# Research on Ambient-Temperature Passive Magnetic Bearings at the Lawrence Livermore National Laboratory

R. F. Post
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L. S. Tung

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## ABSTRACT

Research performed at the Lawrence Livermore National Laboratory on the equilibrium and stability of a new class of ambient-temperature passive bearing systems is described. The basic concepts involved are: (1) Stability of the rotating system is only achieved in the rotating state. That is, disengaging mechanical systems are used to insure stable levitation at rest (when Earnshaw's theorem applies). (2) Stable levitation by passive magnetic elements can be achieved if the vector sum of the force derivatives of the several elements of the system is net negative (i.e. restoring) for axial, transverse, and tilt-type perturbations from equilibrium. To satisfy the requirements of (2) using only permanent magnet elements we have employed periodic "Halbach arrays." These interact with passive inductive loaded circuits and act as stabilizers, with the primary forces arising from axially symmetric permanent-magnet elements. Stabilizers and other elements needed to create compact passive magnetic bearing systems have been constructed. Novel passive means for stabilizing classes of rotor-dynamic instabilities in such systems have also been investigated.

## INTRODUCTION

This article briefly summarizes studies made at the Lawrence Livermore National Laboratory on a new concept: the ambient-temperature passive magnetic bearing. These studies aim at developing a new class of bearing/suspension systems for high-speed rotating machinery. These bearings would not only be more energy-efficient than present bearings, but also would not require lubrication or maintenance for the service life of the equipment.

Rotating machines are indispensable elements of modern civilization. Essential to all such machines are their bearings. In virtually every such machine mechanical bearings are used, involving lubrication, wear, energy losses, and finite life.

The presence of lubricated bearings is ubiquitous in industry. Virtually every electric motor in every manufacturing facility, chemical plant, or oil refinery relies on such bearings, and their maintenance and replacement and the lubricants involved cost U. S. industry millions of dollars per year. In addition to these direct costs are costs associated with the reduced efficiency of such motors owing to bearing friction losses. For example, the oil-lubricated bearings for a 50 hp electric motor typically dissipate approximately 1.0 percent of the input electrical power. For a continuously operating motor the annual electrical cost of this level of bearing friction loss (at $.05/kWhr) would amount to nearly $200. Over the lifetime of the motor this added energy cost would amount to nearly as much as the initial cost of the motor, assuming a 10-year service life. Taking industry as a whole it can be estimated that their annual energy costs are increased by an amount in excess of $200 million from electric motor bearing losses alone. It would be very significant, both from an energy efficiency standpoint, and from

R. F. Post, D. D. Ryutov, J. R. Smith, L. S. Tung, Lawrence Livermore National Laboratory, P. O. Box 808, Livermore, CA 94550

the standpoint of reduced maintenance and reduced waste,  if it was possible to replace such bearings with non-contacting, near-zero friction, maintenance-free bearings.   If perfected, passive magnetic bearings would seem to represent an almost ideal replacement for the mechanical bearings in many types of industrial electrical machinery.

In search of this goal we have developed theory and performed experiments defining a new breed of magnetic bearing systems: the ambient-temperature passive magnetic bearing,  a goal thought by many to be impossible to achieve ("the Earnshaw Theorem problem").  As must be expected in the course of exploring a new concept there are key research issues that must be investigated en route to commercially viable systems.

## BACKGROUND

The most fundamental of the issues that must be addressed by any magnetic suspension system goes by the name of Earnshaw's theorem.  Published in 1839, Earnshaw's theorem (as updated to include magnetic forces) asserts the impossibility of stably levitating an object, using only magnetic fields derived from fixed currents or permanent magnets.  The reality of this theorem will be made apparent to anyone who tries to levitate one permanent-magnet disc above another, using the repulsive force between them.  The upper disc will inevitably either slide sideways or flip over and be immediately attracted to the lower magnet.

