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The Discounted Cash Flow (DCF) and Revenue Requirement (RR) Methodologies in Energy Cost Analysis

Doan L. Phung

institute for energy analysis

Oak Ridge Associated Universities

MASTER

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Research Report

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Doan L. Phung
Institute for Energy Analysis

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Oak Ridge Associated Universities
Oak Ridge, Tennessee 37830

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Abstract

Of the many cost analysis methods employed, two are most frequently used for the comparison of alternative energy technologies: these are the discounted cash flow (DCF) method and the revenue requirement (RR) method. The former is more favored by unregulated industries which do not know but must estimate in advance how much revenue their products can generate in the competitive marketplace. The latter is favored by regulated industries which know with some certainty the maximum allowable return on their invested capital.

It is shown in this paper that the two methods are based on the same financial principles and that one can lead to the other consistently. Furthermore, the discount rates to be used in various forms of their formulation are interrelated and depend only on the cash flow streams that are included in the formulation.

In the comparison of energy costs between alternative future technologies, the RR method is almost universally used even though the DCF method is often claimed. The paper shows that a consistent pricing policy can be arrived at by any of the formulations when the proper cash flows, discount rate, and escalation rate of the prices are properly accounted for.

The DCF and RR formulations are valid under both inflationary and noninflationary conditions. The only requirement is that when inflation is internalized in one or more parameters of the formulations, all other parameters and the results must reflect the same inflation rate; otherwise, the analysis is no longer consistent.

An example is given to illustrate the relationship between the DCF and RR formulations and their behavior in an inflationary environment.

THE DISCOUNTED CASH FLOW (DCF) AND REVENUE REQUIREMENT (RR) METHODOLOGIES IN ENERGY COST ANALYSIS

INTRODUCTION

Both the DCF and RR methods are based on two fundamental principles: (a) money has a time value — a dollar today being more valuable than a dollar many years from today, and (b) a venture is solvent when receipts and disbursements balance out — the balancing process being predicated on the equality of the present worths of net cash inflows and net cash outflows. The difference between the methods is in the manner they are used, or in the quantity they are supposed to compute.

The DCF method (in this paper assumed to be synonymous with the Internal Rate of Return method) starts out with all known cash flows (such as in a completed project), then attempts to look for the discount rate r which allows the inflow and outflow streams to be equivalent. Such a resultant rate r is also called the DCF rate and is interpreted as the opportunity cost of capital invested in the venture. A direct comparison of such a rate with the owner's experience and/or criteria will help determine whether the project is financially desirable. In particular, when two alternatives are to be compared, the one that yields a higher r is the more attractive.

The RR method starts out at the opposite end. It assumes the owner's exact expectation of return on his capital and then proceeds to compute the minimum revenue he must obtain by selling the products. This minimum revenue should be enough to cover all of the owner's operating costs, all taxes, his return requirement, and the recovery of his capital. The resultant minimum revenue can then be converted to the minimum sale price of the product. A judgment can next be made to see whether such a minimum "cost of production" price can survive the marketplace. In particular, the alternative that leads to a lower cost of production is the financially more attractive alternative, assuming that all other factors such as time frame, quality, and quantity of the product are the same.

The purpose of this paper is to provide a clear formulation for the DCF and RR methods in the cost analysis of energy. While the DCF formulation is a simple mathematical statement of the principles regarding business solvency and the time value of money, three logical derivatives are provided for the formulation of the RR methods. The effect of inflation on the cost analysis of energy is treated for the case when cash flows and costs of money track inflation in a simple manner. Among salient conclusions are the following:

- The DCF formulation leads naturally to the RR formulation and vice versa.
- The DCF resultant rate r has several meanings, depending on what kinds of cash flows are included. When no operating costs and taxes are involved, r is the interest rate for the use of capital. When the tax stream is excluded, r is the before-tax rate of return r_{BT}. When all cost streams are included, r is the after-tax nominal rate of return.

- The current discount rate in the RR formulation is the nominal after-tax rate of return on capital less the tax deductibility effect of debt. It is also a special DCF rate when tax savings on bond interest are not counted as a cash flow in the DCF formulation.

- When inflation is internalized in the cost of money, all cash flow streams must also reflect inflation.

THE DISCOUNTED CASH FLOW (DCF) METHOD

The DCF Formulation

The DCF method is also called the internal-rate-of-return method, the receipts-versus-disbursements method, the investor's-rate-of-return method, and the profitability-index method.[1,2] "Cash flow"* is defined as the movement of money, either into the project (called revenues) or out of the project (called disbursement). The following cash flows are characteristic of any venture. (Symbols used in this paper are listed in Table 1).

Disbursements (cash outflows)

- Beginning-of-project investment, I_o, and subsequent year investment, I_i (i=1, M)

- Annual operating costs, including startup cost, fuel, and operations and maintenance, O_i (i=0, M)

- Annual income tax, T_i (i=1, M)

- Annual ad-valorem expenses, Π_i (i=1, M)

Receipts (cash inflows)

- Revenue as a result of product sale, R_i (i=1, M)

- End-of-life salvage value, assumed negligible here for simplicity (if not zero, it can be identified as $-I_M$).

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*In colloquial business terminology, "cash flow" is frequently understood as the *net* cash inflow (e.g., "Company X has cash flow problems".)

TABLE 1: SYMBOLS USED IN THIS PAPER

B = bond interest payment, $B_i = r_b f_b V_i$

C = unit cost of product (dollars/unit)

$\overline{C}$ = levelized unit cost of product

$\overline{C}_{BT}$ = levelized unit cost of product, using r_{BT} as discount rate

$\overline{C}_x$ = levelized unit cost of product, using x as discount rate

$\overline{C}_r$ = levelized unit cost of product, using r as discount rate

${}_uC_o$ = starting cost value of an increasing curve with u as the escalation rate

${}_xC_o$ = same as above but with x as escalating rate

CRF(r,M) = capital recovery factors for rate r in M periods;

$$CRF(r,M) = \frac{r}{1-(1+r)^{-M}}$$

D = depreciation charge

D^B = book depreciation charge

D^T = tax depreciation charge

E = amount of energy (product) produced annually

I = capital investment

M = service life of project (years)

N = net cash inflow

N' = pseudo net cash inflow (does not include tax benefit due to bond interest)

${}_oN_i'$ = a portion of I_o (used only in algebra, not an important quantity)

O = operating costs (fuel and operations & maintenance)

$PWRR_r$ = present worth of revenue requirement, using r as discount rate

-continued-

TABLE 1: SYMBOLS USED IN THIS PAPER (continued)

$PWRR_x$ = present worth of revenue requirement, using x as discount rate

$PWRR_{BT}$ = present worth of revenue requirement, using r_{BT} as discount rate

Π = ad-valorem costs

R = gross revenue annually (dollars)

R^K = annual revenue needed to cover capital costs

SFF(r,M) = sinking fund factor for rate r in M periods;

$$SFF(r,M) = \frac{r}{(1+r)^M - 1}$$

T = income taxes

V = unrecovered value of the investment (the amount of investment on which a return must be paid)

i = time index, using end-of-year convention except for book value when beginning of year is implied. Thus I_o is the original investment, V_1 is the book value as of the beginning of year 1.

f_b = fraction of capital that is debt (bond)

f_s = fraction of capital that is equity (stock) $f_s = 1 - f_b$

r = nominal after-tax rate of return on capital

r_{BT} = before-tax rate of return on capital

r_{HM} = home mortgage cost of money

r_b = rate of return to bond

r_s = rate of return to stock

${}_ur$ = nominal after-tax rate of return on capital in an inflationary environment

${}_ur_{BT}$ = same as r_{BT}, but in an inflationary environment

τ = effective income tax rate

x = effective after-tax rate of return

${}_ux$ = same as x, but in an inflationary environment

y = escalation rate used in general; it could be the inflation rate, u; it could also be the cost of money, x

The basic objective of the DCF method is to find a discount rate (rate of return) such that the present worth of all cash outflows is equal to the present worth of all cash inflows. In mathematical terms, this statement is equivalent to finding r in the following equality:

$$\sum_i \frac{R_i}{(1+r)^i} = \sum_i \frac{I_i}{(1+r)^i} + \sum_i \frac{O_i+\Pi_i}{(1+r)^i} + \sum_i \frac{T_i}{(1+r)^i} \qquad (1)$$

(All summations over time are for i between 0 and M. Some cash flow data can be zero, e.g., T_o=0.)

