

LA-UR- 97-4072

CONF-980128-~

*Title:*

CHARACTERIZING RELIABILITY IN A PRODUCT/  
PROCESS DESIGN-ASSURANCE PROGRAM

RECEIVED  
FFR 02 1998  
OSTI

*Author(s):*

William J. Kerscher, Delphi-E/GM  
Jane M. Booker, TSA-1  
Thomas R. Bement, TSA-1  
Mary A. Meyer, TSA-1

*Submitted to:*

International Symposium on Product Quality &  
Integrity  
January 19-22, 1998  
Anaheim, CA

19980402 050



**Los Alamos**  
NATIONAL LABORATORY

Los Alamos National Laboratory, an affirmative action/equal opportunity employer, is operated by the University of California for the U.S. Department of Energy under contract W-7405-ENG-36. By acceptance of this article, the publisher recognizes that the U.S. Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or to allow others to do so, for U.S. Government purposes. The Los Alamos National Laboratory requests that the publisher identify this article as work performed under the auspices of the U.S. Department of Energy.

Form No. 839-R  
ST 2629-10-91

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

*dy*

MASTER

## **DISCLAIMER**

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

# Characterizing Reliability in a Product / Process Design-Assurance Program

William J. Kerscher III • Delphi Energy & Engine Management Systems • Flint

Jane M. Booker • Los Alamos National Laboratory • Los Alamos

Thomas R. Bement • Los Alamos National Laboratory • Los Alamos

Mary A. Meyer • Los Alamos National Laboratory • Los Alamos

**Key Words:** Reliability prediction, Bayes analysis, Expert judgment, Reliability growth management, Reliability growth model, Reliability demonstration, Reliability growth testing, Design assurance, Product assurance

## *SUMMARY & CONCLUSIONS*

Just as estimates of cost and program timing are critical factors to be known and monitored during a new product development program, so too is the reliability perspective. The reliability estimate and the uncertainty of that estimate are an excellent way to provide this perspective. Moreover, it is possible to develop realistic reliability estimates at the beginning of a new product program even though hardware is not available, because a considerable amount of knowledge exists in the experience base of engineers, etc. This knowledge is elicited in the form of expert judgment. Further, during the course of the development program much information will become available at different levels of the system (e.g., component, subsystem, system), from different sources (e.g., customer, supplier), and regarding different points in product life (e.g., test time). This information will also become available at different calendar times, and it may range from completely quantitative (e.g., test data) to totally qualitative (e.g., expert judgment) information. Fortunately, it is possible to provide order to all of this diverse information so that it may be consolidated as it occurs. The results may then be used to not only provide a reliability perspective of the program at any point in time, but also to provide steerage to the development team with regard to how to drive reliability higher and / or reduce the uncertainty in reliability. The challenge has been to develop a framework for this perspective which is physically and mathematically sound, but which is flexible enough to accommodate all of the diverse information that becomes available, and responsive enough to provide timely results which support the development process. The information updating approach which is rooted in Bayesian statistics is suggested as a key methodology which is directly applicable to this problem. This paper describes an approach to reliability modeling that encompasses the impact of both product and manufacturing process design on the distribution (characterizing the uncertainty) of reliability over time. It further describes the elicitation of expert judgment which is used to quantify the initial reliability estimate, including

uncertainty. Finally, it describes a Bayesian updating approach which is applicable throughout the development program, and which accommodates a wide variety of possible new information. Although the model is rigorous in its execution, a user friendly approximation is also described which may be useful to the product development team for purposes of test and validation planning.

## *1. INTRODUCTION*

Over the years many advancing techniques in the area of reliability engineering have surfaced in the military sphere of influence, and one of these techniques is Reliability Growth Testing (RGT). Private industry has reviewed RGT as part of the solution to their reliability concerns, but many practical considerations have slowed its implementation. It's objective is to demonstrate the reliability requirement of a new product with a specified confidence. This paper speaks directly to that objective but discusses a somewhat different approach to achieving it. Rather than conducting testing as a continuum and developing statistical confidence bands around the results, this Bayesian updating approach starts with a reliability estimate characterized by large uncertainty and then proceeds to reduce the uncertainty by folding in fresh information in a Bayesian framework.

In the traditional military context a product would be developed (or an existing product modified), and then the product would be put on test. The typical long-term test was designed to statistically demonstrate a reliability requirement at a specified confidence. This product was then delivered to the military services with demonstrated reliability as part of the deliverables package. The fact that the test involved additional time, cost and resources was deemed to be acceptable. In the industrial setting, however, these drawbacks can become acute, and in many cases deter the use of this traditional approach. Also, although not planned, it is possible for the end of a development program to approach the scheduled start of volume production. Reliability growth testing at this point is seen not only as an additional amount of

time in the development program, but also as a holding item before production may begin.

