SSCL--325

DE91 004681

SSCL-325

# LONG TERM PREDICTION AND THE SSC

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September 1990



* Operated by Universities Research Association, Inc., for the U.S. Department of Energy under Contract No. DE-AC02-89ER40486

# LONG TERM PREDICTION AND THE SSC.

NOTES FOR WORKSHOP ON NONLINEAR PROBLEMS IN FUTURE PARTICLE ACCELERATORS, CAPRI, ITALY, APRIL, 1990.


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## ABSTRACT


Successful operation of the Superconducting Supercollider (SSC) will depend on the stable circulation of particles for tens of millions of turns around the rings, in the presence of small nonlinear deflecting fields. One design challenge is to set specifications for the maximum allowable field imperfections of this sort, consistent with the required stability. Another challenge is to plan for the inclusion of field compensating elements that will ameliorate the effects of errors. The "tools" available for projecting the long term stability are theoretical, both analytic and numerical, and experimental. These aspects are reviewed.


**Introduction.** In this paper some of the issues that have arisen in anticipating the long term stability of particles in the SSC will be reviewed. Since much of the work involves a lot off detail that is difficult to include in an overview the discussion here will be rather qualitative. Furthermore it will be somewhat disjointed, amounting to a miscellany of brief discussions of problems, methods, results, conclusions, philosophy and so on. Where possible reference will be made to detailed reports. The topics to be discussed are:

- Operational issues and nonlinear effects that jeopardize performance.
- A progression from complete predictability to complete unpredictability.
- "Proofs" of particle stability.
- "Experimental" mathematics.
- Limitations of "restoring" symplecticity.
- "Exact" tracking.
- Long term tracking results.
- Accelerator modeling and routine error correction procedures in the SSC.
- Beam dynamics experiments.
- Parallel computation.

**Operational issues and nonlinear effects that jeopardize performance.** The SSC will be used to accelerate protons to an energy of 20 TeV. This will be performed in a sequence of accelerators starting with a linac, followed by boosters (of low, medium, and high energy) and finally the collider itself. The collider consists of two rings, one above the other, with counter-rotating protons. In this sequence there are many questions of dynamics that might be appropriate for study at a workshop on nonlinear dynamics. There are questions of large amplitude particle loss, resonant extraction, particle density dilution due to filamentation, adiabatic invariance, effects of resonances, chaos, turbulence and so on. All of these things relate to all accelerators in the chain. There is, however, one link in the chain that is considered to be the weakest one; that is where studies have mainly been concentrated and it is what will be emphasized in this report.

This most critical phase in the filling cycle occurs in the period during which previously injected protons are circulating in the collider at low energy (2 TeV), waiting for the injection of all the other protons to be completed. At that energy the beam has not yet benefited from the adiabatic reduction of transverse size that will accompany its acceleration to 20 TeV. As a result large amplitude particles, if they are ever to be lost, will be lost at that time. Traveling at the speed of light for, say, an hour, the protons travel $10^{12}$ meters, circulating some $10^7$ times around the collider. In order not to be lost the particles must remain within the "physical aperture" defined by the vacuum tube, whose bore radius is about 2 centimeters, or by the occasional mask or instrument that may intrude slightly into the chamber. With the a typical transverse amplitude being 1 millimeter this appears not to be very difficult to achieve. However, the coils carrying the currents that generate the dipole (i.e. uniform) magnetic field that bends the protons can, for reasons of cost, be only slightly larger in radius than the bore tube, and that makes it difficult to achieve satisfactory field uniformity. The effect of nonuniform fields is to make a "magnetic aperture" or "dynamic aperture" outside which particles are lost because the motion is unstable. For the SSC, with anticipated field quality, the dynamic aperture may be about one centimeter. Some beam quality degradation can be anticipated for a beam that circulates very long in the outer portion of this region; for that reason an even smaller "linear aperture" can be defined within which the beam circulates without degradation. This will be about 5 millimeters.

In the jargon of the field, the field uniformity is analysed by expanding the field in a (complex) "multipole" power series. After the dipole term comes a quadrupole term that describes linear transverse variation of the field. This component is a nuisance, because it alters the focusing properties, and the horizontal-vertical coupling properties of the lattice (i.e. the totality of elements making up the ring.) It is, however, only a nuisance, as it is reasonably straightforward to compensate for it "operationally". This term is used to describe compensation schemes that rely only on measurements that can be reliably performed using the beam itself, without any knowledge of the magnetic fields of the magnets in the ring. From a theoretical point of view the quadrupole fields can also be regarded as fairly innocuous as they leave the equations of motion linear.

