j \vec{V}_{fj}] - \Sigma \Gamma_j + \frac{\partial}{\partial t} [\phi_f \Sigma C_{ijf} \rho_j S_{jf}] = 0 \tag{19}$$

2.4 EQUATION OF MOTION

Assuming Darcy's law is valid in the rock matrix the governing flow equations in the primary pores are

$$\vec{V}_{1j} = - \frac{K_1 k_{r1j}}{\mu_j} [\nabla P_{1j} - \rho_j g \nabla D] \tag{20}$$

K_1 is the absolute permeability of the primary rock matrix, k_{r1j} is the relative permeability of the primary rock matrix to the j'th fluid phase, P_{1j} is the pressure of the j'th phase in the primary rock matrix, D is depth, g is the acceleration of gravity and μ_j is the viscosity of the j'th phase.

To develop the equation of motion for flow in the fractures, the flow of a single phase fluid is investigated and the results generalized for multi-phase flow. For a given fracture transporting a Newtonian fluid, the Navier-Stokes equations govern the flow. Assuming flow in the fractures can be approximated as incompressible flow, the governing Navier Stokes equation for single phase flow is

$$\rho_f \frac{dV_f}{dt} = \rho_f g - \nabla P_f + \mu_f \nabla^2 \vec{V}_f \tag{21}$$

$$\frac{d\vec{V}_f}{dt} = \frac{\partial \vec{V}_f}{\partial t} + (\vec{V}_f \circ \nabla) \vec{V}_f$$

If we assume the flow in the individual fractures can be approximated as flow within porous walled conduits of hydraulic radius r_H, and the cross-sectional dimensions of a fracture are smaller than the length of a fracture segment, the velocity $\vec{V}_f$ will be approximately parallel to the fracture orientation [14-17], and hence,

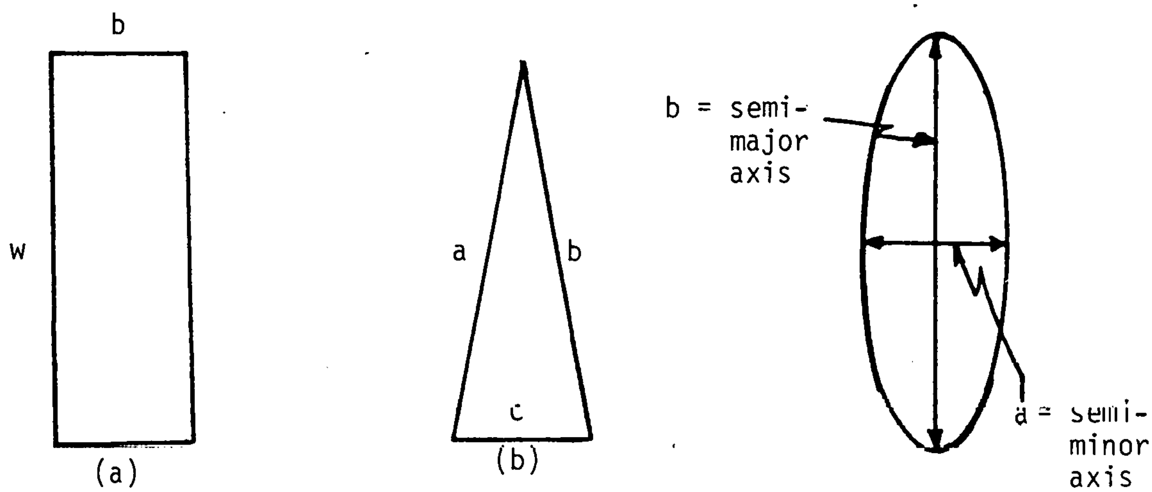
$$\vec{V}_f = \left| V_f \right| \vec{\Omega} \tag{22}$$

where $\vec{\Omega}$ is a unit vector in the direction of the fracture orientation. Approximating the flow within the fractures as Poiseuillian then,

$$\nabla^2 \vec{V}_f \approx - \left[\frac{8U_\circ(\vec{\Omega})}{r_H^2}\right] \lambda \tag{23}$$

$U_\circ$ ($\vec{\Omega}$) is the magnitude of the average velocity of the flow over the cross section of an individual fracture of hydraulic radius r_H and λ is a parameter introduced to take into account entrance and exit effects. Warren and Root [8], Lew and Fung [14], and Duguid and Lee [15] introduced such a parameter to take into account branching or joining or different fractures. The specific form of λ would be quite complex but as it turns out the exact formulation of this parameter is not necessary to derive the governing macroscopic flow equations for the fractures. By expressing $\nabla^2 \vec{V}_f$ as a function of hydraulic radius, various geometrical cross sections for the fracture can be investigated. As pointed out by White [37], one must exercise caution when using the hydraulic radius concept since it is an approximation to an exact solution. Fortunately, for the dimensions involved in fracture flow the hydraulic radius concept should represent the flow phenomena reasonably well. Figure 2.1 gives the hydraulic radii for a few probable fracture geometric cross sections. For fracture flow it is generally true that, for the geometries given in Figure 2.1 $w \gg b$ for a parallelipiped, $a \approx b \gg c$ for a triangle, and $b \gg a$ for an ellipse.