Probably the most significant negative factor, however, regards the organizational environment that design engineers are asked to work within. Not atypically, the reliability growth test may be the first large scale organized development test to be conducted on the new product design. The results typically identify several weak spots / failures in the design, which should be expected. The reliability growth test, however, has been organized to demonstrate the desired reliability, and do it efficiently, by organizing the test around an anticipated few or no failures. The result is a triple blow to the design program. First, it demonstrates that the desired reliability has not been achieved. Second, it demonstrates it with statistical confidence, and finally, it may produce this result near to the scheduled start of volume production, which dictates the choice of shipping defective product or delaying the start of production. Perceptive program managers who recognize the deficiency of their product in the area of reliability naturally tend to resist demonstrating the fact without sufficient time to respond. All of these factors tend to work against the implementation of traditional reliability growth testing in an industrial setting.

There is a definite need, however, for an understanding of the reliability perspective of a new product during its development program. Identifying the uncertainty in the reliability estimates, early enough in the development cycle for corrective action to be organized by the development team, can be a powerful factor in the drive for high reliability. The Bayesian updating approach is suggested as a methodology which is directly applicable to this problem.

The following notations are used :

$R_i$	reliability characterization of a system , estimated at time step, i.
$f(R_i)$	probability distribution function of $R_i$ , representing the uncertainty in system reliability.
$\lambda$	failure rate for a component, subsystem or system (e.g., failures per vehicle per scaled unit of time) and scale parameter of the Weibull distribution.
$\beta$	slope or shape parameter of the Weibull distribution.
$R(t)$	reliability from a two-parameter Weibull distribution.
$\Gamma(n)$	gamma function, which is the $\int x^{(n-1)} e^{-x} dx$ from 0 to 1.
$\theta$	parameter of interest.

## 2. OVERVIEW OF RELIABILITY UPDATING METHODOLOGY

The reliability of the product (including the manufacturing process) at any given point in time or at any given step in the overall product / process design assurance program is hereby referred to by the term reliability characterization. "Reliability characterization" refers to both

the functional calculation of the reliability (point estimate value) and the uncertainty (usually represented by a distribution function) that accompanies that reliability value. Reliability values can be calculated from formulas or models, such as a reliability block diagram.

Either the reliability calculation and/or its uncertainty distribution can change due to any change occurring during the product / process design assurance program (Ref. 1). Such changes could include the availability of new information or new data from tests, vendors or corrective actions. New terms could be added to the model such as a new component in the system or a new failure mode. Changes can occur in the design phase and / or the manufacturing process which can affect the reliability value and / or its associated uncertainty.

Once a change occurs anywhere in the process, a new step (i) occurs and a new reliability,  $R_i$ , is calculated along with a new uncertainty distribution,  $f(R_i)$ . The tracking of  $R_i$  and  $f(R_i)$  over time is one method of monitoring how the changes in reliability are approaching the target value, as part of the validation effort.

Calculating  $R_i$  and  $f(R_i)$  requires combining information from various sources, e.g., test data, product data, engineering judgment, specifications / requirements, data from similar components, and data at different levels (component, system, etc.). This information may also be generated at a supplier or customer as well as the parent company. Re-evaluating  $R_i$  and  $f(R_i)$  in light of new information requires methods for incorporating new information with existing information, e.g., adding new test data or accommodating design changes.

With each evaluation of  $R_i$  and  $f(R_i)$ , gaps in the current state of knowledge become apparent, providing the basis for a strategy for deciding where to devote future testing and analysis resources. If a gap from poor information results in large uncertainty in one important area (e.g., unknown performance of a component), then time / effort should be devoted to understanding why and where additional information can be useful in improving the reliability and / or reducing the uncertainty. Understanding the effects of changes in the design, in uncertainties, and test results can come from re-calculating the reliability characterization. Also, by providing an approximation of this information in a user friendly form, that may be easily understood and manipulated by the product / process design community, a powerful tool in developing optimized test and validation plans may be organized. Users can experiment with anticipated or theoretical design, reliability, and uncertainty changes. For example, they may ask "what happens to  $R_i$  and  $f(R_i)$  if component A is replaced by component B, or if 10 successful new tests are performed on subsystem D ?" This methodology was evaluated on a pilot program in the automotive industry, hereafter referred to as the "pilot", the results of which are the subject of this paper.

### 3. FRAMEWORK

One of the first activities of an organized reliability program is the construction of a reliability logic flow diagram (e.g. reliability block diagram, success tree) of the product under development. The framework of the reliability characterization involves selecting a mathematical model of that logic flow diagram, making an initial estimate of the parameters identified in the model, and organizing a methodology for updating the model as new information becomes available. Section 4 describes the Weibull functions selected to model the product reliability, Section 5 describes the elicitation of expert judgment which is used to develop the initial (or prior information-based) estimate, and Section 6 describes the Bayesian methodology utilized to update the model.