It is higher multipoles, also known as the nonlinear multipoles, that threaten to cause the loss of large amplitude particles. These elements cause the equations describing the transverse motion (betatron motion in the jargon) to be nonlinear. These (relativistic) Newton's Law equations are what make the understanding of accelerators depend on nonlinear dynamics, the subject of the Workshop. It can even be said that accelerators make the best laboratory for the experimental investigation of Hamiltonian Mechanics, for which freedom from loss is the *sine qua non*. For all practical purposes the betatron oscillations are lossless, with damping times of the order of $10^{10}$ oscillation periods.

The leading nonlinear multipoles are sextupoles and octupoles. These can be present in the lattice both intentionally and unintentionally. Before proceeding to discuss the latter, which are somewhat the more troublesome, we mention two intentional uses as "medicine" to cure various lattice sicknesses. Sextupoles are used intentionally to reduce the chromaticity ( i.e. the momentum dependence of the focusing.) Octupoles make the focusing depend on amplitude, which is sometimes exploited to improve current-dependent behavior. These medicines have side-effects, the main one being the loss of large amplitude stability. But since they are present intentionally, the "dose" is known and can be controlled, so that this source of nonlinearity can be handled confidently in a large proton accelerator such as the SSC. For electron machines, in which very strong focusing (leading to large chromaticity) is used to reduce the transverse beam size, this is less true. Even for a large proton accelerator like the SSC there is a kind of design trade-off situation in which stronger focusing can be used to desensitize the lattice to unintentional multipoles: this makes the intentionally present nonlinearities not entirely negligible.

It is unintentional elements that represent the greatest hazard. The strength of each of these multipoles can be expressed as the sum of a *systematic* part, which is the mean strength, $a_i$ or $b_i$, of the particular multipole for all magnets of the particular type, and a *random* part, centered on zero, that is characterized by an r.m.s. deviation, $\sigma_{ai}$ or $\sigma_{bi}$. These two types of error lead to quite different qualitative behavior. Which of the two is the more serious is largely a quantitative matter, but it is also somewhat subjective as it depends on the credibility of planned compensation schemes. In constructing the accelerator magnets great care is taken trying to make the magnetic fields uniform and identical within the economic constraint that the magnets not be too expensive. When this is done with SSC magnets typical deviations from uniformity are a few parts in ten thousand, at a point one centimeter from the magnet axis.

Systematic field errors can be caused by errors in the design location of the current carrying elements. Since this leads to excitation independent nonuniformity, it is not too serious. Far more serious are the fields caused by persistent currents in the superconducting coils. Not only are these excitation dependent but also they exhibit hysteretic dependence on previous excitation history. The leading effect of systematic field errors is to make the betatron frequencies (they are usually called "tunes") depend on momentum. The degree to which this can be compensated operationally has been studied[2] and continues to be an important issue. The tune shifts caused by systematic multipoles are directly measureable and can be used to set the strengths of compensating elements. This leads to the desirable situation that adjusting the strengths of compensating elements only tends to become difficult when they no longer need adjustment. Unfortunately systematic multipoles also cause the tunes to be amplitude dependent, and the operational procedure mentioned in the previous sentence does not work well in suppressing this effect.[3]

Random multipoles are caused by magnet-to-magnet variation in the manufacturing process. This places a high premium on using uniform coil materials, accurately manufactured parts, reproducible jigging and so on, all tending to increase costs. The leading effect of random multipoles is to violate the invariance of the "Courant Snyder invariant". In a linear lattice this quantity, which could also be called the betatron amplitude, does not vary from turn to turn.[4] Nonlinear elements cause this quantity to vary, with the fractional variation being called the "smear". Empirically it is found that, though smear of a few percent is acceptable, values greater than five or ten percent tend to be accompanied by resonances, chaos, density dilution and particle loss.