In figure 2.1 dimension b for the parallelipiped, dimension c for the triangle, and dimension a for the ellipse represent the aperture of the fracture. In general the aperture of a fracture is small compared to the



PARALLELIPIPED TRIANGLE ELLIPSE

$$r_H = \frac{wb}{2(w+b)} \qquad r_H = \frac{1}{4}\sqrt{\frac{(b+c-a)(a+c-b)(a+b-c)}{a+b+c}} \qquad r_H = \frac{ab}{\sqrt{2(a^2+b^2)}}$$

FIGURE 2.1

HYDRAULIC RADII OF PROBABLE FRACTURE GEOMETRIC CROSS SECTIONS other dimensions of the fracture. Hence, for each case in figure 2.1, the following approximations can be made.

$$\square\ r_H \simeq \frac{b}{2}\ ,\ \triangle\ r_H \simeq \frac{c}{4},\quad \bigcirc\ r_H \simeq \frac{a}{\sqrt{2}} \qquad (24)$$

In the fractured medium there are many fractures, and an average over the distribution of these fractures must be performed before equation 21 will apply to the entire medium. To describe the fracture distribution it is necessary to introduce a function $f(\vec{r}, \vec{\Omega})$ which statistically represents the number of oriented fractures per unit area per unit solid angle. $f(\vec{r}, \vec{\Omega})$ is a vector function of position $\vec{r}$, and direction $\vec{\Omega}$ in order to characterize the inhomogeneity and anisotrophy of the fractures. Reference to Figure 2.2 may help to clarify the nature of the function $f(\vec{r},\vec{\Omega})$ which we choose to call the fracture matrix function. In figure 2.2 ΔA is an incremental area perpendicular to the vector $\vec{\Omega}$. The solid angle subtended by ΔA is designated as $d\Omega$. According to the definition of $f(\vec{r}, \vec{\Omega})$, the number of fractures of ΔN which are oriented within $d\Omega$ and within direction $\vec{\Omega}$ is given by,

$$\Delta N = f(\vec{r}, \vec{\Omega})\ \Delta A\ d\Omega \qquad (25)$$

The number of fractures passing through area ΔS with unit vector $\vec{\omega}$ is

$$\Delta N'(\vec{r}, \vec{\Omega}, \vec{\omega}) = f(\vec{r}, \vec{\Omega})\ d\Omega\ \vec{\Omega} \cdot \Delta S\vec{\omega} \qquad (26)$$

Since we desire the total number of fractures passing through ΔS, then we must integrate over the solid angle $d\Omega$. Now,

$\frac{\Delta N}{\Delta S}$ = total number of fractures per unit area passing through an incremental area with unit normal $\vec{\omega}$

Hence, we can define $N(\vec{r}, \vec{\omega})$ as the hemispherical number density on an area located at $\vec{r}$ with unit area vector $\vec{\omega}$.