There are three major difficulties with the DCF formulation. First, the solution for r requires several trial-and-error computations. Sometimes a solution does not exist (when the unrecovered capital investment at a particular time is negative, meaning that there is no investment at that time). Sometimes several solutions are possible [when the net cash flow (R_i-O_i-Π_i-T_i) changes sign several times during the life of the project]. With the aid of modern computers or electronic calculators, the solution of Equation 1 is not as cumbersome as it once was.

The second difficulty with Equation 1 is the need for all data to be available. This is only possible when the entire history of the project is known, such as for a venture already completed. To look forward at a project, it is hard enough to estimate data for the I_i, O_i, Π_i streams, but it is almost impossible to predict the R_i and from there the T_i streams. For an unregulated industry, the ability to predict the R_i stream in a competitive marketplace makes the whole difference between success and failure of the venture.

The third difficulty with Equation 1 is the tax cash flow stream T_i, which is not an independent stream by itself. When R_i, I_i, O_i, and Π_i are known, the stream T_i is determined by

$$T_i = \tau(R_i - O_i - \Pi_i - D_i^T - r_b f_b V_i) \qquad (2)$$

where D_i^T is the depreciation charge for tax purposes, V_i is the outstanding unrecovered capital investment, f_b is the fraction of debt financing, and r_b is the debt interest rate. The determination of D_i^T can be made when the depreciation schedule is known (for example, straight-line schedule, sum-of-the-years digits schedule).

Substituting Equation 2 into Equation 1, one has an alternative expression for the DCF formulation:

$$\sum_i \frac{R_i}{(1+r)^i} = \frac{1}{1-\tau} \sum_i \frac{I_i}{(1+r)^i} + \sum_i \frac{O_i + \Pi_i}{(1+r)^i} - \frac{\tau}{1-\tau} \sum_i \frac{D_i^T + r_b f_b V_i}{(1+r)^i} \qquad (3)$$

Equation 3 highlights the basic difficulty with the DCF formulation. Unless the tax stream T_i in Equation 1 is known, as in the case of a completed project, it is dependent not only on R_i, O_i, Π_i, but also on the depreciation schedule and the interest payment on the debt portion of the unrecovered capital. The stream V_i is simply determined when book and tax depreciation are the same and when $I_i = 0$ for $i \neq 0$; otherwise it is not readily available. A general expression for V_i will be presented later in the formulation of the RR method.

Special Cases of the DCF Formulation and the Meaning of the DCF Rate r

1. Home Mortgage Loan

If an amount of money, I_o, is loaned out at time i=0 and a level revenue of $\overline{R}$ is expected at the end of each period up to M, and if no operating and ad-valorem costs are expected then Equation 1 takes on the form

$$\sum_{i}^{M} \frac{\overline{R}}{(1+r_{HM})^{i}} = I_o \tag{4}$$

or

$$\overline{R} = I_o \ CRF \ (r_{HM}, M)$$
$$= I_o \ [r_{HM}+SFF(r_{HM}, M)]$$

where r_{HM} is the interest on a home mortgage loan, CRF $(r_{HM}, M) = [r_{HM}]/[1-(1+r_{HM})^{-M}]$ is the capital recovery factor for interest r_{HM} over M periods, and SFF $(r_{HM}, M) = [r_{HM}]/[(1+r_{HM})^{M}-1]$ is the sinking-fund factor for interest r_{HM} over M periods.

2. Cost of Capital After Taxes

As O_i's are out-of-pocket, tax-deductible items which can be subtracted directly from R_i without altering the significance of

Equation 1, one can write

$$R_i^K \equiv R_i - O_i - \Pi_i$$

and

$$\sum_i \frac{R_i^K}{(1+r)^i} = \sum_i \frac{I_i}{(1+r)^i} + \sum_i \frac{T_i}{(1+r)^i}$$

This expression is used when only the capital aspect of the venture is considered. The resultant DCF rate r is, of course, the same as that of Equation 1 if all cash flow streams are known and if R_i^K is defined as the "revenue needed to cover the costs of capital including taxes."

If a constant rate $\bar{\phi}$ is defined such that $\bar{\phi} I_o$ is the level revenue stream sufficient to cover all capital related costs, then

$$\sum_i \frac{\bar{\phi} I_o}{(1+r)^i} \equiv \sum_i \frac{R_i{}^K}{(1+r)^i}$$

or

$$\bar{\phi} = \mathrm{CRF}(r,\ M) \left(\sum_i \frac{I_i}{(1+r)^i} + \sum_i \frac{T_i}{(1+r)^i} \right)$$

In this case, $\bar{\phi}$ is called the annual capital charge rate or capital fixed charge rate (ad-valorem charges being included in operating costs).

When the capitalization structure consists of a bond fraction f_b with bond rate r_b, and a stock fraction f_s with stock rate r_s, then the nominal cost of money is r, where:

$$r = r_b f_b + r_s f_s \ ; \quad f_s = 1 - f_b$$

where r is known, r_s is computed by:

$$r_s = \frac{r - r_b f_b}{f_s} \tag{6}$$

The objective of the venture owner is of course to achieve a high r_s by both achieving a high r and arranging for a high debt fraction.

3. Cost of Capital Before Income Taxes

Equation 5 can further be rewritten in two ways:

$$\sum_i \frac{R_i^K - T_i}{(1+r)^i} = \sum_i \frac{I_i}{(1+r)^i} \tag{7}$$

or

$$\sum_i \frac{R_i^K}{(1+r_{BT})^i} = \sum_i \frac{I_i}{(1+r_{BT})^i} \tag{8}$$

(where r_{BT} will be clarified below).

Equation 7 is similar to the simple home mortgage loan of Equation 4 except in this case it is a business venture with revenues reduced by income taxes. It is a restatement of the basic property of the DCF in the following context: "The present worth of net revenues (after taxes and all other expenses) is equivalent to the present worth of all investments." The DCF rate r that allows such equivalency is the opportunity cost of money in the venture.

Equation 8 conveys a different message. The revenue stream in this equation is the revenue before tax payment. Thus a different DCF rate, $r_{BT} > r$, is the solution for the equivalency statement, "The present worth of revenues before tax payment is equivalent to the present worth of all investments." In this case, r_{BT} is the "before tax" discount rate. If the project is subject to an effective tax rate of τ on equity earning, then r_{BT} and r are related as follows:

$$r_{BT} = r_b f_b + r_s f_s + \frac{\tau}{1-\tau} r_s f_s = r + \frac{\tau}{1-\tau} r_s f_s \qquad (9)$$

The return on equity can then be found from r_{BT}:

$$r_s = \frac{(1-\tau)(r_{BT} - r_b f_b)}{f_s} \qquad (10)$$

The Bureau of Mines has used the "before tax" DCF rate in many of its analyses on the cost of coal and oil shale.[3,4,5] Equation 9 shows that r_{BT} must be higher than r, which ranges between 10 percent and 20 percent for most businesses in the current economic environment. A selection of too low a r_{BT} will result in a low R_i stream which in turn predicts a low unit product cost.

THE REVENUE REQUIREMENT (RR) METHOD

Intent of the RR Method

The RR method is also called the "minimum" revenue requirement method because its purpose is to determine the minimum revenue that can cover all costs, including a minimum acceptable (or allowed) return on capital investment. This method assumes that the cost of money is known (e.g., stock rate, stock fraction, bond rate, bond fraction, taxes) and that the revenue stream R_i and/or its present worth is to be determined. The knowledge of R_i for every year i will lead to the determination of the "bare-bones" sale price of the product if the quantity of production, E_i, is known. These bare-bones sale prices are subsequently put in perspective with respect to the perceived marketplace to help judge whether the venture is economically feasible. In particular, if two similar ventures are to be compared, the one that yields a lower bare-bones sale price of the product is the more attractive venture.