### 4. DESCRIPTION OF WEIBULL MODELS

The concept of the hazard function of a manufactured product being made up of definable portions such as infant mortality, useful life, and wearout, has long been postulated (Ref. 2). It is further suggested here that the "infant mortality" is mainly due to the latent defect sub-population generated during the manufacturing process, and the "useful life" portion is primarily due to latent design defects which manifest themselves over the life of the product. "Wearout" is the third sub-population of parts which fail due to failure modes associated with operating the product beyond its useful life. Good engineering practice has long held that wearout failure modes should be identified during the development process, and that those failure modes that cannot be designed out should at least be designed to occur beyond the useful life of the product. For the purposes of this paper any wearout failure modes are assumed to occur beyond useful life, and are not, therefore discussed here. The approach to identifying and addressing the latent defects in the first two sub-populations is not as well established, although that is in fact the objective of a comprehensive design assurance program (Ref. 3). A first helpful step in identifying those latent defects is the establishment of a reliability model. Fig. 1 shows a portion of the reliability logic flow diagram used in the pilot program. The section shown is in the form of a success tree diagram.

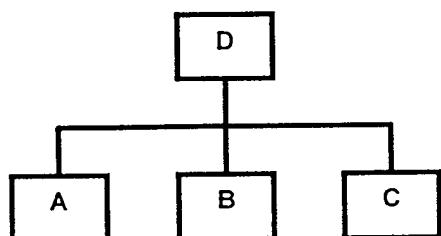


Figure 1. Reliability Logic Flow Diagram

A reliability model of the logic flow diagram of a new product design must be physically appropriate and mathematically correct in order to make its application useful during the development program. Of equal importance, the model and its usage must be culturally acceptable to the organization using it. Neither of these requirements is a small challenge. With regard to the first requirement, it is suggested here that the Weibull distribution may be well suited to the task. Specifically, the two parameter Weibull may be used to model both the defect sub-population due to the manufacturing process, as well as the defect sub-population due to the product design (Ref. 2). The total distribution is the combination of the two sub-populations (or three sub-populations in the presence of wearout phenomena). With regard to the second requirement, this model of the reliability perspective well suits the implicit understanding of the design and manufacturing community, given their awareness of the "bathtub curve", and therefore may be culturally acceptable.

The two-parameter Weibull expression for reliability is given in eq (1).

$$R(t) = \exp(-\lambda (t)^\beta) \quad (1)$$

This version of the Weibull separates the two parameters and often simplifies the algebra and the subsequent Bayesian manipulations (Ref. 4). The challenge is to identify the two parameters;  $\beta$  (the slope) and  $\lambda$  (the failure rate per scaled unit of time). First, it is suggested that an initial estimate of the slope parameter be determined from the previous performance of similar products. This may be a much more reasoned estimate than might be first thought. If a manufacturing organization is in the business of producing a particular "family" of products, then the reliability performance of all of the products produced may be similar. This situation is typical of some companies. Consequently, a reasonable estimate of the slope parameter of the new product may be identified based on the established performance of previous members of the same "family" of products. The slope parameter,  $\beta$ , of the product used in the pilot was estimated to be 0.75 from prior history of the product "family". Second, an initial estimate of the failure rate of the distribution may be made from whatever information is available relative to previous designs. Typically, at the outset of a new product development program actual test data is very limited due to the absence of hardware. Considerable information about the potential reliability is available, however, in the form of expert judgment possessed by members of the product development team. For instance, it may be possible to estimate the reliability of the product at a particular time of the product's life. Expert judgment (Sec. 5) was used in this way to estimate these various failure rates,  $\lambda$ , and their uncertainty, for the pilot. Table 1 lists the two parameters,  $\beta$  and  $\lambda$ , of the components in the example: A, B, and C. No information was elicited for the subsystem D. This defines the sub-population due to latent design defects.

A somewhat similar approach may be taken with respect to the latent defect sub-population generated by the manufacturing process. Again, a two-parameter Weibull distribution may be used as a model. The physical situation here, however, is different enough to argue for a different approach to estimating the parameters. When mistakes are made in the manufacturing process, they certainly can be in matters of degree (e.g., solder bath temperature). More typical, however, are mistakes that tend to be very significant, such as putting parts together upside down or leaving parts out of an assembly. These latent manufacturing defects tend to manifest themselves relatively quickly, if not immediately. In this approach, if the fraction of the sub-population that would fail almost immediately, and the maximum life of the sub-population could be estimated, then sufficient information would exist to estimate the two parameters directly. Simply put, this would reduce the determination of the two Weibull parameters for the manufacturing process to understanding the reliability of the sub-population at two different points in time, specifically, near time zero, and at the time at which all of the elements of the sub-population have failed. (Indeed, any two estimated points may be used for this determination). Finally, an estimate may be made, with uncertainty, regarding the fraction that this manufacturing defect sub-population is to the total population (very small in the example). This fraction, together with a reliability based on the Weibull parameters just discussed, is used to determine the impact on field reliability relative to manufacturing caused failures. In the pilot, expert judgment (Sec. 5) was used in this way to make these estimates, and Table 1 lists the two parameters,  $\beta$  and  $\lambda$ , of the components in the example: A, B, and C. Again, no information was elicited for the subsystem D, whose reliability is defined by the logic flow diagram (Fig. 1) and the reliabilities of components A, B, and C. This defines the sub-population due to latent manufacturing defects.