**A progression from complete predictability to complete unpredictability.** As the amplitude of transverse betatron oscillations of protons in an accelerator is increased from small to large values, bands distinguished by qualitatively very different behavior are encountered. At all amplitudes the local motion of an individual particle is governed by an extremely simple equation — an ordinary differential equation, Newton's Law (relativistic). Nevertheless a full spectrum of possibilities has come to be expected for Hamiltonian oscillators. An early work by Hénon,[5] written with no reference whatsoever to accelerators anticipated much of what has by now been abundantly observed. In this section the characteristic features are described.

At small amplitudes the particle behavior is perfectly linear and accurately predictable. The transverse displacement, say $x$, at a particular point in the lattice varies sinusoidally in a range $\pm a_x(s)$. Because the focusing strength varies, $a_x(s)$ is a function of the longitudinal coordinate $s$. When $a_x$ is generalized to account for this "parametric" variation, it becomes the Courant-Snyder invariant.[4] In the small amplitude region that quantity is truly constant.

As $a_x$ is increased, nonlinear forces begin to have an effect. There is a large region in which they can be treated perturbatively. Though $a_x$ is not quite invariant, its variation can be accurately predicted, for example by the introduction of "distortion" functions. The magnitude of the nonlinearity can be quantified by the smear, defined above. If the amplitudes $x, \beta dx/ds$ are recorded, turn-by-turn, in a Poincaré phase space plot taken at a fixed point in the lattice, they lie on a smooth, "regular", if somewhat distorted, closed curve. (The factor $\beta$ is chosen to make this curve approximately circular.) This is the region in which ordinary accelerator operations live. Particles that start close together, stay close together, or at least diverge reasonably slowly. This makes the motion predictable, at least in the sense that $a_x$ stays constant — tune difference will cause the angle in phase space to diverge eventually.

Already in the region just described, if one looks closely enough, tiny regions can be observed in which the particle tunes exhibit "tune entrainment" onto rational, resonant values. Thin, almost circular, chains of resonance islands are observed. All particles on one of these islands exhibit exactly the same tune. As $a_x$ is increased the the radial extent of these resonance islands become great enough that the islands can not be overlooked. In some areas of this region, if one looks closely enough, chaotic motion is observed. Close-together chaotic trajectories diverge quickly, making them unpredictable.

At larger amplitudes the island chains tend to overlap and the chaotic regions become greater and greater, eventually leaving little area for regular motion. In this region the motion of individual particles is completely unpredictable, and more nearly resembles a random walk in the Poincaré plot. Toward the outside of the region particles are lost; whether they mathematically diverge to infinity, or simply grow to an amplitude at which they strike some obstacle, is equivalent from the point of view of accelerator physics. In this outer region there can still be predictability of the evolution of collections of particles, but only in the sense of diffusion. Such a description would use partial differential equations to govern the time evolution of particle distributions.

All features mentioned in this section have been observed in accelerators. Some of the observations will be described in later sections.

**"Proofs" of particle stability.** This section and the next will be partly philosophical, partly facetious, partly iconoclastic, but serious nonetheless. They relate precisely to the most important question that can be posed at a workshop on theory and accelerators: to what extent can theory prove in advance that an accelerator will work? In particular, will particles circulate stably?

As a matter of sociology, all that is required of a "proof" of stability is to persuade providers of funds to provide adequate support to enable the accelerator to be built. This means that the "standard of proof" becomes greater and greater as the accelerators become more expensive. Furthermore, as the machines get larger and larger, the apertures get smaller and smaller. In the previously introduced multipole series, higher and higher order terms become important.

This make the "proof" harder and harder. Fortunately, the mathematical tools become more powerful, and so do the computers, so the task is not necessarily hopeless.

When Lawrence contemplated building a cyclotron he had only himself to persuade that the particles would circulate stably. He knew that the solution of Newton's differential equation for a charged particle in a uniform magnetic field was a well-bounded curve, namely a circle. QED an adequate "proof" of stability. He was not troubled by questions of focusing and quadrupoles. It is just as well, as he might have been dissuaded from proceeding by the fact that there was no vertical stability. Because of the large aperture and the small number of revolutions required, this lack of stability was not serious.

Transverse stability was first analysed for the betatron. After appropriate linearization of the equations of motion, they became simple harmonic. "Proof" of stability then amounted to demonstrating that the transverse force had the correct sign to be a "restoring" force. In that case the solutions of the differential equations are sines or cosines. Because those functions are bounded by $\pm 1$ the motion is bounded. QED.