Figure 1 illustrates the exact intent of the RR method. The minimum revenue R_i in the year i should be enough to cover all costs, which include operating costs, ad-valorem taxes, depreciation charge, return on the outstanding capital tied up in the project, and income taxes. Using the symbols previously introduced and further listed in Table 1, the mathematical expression for this statement is

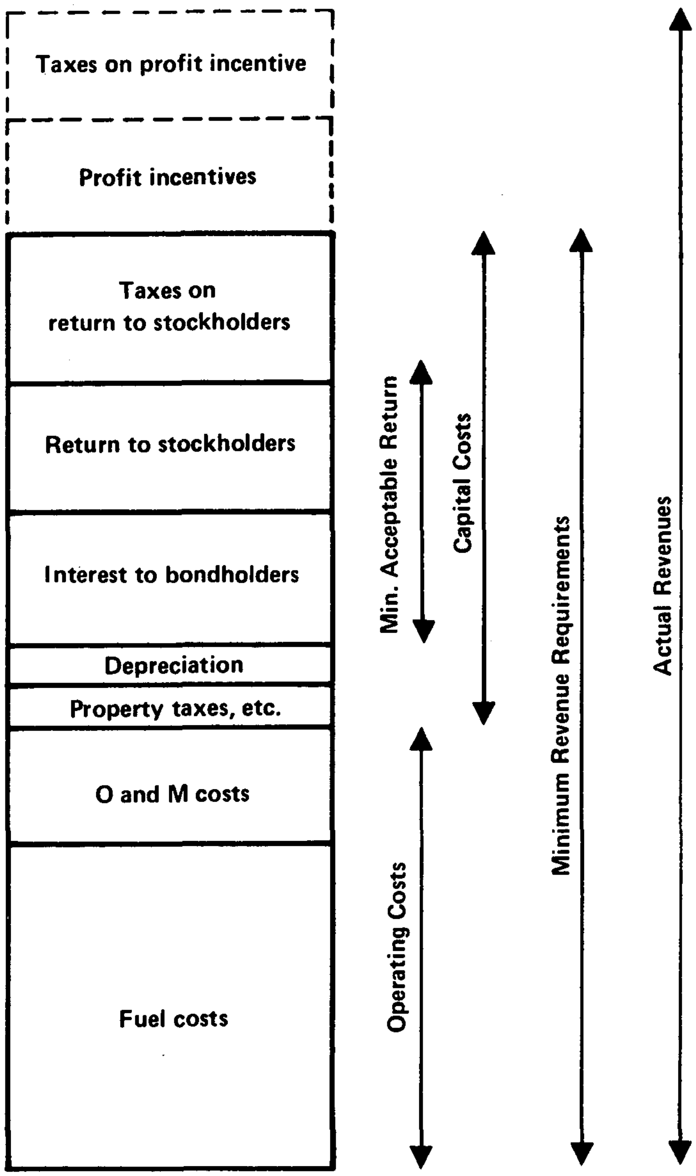
$$R_i = O_i + \Pi_i + D_i + rV_i + T_i \tag{13}$$

where r is the nominal cost of capital determinable by the financial structure of the project.

$$r = r_b f_b + r_s f_s \; ; \quad f_s = 1 - f_b \tag{14}$$

FIGURE 1

EXACT INTENT OF REVENUE REQUIREMENT METHOD



It is noted that all quantities in Equations 13 and 14 are known, with the exception of V_i and T_i, which are not independent quantities by themselves but can be expressed in terms of other quantities. V_i is the outstanding book value of the project for which a return of rV_i must be earned by the end of the year i, and T_i is the tax payment which is determined by the tax rate and by the taxable income. We shall present below three alternatives derivations for the RR formulation which permit the calculation of the present worth of revenue requirements on the basis of readily accessible quantities, namely, I_i, O_i, Π_i, x and τ.

Derivation 1: Year-to-Year Iteration of V_i and T_i

This derivation assumes that the outstanding book value, V_i, is reduced each year by a depreciation charge but increased by an additional investment. In addition, all cash resulting from tax depreciation charge is assumed available to pay back stockholders and bondholders ratably. A case for different tax and book depreciation charges is considered in Derivation 2.

At the beginning of year 1, the outstanding book value is $V_1 = I_o$. By year end, revenue is R_1; operating cost, O_1; ad-valorem taxes, Π_1; depreciation charge, D_1^T; return to bondholders, $r_b f_b I_o$; return to stockholders, $r_s f_s I_o$; and tax, T_1. In terms of other quantities, T_1 can be expressed as

$$T_1 = \tau(R_1 - O_1 - D_1^T - \Pi_1 - I_o r_b f_b) \qquad (15)$$

where the depreciation charge D_1^T is

$$D_1^T = R_1 - O_1 - \Pi_1 - I_o r_b f_b - I_o r_s f_s - T_1 \qquad (16)$$

At the beginning of year 2, the depreciation charge D_1^T is distributed ratably to stock- and bondholders, but an investment I_1 may be made by them for facility improvement. The outstanding book value is, using Equations 15 and 16,

$$\begin{aligned} V_2 &= I_o + I_1 - D_1^T \\ &= I_o[1 + (1-\tau)r_b f_b + r_s f_s] + I_1 + (1-\tau)(O_1+\Pi_1) \\ &\quad - \tau D_1^T - (1-\tau)R_1 \end{aligned}$$

By continuing this iterative process for V_3, V_4,... V_j, one obtains the expression V_j for the year j as follows:

$$\begin{aligned} V_j = & \sum_{i=0}^{j-1} I_i(1+x)^{j-i-1} + \sum_{i=0}^{j-1} (1-\tau)(O_i+\Pi_i)(1+x)^{j-i-1} \\ & - \tau \sum_{i=0}^{j-1} D_i^T(1+x)^{j-i-1} - (1-\tau) \sum_{i=0}^{j} R_i(1+x)^{j-i-1} \end{aligned} \qquad (17)$$

where the quantity x has been defined as

$$x = (1-\tau)r_b f_b + r_s f_s \; ; \quad f_s = 1-f_b \qquad (18)$$

Equation 17 is the "exact" book value of the project at the beginning of year j. It is intimately related to the financial history of the project.

A boundary condition is that at the end of the project, the book value or V_{M+1} must be equal to zero. Thus, by setting V_{M+1} of Equation 17 equal to zero, one has

$$0 = \sum_i I_i(1+x)^{M-i} + (1-\tau)\ \Sigma(O_i+\Pi_i)(1+x)^{M-i}$$

$$- \tau \sum_i D_i^T(1+x)^{M-i} - (1-\tau) \sum_i R_i(1+x)^{M-i} \tag{19}$$

Or, after shifting terms and dividing by $(1+x)^M$, one obtains

$$\sum_i \frac{R_i}{(1+x)^i} = \frac{1}{1-\tau} \sum_i \frac{I_i}{(1+x)^i} + \sum_i \frac{O_i+\Pi_i}{(1+x)^i} - \frac{\tau}{1-\tau} \sum_i \frac{D_i^T}{(1+x)^i} \tag{20}$$

where all summations are from i=0 to i=M. Some values of the cash flow stream, notably R_o, Π_o, D_o^T, are zero when i=0.

Equation 20 is the basic formulation of the RR methodology. It states that the present worth of revenue requirements, using x as discount rate, can be readily found by using known quantities. When two similar alternatives are to be compared, the one that has a lower present worth of revenue requirements is the more financially attractive alternative.

Derivation 2: Both Book and Tax Depreciation Are Used

If the book value during the year i is dependent only on the book depreciation schedule, then it can be written as

$$V_i = \sum_{j=0}^{i-1} (I_j - D_j^B) \qquad (21)$$

The income tax is still computed on the basis of a tax depreciation schedule as shown in Equation 15. Substituting Equations 15 and 21 into the RR statement of Equation 13, one has

$$(1-\tau)R_i = (1-\tau)(O_i + \Pi_i) + (r - \tau r_b f_b) \sum_{j=0}^{i-1} (I_j - D_j^B) + D_i^B - \tau D_i^T \qquad (22)$$

Note that $r - \tau r_b f_b$ is exactly the value of x as defined in Equation 18.