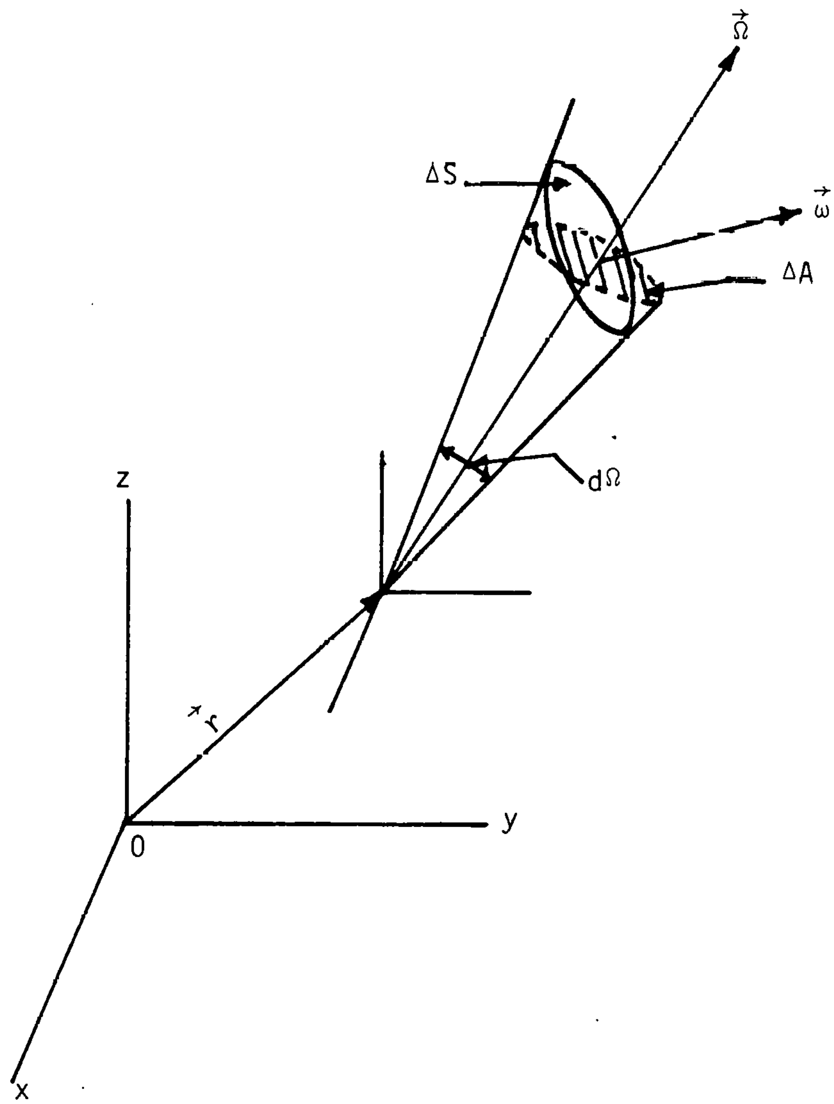


FIGURE 2.2

GEOMETRY SHOWING RELATION

BETWEEN $\vec{\Omega}$ AND $\vec{\omega}$

$$N(\vec{r},\vec{\omega}) = \int_0^{2\pi} f(\vec{r}\ \vec{\Omega})\vec{\Omega}.\vec{\omega}\ d\Omega_\omega \tag{27}$$

In equation 27, $d\Omega_\omega$ is the solid angle about $\vec{\omega}$.

Recalling that for flow in the fractures the velocity $\vec{V}_f$ will be approximately parallel to the fracture orientation, we only need to consider the component of equations 21 in the direction $\vec{\Omega}$. Therefore, equations 21 can be written in the following form:

$$\rho_f\ [\frac{\partial V_f}{\partial t} + (V_f \circ \nabla)\ V_f]\ \vec{\Omega} = (\rho_f \vec{g}\cdot\vec{\Omega})\vec{\Omega} - (\nabla P_f \circ \vec{\Omega})\vec{\Omega} + [-\ \frac{8\mu_f}{r_H^2}\ U_\circ\ (\vec{\Omega})\lambda]\vec{\Omega} \tag{28}$$

Solving for $U_\circ\ (\vec{\Omega})\vec{\Omega}$ we obtain

$$U_\circ(\vec{\Omega})\vec{\Omega} = -\{\rho_f[\frac{\partial V_f}{\partial t} + (V_f.\nabla)\ V_f]\vec{\Omega} + (\rho_f\vec{g}\cdot\vec{\Omega})\vec{\Omega} - (\nabla P_f \circ \vec{\Omega})\vec{\Omega}\}\ \frac{r_H^2}{8\mu_f\lambda} \tag{29}$$

Assuming $(V_f.\nabla)\ V_f$ is small compared to $\frac{\partial V_f}{\partial t}$ equation (29) reduces to

$$U_\circ(\vec{\Omega})\vec{\Omega} = \{-\rho_f[\frac{\partial \vec{V}_f}{\partial t} - \vec{g}] - \nabla P_f\}\ \frac{r_H^2}{8\mu_f\lambda} \circ \vec{\Omega}\vec{\Omega} \tag{30}$$

The macroscopic value of the velocity of the fluid in the fractures is

$$\vec{V}_f(\vec{r}) = \frac{1}{\tau}\int_{\tau_f} U_\circ\ (\vec{\Omega})\vec{\Omega}\ d\tau \tag{31}$$

τ is a small spherical volume in space which is chosen in such a manner that the radius of τ is much smaller than the characteristic length of the system so that the change of the macroscopic variable within τ can be neglected. Furthermore, τ is large enough such that its radius is much smaller than the characteristic length of the fracture but is greater than the largest dimension of the fracture cross section area. That is, the volume τ is large enough to contain a statistically significant number of fractures. τ_f is that portion of τ occupied by the fluid which fills the void spaces.