Historically, in the development of accelerators, the terror of nonlinearity first became prevalent just after the discovery of the alternating gradient synchrotron (AGS) principle. This principle, already referred to above,[4] made it possible to focus particles into beams that could be contained in more slender toroidal vacuum chambers than had previously been thought posssible. This permitted the magnets to be much lighter and cheaper. The theory was the first to rely on transfer maps: with only linear forces being contemplated they were called transfer matrices. Evolution of a phase point in the Poincaré plot could be obtained by iterating the once-around transfer matrix. Stability could be related to the eigenvalues of the transfer matrix. Obviously the existence of any eigenvalue of magnitude greater than 1 results in unbounded motion. Symplecticity (intruding into accelerator physics for the first time) assured that the eigenvalues came in reciprocal pairs, and that the trace divided by two gives the cosine of the tune. Stable motion requires that the tune be a real angle. "Proof" of stability amounted then to showing that the trace was bounded by $\pm 2$. QED. Though the AGS principle was invented at Brookhaven, a weak focusing electron synchrotron at Cornell, was the first to prove out the principle experimentally. Already in construction at the time, as a weak focusing machine, it was immediately converted to alternating gradient when the AGS principle was propounded. Shortly thereafter the possibility that nonlinear resonances would make it fail became a real concern. The successful commissioning of the synchrotron laid that fear to rest and (at least at Cornell) removed the incentive to study nonlinear effects for a considerable period of time. The fact that the magnetic aperture was much greater than the physical aperture justified the neglect of nonlinearity.

These examples have been intended to show that proofs of stability in advance of construction have not typically been ironclad. The determination to go ahead has been based more on experience, intuition, and self-confidence than on mathematical rigor. Successful operation has been the only really valid proof. For the next generation of accelerator, SSC or LHC, we are expected to do better, even though the problem has become harder. It may be more realistic to demand that theory veto designs that certainly will not work than that it provide designs guaranteed to work.

At least three theoretical approaches have been taken to proving that particles will be stable for long times. One is true analytic theory such as that of KAM, Arnold and Nekoroshev. This work yields mathematically rigorous results; if the assumed conditions are satisfied the conclusions cannot be doubted. Unfortunately, insufficient progress has been made in weakening the conditions to make them apply to realistic situations, so the results are not yet powerful enough to contribute to the design dialog. The other two appproaches are numerical. Both are conjectural in that they make less than fully substantiated simplifications in order to make the equations tractable. In one approach plausible, but uncontrolled mathematical approximations are introduced as needed to permit solution. In the other, exact mathematics is applied to a simplified model, the idealization of which is at the physics level. In both cases the credibility can be enhanced by showing that weakening the assumptions does not invalidate the conclusions.

It is clear that neither approach can yield a mathematically water-tight proof that protons will circualate stably for $10^7$ turns in the as-built accelerator. Different people will attach different credibility to the two approaches. I will shamelessly indicate my bias by attaching the derogatory name "experimental mathematics" (an oxymoron?) to one approach and the congratulatory name "exact tracking" to the other.

These calculations have an influence on design decisions during the accelerator planning phase. Unfortunately errors made can lead either to excessive pessimism (and hence cost extravagance) or excessive optimism (and hence excessive risk). To the extent that computational errors give spurious diffusion they will lead to spurious transverse growth and undue pessimism. The use of too coarse granularity may also enhance damaging resonances and lead to excessive pessimism. On the other hand, calculations are always, to some extent, idealized and this, almost certainly, tends to give too optimistic results. In practice, at least for the SSC, uncertainty in the magnitudes of errors expected for yet-to-be-built magnets results in as much uncertainty as do these theoretical tracking uncertainties. A two or threefold reduction in those errors would render many tracking studies superfluous.

**"Experimental mathematics".** This section starts with a digression. Much of what is called Physics has developed as follows. With curiosity piqued by some observation, a physical model is formed, based on intuition, guesswork, experience, idealization and so on, and the model is converted to equations. If these equations are neither elegant nor tractible the model is discarded or revised. Otherwise, using valid mathematics and controlled (i.e. known to not change the answer too badly) approximation, the implications of the model are worked out. If the predictions disagree with the observations the model is discarded or revised, or sufficient doubt is cast upon the original observation that it is repeated. If the predictions agree everyone celebrates, new experiments are suggested, new results predicted and the process repeated. If the model has a few successes and no failures, it comes to be accepted as correct, at least in areas close to where it has been tested. There is nothing experimental about the mathematics that enters this sequence.