In connection with his pioneering work on the ultra-centrifuge, physicist Jesse Beams and co-workers at  the University of Virginia were apparently the first to develop, almost 60 years ago, a practical means of circumventing Earnshaw's theorem in supporting centrifuge rotors magnetically (Holmes, 1937).  They introduced the idea of using  photo-electric sensors, feedback circuits, and control coils to achieve stable levitation of a spinning object.  Since this early work an industry based on the principle of the "active" magnetic bearing has grown up to supply magnetic bearings to industry for applications where the advantages of such bearings outweigh their disadvantages: complexity and high cost.

With the advent of high-temperature superconductors another approach to the problem became feasible: The "flux pinning" characteristic of the HTSC can be used to stably levitate a rotating permanent magnet above a slab of the superconductor.   Argonne National Laboratory has done pioneering work on this approach.  One important result of their work has been to demonstrate passive magnetic systems having an extremely low equivalent friction loss (Weinberger, et. al., 1991).  Argonne is looking into applications of their technology for the levitation of large flywheel rotors.  It seems, however, doubtful that their technique can be applied to garden-variety rotating machinery, owing to cost considerations and the necessity for continuously operating cryogenic systems.

Beginning about 1991 some internally funded work was begun at the Lawrence Livermore National Laboratory on a totally different approach:  the ambient-temperature passive magnetic bearing (or, more accurately,  magnetic bearing system).  This work was initiated in support of the development of a new class of modular flywheel energy storage systems: "electromechanical batteries"  - EMBs - (Post, Fowler, and Post, 1993).   So far as we are aware no other U S research group is investigating the research issues associated with ambient-temperature passive magnetic bearings.  The likely reason for this situation is that researchers have accepted the widely held belief that Earnshaw's theorem cannot be evaded without the use of active controls or of superconductors.  What we have shown in our work so far is that for dynamic, as opposed to static, rotating systems, Earnshaw's theorem can be circumvented by using electrodynamic forces arising from purely passive elements.


## AN "EXISTENCE PROOF" THAT EARNSHAW'S THEOREM CAN BE EVADED

That the exploitation of dynamic effects represents a way to circumvent Earnshaw's theorem can be seen in a recently marketed magnetic toy, a top called the "Levitron".    It consists of a small disc of ceramic permanent-magnet material through a hole in which a thin wooden dowel, pointed at the bottom, is passed.  The "base" above which the top is to be

levitated contains a square block, of permanent-magnet material. This block is magnetized in a non-axially symmetric pattern so as to create a region a few centimeters above the block where the intensity of the force field, as averaged azimuthally, is a minimum for both lateral and vertical displacements. The field configuration thus resembles the multi-pole "magnetic well" configurations employed in magnetic fusion research to stably confine a hot plasma.

In this field the top, if its axis is vertical and if located at the right distance above the base, is at a potential minimum for lateral and vertical displacements. If not spinning, it would be unstable, as predicted by Earnshaw's theorem, but in this case only against tilt-type displacements. However, with sufficient spin gyroscopic effects will keep the top from tilting. The top will remain stably levitated in mid-air above the base for minutes, until air friction slows it to the point that the gyroscopic moment is too low to overcome the tendency toward tilting. (When we spun it up and then placed a vacuum bell jar over it and rough-pumped it down to a modest vacuum, it spun stably for half an hour!)

The stable rotational state exhibited by this simple toy is an "existence proof" of a stable passively levitated rotating system, i.e., one not limited by Earnshaw's theorem, and one that employs only ambient-temperature passive magnetic elements. Unfortunately, the Levitron concept does not appear to represent a practical approach to a passive bearing for most industrial applications. For practical applications, therefore, our approach involves a different variety of dynamic effects to circumvent Earnshaw's theorem, as described below.