$$\bar{\vec{V}}_f(\vec{r}) = \frac{r_H^2}{8\mu_f \lambda \tau_f} \int_{\tau_f} [-\rho_f(\frac{\partial \vec{V}_f}{\partial t} - \vec{g}) - \nabla P_f] \cdot \vec{\Omega}\vec{\Omega}\, d\tau \qquad (32)$$

Equation 32 can be transformed to an integral over the solid angle $d\Omega$ to arrive at

$$\bar{\vec{V}}_f(\vec{r}) = [- \frac{r_H^2 A}{16\mu_f \lambda} \oint f(\vec{r},\vec{\Omega})\vec{\Omega}\vec{\Omega} d\vec{\Omega}] \cdot \{\rho_f (\frac{\partial \vec{V}_f}{\partial t} - \vec{g}) + \nabla P_f\} \qquad (33)$$

It can be shown that the integral on the right hand side of equation 32 integrates to the following form:

$$\bar{\vec{V}}_f + - \frac{\bar{\bar{K}}_f}{\mu_f} \cdot [\nabla \bar{P}_f + \rho_f (\frac{\partial \vec{V}_f}{\partial t} - \vec{g})] \qquad (34)$$

$\bar{\bar{K}}_f$ is the intrinsic permeability tensor for the fractures and is given by,

$$\bar{\bar{K}}_f = \frac{A r_H^2}{16\lambda} \oint f(\vec{r},\vec{\Omega}) \vec{\Omega}\vec{\Omega}\, d\vec{\Omega} \qquad (35)$$

A is the cross-sectional area of the fracture conduit. A factor 1/2 is introduced to account for the fact that each fracture gets counted twice in the integration over a whole solid angle of 4π. Inspection of equation 35 reveals that in general $\bar{\bar{K}}_f$ is a second rank symmetric tensor. Equation 34 is a generalized Darcy law similar to those given by Bear [28] and Collins [36]. Assuming $f(\vec{r},\vec{\Omega})$ and λ are known, expressions for $\bar{\bar{K}}_f$ can be determined. Although in principle $f(\vec{r},\vec{\Omega})$ and λ can be determined, an experimental program would be required to determine their precise relationships. In view of this fact we recognize that $\overline{\overline{K}}_f$ is a second rank symmetric tensor whose general form in a x,y,z coordinate system is:

$$\overline{\overline{K}}_f = \begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix} \tag{36}$$

where $K_{xy} = K_{yx}, K_{xz} = K_{zx}$, and $K_{yz} = K_{zy}$

For two dimensional flow through a fractured medium where the fractures are randomly distributed in space but have a constant orientation, and following the pricedure outlined by Wilson and Witherspoon [1], one can show that

$$\begin{aligned} K_{xx} &= K_f \operatorname{Cos}^2 \alpha \\ K_{xy} &= K_f \operatorname{Sin} \alpha \operatorname{Cos} \alpha \\ K_{yy} &= K_f \operatorname{Sin}^2 \alpha \\ K_{xz} &= K_{zz} = 0 \end{aligned} \tag{37}$$

K_f is the fluid conductivity for a single fracture conduit and α is the fracture orientation. For the case of a parallelipiped, triangular or an elliptical conduit, the fluid conductivity can be expressed as

$$K_{\square} = \frac{r_H{}^2}{6\mu_f} \;,\; K_{\triangle} = \frac{}{4\mu_f}\left(\frac{c}{a}\right) r_H{}^2,\; K_{0} = \frac{r_H{}^2}{2\mu_f} \tag{38}$$

The hydraulic radii r_H are as defined in Figure 2.1.

To extend the generalized Darcy's law given by equation 34 to the flow of multiphase fluids requires some fundamental assumptions. Following a suggestion by Collins [36] and Bear [28], the most reasonable postulate is to assume that the relative permeabilities are independent of direction in the medium. Therefore, the relative permeabilities k_{rjf} are,

$$
\begin{aligned}
k_{rwf} &= \left(\frac{K_{1w}}{K_1}\right)_f = \left(\frac{K_{2w}}{K_2}\right)_f = \left(\frac{K_{3w}}{K_3}\right)_f \\
k_{rof} &= \left(\frac{K_{1o}}{K_1}\right)_f = \left(\frac{K_{2o}}{K_2}\right)_2 = \left(\frac{K_{3o}}{K_3}\right)_f \qquad (39) \\
k_{rgf} &= \left(\frac{K_{1g}}{K_1}\right)_f = \left(\frac{K_{2g}}{K_2}\right)_f = \left(\frac{K_{3g}}{K_3}\right)_f
\end{aligned}
$$

K_1, K_2 and K_3 are the principal permeabilities and the subscript f implies fracture values.