By experimental mathematics I mean the following. Suppose that, guided by the need to solve them, the equations are transformed, or beautified, or emasculated, or whatever, but simplified sufficiently to be solveable. These manipulations, though possibly guided by considerations of elegance and judgment, are uncontrolled in the sense that their effect on the answer is not known. Suppose also that, in some restricted domain, the equations can be solved by legitimate mathematics. Such a solution in a restricted domain can then play a role much like observations do in physics. If the simplified solution disagrees with the known correct solution then the simplified solution is certainly wrong and must be rejected. If it agrees in the restricted domain, it may well also be valid in a wider domain. As "science" this is just as valid as was the physics of the previous paragraph. But it is not mathematics; or rather, when applied outside the restricted domain it must not be given the same credibility that true mathematics deserves.

A simple, and quite common, instance of this line of reasoning occurs in long time prediction, for example using truncated transfer maps. Here the restricted domain is short times and the extended domain is long times. After a one-turn transfer map has been generated the computer can economically iterate it to predict far into the future, say for $10^7$ turns. Based on agreement with a presumably reliable method for short times, say $10^3$ turns, one may be tempted to claim that the long time prediction is valid, but that would be fallacious. It is almost as valid to claim, based on the reliable method's having shown stability for $10^3$ turns, that the motion is stable forever.

That is not to say that either prediction is wrong; most of accelerator physics has been based on the proposition that stability of equations truncated to linear order assures adequate stability and that has not been found to be seriously wrong yet.

**Limitatations of "restoring" symplecticity.** Since the particle motion is Hamiltonian, it is necessarily also symplectic. Unfortunately, approximation methods, especially truncation, tend to violate symplecticity. At some level that makes them wrong and unreliable for long time prediction. The simplest example, for one dimensional motion, would have the determinant of the transfer matrix not quite unity, say a bit greater than one. That would lead to inexorable amplitude growth and eventual instability on a sufficiently long time scale. There are ways to restore symplecticity, but they are not unique. Hagel and Zotter[6] give an example in which it is possible to restore symplecticity but only at the cost of failing to conserve energy.

There is however a school of thought[7] saying that long time stability of particles in accelerators and symplecticity are so tightly intertwined that the long time essentials of the motion will be preserved if the description is symplectic, even if it is only approximate. This optimism is perhaps based on the fact that "microscopic" (also known as local) particle densities are preserved at all amplitudes. For linear beam transport systems this has been a powerful guiding principle. It can be carried over to nonlinear systems by restricting the form of nonlinearites. The idea can be explained along with the maxim that "kick codes are automatically symplectic". Here a kick is an infinitely thin element that administers a deflection that depends on the transverse coordinate but not the slope. Phase space coordinates $x_-, x'_-$ before and $x_+, x'_+$ after such a kick are related by

$$\begin{aligned} x_+ &= x_- \\ x'_+ &= x'_- + f(x_-) \end{aligned} \qquad (1)$$

where $f(x_-)$ is any function, linear or otherwise, of $x_-$ but not $x'_-$. The Jacobean of this transformation is identically equal to 1, and as a result the phase space density is conserved at all amplitudes.

Other quantities are at least as important as local densities for characterizing transverse particle beam distributions: particularly global quantities like emittance (r.m.s. size), and "tails" of the distributions. It has already been shown that symplecticity assures conservation of local densities. A paper by Dragt et al.[8] discusses the conservation of emittance and other moments. For linear transformations there are powerful results assuring the conservation of emittance, but results are sparse in the face of nonlinearity. Actually it is the growth of tails on particle distributions that will lead to loss of particles when the tail particles hit a nearby obstacle. Unfortunately, conservation of microscopic phase space density does not prevent the growth of tails. In the well known process of resonant extraction from an accelerator particle escape to "infinity" along an escaping separatrix, preserving microscopic phase space density all the while. Furthermore the existence of filamentation can cause macroscopic phase space density dilution even with conservation of microscopic density. These effects lead naturally to the growth of tails on the particle distribution function, and those lead naturally to beam loss.