## DESCRIPTION OF THE ELEMENTS OF THE NEW SYSTEM

As an aid in understanding the physics problems to be investigated we list the two main concepts associated with the Livermore ambient-temperature passive magnetic bearing approach. These are:

(1)      It is sufficient in the applications intended if stability is only achieved in the rotating state. That is to say, a centrifugally disengaging mechanical system can be used to insure stable levitation at rest (when Earnshaw's theorem applies). This relaxation of requirements opens up the possibility of using dynamic effects to achieve stability, a possibility not included in the assumptions made in deriving Earnshaw's theorem.

(2)      Stable levitation can be achieved if the vector sum of the force derivatives of the several elements of the bearing system, for axial, radial, and tilt-type perturbations from equilibrium, is net negative (i.e., restoring). In this way it is possible to achieve Earnshaw-stable levitation using a system composed of multiple elements, no one of which is by itself stable against all three of these perturbations. The achievement of stability then becomes a quantitative matter, where the destabilizing tendency, along a given axis, of one magnetic element is paired off against the (greater) stabilizing tendency of another element along that axis, and so forth (Baldwin, Fowler, and Post, 1993).

One result of the theory: If only axially symmetric permanent-magnet elements are employed, in either an attracting or a repelling mode, there is a relationship between their force derivatives, as follows:    For lateral displacements from equilibrium as compared to axial displacements, the force derivatives are in relationship to each other's magnitude either as  -1 to +2, or +1 to -2. That is, if attracting,  the relative force derivative for radial displacements is -1 (stable, i.e., restoring), but +2 (unstable, and of twice the magnitude) for axial displacements. If repelling, the opposite applies, being unstable, i.e., +1, for radial displacements, but stable, i.e., -2, so with twice the magnitude,  for axial displacements. The 2:1 ratio of the magnitudes of the force derivatives in both the above cases is the result of the fact that in an axially symmetric system the force arising from a transverse displacement is the average of a cosine function, whereas in the axial direction no such averaging is required.

Given the situation just described the best that can be done with an arbitrary combination of such axially symmetric elements is to achieve a metastable state. This is the state that one would achieve by exactly pairing attracting and repelling bearing elements so that the plus and minus force derivatives canceled, leaving a neutrally stable state, a situation of no practical interest. To resolve this problem at least one new element must be introduced, one where the ratio of force derivatives deviates (in a favorable way) from that of axially symmetric elements.

This insight provided the starting point for the development of our concepts, embodying new types of non axially symmetric systems.

One such new element that we are exploiting, is the periodic permanent-magnet array known as a "Halbach array", named after the Lawrence Berkeley Laboratory physicist, Klaus Halbach, who invented this configuration for use in focusing particle beams (Halbach, 1985).

Our principal use of the Halbach array is to employ it to interact with an inductively loaded periodic array of circuits to produce a stabilizing force derivative along an intended coordinate (i.e., either radial or axial), without an accompanying destabilizing force derivative for orthogonal displacements. The inductive loading minimizes the power losses in the circuits, while, through shifting the phase of the current by nearly 90 degrees at operating speeds, it maximizes the repelling force between the magnet array and the array of circuits. Figure 1 (left) shows a schematic drawing of a cylindrical Halbach array and the circuit array. This particular embodiment of the idea addresses the problem of providing a radial stabilizing force, as it exhibits a negative force derivative (positive stiffness) for displacements transverse to the axis of rotation. Figure 1 (right) shows a comparison with experiment of the theoretically predicted force exerted by a small array.