Dividing both sides of Equation 22 by $(1+x)^i$ and summing over all i's, one has

$$\sum_i \frac{R_i}{(1+x)^i} = \sum_i \frac{O_i + \Pi_i}{(1+x)^i} + \frac{1}{1-\tau} \sum_i \frac{x \sum_{j=0}^{i-1} (I_j - D_j^B)}{(1+x)^i} + \frac{1}{1-\tau} \sum_i \frac{D_i^B - \tau D_i^T}{(1+x)^i} \qquad (23)$$

The double summations can be listed term by term and can be reduced as follows:

$$\sum_i \frac{x \sum_{j=0}^{i-1} I_j}{(1+x)^i} = \sum_{i=0}^{M-1} \frac{I_i}{(1+x)^i} - \frac{1}{(1+x)^M} \sum_{i=0}^{M-1} I_i$$

$$\sum_i \frac{x \sum_{j=0}^{i-1} D_j^B}{(1+x)^i} = \sum_{i=0}^{M-1} \frac{D_i^B}{(1+x)^i} - \frac{1}{(1+x)^M} \sum_{i=0}^{M-1} D_i^B$$

With these expressions, Equation 23 reduces to

$$\sum_i \frac{R_i}{(1+x)^i} = \frac{1}{1-\tau} \left(\sum_i^{M-1} \frac{I_i}{(1+x)^i} + \frac{1}{(1+x)^M} \sum_i^M D_i^B - \frac{1}{(1+x)^M} \sum_{i=0}^{M-1} I_i \right) - \frac{\tau}{1-\tau} \sum_i^M \frac{D_i^T}{(1+x)^i} + \sum_i^M \frac{O_i+\Pi_i}{(1+x)^M} \quad (24)$$

Equation 24 is identical to Equation 20 if the quantity

$$\sum_i^M D_i^B - \sum_i^{M-1} I_i$$

is interpreted as the investment at the beginning of the last year, and is represented by the symbol I_M. The derivation is advantageous because even though both book and tax depreciation are considered, only tax depreciation is important to the revenue requirement.

Derivation 3: Net Cash Inflow As Basis

This derivation is based on a common perception of financial solvency from the viewpoint of the owner.

As a first step, let us consider a project with 100 percent equity. Let r be the rate of return (same as r_s in this case) the owner requires of the investment. He perceives that at the end of each year i, he should receive a net cash inflow N_i such that the present worth of all such cash inflows, using r as discount rate, should be the same as the total disbursement I_o:

$$I_o = \sum_i \frac{N_i}{(1+r)^i} \tag{25}$$

The net cash inflow N_i is the amount left in hand of the owner after the gross revenue R_i has been used to pay for operating costs O_i, ad-valorem tax Π_i, income tax T_i, and any additional investment required to keep the project in operation.

$$N_i = R_i - O_i - \Pi_i - T_i - I_i \tag{26}$$

But the income tax T_i is expressible in terms of R_i, O_i, Π_i and the tax depreciation D_i^T (there is no bond interest payment in this case).

$$T_i = \tau\ (R_i - O_i - \Pi_i - D_i^T) \tag{27}$$

Combining Equation 25, 26, and 27, we have:

$$I_o = (1-\tau) \sum_i \frac{R_i}{(1+r)^i} - (1-\tau) \sum_i \frac{O_i+\Pi_i}{(1+r)^i} - \tau \sum_i \frac{D_i^T}{(1+r)^i} - \sum_i \frac{I_i}{(1+r)^i} \tag{28}$$

where the summations starts with i=1 and ends with i=M. At i=0, all cash flows are zero with the exception of I_o and possibly O_o, and the summations can be extended to i=o with increased generality. After switching sides and dividing by $(1-\tau)$, we have:

$$\sum_{i=0}^{M} \frac{R_i}{(1+r)^i} = \frac{1}{1-\tau} \sum_{i=0}^{M} \frac{I_i}{(1+r)^i} + \sum_{i=0}^{M} \frac{O_i+\Pi_i}{(1+r)^i} - \frac{\tau}{1-\tau} \sum_{i=0}^{M} \frac{D_i^T}{(1+r)^i} \qquad (29)$$

Note that Equation 29 is the same as 20 when there is no bond component in the capitalization.

When the owner also finances the investment with debt (fraction f_b, return r_b), he is still obligated to generate enough cash inflow each year to pay for the cost of capital $(r=r_bf_b+r_sf_s)$ and the recovery of capital (through depreciation). However, interest payment to bondholders is tax deductible. The owner can perceive that his cost of capital is only $x = r - \tau r_bf_b$ of which he pays r_sf_s to the equity holders and $(1-\tau)r_bf_b$ to the bondholders who are further paid τr_bf_b directly from the amount set aside for income tax and bond interest deduction payment, T_i'. Since the real tax payment is $T_i = \tau(R_i - O_i - \Pi_i - D_i^T - r_bf_bV_i)$, the amount set aside for income tax <u>and</u> bond interest deduction is

$$T_i' = T_i + \tau r_bf_bV_i = \tau(R_i - O_i - \Pi_i - D_i^T) \qquad (30)$$

The net cash inflow from the owner's viewpoint is

$$N_i' = N_i - \tau r_bf_bV_i = R_i - O_i - \Pi_i - T_i' - I_i \qquad (31)$$

And the condition of solvency expressed in Equation 25 becomes:

$$I_o = \sum_i \frac{N_i'}{(1+x)^i} \qquad (32)$$

Combining Equations 30, 31, and 32, and using the same arguments concerning the extension of the summations to i=0, one has:

$$\sum_i \frac{R_i}{(1+x)^i} = \frac{1}{1-\tau} \sum_i \frac{I_i}{(1+x)^i} + \sum_i \frac{O_i+\Pi_i}{(1+x)^i} - \frac{\tau}{1-\tau} \sum_i \frac{D_i^T}{(1+x)^i} \qquad (20)$$

This derivation includes bond financing in the project and results exactly in the RR formula arrived at previously.

An advantage of the above derivation is its simplicity. A disadvantage is the lack of rigor in the use of r (when there is no bond financing) and x (when there is bond financing) as discount rates.

An insight into the use of x (and hence r) as discount rate for present worth calculations can be obtained as follows. Let us divide the original investment I_o into M unequal portions, say ${}_oN_1'$, ${}_oN_2'$, ..., ${}_oN_i'$, ..., ${}_oN_M'$. Let us further require that each of these portions are invested in the firm such that by the end of the first year, ${}_oN_1'$ should grow into the cash inflow N_1'; at the end of the second year, ${}_oN_2'$ should grow into N_2', and so forth. If such an operation is carried out, the project is solvent because the investors can realize a rate of return r and can recover the capital as well, simply by using the net cash inflow N_1', N_2', ..., to pay for capital recovery, return $r_s f_s V_i$ to equityholders, and return $(1-\tau)r_b f_b V_i$ to bondholders. (An additional $\tau r_b f_b V_i$ is paid to bondholder from T_i'.)

Let us consider the portion ${}_oN_i'$. At the end of the first year, it earns an interest rN_i' but has to pay $\tau r_b f_b N_i'$ in tax because $r_b f_b N_i'$ is not paid to bondholders at the end of this first year and hence is not tax deductible. Thus by the end of the first year, ${}_oN_i'$ accumulates into a cash-in-hand of ${}_1N_i'$ where

$$\begin{aligned} {}_1N_i' &= {}_oN_i' + r\,{}_oN_i' - \tau r_b f_b\,{}_oN_i' \\ &= (1 + r - \tau r_b f_b)\,{}_oN_i' \\ &= (1 + x)\,{}_oN_i' \end{aligned}$$

By continuing the reasoning this way, by the end of the second year the cash-in-hand corresponding to the investment portion of ${}_oN_i'$ is $(1 + x)^2{}_oN_i'$, and so on. By the end of the year i, the cash-in-hand is:

$${}_iN_i' = (1 + x)^i\,{}_oN_i'$$

The premise is that the portion of ${}_oN_i'$ of the original capital must grow to the cash-in-hand N_i' by the end of the year i

$$N_i' \equiv {}_iN_i' = (1 + x)^i\,{}_oN_i'$$

or the relationship between N_i and its present value (at year i = 0) is established:

$$ {}_oN_i' = \frac{N_i'}{(1 + x)^i} $$

Since $I_o = \sum_i {}_oN_i'$ by definition, and since N_i' can be found by Equation 31, Equation 32 is valid and the result, Equation 20, will then follow.