This approach to estimating the reliability performance of manufactured products may fit well within the culture of many manufacturing organizations. Because of the nature of the business and the manufacturing operations, knowledgeable individuals may be able to make reasoned estimates as to both the total amount of defective product that will surface due to these latent manufacturing defects, and

also the proportion of those that will fail almost immediately (e.g. prior to the next manufacturing operation). Many individuals may be comfortable with this representation because a vertical assembly operation compartmentalizes several of its manufacturing steps (including different manufacturing locations), and typically maintains data on this sort of information.

Once the individual distributions for the latent design and manufacturing defects have been identified, they may be combined to produce the distribution representative of the whole component or subsystem. All of the individual distributions of the individual elements may then be combined according to the reliability logic flow diagram to form the distribution representative of the entire product. This is the approach taken on the pilot program being discussed here. (Again, if a wearout distribution were also being considered it would also be combined at this point to produce the total distribution representative of the product). Estimates of reliability (including uncertainty) can then be calculated using eq (1) at various points in time for predicting the long term performance.

##### 5. ELICITATION OF EXPERT JUDGMENT

To obtain an initial overall reliability estimate,  $R_0$ , of the entire logic flow diagram, estimates of component and subsystem reliability's (with uncertainties) were elicited from teams of subject matter experts. The experts had been previously selected by their managers and peers as being knowledgeable of their subsystem or component. The elicitations were first conducted on those working on the product design and then on those working on the manufacturing process.

The experts were not asked to estimate reliabilities, per se, but allowed to provide their estimates about component, subsystem and system performance in terms familiar to them. (This approach and its benefits are described in further detail in Ref. 5). For example, the experts in the design process gave their estimates as incidents per thousand vehicles (IPTV), while those familiar with the manufacturing process gave their estimates as parts per million (PPM). As

Table 1. Weibull Parameters for Design and Manufacturing Models  
and  
Initial Reliability Estimates at 12 Months and 100,000 Miles

	Parameters				Reliability $R_0$							
	Design		Manufacturing		12 Month		100,000 Miles					
	$\beta$	$\lambda$	$\beta$	$\lambda$	5	50	95	5	50	95	Percentiles	
Component A	0.75	0.00001	0.14	5.17		0.9996	0.9999	1	0.9993	0.9999	1	
Component B	0.75	0.00002	0.43	9.94		0.9989	1	1	0.9986	0.9999	1	
Component C	0.75	0.001	0.42	4.18		0.976	0.9989	0.9999	0.8829	0.9952	0.9997	
Subsystem D	~	~	~	~		0.9723	0.9985	0.9998	0.8794	0.9944	0.9994	

part of their estimates, the experts were asked to give a very brief explanation of their reasoning. In addition, the experts provided ranges on their estimates, which were used to represent the uncertainty and ultimately formulate  $f(R_i)$ .

The results from the design elicitations were presented to all of the participating experts for their review and reconciliation across the entire system. This information was then used to calculate the  $\beta$  and  $\lambda$  parameters for design and manufacturing as given in Section 4. The uncertainty expressed in the expert elicitations was transformed into distributional information in the mathematical model. Reliabilities were then calculated using eq (1), with subsystem and system estimates being calculated using the reliability logic flow diagram and numerical sampling techniques. The results included reliabilities in distributional form (reflecting the uncertainty) for components, subsystems and the system at various times. The results for the initial reliability  $R_0$  at 12 months and 100,000 miles are summarized in Table 1. For instance, the median reliability of subsystem D at 12 months was estimated to be 0.9985, with the 5 th and 95 th percentile reliability estimated at .9723 and .9998 respectively.

Subsequent information, including new test data, is reflected in subsequent values of  $R_i$  and  $f(R_i)$  as described in Section 6. In this way reliability may be monitored over time (reliability growth), and plans formulated accordingly.

#### 6. DESCRIPTION OF UPDATING METHODOLOGY

As discussed previously, the system under study needs to be defined and characterized with a logical structure, such as a reliability block diagram or success tree. As part of the diagram, how the blocks interact / connect is specified as are any levels within the blocks (e.g., component, subsystem and system). These interrelations of the blocks will determine how the reliability is to be calculated at various levels. For instance, if the components within a block (A, B, and C in the example in Fig. 1) are all in series, the block (subsystem) reliability is the product of the reliabilities of the components.

It should be noted that information about failure modes of various blocks, and their apportionment, can also be

elicited during the initial characterization. This may become important later when tests are planned or performed on a subset of failure modes.