It is even possible to use opposing Halbach arrays, or geometrically configured circuits, in such a way that there is near-zero current in the circuits at equilibrium (thus vanishingly small losses) nevertheless accompanied by large stabilizing force derivatives. The physics insight here is that achieving stable levitation or constraint along a given axis requires only that the potential function along that axis should have a minimum at the equilibrium point: At that point the net force is zero. A restoring force is only required when a deviation from equilibrium occurs.
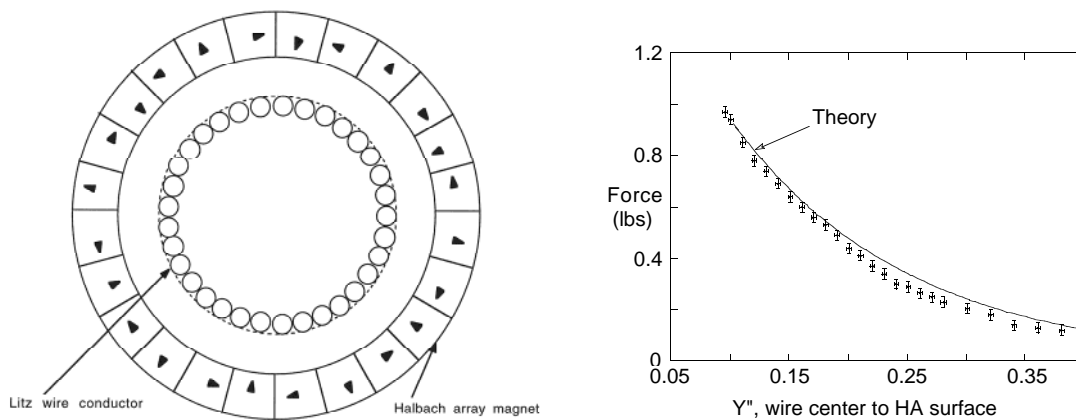


Figure 1: Schematic drawing of Halbach array stabilizer and a comparison between theory and experiment for a small stabilizer.

Coupled with the use of inductively loaded circuits as elements in a suspension/bearing system we employ axially symmetric permanent-magnet elements to produce the main levitation and/or centering forces. These latter elements are designed so as to produce prescribed values of levitation forces and force derivatives. When combined with the dynamic elements the complete assembly comprises a system that possesses a stable force equilibrium state when rotating. Disengaging mechanical bearings are used to sustain the equilibrium when the system is at rest, or when rotating below a low transition speed (typically of order a few hundred to a few thousand revolutions per minute).

Recently a US patent has issued (Post, 1996) that describes several embodiments of the ambient-temperature passive magnetic bearing based on the concepts described above.

## THE SECOND HURDLE: ROTOR-DYNAMIC INSTABILITIES

Described above is how we employ dynamic effects to solve the equilibrium problem posed by Earnshaw's theorem. However, in high-speed rotating systems there exists an entirely different class of stability problems: rotor-dynamic instabilities. The driving energy for such instabilities is the presence of the ordered kinetic energy of the rotating system. As in Murphy's Law, if the rotating system can find a way to convert its ordered rotational energy into disordered motion it will do so, with sometimes spectacular effect.

In systems employing mechanical bearings the stabilization of rotor-dynamic instabilities, a problem perennially encountered since the last century, has required untold effort. Until recently the approach has been largely empirical, involving a variety of mechanically based stabilizing elements. Among these are dampers using viscous forces in oil films, dissipative bearing mounts, etc. In systems employing mechanical bearings the analysis of stability is further complicated by the non-linearity of the elements involved.

TRANSVERSE WHIRL INSTABILITIES

To illustrate the nature of the rotor-dynamic instabilities we will use as an example a ubiquitous one: a "whirl" mode. The reason such instabilities are omni-present is that they arise from an almost universal source: the existence of displacement-dependent torques. This type of whirl instability involves only displacements that are transverse to the axis of rotation (and therefore the whirl motion does not involve gyroscopic effects).

The situation is as follows: Transverse displacements result in forces acting on the center-of-mass of the rotating object. One is the restoring force exerted by the positive stiffness (negative force derivative) of the bearing, keeping the system centered. (In a simple journal bearing this force is supplied by the oil film that exists between the elements.) Always present in mechanical systems is the torque caused by frictional drag. Such drag torques depend on displacement, increasing on the side where the rotating element comes closer to the stationary part of the bearing and decreasing on the opposite side. Summing the vectors representing these forces we see that the resultant vector is no longer directed radially inward (which would imply simple harmonic motion at a frequency determined by the stiffness and the mass of the rotating object), but is instead directed so as to put the rotating element into a whirling motion in the form of an exponentially growing spiral (until limited by physical contact with the stationary element). In worst-case scenarios whirl instabilities of this type can destroy the bearings and/or the rotating object.