RELATIONSHIP BETWEEN THE DCF AND RR FORMULATIONS

The DCF method allows the analyst to solve for the internal rate r, the rate of return on capital for the specific venture under consideration. Such a solution is found by many trial-and-error computations or by graphs. The objective is to find a rate r such that both sides of Equation 1 or 3 are equal

$$ PWRR_r = \sum_i \frac{R_i}{(1+r)^i} = \sum_i \frac{I_i}{(1+r)^i} + \sum_i \frac{O_i+\Pi_i}{(1+r)^i} + \sum_i \frac{T_i}{(1+r)^i} \qquad (1) $$

or

$$ PWRR_r = \sum_i \frac{R_i}{(1+r)^i} = \frac{1}{1-\tau} \sum_i \frac{I_i}{(1+r)^i} + \sum_i \frac{O_i+\Pi_i}{(1+r)^i} - \frac{\tau}{1-\tau} \sum_i \frac{D_i^T}{(1+r)^i} - \frac{\tau}{1-\tau} \sum_i \frac{r_b f_b V_i}{(1+r)^i} \qquad (3) $$

The RR method allows the analyst to compute the present worth of revenue requirements when the minimum acceptable return on capital (r) is counted as a cost. From r, a tax-adjusted x can be found such that

$$PWRR_x = \sum_i \frac{R_i}{(1+x)^i} = \frac{1}{1-\tau} \sum_i \frac{I_i}{(1+x)^i} + \sum_i \frac{O_i+\Pi_i}{(1+x)^i} - \frac{\tau}{1-\tau} \sum_i \frac{D_i^T}{(1+x)^i} \qquad (20)$$

The remarkable similarity between Equations 3 and 20 indicates that if the last term (tax saving on bond interest payment) of Equation 3 is dropped, the resultant DCF rate of Equation 3 would be exactly x. This is just another special case of the DCF formulation, similar in character to the cases discussed earlier concerning home mortgage interest rate r_{HM} and rate of return before taxes r_{BT}. Thus x, the tax-adjusted cost of money, is the result of the DCF condition when tax savings due to bond interest payment is omitted as a cash flow stream in the DCF formulation. Since the last term in Equation 3 is negative, one should expect that x is smaller than or equal to r. Equations 14 and 18 show that this is indeed true.

Algebraically, it can be easily shown that Equation 3 naturally leads to Equation 4. In order to do this, one simply substitutes the value of V_i in Equation 17 into Equation 3 and carries out the necessary algebraic manipulations. The algebra is provided in Appendix I.

The present worth of a stream of revenue does not have to be an absolute number because it depends on the discount rate chosen for discounting calculations. $PWRR_r$ in Equation 3 is certainly smaller than or equal to $PWRR_x$ in Equation 20 because $x \leq r$. Therefore, Equations 3 and 20 are equivalent identity expressions but not the same. Their relationship is that the resultant DCF rate r from Equations 1 or 3 is related to x through Equations 14 and 18; vice versa, the revenue stream R_i as computed by Equation 20 also satisfies the DCF condition of Equations 1 or 3.

TREATMENT OF INFLATION

Inflation does not present any constraint in the formulation of Equations 1 and 20. The only implicit assumption is that both the cash flows and the cost of money must reflect the economic condition (inflationary or noninflationary) prevailing at the time the venture is undertaken.

Today's economy has an inherent inflationary character. When one speaks about a 9 percent rate of return on bonds or a 14 percent rate of return on equity, one has in fact assumed an inherent inflation rate of approximately 5 percent. Failure to consistently observe inflation is a frequent pitfall in energy cost analysis. For example, the practice in the late 1950s (when inflation was mild) was to construct the cash flow streams in Equation 1 such that they would stay level for the entire project life. When such constant-dollar cash flows are used, it is to be expected that the resultant DCF rate is the opportunity cost of money in a noninflationary environment. To inadvertently compare such a rate, of about 8-10 percent, to today's commonly encountered rate of 10-20 percent is to underestimate the attractiveness of the venture. Similarly, when Equation 20 is used, an input for x must be provided. A high rate (say from 8-16 percent) is the one that reflects an inherent inflation rate, and the input for I_i, O_i, Π_i must be adjusted to increase with respect to time; otherwise, $PWRR_x$ would be underestimated, a sure prelude to unprofitability. Under the present tax regulations, D_i^T is only dependent on depreciation schedules (e.g., straight line, sum-of-the-years digits, double declining, etc.). When inflation is prevailing (large x), the cash streams I_i, O_i, Π_i could be properly adjusted to reflect it, but D_i^T could not due to current tax regulations. This would slightly underestimate the R_i stream, resulting in unreal pricing of the products. The losers are the people who receive D_i for the recovery of capital, since that amount can no longer purchase new equipment necessary to replace the old. The situation is further complicated, however, because the bondholders and stockholders also receive higher returns in an inflationary environment.

PRICING POLICY:
DECREASING, INCREASING, AND LEVELIZED COST OF PRODUCT

If E_i is the total production in the year i and C_i is the product unit cost, then the relationship between C_i and R_i is

$$C_i = \frac{R_i}{E_i}$$

In the DCF formulation of Equation 1, R_i is given; therefore C_i is given when E_i is known. However, since R_i must be calculated in the RR method, a systematic way to interpret C_i appears necessary.

The exact definition of revenue requirement as expressed in Equation 13 indicates that, for a relatively constant annual production ($E_i \approx$ constant), the unit cost decreases with respect to time. This is because the book value of the project decreases, and therefore the required return on investment and its associated taxes also decrease. Obviously, a pricing policy that decreases prices with respect to time is unrealistic and socially inequitable, particularly when inflation is inherent in the economy (technological innovation and competition forces are not included here).

A simple and frequently used concept is to represent the bare-bones prices by a level value called the levelized cost. The levelized cost $\overline{C}$ is defined as that constant unit cost to be charged for the product throughout the life of the venture, such that the hypothetical revenue stream has the same present worth as the revenue stream. The only requirement is that the same discount rate be used for both present worth calculations. From Equations 1, 8, and 20, we have

$$\sum_i \frac{\overline{C}_{BT} E_i}{(1+r_{BT})^i} \equiv \sum_i \frac{R_i}{(1+r_{BT})^i} = PWRR_{BT} \qquad (33a)$$

$$\sum_i \frac{\overline{C}_r \, E_i}{(1+r)^i} \equiv \sum_i \frac{R_i}{(1+r)^i} = PWRR_r \tag{33b}$$

$$\sum_i \frac{\overline{C}_x \, E_i}{(1+x)^i} \equiv \sum_i \frac{R_i}{(1+x)^i} = PWRR_x \tag{33c}$$

Or, when E is a constant (level annual production)

$$\overline{C}_{BT} = CRF\ (r_{BT},\ M) \quad PWRR_{BT} \tag{34a}$$

$$\overline{C}_r = CRF\ (r,\ M) \quad PWRR_r \tag{34b}$$

$$\overline{C}_x = CRF\ (x,\ M) \quad PWRR_x \tag{34c}$$

It is simple to demonstrate numerically that $\overline{C}_{BT}$, $\overline{C}_r$, and $\overline{C}_x$ are practically the same. This is because the bare-bones price of product (levelized) should be a unique value whether one uses before-tax, nominal, or effective after-tax cost of money as discount rate. When inflation is implied, values in Equation 34 also imply inflation.

One should note that of the three expressions in Equation 34, only $\overline{C}_x$ is readily computable from Equation 20.

While $\overline{C}$ is convenient for use as a yardstick to compare the economic attractiveness of similar but competing alternatives, it is hard to place it in perspective with reality. When this is the purpose, a new value ${}_yC_o$ can be defined such that it represents the base unit cost at a base time i=0; the unit cost at any later time is found from ${}_yC_o$ by multiplying it with an escalation factor based on y. The only requirement of this increasing revenue stream is that its present worth must be equivalent to $PWRR_x$. Thus one has

$$\sum_i \frac{{}_yC_o(1+y)^i}{(1+x)^i} = \sum_i \frac{R_i}{(1+x)^i} \tag{35}$$

where y is the escalation rate.

Two values of y are of special interest. When y is taken as the inflation rate of the economy, then ${}_uC_o$ is the starting unit price for an increasing pricing curve that is in step with inflation.

$${}_uC_o = \text{CRF }(\gamma,\ M)\ \sum_i \frac{R_i}{(1+x)^i} = \frac{\text{CRF}(\gamma,\ M)\,\text{PWRR}_x}{E} \tag{36}$$

where $1 + \gamma \equiv (1+x)/(1+u)$.