Using equations (39) in equations (34) yields the final form of the flow equations for the fractures

$$\vec{V}_{fj} = \frac{-k_{rjf}\overline{\overline{K}}_f}{\mu_j}\cdot\left[\nabla P_{fj} + \rho_j\left(\frac{\partial \vec{V}_{fJ}}{\partial t} - g\nabla D\right)\right] \qquad (40)$$

where j = phase

For simplification purposes, the bars denoting average values have been dropped, however, it is understood that average values are implied. In addition, the gravity term has been re-written to make it consistent with equations 20 for flow in the primary rock matrix.

2.5 SUMMARY OF THE GOVERNING EQUATIONS

To summarize, the following equations can be used to describe the flow of a multiphase/multicomponent fluid through an anisotropic fractured porous medium.

Porosity Equations:

$$\frac{\partial \phi_1}{\partial t} = \phi_1 c_f \frac{\partial P_f}{\partial t} - (1-\phi_1)\, c_1 \frac{\partial P_1}{\partial t} \qquad (14)$$

$$\frac{\partial \phi_f}{\partial t} = \phi_f c_1 \frac{\partial P_1}{\partial t} - (1-\phi_f) c_f \frac{\partial P_f}{\partial t} \qquad (15)$$

Continuity Equations:

$$\nabla \cdot [\Sigma C_{ij1}\rho_j \vec{V}_{1j}] + \Sigma\Gamma_j + \frac{\partial}{\partial t}[\phi_1 \Sigma C_{ij1}\rho_j S_{j1}] = 0 \qquad (18)$$

$$\nabla \cdot [\Sigma C_{ijf}\rho_j \vec{V}_{fj}] - \Sigma\Gamma_j + \frac{\partial}{\partial t}[\phi_f \Sigma C_{ijf}\rho_j S_{jf}] = 0 \qquad (19)$$

Motion Equations:

$$\vec{V}_{ij} = \frac{-K_1 k_{rij}}{\mu_j}[\nabla P_{1j} - \rho_j g \nabla D] \qquad (20)$$

$$\vec{V}_{fj} = \frac{-k_{rfj}}{\mu_j}\overline{\overline{K}}_f \; [\nabla P_{fj} + \rho_j (\frac{\partial \vec{V}_{fj}}{\partial t} - g\nabla D)] \qquad (40)$$

To complete the formulation of the governing equations an expression for the fluid interaction term must be developed. This is accomplished in the next section.

The basic equations derived do not include sources and/or sinks to represent injection or producing wells. These can be included by the addition of such terms to these equations without any particular difficulty.

2.6 FLUID INTERACTION TERM - Γ_j

For a single phase fluid flowing in a fractured porous media Duguid and Lee [15] show that Γ can be expressed as,

$$\Gamma = \frac{\beta}{\mu}[(P_1 - P_f) + 2\sum_{n=1}^{\infty} (-1)^n P_1 e^{-B} - 2\sum_{n=1}^{\infty} P_f e^{-B}] \qquad (41)$$

where $B = \dfrac{K_1 n^2 \pi^2 t}{2\ell^2 \mu \phi_1 c}$, $\beta = \dfrac{4K_1 \phi_f \rho}{\pi d \ell}$

In equation 41, t is time, ℓ is the characteristic half length of a fracture, c is the single phase fluid compressibility, d is the half width of the fracture aperture and the other variables are as defined previously. Since there are several phases flowing simultaneously, the flux of component C_{ij} into the fracture from the rock matrix is

$$C_{ij}\rho_j V_{ij}\Big|_{\ell} = \frac{C_{ij\,j}\,K_1 k_{r1j}}{\mu_j}\,\nabla P_{1j}\Big|_{\ell} \qquad (42)$$

which is Darcy's law evaluated at the surface of the fracture. In applying Darcy's law to formulate an expression for Γ from equation 42, a relationship must be obtained between an integration over the fracture surface and an integration over the fracture volume.