Whirl instabilities can also be caused by other situations than the ones just described. For example, displacement-dependent torques from aerodynamic forces, from armature-stator interactions in electric machinery, and even effects arising from internal mechanical hysteresis, can cause whirl instabilities. Analyses of the role of internal mechanical hysteresis in producing whirl instabilities are given, for example, in works by Thomson, Younger and Gordon, (1977). and by Bucciarelli (1982) and in references cited in these reports.

The absence of whirl instabilities in well-designed rotating systems is not because of luck, but it is because the designer has used proven damping means to suppress them. The relevance of these considerations to the present case is that when we create bearing systems with very small losses a correspondingly small displacement-dependent torque (for example arising from eddy currents) may be enough to tip the balance in the direction of whirl instabilities. A detailed understanding of this new situation (the passive magnetic bearing) is required, which involves effects not encountered in ordinary systems. We cannot here assume the solution to these problems that has been adopted by the designers of active magnetic bearings. In their case the answer has been to add increasing sophistication to the sensors and circuitry, involving decreasing the response time and adding "smarter" control loops, so that the feed-back systems sense incipient whirl motion and suppress it. The price paid is one of increased cost and complexity and decreased reliability.


STABILIZATION OF TRANSVERSE WHIRL MODES

By contrast with the approach taken in the active magnetic bearing, the approach we have taken with the passive bearing is to "go back to first principles." That is, to utilize theory to predict the conditions required to insure that whirl instabilities do not arise in the first place, and then find ways of implementing the requirements of theory with passive elements. We have identified two generically different means to accomplish this end. The first of these is electromagnetic damping, for example, using eddy currents induced in a conducting surface. An equation giving the criterion for stabilization by this means is given in the appendix for the case of a velocity-dependent damping force. Note that, by contrast with mechanical dampers, the damping means here is "inertialess." That is, the forces exerted, being electromagnetic in nature, are not modified by the inertia of the elements involved. In all mechanical systems the inertial response of the various components of the damper introduces a frequency dependence that can progressively vitiate the damping effect as the rotation speed increases. Such inertial

effects can become particularly marked in systems with mechanical bearings operating at high rotation speeds, and can as a result severely constrain their design.

The second stabilizing technique for transverse whirl modes, is that of introducing anisotropy into the bearing stiffnesses. Previously untested in this context, this technique is predicted to strongly suppress transverse whirl modes (Gunter, 1966). Use of this technique is made possible here by our use of non axially symmetric systems. Anisotropy can be introduced through the Halbach-array stabilizing elements, for example. By definition, this mode of stabilization cannot be applied in a magnetic bearing system that employs only axially symmetric elements.

An equation giving the criterion for suppression of transverse whirl by anisotropy is given in the appendix. The criterion defines the anisotropy ratio (ratio of stiffnesses along two axes perpendicular to each other) required for stabilization to the magnitude of the displacement-dependent torque that drives the whirl motion. If the inequality is satisfied theory predicts that transverse whirl will not develop, within the limitation of the assumption of linearity that was made in deriving the theory. The results of the analysis have been confirmed by programming the equations of motion involved and solving them on a computer.


TILT-WHIRL INSTABILITIES

There is another class of whirl instabilities, one involving gyroscopic effects, that can also occur in special circumstances. This could be called a "tilt-whirl" mode, since it involves a bi-conical whirling motion of the rotating object, with opposite ends of the rotor being 180 degrees out of phase with each other. Theory (Ryutov, 1995) indicates that this mode requires the satisfaction of different criteria for its stabilization than the transverse whirl mode. A new result from Ryutov's theory (one we have not found in the literature) relates to a novel means of suppressing this mode, one that we can implement in our passive bearing geometry. The concept is the following one:

If the usual geometry of a bearing-shaft assembly (that is, the shaft rotates inside of a stationary bearing element) is inverted, so that a hollow shaft rotates outside a stationary bearing element, then displacement-dependent drag forces are found from the theory to be stabilizing rather than destabilizing for the tilt-whirl mode. The theory indicates the possibility of satisfying the conditions for stabilizing both transverse and tilt-whirl modes by utilizing a suitable combination of geometry and anisotropy.