Pooling data from different sources or of different types (e.g. tests, process capability studies, engineering judgment) is usually done with methods that combine the distribution functions associated with the various information sources. Bayes Theorem offers one mechanism for such combination. Bayesian pooling combines information with the following structure: the existing information (data) forms a distribution, called the likelihood. That likelihood distribution is formed from the data / information symbolized by the random variable,  $x$ , and it has characteristics (i.e. parameters), such as a mean. That parameter(s) is not considered a fixed quantity but instead, has its own probability distribution, called the prior. The prior is combined with the likelihood using Bayes Theorem to form the resulting or posterior distribution. Bayes Theorem is used to calculate the posterior distribution,  $g(\theta|x)$ , from the likelihood distribution,  $f(x|\theta)$  as:

$$g(\theta|x) = [f(x|\theta) g(\theta)] / \int f(x|\theta) g(\theta) d\theta \quad (2)$$

where  $g(\theta)$  is the prior distribution on the parameter of interest,  $\theta$ . Bayesian combination is often referred to as an updating process, where new information is combined with existing information.

Simulation methods are often used to combine or propagate uncertainties (represented as distribution functions) through the logic flow diagram, as well as accomplishing the Bayesian combination itself. This is the approach taken with this pilot project. The range and nominal estimates provided through the expert elicitation are used to form empirical distribution functions for reliability (initial reliability characterization) for each item in the logic flow diagram. Monte Carlo simulation is used to propagate reliability characterizations through the various levels of the diagram, with the accuracy being dependent on the number of simulations. The posterior distributions resulting from the simulation are empirical in form, meaning they are not fit to any particular distribution (e.g., a beta) or distribution family. It is not necessary to develop prior information for

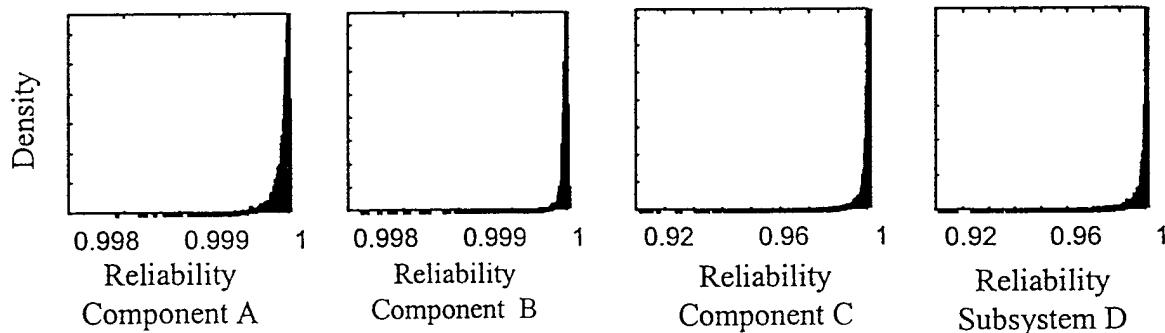


Figure 2. Reliability Prior Distributions @ 12 Months

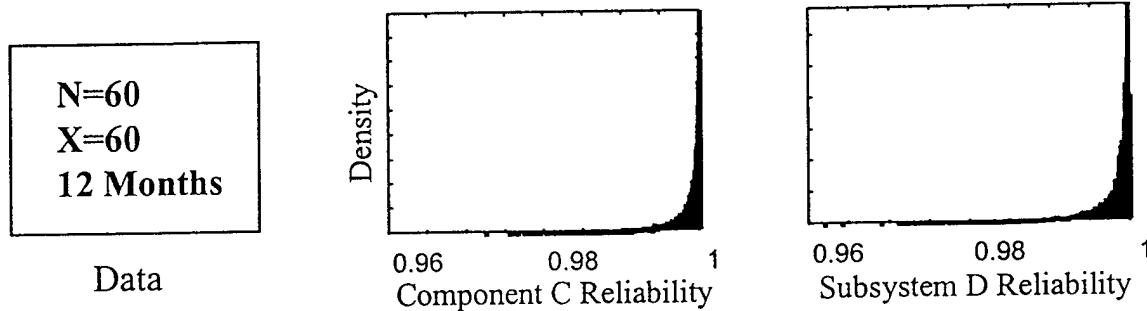


Figure 3. Reliability Posterior Distributions @ 12 Months

subsystems above the component level. These are available by combining the reliability characterizations from the levels below. However, if there is information on these subsystems, the reliability characterization from that information can be combined with the distribution from levels below using methods in Refs. 6, 7, and 8. More importantly, test data and other new information can also be added to the existing reliability characterization at any level and / or block (e.g. system, subsystem, component). This data may be applicable to the entire block, or only to a single failure mode within the block. This process is presented in detail in Ref. 8 for series systems and in Ref. 7 for series / parallel systems.