**"Exact" tracking.** It is clear that the exact representation of every magnetic element of a complicated accelerator lattice is impossible. Nevertheless, for the sake of argument, let us say we have such a lattice description. In that case the numerical method described so far could be called "approximate tracking in an exact lattice". We now describe an alternative method that can be called "exact tracking in an approximate lattice". In this approach the physical model of the lattice is simplified by using only elements for which exact analytical formulas can be derived. This is the approach taken by the accelerator modeling code TEAPOT.[9]

**Aside:** in this paper issues of round-off error in computers are not addressed. "Exact" and "to machine precision" will be used synonymously, and both will mean that any analytic expressions in the computer code are the same expressions that would be said to give an exact analytic solution of the equations of motion.

The only lattice elements permitted in this code are elements through which particle trajectories can be traced exactly. In practice this restricts the description to thin elements and drifts — the acronym stands for Thin Element Accelerator Program for Optics and Tracking. This does not, however, preclude really quite accurate descriptions, as thick elements can be broken up into "adequately" thin elements. From this point of view the code is just a numerical differential equation solver, or, as it can be called, a symplectic integrator. For small accelerators or for tracking a small number of turns in a large accelerator this is one way the code is in fact used.

For a workshop emphasizing particle stability in future particle accelerators a different emphasis is appropriate. Using thin elements only, one can design an ideal, better-than-true-life accelerator. Given a choice between a zero length and a thick element having the same length-strength product (a physical impossibility unfortunately) the accelerator designer would choose the thin element because it permits simpler, more flexible, design. Using these ideal elements, a lattice that, compared to the true SSC, has the same gross optics, the same dispersion supressors, the same intersection region optics, the same correction elements, and so on is designed. One can then inquire about the long time stability of particles in the better-than-true-life accelerator. This one does by the exact tracking of particle trajectories, which is, by construction, possible. Furthermore it is easy to model realistic deviations from perfection, such as multipole field errors and element misalignment, without giving up the exact tracking capability. If performance of the idealized lattice is unsatisfactory, then it is assumed that the design is insufficiently conservative. If performance is satisfactory, then the design is tentatively accepted, possibly with a safety factor being included, and attention is turned to other potential failure modes. Except for some recent work on long time stability[10] most modeling of the effects of and compensation of imperfections of the SSC have taken this approach.

In the idealized model there are formally only two elements, field-free drift sections and thin element multipoles. The latter are, differentiated into dipoles, quadrupoles, solenoids, markers, etc. on the basis of the numerical values of the multipole coefficients but they are formally equivalent as far as the code is concerned. Propagation through drifts in the computer amounts to solving for the intersection of a straight line and a plane. Though this is entirely elementary, trigonometric functions appear that make even drift regions "nonlinear". Of course, in the limit of paraxial trajectories the equations degenerate into linear equations. Thin elements are also simple though, like drifts, they are not trivial. For example they are not equivalent to the "kicks" described by Eq. (1) . It was emphasized there that the deflection in slope depends on the transverse displacement, but not on the slope itself—that was what seemed to be required to preserve symplecticity. Though this is a good approximation for paraxial trajectories it is not actually correct, even in the limit of zero length elements, as the reader can verify, or look up in reference.[9] Though this might appear to indicate that the exact motion is not symplectic, it really only shows that symplecticity is subtle, as the exact solution is necessarily symplectic.

**Long term tracking results.** Detailed results are given in reports by reports by Yan[11] and by Ritson[7] in presentations at this workshop, and in many SSC reports, such as the SSC Conceptual Design Report[12] and the SSC Suplementary Conceptual Design Report.[13]

Results tend to have the following qualitative features. Particles launched with large amplitudes are lost after a small number of turns, say less than 100, while small amplitude particle oscillate sinuisoidally, for as long as one has the patience to observe them. There is an intermediate, roughly, but not perfectly, circular band, perhaps ±20 percent in phase space radius, for which the ultimate fate of any particular particle can only be determined by tracking it for as many as a million turns or more. In more physical terms, it appears to be necessary to track particles for ten or so periods of the longest natural period of the system to achieve selectivity in the determination of whether or not a particle is stable. For the collider, with a synchrotron oscillation period of 1000 turns, this rule of thumb anticipates appreciable evolution continuing for 10,000 turns.