When y is taken as x, ${}_xC_o$ is the starting unit price for an increasing pricing curve, with its escalation rate being the effective cost of money.

$${}_xC_o = \frac{1}{M}\ \sum_i \frac{R_i}{(1+x)^i} = \frac{\text{PWRR}_x}{ME} \tag{37}$$

Both Equations 36 and 37 have been applied in cost analyses of energy technologies.[6] They must be carefully interpreted when the costs of diverse technologies are compared.

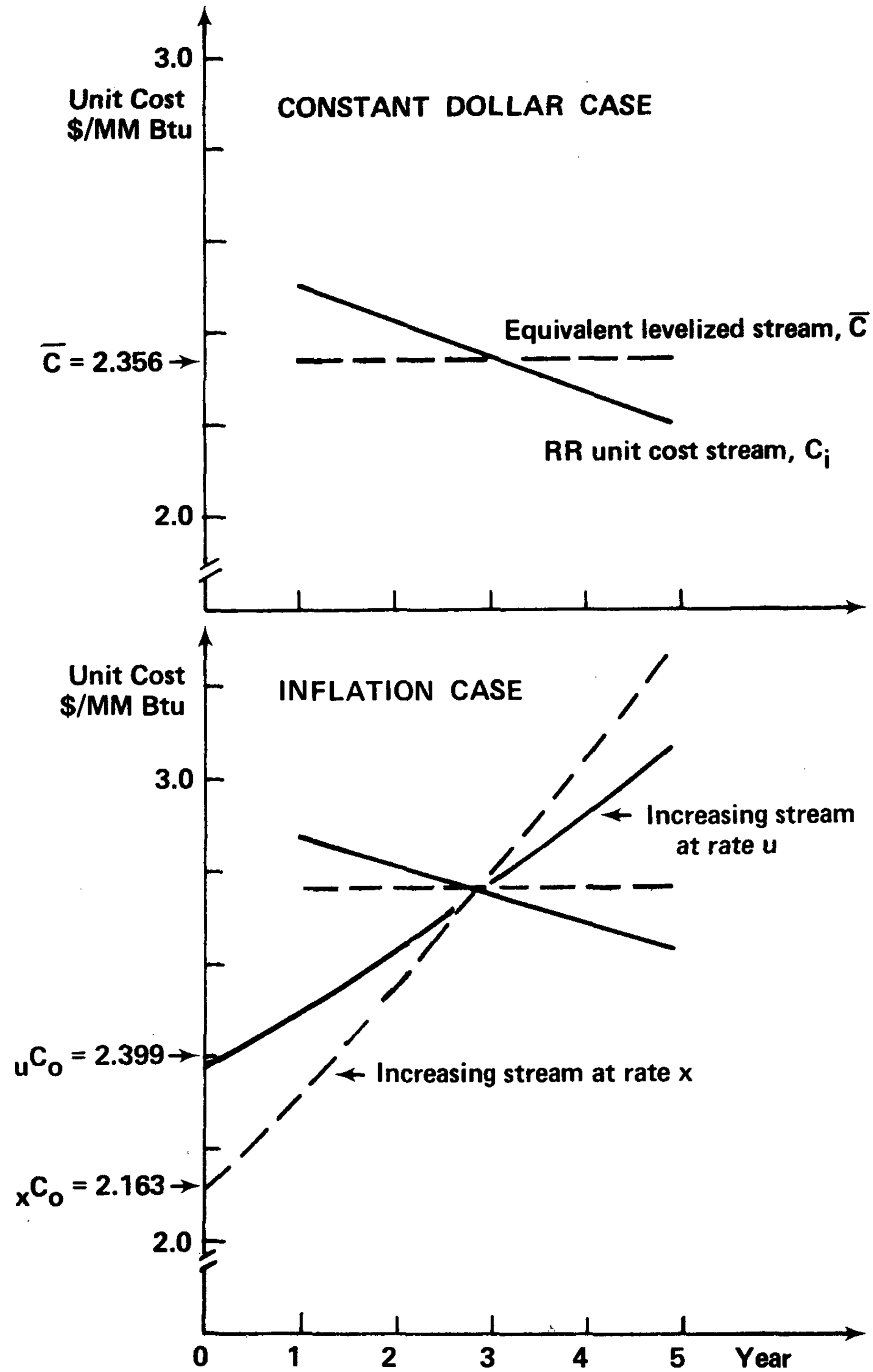
Figure 2 illustrates the relationship between C_i, $\overline{C}$, ${}_uC_o$, and ${}_xC_o$.

NUMERICAL EXAMPLE

In 1980, an energy venture needs a million dollars (1980 dollars) of investment and is expected to produce 250,000 million Btu (MMBtu)

FIGURE 2

EQUIVALENCE BETWEEN DECREASING, INCREASING, AND LEVELIZED PRICING POLICIES



of energy each year for five years. After the fifth year, it is closed down with no salvage value. This example shows how to determine the annual minimum sale price of energy from several points of view. Additional assumptions are as follows:

	In Constant 1980 Dollars	With 5 Percent Inflation
Investment Tax Credit	none	none
Effective Tax Rate	0.5	0.5
Operating & Ad-Valorem Costs	$330,000	$330,000 (1.05)^i (i=1, 5)
Debt Fraction	0.5	0.5
Debt Interest Rate	3%/year	8.15%/year
Equity Return Rate	8%/year	13.4%/year
Depreciation Schedule	straight line	straight line

Determination of RR Cash Flows

Because the revenue cash flow is not known although the cost of money is, the RR method is used to determine all cash flows. Table 2a illustrates how this is determined for the constant-dollar case, and Table 2b for the case with 5 percent inflation. Note that in both tables, the objective is to determine the T_i and R_i streams; but in order to do so, other values such as D_i, V_i, $r_b f_b V_i$, and $r_s f_s V_i$ must

TABLE 2

YEAR-BY-YEAR DETERMINATION OF CASH FLOWS BASED ON THE REVENUE REQUIREMENT CONDITION

Year	I_i	D_i	V_i	$r_b f_b V_i$	$r_s f_s V_i$	T_i	$D_i + rV_i + T_i$	$O_i + \Pi_i$	R_i	C_i
Table 2a: Constant-Dollar Case										
0	1,000,000	0	0	0	0	0	0	0	0	—
1	0	200,000	1,000,000	15,000	40,000	40,000	295,000	330,000	625,000	2.500
2	0	200,000	800,000	12,000	32,000	32,000	276,000	330,000	606,000	2.424
3	0	200,000	600,000	9,000	24,000	24,000	257,000	330,000	587,000	2.348
4	0	200,000	400,000	6,000	16,000	16,000	238,000	330,000	568,000	2.272
5	0	200,000	200,000	3,000	8,000	8,000	219,000	330,000	549,000	2.196
Table 2b: Inflation Case										
0	1,000,000	0	0	0	0	0	0	0	0	—
1	0	200,000	1,000,000	40,750	67,000	67,000	374,750	346,500	721,250	2.885
2	0	200,000	800,000	32,600	53,500	53,500	339,600	363,825	703,425	2.814
3	0	200,000	600,000	24,450	40,200	40,200	304,850	382,016	686,866	2.747
4	0	200,000	400,000	16,300	26,800	26,800	269,900	401,117	671,017	2.684
5	0	200,000	200,000	8,150	13,400	13,400	234,950	421,173	656,123	2.624

also be determined. Further note that the depreciation stream D_i is the same for both the constant-dollar case and the inflation case because existing tax regulations do not distinguish between the two.

The Appropriate Cash Flows Do Satisfy the DCF Condition

DCF before-tax discount rate (r_{BT}). The appropriate cash flow streams for the DCF condition are R_i (cash inflow) and I_i, O_i, and Π_i (cash outflows). The before-tax discount rate is defined by Equation 9 and is 9.5 percent for the constant-dollar case and 17.475 percent for the inflation case. Tables 3a and 3b show that Equation 8 is satisfied.