In general, the initial reliability characterization  $R_0$ , is developed from expert judgment and is referred to as the native prior distribution. During the course of the development program data may be developed regarding each element (e.g. system, subsystem, component) and this would be used to form data distributions (sometimes called likelihood distributions). All of the distribution information in the items at the various levels must be combined up through the logic flow diagram, to produce a final estimate of the reliability and its uncertainty at the top, or system, level. Three different combination methods are used:

- For each prior distribution that needs combining with a data distribution, Bayes Theorem is used and a posterior distribution results.
- Posterior distributions within a given level are combined according to the logic of the logic flow diagram to form the induced prior distribution of the next higher level.

- Induced prior and native prior distributions at the higher levels are combined within the same item using a method in Ref. 8 to form the combined prior (for that item) which is then merged with the data (for that item) via method 1. This approach is continued up the diagram until a posterior distribution is developed at the system level.

As more data becomes available and incorporated into the reliability characterization through the Bayesian updating process, this data will tend to dominate over the effects of the initial estimate developed through expert judgment. In other words,  $R_i$  formulated from many test results will look less and less like  $R_0$  from expert estimates.

A single update from our pilot example will be helpful to illustrate. Fig. 2 shows the probability distributions of reliability at 12 months for the components and subsystem in the example at a certain point during the development program. Note that there is considerable uncertainty around component C which is reflected in subsystem D (note also the difference in x-axis scales). In our pilot example, 60 samples of component C were tested for 12 months with no observed failures, and this was treated as an update event. Fig. 3 shows this data and the resulting posterior distribution of component C after the Bayesian update. Note how the additional data works to reduce the uncertainty around the estimate. Fig. 3 also shows how this additional testing is reflected as reduced uncertainty at the subsystem level D. A numerical summary of the Bayesian update is shown in Table 2.

Table 2. Prior and Posterior Reliability Distributions (Testing of Component C)

Percentiles	Prior R0						Posterior R1					
	12 Month			100,000 Miles			12 Month			100,000 Miles		
	5	50	95	5	50	95	5	50	95	5	50	95
Component A	0.9996	0.9999	1	0.9993	0.9999	1				Same		
Component B	0.9989	1	1	0.9986	0.9999	1				Same		
Component C	0.976	0.9989	0.9999	0.8829	0.9952	0.9997	0.9908	0.9992	0.9999	0.9599	0.9964	0.9997
Subsystem D	0.9723	0.9985	0.9998	0.8794	0.9944	0.9994	0.9887	0.9989	0.9998	0.957	0.9957	0.9994

This methodology was used throughout the pilot activity to provide estimates of reliability with uncertainty for all components, subsystems, and the system at various operating times. The median system reliability and lower 90 % confidence limit were also plotted against calendar time (as update events occurred) to track progress and demonstrate reliability growth as shown in Fig. 4. The individual data points correspond to the initial reliability characterization  $R_0$

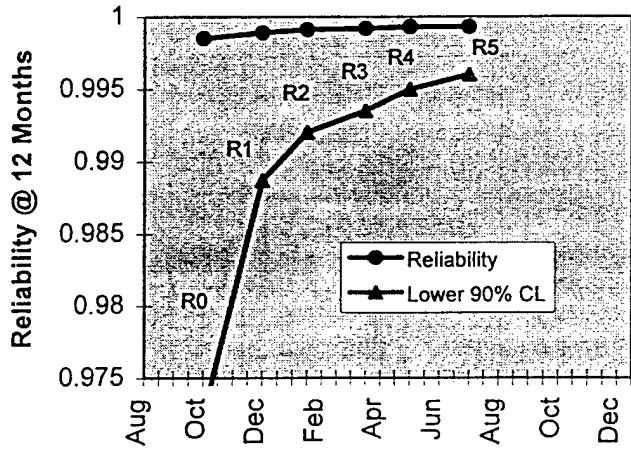


Figure 4. Reliability Growth Diagram

and the events associated with the updates  $R_i$ . This plot captures the results of the design teams' early efforts to improve reliability, but the power of the approach is the roadmap developed which may be used by the team to organize their planning to achieve higher reliability.

#### 7. A USEFUL APPROXIMATION

While the methodology described in Section 6 does not require  $f(R_i)$  to conform to any particular distributional form or family, a useful approximation which sometimes may be helpful for planning purposes can be organized around the beta and binomial distributions, eq (3) and eq (4) respectively.

$$\text{Beta } (a, b) = \{ \Gamma(a+b) / [ \Gamma(a) \Gamma(b) ] \} \times x^{a-1} (1-x)^{b-1} \quad (3)$$

$$\text{Binomial } (n, p) = \frac{n!}{x!(n-p)!} p^x (1-p)^{n-x} \quad (4)$$

The beta distribution is the conjugate prior distribution for the

binomial parameter,  $p$ , (Ref. 7) and can in some cases be used to approximate the empirical distribution (resulting from the simulation) of the  $R_i$ . The beta is often well-suited for representing possible values for  $p$  because it ranges between 0 and 1, and in addition, it is an extremely flexible distribution with many possible shapes (e.g., symmetric, asymmetric, unimodal, uniform, u-shaped, or J-shaped). Its usefulness derives from the fact that the two parameters of the beta in eq (3),  $a$  and  $b$ , are sometimes referred to as the *pseudo successes* and *pseudo failures*, respectively. This calls to mind the image of a *pseudo test*, where  $a + b$  equals the number of pseudo tests.