It can be shown [15] that for any arbitrary scalar function $Q(\vec{r},\vec{\Omega})$

$$\int_{\tau_f} Q\,(\vec{r},\vec{\Omega})d = \frac{\tau}{2}\oint AQ\,(\vec{r},\vec{\Omega})\;f\,(\vec{r},\vec{\Omega})\;d \qquad (43)$$

where A = cross section of the fracture conduit

For $Q\,(\vec{r},\vec{\Omega}) = 1$

$$\int_{\tau_f} d\tau = \frac{\tau}{2}\oint A\;\Gamma\,(\vec{r}\;\vec{\Omega})\;d\Omega \qquad (44)$$

From the definition of porosity

$$\int_{\tau} \phi_f d\tau = \int_{\tau_f} d\tau \qquad (45)$$

Hence,

$$\int_{\tau} \phi_f d\tau = \frac{\tau}{2}\oint A\;f\,(\vec{r}\;\vec{\Omega})\;d\Omega \qquad (46)$$

Now, the total surface area of a fracture is

$$\int_{S_f} dS = \frac{\tau}{2}\oint \hat{P}\ f(\vec{r},\vec{\Omega})\ d\Omega \tag{47}$$

where $\hat{P}$ = perimeter of a fracture with cross section area A

Combining equations 46 and 47 we obtain

$$\int_{S_f} dS = \frac{\hat{P}}{A}\int_{\tau} \phi_f\ d\tau \tag{48}$$

From equation 42 and 48 the mass flux of component C_{ij} into the fracture is written as

$$\int_{S_f} C_{ij}\rho_j V_{1j}\Big|_{\ell}\ dS = \frac{\hat{P}}{A}\int_{\tau} \phi_f\ C_{ij}\rho_j\ V_{1j}\Big|_{\ell}\ d\tau \tag{49}$$

But the mass flux into the fracture is also given by

$$\int_{\tau} \Gamma_j d\tau = \int_{S_f} C_{ij}\rho_j V_{1j}\Big|_{\ell}\ dS \tag{50}$$

Hence, combining equations 49 and 50

$$\int_{\tau} \Gamma_j d\tau = \frac{\hat{P}}{A}\int_{\tau} \phi_f\ C_{ij}\rho_j V_{1j}\Big|_{\ell}\ d\tau \tag{51}$$

The only way these two integrals can be equal is for their integrands to be equal, hence

$$\Gamma_j = \left(\frac{\hat{P}}{A}\right)\phi_f C_{ij}\rho_j V_{1j}\Big|_{\ell} \tag{52}$$

By definition $\frac{\hat{P}}{A} \equiv \frac{1}{r_H}$, the hydraulic radius. Therefore, equation 52 becomes

$$\Gamma_j = \left.\frac{\phi_f C_{ij} \rho_j V_{lj}}{r_H}\right|_{\ell} \tag{53}$$

Equation 53 is identical to that derived by Duguid and Lee [15] for a single phase fluid if we set $C_{ij}=1$. Hence, each phase obeys the same equations at the surface of the fracture as does the single phase case presented by these authors. Therefore, the fluid interaction equation derived by Duguid and Lee (equation 41) can be generalized for the multiphase case. Hence, generalizing equation 41 we obtain the fluid interaction term Γ_j for each phase.

$$\Gamma_j = \frac{\beta_j}{\mu_j}\left[(P_{lj} - P_{fj}) + 2\sum_{n=1}^{\infty}(-1)^n \; P_{lj} e^{-B_j} - 2\sum_{n-1}^{\infty} P_{fj} e^{-B_j}\right] \tag{54}$$

where $\beta_j = \frac{K_1 k_{rlj} \phi_f C_{ij} \rho_j}{r_H \ell}$, $B_j = \frac{K_1 k_{rlj} n^2 \pi^2 t}{2\ell^2 \mu_j \phi_1 \; c_1}$

In equation 54 the parameters are as defined previously. If we assume the fluid interaction term can be approximated by its steady state value, then

$$\Gamma_j = \frac{\beta_j}{\mu_j} (P_{lj} - P_{fj}) \tag{55}$$

Equation 54 (or 55) completes the formulation for general multiphase flow in fractured porous media.

2.7 REQUIRED AUXILLARY EQUATIONS

Combining equations 14, 15, 18, 19, 20, 40, 54 (or 55) constitutes a system of equations which can be solved assuming rock matrix, fracture matrix and fluid properties are known. In addition to these additional equations the following functional or algebraic equations are required.