DCF nominal after-tax discount rate (r). The appropriate cash flow streams for the DCF condition are R_i (cash inflow) and I_i, O_i, Π_i, and T_i (cash outflows). The nominal after-tax discount rate is defined by Equation 14 and is 5.5 percent for the constant-dollar case and 10.775 for the inflation case. Tables 4a and 4b demonstrate that the DCF condition of Equation 1 is satisfied.

DCF effective after-tax discount rate (x). The appropriate cash flow streams are R_i (cash inflow), I_i (investment), O_i and Π_i (operating and ad-valorem costs), and D_i (depreciation). The appropriate discount rate is x as defined in Equation 18. Tables 5a and 5b demonstrate that identity 20 (or Equation 3 without the bond interest term) is satisfied.

TABLE 3

DEMONSTRATION THAT THE APPROPRIATE RR-DETERMINED CASH FLOWS SATISFY THE DCF CONDITION <u>BEFORE</u> <u>TAX</u>

$$r_{BT} = \frac{r_s f_s}{1-\tau} + r_b f_b$$

Year	Cash Inflow R_i	Cash Outflow Not Including Taxes I_i		Cash Outflow Not Including Taxes $O_i + \Pi_i$
Table 3a: Constant-Dollar Case, r_{BT} = 9.5 Percent				
0	0	1,000,000		0
1	625,000	0		330,000
2	606,000	0		330,000
3	587,000	0		330,000
4	568,000	0		330,000
5	549,000	0		330,000
Present Worth At r_{BT} = 9.5%	2,267,103.90 =	1,000,000	+	1,267,103.90
Table 3b: Inflation Case, r_{BT} = 17.475 Percent				
0	0	1,000,000		0
1	721,250	0		346,500
2	703,425	0		363,825
3	686,866	0		382,016
4	671,017	0		401,117
5	656,123	0		421,173
Present Worth At r_{BT} = 17.475%	2,192,947.73 ≈	1,000,000	+	1,193,092.66

(within 0.007%)

TABLE 4

DEMONSTRATION THAT THE APPROPRIATE RR-DETERMINED CASH FLOWS SATISFY THE DCF CONDITION

$$r = r_b f_b + r_s f_s$$

	Cash Inflow		Cash Outflow Including Tax				
Year	R_i		I_i		$O_i + \Pi_i$		T_i
Table 4a: Constant-Dollar Case, r = 5.5 Percent							
0	0		1,000,000		0		0
1	625,000		0		330,000		40,000
2	606,000		0		330,000		32,000
3	587,000		0		330,000		24,000
4	568,000		0		330,000		16,000
5	549,000		0		330,000		8,000
Present Worth At r = 5.5%	2,515,334.32	=	1,000,000	+	1,409,193.88	+	106,140.44
Table 4b: Inflation Case, r = 10.775 Percent							
0	0		1,000,000		0		0
1	721,250		0		346,500		67,000
2	703,425		0		363,825		53,500
3	686,866		0		382,016		40,200
4	671,017		0		401,117		26,800
5	656,123		0		421,173		13,400
Present Worth at r = 10.775%	2,568,598.11	≈	1,000,000	+	1,409,193.88	+	159,485.90

(within 0.003%)

TABLE 5

DEMONSTRATION THAT THE APPROPRIATE RR-DETERMINED CASH FLOWS SATISFY THE DCF CONDITION,

EQUATION 20, $x = (1-\tau)\, r_b f_b + r_s f_s$

Table 5a: Constant Dollar Case

Year	Revenue Stream R_i	Investment I_i	Operating & Ad-Valorem $O_i + \Pi_i$	Depreciation D_i
0	0	1,000,000	0	0
1	625,000	0	330,000	200,000
2	606,000	0	330,000	200,000
3	587,000	0	330,000	200,000
4	568,000	0	330,000	200,000
5	549,000	0	330,000	200,000
Present Worth Using x = 4.75%	2,566,742.92	1,000,000	1,438,655.10	871,912.18

Equations 3 and 20

$$2{,}566{,}742.92 = \frac{1{,}000{,}000}{0.5} + 1{,}438{,}655.10 - \frac{0.5}{0.5}\,(871{,}912.18)$$

Table 5b: Inflation Case

Year	Revenue Stream R_i	Investment I_i	Operating & Ad-Valorem $O_i + \Pi_i$	Depreciation D_i
0	0	1,000,000	0	0
1	721,250	0	346,500	200,000
2	703,425	0	363,825	200,000
3	686,866	0	382,016	200,000
4	671,017	0	401,117	200,000
5	656,123	0	421,173	200,000
Present Worth Using x = 8.7375%	2,704,031.74	1,000,000	1,487,458.67	783,257.78

Equation 20

$$2{,}704{,}031.74 = \frac{1{,}000{,}000}{0.5} + 1{,}487{,}458.67 - \frac{0.5}{0.5}\,(783{,}257.78)$$

(within 0.006%)

Selection of Discount Rate

It is thus amply clear that the cash flows as exactly determined by the RR method also satisfy the DCF condition. For each set of cash flow streams (before tax, nominal after tax, and effective after tax), one and only one discount rate is appropriate. Conversely, when a discount rate is specified, only one identity between members of a specific set of cash flows holds.

When the cash flow streams R_i, I_i, O_i, Π_i, T_i are given, the solution of Equation 1 would give the nominal after-tax cost of money r. Furthermore, depending on the nature of the cash flows, r can be interpreted as including or excluding inflation. When only R_i, I_i, O_i, and Π_i (but not T_i) are available, the solution of Equation 8 gives the before-tax cost of money, r_{BT}. Finally, when R_i, I_i, O_i, Π_i, and D_i are given, the solution of Equation 20 gives the effective after-tax cost of money x. The nature of r_{BT} and x with respect to inflation depends on the nature of the cash flows from which they are calculated.

When comparing the cost of energy technologies, the reference frame is usually a long time (e.g., 30 years) in the future. The cost of money (e.g., bonds, stocks) are commonly specified, and the costs of production are to be calculated. Investments, operating costs, ad-valorem costs, and depreciation charges are also specified, but the income taxes are usually not known except by a rate. In this case, Equation 20 is the most appropriate equation that permits the computation of the bare-bones cost of production.

Bare-Bones Cost of Production

The levelized unit cost of production $\overline{C}$ is computed based on the definition of equivalence between revenue streams. Table 6 demonstrates that no matter what discount rate is used (as long as the appropriate cash flow streams are included), the levelized cost of energy is around 2.356$/MMBtu in constant 1980 dollars or 2.764$/MMBtu in then-current dollars (between 1981 and 1985).

Base-Year Cost of Production

While the figure 2.356$/MMBtu in constant 1980 dollars is easy to understand, the figure 2.764$/MMBtu in then-current dollars can be interpreted as equivalent to 2.40$/MMBtu in 1980 and increasing each year at the inflation rate of 5 percent per year (2.52 in 1981, 2.646 in 1982, 2.778 in 1983, 2.917 in 1984, and 3.063 dollars/MMBtu in 1985). These figures are computed in accordance with Equation 36.

With the assumption that every year the bare-bones sale price can be increased at a rate of x = 8.7375 percent, then Equation 37 holds, and the base-year (1980) price is 2.163$/MMBtu.

TABLE 6

LEVELIZED BARE-BONES PRICE OF ENERGY (COST OF PRODUCTION)

Mode of Analysis	Discount Rate	Capital Recovery Factor	Present Worth of Revenues ($)	Levelized Bare-Bones Price ($/MMBtu)
Constant Dollars				
•Before tax	$r_{BT}=0.095$	0.2604364	2,267,103.90	2.362
•Nominal after tax	$r=0.055$	0.2341764	2,515,334.32	2.356
•Effective after tax	$x=0.0475$	0.2293810	2,566,742.92	2.355
Inflated Dollars				
•Before tax	${}_ur_{BT}=0.17475$	0.3159832	2,192,947.73	2.772
•Nominal after tax	${}_ur=0.10775$	0.2690406	2,568,598.11	2.764
•Effective after tax	${}_ux=0.087375$	0.2553438	2,704,031.74	2.762

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REFERENCES

1. Paul H. Jeynes (1968). Profitability and Economic Choice, The Iowa State University Press, Ames, Iowa, p. 339.

2. Paul E. DeGarmo (1969). Engineering Economy, 4th ed., 4th printing, MacMillan Company, Toronto, Ontario, Canada.

3. U.S. Bureau of Mines Information Circular 8689 (1975). "Basic Estimated Capital Investment and Operating Costs for Underground Bituminous Coal Mines," Process Evaluation Group, Morgantown, West Virginia.