**INTRINSIC CAPABILITIES OF THE PASSIVE BEARING SYSTEM**

As the example of the Levitron top shows, it is not enough to demonstrate an Earnshaw-stable situation of a passive bearing element in order to have confidence that the end result of an investigation can be applied practically. In the case of the ambient-temperature passive bearing system, theory shows that levitation forces and bearing stiffnesses of the magnitude needed for a wide variety of practical applications can be achieved. We are aided in this by the ongoing developments in high-field permanent-magnet material, particularly of the Neodymium-Iron-Boron variety, where material with remanent fields in excess of 1.4 Tesla is now commercially available. Note that a field of 1.0 Tesla (a level of working field obtainable using NdFeB permanent-magnet material) is capable of supporting a weight of 4 kilograms per square centimeter of magnet pole area. Thus the attractive force generated by a field of 1.0 Tesla between circular pole faces only the size of a US quarter is of order 180 Newtons (18 kilograms), certainly a "practical" level of levitating force.

In addition to the generation of levitating forces, we have employed variations in the geometry of the permanent magnet elements to control the quantitative value of their force derivatives (stiffnesses). For example, by adjusting the ratio of the radii of two permanent magnet elements (in the form of thin discs) it is possible to vary the ratio of the force derivative to the force over a wide range. As an example the graph in Figure 2 (Post, 1996) shows the

locus of those radius ratios as a function of separation between the magnets such that the force derivative is <u>zero</u>, while the force is non-zero. Crossing the boundary of this locus line the force derivative changes sign, while the force does not.
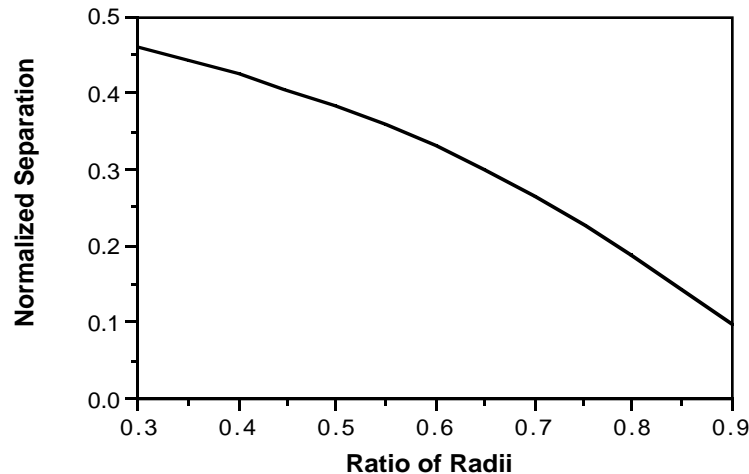


Figure 2. Locus of ratio of radii of thin-disc magnets <u>vs</u> separation, for zero stiffness.

Results such as the one just cited concerning the effects of geometry on stiffness have been used in our work to define combinations of permanent magnet elements and Halbach-array stabilizers that together satisfy the earlier-defined criterion for Earnshaw-stable systems. We see that achieving stability is a quantitative matter, one where it is necessary to know the stiffness and drag and mechanical hysteresis parameters of all the elements of the bearing system and the rotating elements that it supports in order to insure stability.

The theory of our systems has shown that it should be possible to achieve radial stiffness values that approach those possible with lubricated mechanical bearing systems, namely stiffnesses of order $10^7$ to $10^8$ Newtons/meter (57,000 to 570,000 lbf/in). That such levels of stiffness are predicted to be possible arises from the use of high-order Halbach arrays in our systems. Since the fields from such arrays have steep spatial gradients, the variation of repelling force with displacement can be made to be correspondingly steep. An equation defining the stiffness of one type of Halbach array element is given in the appendix. It is also possible to still further increase the stiffnesses by employing non-linear resistive elements in the circuits of these bearing elements (Post, 1996).