A useful planning application involves situations where new test data is, or will be, of the form of  $x$  number of successes out of  $n$  number of trials. Such data is binomially (eq (4)) distributed. In a Bayesian reliability formulation, if a beta distribution with parameters  $a$  and  $b$  is considered to be the prior distribution for  $R_0$ , then the posterior distribution for  $R_1$  will also be a beta, with parameters  $a + x$  and  $b + n - x$ . Thus, using the beta formulation may be useful in characterizing the possible value of additional tests. Because the posterior distribution and the prior distribution are both of the beta family, this process could be iterated indefinitely.

For example, the beta distribution shown in Fig. 5 was fit to the prior reliability distribution for component C in Fig. 2 (design portion only). In this case, a beta approximation yielded,  $a = 28.2$  pseudo successes and  $b = 0.22$  pseudo failures (a pseudo test of about 28 samples). New information, in the form of a 12 month test of 60 of these components resulting in zero failures was introduced, and a new predicted posterior beta reliability distribution was determined, also shown in Fig. 5, using the methodology described above. Note that the beta parameters of this predicted posterior distribution are  $a = 88.2$  and  $b = 0.22$ . This is obviously quite similar to the corresponding fitted posterior reliability distribution calculated empirically for component C and also shown in Fig. 5. It is also possible to streamline the calculations of the posterior distribution of subsystem D by using this beta estimate. The power of this approximation, however, lies in simply noting the potential impact of this test (visually or through the beta parameters) and allowing the engineering community to judge the usefulness of this test before it is run.

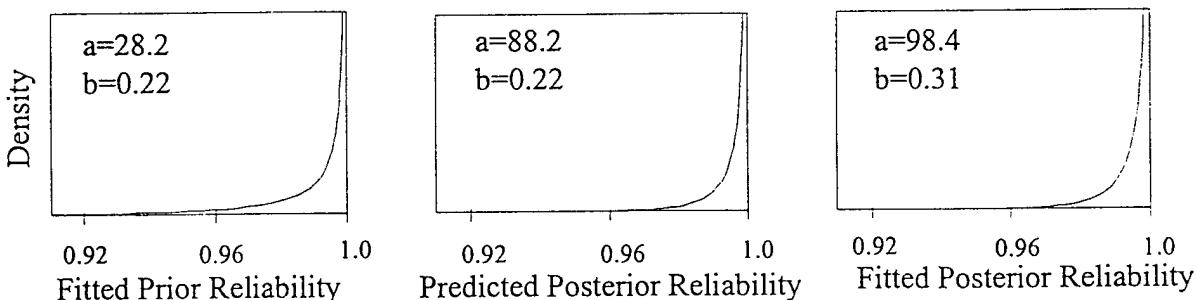


Figure 5. Component C Beta Distributions (Design Failure)

It should be noted that often a beta distribution cannot do an acceptable job of approximating a distribution of  $R_i$  and remain consistent with the intentions of the experts who provided the input. This is especially true when experts state a very high level of reliability with a small but significant possibility that the reliability may be quite low. For example, suppose the most likely reliability is estimated to be 0.997, the best reliability probably does not exceed 0.9995, but because of some uncertainty (e.g., an untested scenario), there is a small possibility that it could be as low as 0.70. Also, it often becomes more difficult and inaccurate to attempt to fit the beta to distributions formed from composite information at higher levels of the model and / or after several updates have introduced significant unrelated data. When the beta does not provide an acceptable approximation, one must rely on using parameters such as the mean and percentiles to characterize the reliability uncertainty distribution as discussed in Section 6.

These examples illustrate cases where new test information or data are introduced to update a reliability,  $R_i$ , to the form  $R_{i+1}$ . The continuous monitoring of  $R_i$  and  $f(R_i)$  is possible as new information or changes become available. Not all changes may be beneficial, as reliability can decrease and / or the uncertainty increase at any given change step,  $i$ . However, by judiciously planning new tests or changes for the purposes of reducing uncertainty and / or improving reliability, the overall trend will indicate such desired results. This overall methodology may prove useful in characterizing the reliability of a new product in its concept stage, updating and reporting on that reliability during the development stage, and planning appropriate future activities which, when accomplished, will achieve a high reliability product.