4. U. S. Bureau of Mines Information Circular 8703 (1976). "Basic Estimated Capital Investment and Operating Costs for Coal Strip Mines," Revision of IC 8661, Process Evaluation Group, Morgantown, West Virginia.

5. U.S. Bureau of Mines (1976). "Economic Analysis of Westinghouse Low-BTU Gasification — Combined Cycle Power Generating System Producing 134.1 MW," ERDA 76-49, by BOM Process Evaluation Group, Morgantown, West Virginia.

6. Robert Skamser (1976). "Coal Gasification Commercial Concepts Gas Cost Guidelines," Report FE-1235-1, prepared for U.S. ERDA, C. F. Braun and Co., Alhambra, California.

7. Manson Benedict. Class notes for course "Economics of Nuclear Power," 1969-to-date Nuclear Engineering Department, Massachusetts Institute of Technology, Cambridge, Massachusetts.

8. D. R. Vondy (1965). "Basis and Certain Features of the Discount Technique." Appendix F in A Comparative Evaluation of Advanced Converters, M. W. Rosenthal et al, ORNL-3686, Oak Ridge National Laboratory, Oak Ridge, Tennessee, 1965.

APPENDIX I

Algebraic Proof that Equation 1 Leads to Equation 20

The DCF method allows the solution for the internal rate r, which is also the rate of return on capital for the specific venture opportunity under consideration. The internal rate is found by trial and error of the following equality, both sides of which are input data

$$PWRR_r = \sum_i \frac{R_i}{(1+r)^i} = \sum_i \frac{I_i}{(1+r)^i} + \sum_i \frac{O_i+\Pi_i}{(1+r)^i} + \sum_i \frac{T_i}{(1+r)^i} \quad (1)$$

$$= \frac{1}{1-\tau} \sum_i \frac{I_i}{(1+r)^i} + \sum_i \frac{O_i+\Pi_i}{(1+r)^i} - \frac{\tau}{1-\tau} \sum_i \frac{D_i}{(1+r)^i}$$

$$- \frac{\tau}{1-\tau} \sum_i \frac{r_b f_b V_i}{(1+r)^i} \quad (3)$$

The RR method allows the computation of the present worth of revenue requirements when the minimum acceptable return on capital is counted as a cost

$$PWRR_x = \sum_i \frac{R_i}{(1+x)^i} = \frac{1}{1-\tau} \sum_i \frac{I_i}{(1+x)^i} + \sum_i \frac{O_i+\Pi_i}{(1+x)^i} - \frac{\tau}{1-\tau} \sum_i \frac{D_i^T}{(1+x)^i} \quad (20)$$

where the discount rate, x, is predetermined by the capital structure (r_b, r_s, f_b, f_s) and by the effective tax rate (τ).

The first step to prove that Equation 1 or 3 will lead to Equation 20 is to write the expression for V_i from Equation 17.

$$V_i = \sum_{j=0}^{i-1} I_j(1+x)^{i-j-1} + (1-\tau) \sum_{j=0}^{i-1} (O_j+\Pi_j)(1+x)^{i-j-1}$$

$$- \tau \sum_{j=0}^{i-1} D_j^T(1+x)^{i-j-1} - (1-\tau) \sum_{j=0}^{i-1} R_j(1+x)^{i-j-1} \qquad (17)$$

in which many terms are zero for j=0.

Substituting Equation 17 into Equation 3 one obtains

$$(1-\tau) \sum_{i=0}^{M} \frac{R_i - \tau r_b f_b \sum_{j=0}^{i-1} R_j(1+x)^{i-j-1}}{(1+r)^i} = \sum_{i=0}^{M} \frac{I_i - \tau r_b f_b \sum_{j=0}^{i-1} I_j(1+x)^{i-j-1}}{(1+r)^i}$$

$$+ (1-\tau) \sum_{i=0}^{M} \frac{(O_i+\Pi_i) - \tau r_b f_b \sum_{j=0}^{i-1} (O_i+\Pi_i)(1+x)^{i-j-1}}{(1+r)^i}$$

$$- \tau \sum_{i=0}^{M} \frac{D_i^T - \tau r_b f_b \sum_{j=0}^{i-1} D_j^T(1+x)^{i-j-1}}{(1+r)^i} \qquad (31)$$

Consider the first sum on the right of 31. The coefficient of I_o is

$$1 - \frac{\tau r_b f_b}{1+r} \sum_{k=0}^{M-1} \left(\frac{1+x}{1+r}\right)^k = \left(\frac{1+x}{1+r}\right)^M$$

where $r = x + r_b f_b \tau$ has been used.

The coefficient of I_1 is

$$\frac{1}{1+r} - \frac{\tau r_b f_b}{(1+r)^2} \sum_{k=0}^{M-2} \left(\frac{1+x}{1+r}\right)^k = \frac{(1+x)^{M-1}}{(1+r)^M}$$

Continuing in this way, one has

$$\sum_{i=0}^{M} \frac{I_i - \tau r_b f_b \sum_{j=0}^{i-1} I_j (1+x)^{i-j-1}}{(1+r)^i} = \frac{1}{(1+r)^M} \sum_{i=0}^{M} I_i (1+x)^{M-i}$$

Similarly, for other terms in Equation 31, bearing in mind that $R_o=0$ and $D_o=0$, one has

$$(1-\tau) \sum_{i=0}^{M} \frac{R_i - \tau r_b f_b \sum_{j=0}^{i-1} R_j (1+x)^{i-j-1}}{(1+r)^i} = \frac{(1-\tau)}{(1+r)^M} \sum_{i=0}^{M} R_i (1+x)^{M-i}$$

$$(1-\tau) \sum_{i=0}^{M} \frac{(O_i+\Pi_i) - \tau r_b f_b \sum_{j=0}^{i-1} (O_j+\Pi_j)^{i-j-1}}{(1+r)^i} = \frac{1}{(1+r)^M} \sum_{i=0}^{M} (O_i+\Pi_i)(1+x)^{M-i}$$

$$-\tau \sum_{i=0}^{M} \frac{D_i^T - \tau r_b f_b \sum_{j=0}^{i-1} D_j^T (1+x)^{i-j-1}}{(1+r)^i} = \frac{-\tau}{(1+r)^M} \sum_{i=0}^{M} D_i (1+x)^{M-i}$$

Thus, Equation 31 becomes

$$(1-\tau) \sum_{i=0}^{M} R_i (1+x)^{M-i} = \sum_{i=0}^{M} I_i (1+x)^{M-i} + (1-\tau) \sum_{i=0}^{M} (O_i+\Pi_i)(1+x)^{M-i}$$
$$- \tau \sum_{i=0}^{M} D_i (1+x)^{M-i}$$

By dividing both sides by $(1+x)^M$, one proves that Equation 1 leads to Equation 20 consistently. Conversely, by reversing the algebraic process, one can also claim that Equation 20 leads to Equation 1 consistently.

A natural question is which one of $PWRR_r$ and $PWRR_x$ represent the "correct" present worth of revenue requirements? The answer is neither and both. "Neither" because the concept of "present worth" depends on the perception of the opportunity value of money, and hence depends on individuals and/or circumstances. "Both" because each quantity, when specified with the discount rate, can reflect well on the required revenue stream R_i.

If one is forced to choose a quantity to reflect the "present worth of revenue requirements", $PWRR_r$ is probably a good choice. This is because Equation 1 is the simplest statement of the DCF principles with cash inflow and outflow streams both realistic and well understood. In addition, the discount rate r is also realistic, something that can be obvious to the owner of the venture. However, as pointed out earlier, $PWRR_r$ cannot be obtained for a project that has not drawn to a conclusion. On the other hand, the revenue requirements of a project that will be operational in the future can be clearly specified, and $PWRR_x$ can be computed. From Equations 28 and 29, and assuming that E_i is level for all i's, then the relationship between $PWRR_r$ and $PWRR_x$ is:

$$PWRR_r = PWRR_x \frac{CRF\ (x,M)}{CRF\ (r,M)} \qquad (32)$$

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P.O. Box 117
Oak Ridge, TN 37830