Another practical requirement for a proposed magnetic bearing system that is intended in the long run to enhance the efficiency of rotating machinery is that its drag torque (equivalent-friction losses) should be low. Although it must be better quantified by actual measurements, preliminary estimates of the drag torques of our ambient-temperature bearing elements indicate that their losses should be very low. When one considers either the power consumption of the amplifiers of active magnetic bearing systems, or the power required to refrigerate HTSC passive bearing elements, it appears that the ambient-temperature passive bearing system might be capable of even lower power losses than either of these approaches.

## SUMMARY

A brief description has been given outlining the concepts and design principles that are being employed at Livermore to develop ambient-temperature passive magnetic bearings. In addition to tackling the "Earnshaw theorem problem," the studies are aimed at finding means to insure stability against whirl-type modes in such systems. With further development it appears

that there may be many applications for such bearings, beyond those that first stimulated their development - the electromechanical battery.

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## APPENDIX

(**A**)   In cartesian coordinates the equations of motion for the center-of-mass for transverse displacements take the form:

$$M\frac{d^2x}{dt^2} = -K_x \ x + \alpha_x \ y - \beta\frac{dx}{dt} \tag{A1}$$

$$M\frac{d^2y}{dt^2} = -K_y \ y - \alpha_y \ x - \beta\frac{dy}{dt} \tag{A2}$$

Here $K_x$ and $K_y$ and $\alpha_x$ and $\alpha_y$ are the stiffnesses and drag coefficients in the x and y directions, respectively, and $\beta$ is the damping coefficient for the eddy-current damper.
    In the case of isotropic stiffness and drag coefficients ($K_x = K_y = K; \alpha_x = \alpha_y = \alpha$), solution of the above equations yields the stabilization by eddy-current dampers:

$$\beta > \frac{\alpha}{\Omega_0} \ \ , \text{stable}, \ \ \ \Omega_0 = \sqrt{\frac{K}{M}} \ \ \text{radians/sec.} \tag{A3}$$

    In the case of anisotropic stiffness and drag coefficients, if no eddy-current dampers are employed, stabilization requires:

$$\frac{K_x}{K_y} < \left[ 1 - \frac{2\sqrt{\alpha_x \alpha_y}}{K_y} \right] \quad , \ \text{stable}, \ K_x < K_y \qquad (A4)$$

**(B)**    The radial force derivative (negative of the stiffness) of a close-packed array of circuits in the form of "window frames" of litz wire positioned across the diameter of a cylinder of radius c (m.) inside an N-pole Halbach array with an inner radius of a (m.) and an outer radius of b (m.) is given by the equation:

$$\frac{dF}{dx} = -1.0 \times 10^7 \left[ \frac{B_0^2 h^2 N}{P} \right] \left( \frac{c}{a} \right)^{2N-1} \quad \text{Newtons/meter} \qquad (B1)$$

where h (m.) is the axial width of the Halbach array, P (m.) is the perimeter of the circuit, and $B_0$ (Tesla)  is the strength of the field at the inner surface of the even-order Halbach array, given by the equation (Halbach, 1985):

$$B_0 = B_r \left[ \frac{N}{N-1} \right] \left\{ 1 - \left[ \frac{a}{b} \right]^{N-1} \right\} \cos^N(\pi/M)[\sin(N\pi/M)/(N\pi/M)] \quad , \ N = 2, 4, 6, \ldots \quad (B2)$$

$B_r$ (Tesla)   is the remanent field of the permanent magnet material; M = 4N is the number of magnet segments (see Figure 1 in the text).  The choice of even-order Halbach arrays  assures that in the undisplaced position there will be flux cancellation within each circuit.