

"IDENTICAL" BANDS IN NORMALLY-DEFORMED NUCLEI

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ABSTRACT

Gamma-ray transitions energies in neighboring odd- and even-mass nuclei for normally-deformed nuclear configurations are analyzed in a manner similar to recent analyses for superdeformed states. The moment of inertia is shown to depend on pair correlations and the aligned angular momentum of the odd nucleon. The implications of this analysis for "identical" superdeformed bands are discussed.

1. INTRODUCTION

Thirty-five years ago Aage Bohr and Ben Mottelson, in a paper<sup>1]</sup> "dedicated to Professor Niels Bohr on the occasion of his seventieth birthday", attributed the reduction of the moment of inertia of deformed nuclei to an attractive, short-range interaction between the constituent nucleons. This seminal paper not only anticipated pair correlations in nuclei<sup>2]</sup> by three years, it also predated the BCS interpretation<sup>3]</sup> of superconductors as electronic correlations by two years. An experimentally-verifiable consequence of nuclear pairing, where only a few valence nucleons participate in the correlations, is a significantly smaller reduction of the moment of inertia (i.e., a larger moment of inertia) for odd-A nuclei than for even-even nuclei. The unpaired nucleon "blocks" the contributions of the orbital of the odd nucleon to the pair correlations, thereby reducing the nuclear superfluidity and increasing the moment of inertia. This effect is quite large, typically 10-40 percent, for rare earth nuclei, because:

**MASTER**

i) only a few nucleons near the Fermi level contribute to pair correlations; and ii) the dependence of the moment of inertia on pair correlations is nonlinear. This effect has been verified many times, see for example, Figure 4-12 of ref. 4].

Therefore, the recent discovery<sup>5]</sup> of superdeformed bands in a sizable number<sup>6,7]</sup> of odd- and even-mass nuclei differing by a single nucleon that have identical gamma-ray transitions energies,  $E_\gamma$ , to a few parts per thousand was shocking! Not only is this two orders of magnitude less than the typical deviations in normally-deformed nuclei, but it also is nearly an order of magnitude less than the variations expected for a *macroscopic* rigid rotor,<sup>8]</sup> in which the moment of inertia should be proportional to  $A^{5/3}$  giving an  $E_\gamma = 1/\mathcal{I} = A^{-5/3}$ . (Note, however, that in a quantum system the moment of inertia often is not proportional<sup>9]</sup> to  $A^{5/3}$ .) Therefore, it seems prudent to revisit the variation of the moments of inertia between neighboring even- and odd- $A$  nuclei having normal deformations,  $\beta = 0.2 - 0.3$ , where large bases of experimental data exist. This is the topic of the present report.

## 2. THE CONDITION FOR "IDENTICAL BANDS"

To understand identical bands in neighboring nuclei it is necessary to consider the energy of a valence particle coupled to an axially-symmetric rotating nucleus. The particle-rotor model<sup>4]</sup> is the appropriate model (see refs. 8,10] for justification).

$$E_I = \epsilon_K + \frac{\hbar^2}{2\mathcal{I}} [I(I + 1) - K^2 + (-1)^{I+1/2} a (I+1/2) \delta_{K,1/2}] \quad (1)$$

$\mathcal{I}$  is the moment of inertia;  $K = \Omega$  is the component of intrinsic angular momentum on the nuclear symmetry axis; and  $\epsilon_K$  is the intrinsic excitation energy (bandhead energy) associated with this configuration.

The decoupling parameter,  $a$ , is given<sup>4]</sup> in the spherical basis ( $n_j$ ) by

$$a = \sum_{n_j} (-1)^{j-1/2} (j+1/2) |c_{n_j}|^2 \quad (2)$$

The decoupling parameter can be related<sup>7]</sup> to the total aligned angular momentum of the unpaired valence particles,  $i$ , by

$$2i = (-1)^{I-1/2} a \quad (3)$$

The gamma-ray transition energy,  $E_\gamma$ , for quadrupole transitions between states of  $I$  and  $I - 2$  is a function of the total angular momentum,  $I$ ; the moment of inertia,  $\mathcal{J}$ ; and the decoupling parameter,  $a$ :

$$\begin{aligned} E_\gamma &= E_I - E_{I-2} \\ &= \frac{\hbar^2}{\mathcal{J}} [2I - 1 + (-1)^{I+1/2} a \delta_{K,1/2}]. \end{aligned} \quad (4)$$

"High-spinners" recognize this equation without the decoupling term (i.e. the last term) as the expression for calculating the kinematic moment of inertia.