The motion of any particular particle can be bizarre. A particle can oscillate for a million turns with amplitude varying over a fairly narrow band and then, in a small number of turns such as 100, develop wild swings and be lost. It is possible to perform "post mortem" analysis of such a particle, in order to determine the cause of death. In one case where this was done carefully it was clear that a resonance involving all three degrees of freedom was responsible. Energy sloshed between the horizontal and vertical degrees of freedom, with little or no coupling into the longitudinal. However, for several cycles of longitudinal oscillation notable distortion appeared at the same longitudinal phase, and the ultimate blow-up appeared at the same phase. These observations suggest that the damaging resonance is a horizontal-vertical coupling resonance, influenced parametrically by the longitudinal oscillation; finally the slow energy modulation familiar field, may give the idea: it is cheaper to balance an automobile wheel by pounding little lead weights between the tire and the rim, than it is to control the dimensional tolerances of all tires and rims; but a fairly sophisticated wheel balancing instrument is needed to tell where to pound the weights.) Only operationally practical methods are used in simulating the performance of compensation schemes, but there is a large degree of subjectivity in assessing credibility. This subjectivity usually derives legitimately from experience on existing accelerators. To some extent uncertainty can be reduced by controlled experimentation on such accelerators. Some were done in connection with experiments to be described below.

Early simulation codes for the SSC were developed by Schachinger and others. They included closed-orbit smoothing, decoupling, tune and chromaticity control (all built into the TEAPOT code) and are described in various reports referred to in reference[12]. Systematic "homing in" on a satisfactory correction scheme is described by Bintinger et al.[3] Recent work, especially in anticipating the effects of malfunctioning beam position monitors, is due to Bourianoff[17] and others. With the collider good field region being a toroid 86 kilometers in circumference and about 1 centimeter across, it is clear that accurate beam steering is required. Root mean square orbit deviations of 0.5 millimeters away from nominal are expected to be achievable. Deviations exceeding that appreciably will lead to performance degradation; perhaps the worst effect is a conspiracy between two bad effects: feed-down due to random closed-orbit deviations in the presence of systematic field errors yields random field errors. As mentioned previously random field errors cause smear and, perhaps, diffusion-like behavior.

For predicting the detailed performance of an accelerator, accurate description of the accelerator parameters is essential. This may seem mundane, but because of the thousands of elements involved it is as important, and as difficult as any other part of the problem. At the SSC, lattice parameters for all accelerators and transfer lines in the complex are maintained in standard format in a master central commercial database (Sybase) planned and implemented by Peggs, Saltmarsh, and Trahern. Global geometric self-consistency and integrity of parameters as they are used for various purposes can be monitored this way.

**Beam dynamics experiments.** Various beam dynamics investigations have been performed using the Tevatron at Fermilab, with the intention of contributing to design decisions for the SSC.[15] Being a superconducting accelerator, the Tevatron is the existing facilty that most resembles the SSC. Furthermore it can be considered as essentially linear, so that nonlinear behavior is dominated by the active controlled addition of known nonlinear elements. Here we just categorize the experimental approaches that have been taken, and what has been learned. Mainly the focus has been on studying nonlinear effects.

**(i) Phenomenological approach.** As has been described above, the level of nonlinearity can be quantified by the quantity called smear which, for a known distribution of nonlinearity, can be detemined without difficulty. What must be determined is how well any accelerator degradation due to nonlinearity correlates with the smear value, and what value of smear can be regarded as tolerable for operations. For these investigations, after "mocking-up" the SSC by turning on nonlinear elements, one can study injection efficiency, damping of injection errors, orbit flattening, storage lifetimes and other operational issues. As smear was raised from the raw machine value somewhat below 1% to about 5%, performance was not greatly impaired, and it was tolerable, though impaired, up to about 10%. this was for a particular distribution of nonlinear elements; the degree to which the degradation is different for different distributions has not, as yet, been determined.

**(ii) Engineering approach.** It has to be assumed tnat practical accelerator operations will be restricted almost entirely to the region in which the effects of nonlinearity can be calculated perturbatively. In this region it is possible to make accurate quantitative comparisons between measured and calculated values of smears, tune shifts and decoherence factors. With accuracies at the several percent level agreement is found. This shows that perturbative calculations can be relied upon for predicting most operational procedures. In this region the uncertainty of prediction will be more due to lack of knowledge of the magnetic fields than to calculational uncertainty.