#### REFERENCES

1. F.L. Hulting, J.A. Robinson, "The reliability of a series system of repairable subsystems: a Bayesian approach", *Naval Research Logistics*, vol 41, 1994, pp 483-506.
2. W.J. Kerscher III, "A reliability model for total field incidents", *Proc. Ann. Reliability & Maintainability Symp.*, 1989 Jan, pp 22-28.
3. W.J. Kerscher III, "System Assurance at AC Rochester", *Proc. Ann. Reliability & Maintainability Symp.*, 1993 Jan, pp 71-76.
4. H.F. Martz, R.A. Waller, *Bayesian Reliability Analysis*, 1982; John Wiley and Sons, Inc.: New York.
5. M.A. Meyer, J.M. Booker, *Eliciting and Analyzing Expert Judgment: A Practical Guide*, 1991; Academic Press: London.
6. H.F. Martz, R.G. Almond, "Using higher-level failure data in fault tree quantification", *Reliability Engineering and System Safety*, 1997, pp 29-42.
7. H.F. Martz, R.A. Waller, "Bayesian reliability analysis of complex series/parallel systems of binomial subsystems and components", *Technometrics*, vol 32, 1990, pp 407-416.
8. H.F. Martz, R.A. Waller, E.T. Fickas, "Bayesian reliability analysis of series systems of binomial subsystems and components", *Technometrics*, vol 30, 1988, pp 143-154.

#### BIOGRAPHIES

Thomas R. Bement  
Los Alamos National Laboratory  
MS F600, Statistics Group (TSA-1)  
Los Alamos, NM 87545 USA

Dr. Bement joined the Statistics Group at Los Alamos in 1975. He received his B.S. degree in mathematics and M.S. degree in statistics from Colorado State University, and his PhD in Statistics from Kansas State University. He has been involved in the nuclear weapons program since joining Los Alamos. During the last five years, his particular area of application has been reliability characterization involving extensive use of expert judgment. He has published in journals such as *Technometrics*, the *Journal of the American Statistical Association*, and the *Journal of Quality Technology*. He was a co-author of the paper receiving the 1992 American Society for Quality Control Brumbaugh Award. He is a member of the American Statistical Association and the American Society for Quality Control.

Jane M. Booker  
Los Alamos National Laboratory  
MS F600, Statistics Group (TSA-1)  
Los Alamos, NM 87545 USA

Dr. Booker joined the Statistics Group at Los Alamos National Laboratory in 1980 and is currently the Group Leader. She received her Bachelor of Science degree in Meteorology / Engineering, her Master of Statistics and Ph.D. in Statistics, all from Texas A&M University. She has 2 years of university teaching experience, 10 years of technical management experience, and over 20 years of consulting experience in scientific and engineering application areas. Her recent research and development efforts have been in the areas of statistical reliability, data pooling methods, and analysis of expert opinion. During her career, she has published in various journals and co-authored a book with Mary Meyer, *Eliciting and Analyzing Expert Judgment: A Practical Guide*, published by Academic Press, London, 1991. She is a member of the American Statistical Association and recipient of the 1995 H.O. Hartley award for service to the Statistics profession.

William J. Kerscher  
Delphi Energy and Engine Management Systems  
4800 S. Saginaw St.  
Flint, Michigan 48501-1360 USA

Mr. Kerscher is responsible for reliability and product assurance activities at Delphi Energy & Engine Management Systems in Flint. He received his Bachelor of Science degree in Electrical Engineering from General Motors Institute and his Master of Science from the Massachusetts Institute of Technology. He is a Certified Reliability Engineer, holds three patents, has been elected a member of Tau Beta Pi, and is currently working on his second masters degree in Reliability Engineering. He has delivered papers on assurance / reliability at the Reliability and Maintainability Symposium, Institute of Environmental Sciences, Product Engineering Technology Conference, Pennsylvania State University Quality Assurance Seminar Series, and the SAE International Congress and Exposition. He has also served as a panel member in these various forums. A member of ASQC, SAE and SRE, Bill has chaired several committees and served as Treasurer, Vice-Chairman, and Chairman of the Mid-Michigan Section of the Society of Automotive Engineers.

Mary A. Meyer  
Los Alamos National Laboratory  
MS F600, Statistics Group (TSA-1)  
Los Alamos, NM 87545 USA

Mary Meyer is a cognitive scientist at Los Alamos National Laboratory. She received her Ph.D. in Ethnology from the University of New Mexico in 1985 for her research into scientists' work-related thinking. During her fifteen years at Los Alamos National Laboratory, she has developed methods for eliciting, documenting, and analyzing expert judgment; she has co-authored a book with Jane Booker, *Eliciting and Analyzing Expert Judgment: A Practical Guide*, published by Academic Press, London, 1991. Her current research focuses on combining probabilistic and anthropological techniques to elicit expert judgment for reliability assessments.

M98003009



Report Number (14) LA-UR-97-4072  
CONF-980128-

\_\_\_\_\_  
\_\_\_\_\_

Publ. Date (11) 1997/0

Sponsor Code (18) OUS, XF

JC Category (19) UC-000, DOE/ER

**DOE**