The saturations in the primary pores and in the fractures must sum to one.

$$S_{j1} + S_{w1} + S_{o1} = 1 \qquad (56)$$

$$S_{gf} + S_{wf} + S_{of} = 1$$

Also, the mass fractions of each phase in the primary pores and fractures must sum to one.

$$\Sigma C_{ig1} = \Sigma C_{io1} = \Sigma C_{iw1} = 1 \qquad (57)$$

$$\Sigma C_{igf} = \Sigma C_{iof} = \Sigma C_{iwf} = 1$$

There are two independent capillary pressure relationships for the primary pores and two for the fractures.

$$P_{g1} - P_{o1} = P_{cgo1}; \qquad P_{gf} - P_{of} = P_{cgof} \qquad (58)$$

$$P_{o1} - P_{w1} = P_{cow1}; \qquad P_{of} - P_{wf} = P_{cowf}$$

The relative permeabilities are functions of saturations.

$$k_{r1j} = F_{1j}(S_{g1}, S_{o1}, S_{w1}) \qquad (59)$$

$$k_{rfj} = G_{1j}(S_{gf}, S_{of}, S_{wf})$$

Densities and viscosities are functions of the phase pressure and composition.

$$\rho_{j1} = F_{2j}(P_{j1}, C_{ij1}), \quad \mu_{j1} = f_{3j}(P_{j1}, C_{ij1}) \qquad (60)$$

$$\rho_{jf} = G_{2j}(P_{jf}, C_{ijf}), \quad \mu_{jf} = f_{4j}(P_{jf\,m} C_{ijf})$$

Finally, for each pair of phases in the primary pores and the fractures, there is a distribution constant for each component which will be a function of pressure, temperature and composition.

$$\frac{C_{ijl}}{C_{ikl}} = \hat{K}_{ijl}(T, P_{jl}, P_{kl}, C_{ijl}, C_{ikl})$$

$$\frac{C_{ijf}}{C_{ikf}} = \hat{K}_{ijf}(T, P_{jf}, P_{kf}, C_{ijf}, C_{ikf}) \qquad (61)$$

In equations 59-60 there are three relationships for each of these equations for the fractures and for the primary rock matrix (i.e. j = oil, water, and gas phase). In equations 61, there are three distribution constants for the primary rock matrix and three for the fractures. If the number of relationships contained in the governing system of differential equations 14, 15, 18, 19, 20 and 40, and the auxiliary relations 56-61 are counted, one will find there are 2 (3N+15). It is obvious that to set up and solve this general system of equations would be a big task. However, it is anticipated that the application of the model to a practical situation would result in considerable simplification as to the number of components which would be carried in each of the phases. Hence, one may desire to develop a program which would consider a CO_2, steam, or polymer flow process and look at the maximum number of components which would have to be considered and develop a general computer code to handle as many flow situations as may be desired. Since the development of a computer program was beyond the scope of the contract work statement, additional comments about the development of a computer code will not be made.

2.8 MATHEMATICAL TECHNIQUES FOR SOLVING THE GOVERNING EQUATIONS:

It is appropriate to briefly discuss various methods which could be employed to solve the governing system of equations. In general the equations which have been developed in the previous section could be solved by several well known numerical schemes. These are a) finite elements b) finite differences, and c) variational methods, to mention a few.

The finite element method is an approximate method of solving differential equations of boundary and/or initial value problems by dividing the domain of interest into many small elements of convenient shapes such as triangles, quadrilaterals, etc. Hence, when irregular boundaries exist in a problem,the finite element approach may offer a distinct advantage over the classical finite difference approach. In spite of this advantage finite element analysis has gained a much larger application in the field of solid body mechanics as opposed to fluid flow problems although in recent years finite element algorithms for solving fluid flow problems have been developed and are beginning to be used on a wider scale. Furthermore, there are several more numerical algorithms available for solving finite difference approximations to a system of differential equations than there are finite element algorithms. However, as more and more complex problems and geometries are encountered in fluid flow problems, the finite element approach may be used more and more. In addition, the finite element method, mathematically speaking, is more sophisticated and complex than its more popular counterpart, finite differences.

The application of finite differences are in principle more simple to comprehend and set up for computer solutions than finite elements. This is particularly true where simple geometries and boundary conditions are specified. In addition, the finite difference approach is a much more time-proven numerical technique for solving fluid flow problems than any of the other numerical schemes [38]. Furthermore, they lend themselves quite readily to the inclusion of probabilistic approaches such as Monte Carlo Techniques. Use of the Monte Carlo Technique is particularly attractive in situations involving a complex reservoir heterogeneity where its speed and inherent stability offer definite advantages over other numerical techniques [39]. Since naturally fractured reservoirs are inherently heterogeneous, the Monte Carlo approach with finite differences may provide an excellent solution approximation to the differential equations which have been derived.

Variational methods are closely related to finite elements and are referred to in the literature as the Rayleigh-Ritz Method and Weighted Residuals Methods. These two variational methods are normally used to derive the finite element algorithms appearing in the literature (Ref. 40).