**(iii) Pure physics approach.** As mentioned before, a functioning proton accelerator can be regarded as a natural laboratory for the experimental study of nonlinear dynamics. Experiments can be conceived of as pure physics without any concern for the degree to which the results will contribute to improved intensity or luminosity or whatever other feature is perceived at the time as being required for improved performance of the device in its primary role as a tool for studying elementary particle interactions. As in the rest of science, one has reasonably high confidence that such investigations will lead to enhanced understanding that will ultimately contribute to the primary goal. Studies at Fermilab[15] in which a "metastable state" of the accelerator was investigated fall into this category. In this state the accelerator acts as a storage ring in which the particles oscillate stably around a fixed pooint that is other than the usual one at the origin. It is one of the stable fixed points accompanying a nonlinear resonance. For a-fifth order resonance – the main case studied – the central trajectory closes on itself only after five turns rather than every turn. Used in this way the accelerator is not quite practical for routine operation, in that the lifetime is of order a few minutes instead of a few hours. With effort it could perhaps be made practical, but probably not superior to the reqular operation, and in any case that is not the point of the investigation. One point *is* that particles oscillating at small amplitude relative to the metastable fixed point are oscillating at large amplitude relative to the normal fixed point. A numerical description of the motion that agrees with observations both at the origin and at the amplitude of the nonlinear resonance can probably be relied upon to be correct throughout the perturbative region which is the region that particles visit in normal operations. Some numerical investigations of this sort have been described above.[16]

Many experiments can be designed to study this metastable state. Some have been performed already and some are planned. Its injection efficiency (production probability) and decay rate can be measured under various conditions. Especially apropriate is to study the influence of oscillatory external forces or modulations, as that should probe the oscillatory characteristics of the state, the simplest of which is the frequency of small oscillation around the metastable fixed point.[18]

Another approach that can be taken is to measure, over long times such as half an hour, the evolution of the transverse beam distribution. Ascribing this evolution to diffusion, solving the diffusion equation with empirically adjusted dependence on amplitude of the diffusion constant, a semi-theoretical, semi-phenomenological description results. Qualitatively the results are consistent with a diffusion constant that vanishes for small amplitudes and becomes large at large amplitudes. Pencil beams remain invariant because all particles have such small amplitudes that diffusion is negligible. Somewhat wider beams *spread* slowly, developing tails, while remaining constant in intensity because no particles are at amplitudes large enough to hit any obstacles and be lost. Beams that are wider yet lose intensity and *shrink* in width; that is because tails grow quickly, allowing particles to reach obstacles and be lost; this removes large amplitude particles from the distribution.

**(iv) Diagnostics approach.** Another experimental activity appropriate to planning for future accelerators is the testing of diagnostic procedures. It has been mentioned above that superconducting magnets have hysteretic magnetic field errors. To control the beam in the presence of such time varying, and somewhat unpredictable forces it will be necessary to have beam property sensors supplying feedback to correctors for stabilization. A loop of this sort that has been successfully tested uses digital Fourier transforms of beam position monitor signals to measure the tunes at two values of the momentum; from this the chromaticity is measured and hence, feeding back to sextupole elements, it is corrected.

**Parallel computation.** For simulating the performance of particle beams in a large accelerator it is necessary to track a large number of particles through a large number of elements. Provided collective effects are not being considered the evolution of each particle proceeds independent of all the others. This makes it natural to exploit multiple computation processors, with one processor per particle. The previously mentioned code ZTRACK was written to exploit the vector capabilities of the Cray to achieve this parallelism. We[19] have now also revised the tracking code TEAPOT to run on a multiple instruction set, multiple data set computer called the Intel Hypercube. Each of many (say 63) processors (also called nodes) has a complete record of the data describing the lattice, and each tracks a single particle. Another node is dedicated to book-keeping and to directing the efforts of the other 63, noting down intermediate results, launching a new particle whenever a particle is lost or finished and so on. Because the individual calculations are so nearly independent, very little communication is necessary, and the individual nodes calculate with about 98% efficiency, compared to their performance on single processor code. For this particular code then, the 64 node Hypercube calculates at about twice the rate of the Cray Y/MP optimized for the same code. These computations have been performed using a Hypercube computer situated at Oak Ridge National Laboratory. Starting in the fall of 1990 a 64 node Hypercube will be available for accelerator simulations at the SSC Laboratory in Dallas.

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12 / 27 / 90