Before a decision is made concerning a solution approach to the system of coupled equations, one should closely assess the overall merits of the various numerical schemes which are available in the literature.

The development of a computer code based upon the equations presented in this chapter would be a rather extensive effort but the resulting simulator which would evolve would make an invaluable tool for analyzing reservoir performance for various enhanced oil recovery processes. In addition it would be an invaluable tool to assist the engineer in designing an EOR project.

CHAPTER III

CONCLUSIONS AND RECOMMENDATIONS

3.1 INTRODUCTION

Multiphase flow in fractured porous media requires considerable more research before the complete flow phenomena is fully understood and adequately described by existing mathematical/physical models. If reasonably accurate projections are to be made concerning enhanced oil recovery from naturally fractured reservoirs, the microscopic processes which are occuring must be analyzed by taking into account the statistical nature of the fractures. Models derived from this viewpoint need additional laboratory experimentation to verify their validity or to improve upon the model described by a set of mathematical equations. To date a fully three dimensional, multiphase simulator for flow through naturally fractured reservoirs has not appeared in the literature. The most complex model to date considers only an oil/water or an oil/gas system and these are limited to two dimensional flow with uniformly spaced fractures which are considerably removed from what one would expect in most reservoirs. Although considerable work has been published on single matrix block phenomena, little or no work has been published on overall flow through a matrix block/fractured system.

3.2 CONCLUSIONS

From the results of this study the following conclusions were reached:

1. The mathematical models in the current literature do not adequately describe multiphase flow in fractured porous media.
2. Little or no information appears in the literature concerning the simulation of enhanced oil recovery processes through

naturally fractured reservoirs.

3. Limited experimental data has been published concerning the measurement of directional permeabilities of naturally fractured reservoirs.

4. None of the models published in the literature have treated the fractures as a general statistical entity to arrive at a model for describing flow through the primary rock matrix and the fractures simultaneously. Most models consider only flow via the fractures.

5. Existing models only treat single phase or at most an oil/water or an oil/gas system for flow through an isotropic rock matrix/fracture system of double porosity.

6. The models which have been developed for water/oil or gas/oil flows in naturally fractured reservoirs have been used to simulate conceptual field examples and have not been used to predict actual field applications of enhanced oil recovery processes in fractured media.

7. The model formulated in this report is the most general model published to date. The model considers multiphase flow through isotropic rock matrix blocks and anisotropic fractures. The equations are general in that they permit time and space variations in all fluid and rock properties. A fluid interaction term is developed which couples the flow through the rock matrix blocks and the fractures.

8. The model developed can be programmed on a digital computer using a finite difference or a finite element approach although the author prefers the former approach since it readily lends itself

to the incorporation of Monte Carlo techniques for handling the terms involving the permeabilities for the primary rock matrix and the permeability tensor for the fractures.

3.3 RECOMMENDATIONS

Specifically the major areas which warrant further study are:

1. Develop a software program utilizing the mathematical model formulated in Chapter II of this report. In this regard emphasis should be placed upon applying the finished computer code to an enhanced oil recovery process such as steam, CO_2 or a polymer.
2. Research into the prediction of fracture porosity, fracture permeabilities, and capillary pressures in naturally fractured rock.
3. Perform laboratory displacement experiments on cores extracted from naturally fractured formations.
4. Investigate the retention, degradation and stability of micellar/polymer fluids in naturally fractured formations.
5. Perform parametric studies utilizing reservoir simulation to investigate the most sensitive parameters associated with multiphase flow through fractured porous media.
6. Research into the microscopic phenomena of displacing hydrocarbons from the primary rock matrix and transport via the fractures.
7. Develop a visual laboratory experiment to observe the displacement of fluids through naturally fractured rocks.
8. Perform post morten simulation studies of existing or previous enhanced oil recovery projects associated with highly heterogeneous or naturally fractured reservoirs. In this regard the

simulator developed utilizing the fracture model formulated in this report could also be used to perform simulation studies of proposed enhanced oil recovery projects.

9. Investigate the effects of fluid movement through a naturally fractured reservoir which have been further fractured by artificial means.

10. Correlation of well logs as a means of investigating rock properties of naturally fractured reservoirs.

The author recognizes that there may be other conclusions which could be drawn concerning multiphase flow through naturally fractured porous media, as well as other equally important research areas which should be addressed. However, within the time allotted to this study the conclusions and recommendations stated previously appear to be reasonable and significant enough to indicate that much research must be forthcoming before naturally fractured reservoir performance can be satisfactorily predicted to warrant the initiation of an expensive enhanced oil recovery project.

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