Thus, the conditions for the existence of "identical bands" in neighboring nuclei for an extended range of angular momenta are: (i) the moments of inertia for the bands must be equal (a result not expected *a priori*); and (ii) the decoupling term must exactly compensate the difference in the angular momentum between even and odd neighbors (integer and half-integer values). The latter condition occurs for  $a = 1$  and  $K = 1/2$  resulting in the identity  $E_\gamma(I + I-2) = E_\gamma(I-1/2 + I-5/2) = E_\gamma(I+1/2 + I-3/2)$ , see Figure 1. This condition provides the explanation<sup>8]</sup> for "identical" bands<sup>5]</sup> in  $^{151}\text{Tb} - ^{152}\text{Dy}$ , and  $^{150}\text{Gd} - ^{151}\text{Tb}$ , since the decoupling parameter of the  $1/2$ -[301] Nilsson orbit involved becomes one in the Pseudo SU(3) limit:

$$a = (-1)^{\tilde{N}} \delta_{\tilde{\Lambda}0}. \quad (5)$$

$\tilde{N}$  and  $\tilde{\Lambda}$  are the pseudo oscillator and pseudo orbital angular momentum quantum numbers, respectively. The quantum numbers for the  $1/2$ -[301] orbital become  $1/2$ -[200] in the pseudo formalism<sup>11]</sup>.

Indeed, equations 3) and 4) also anticipate the "quantization of alignment" discussed in ref. 7]. *If the moments of inertia of neighboring isotopes are equal, then angular momentum quantization*

## Even - Even

## Odd - Mass

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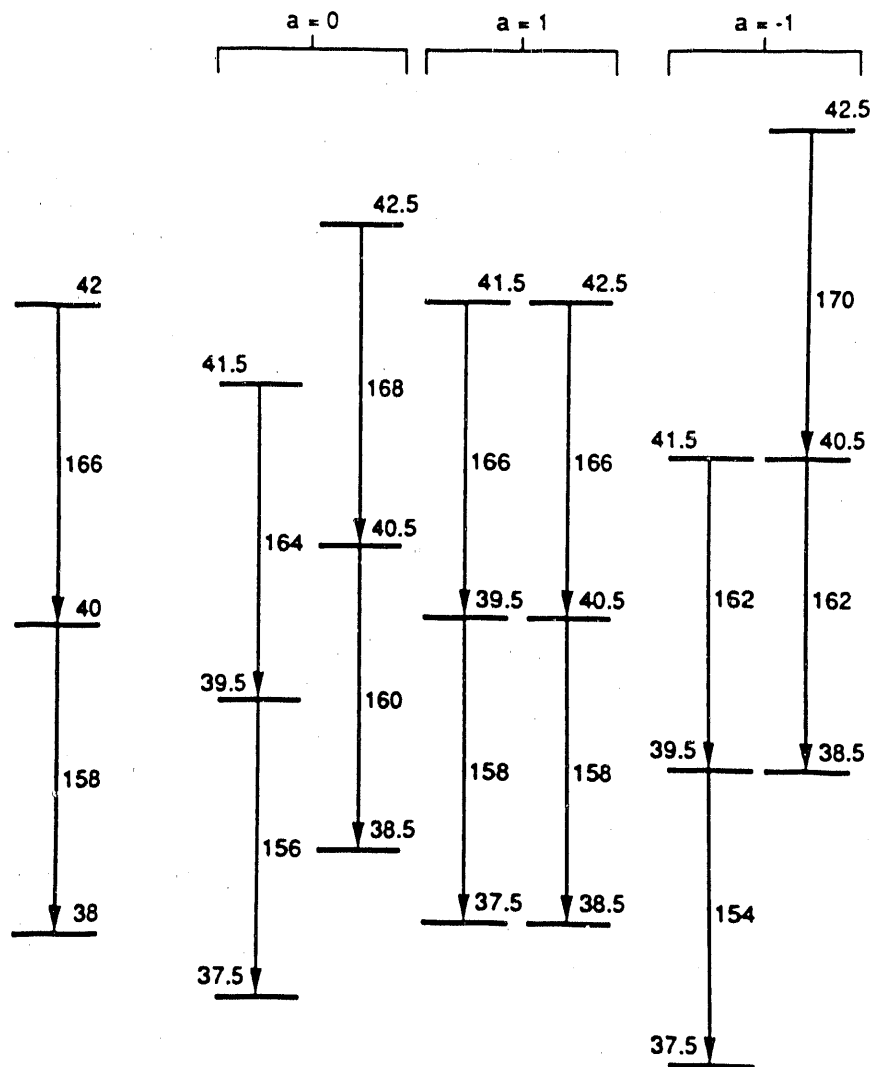


Figure 1: Comparison of the gamma-ray decay sequences near  $I = 40$  for rotating deformed even-even and odd-mass nuclei with the same moment of inertia. For odd-mass nuclei sequences are shown for limiting values of the decoupling parameter,  $a = -1, 0, \text{ and } 1$ . These data are taken from equation 4. The transition energies are given for an arbitrary moment of inertia in units of  $\hbar^2/2\mathcal{J}$ .

leads to "quantized alignments" for those configurations in which  $\Delta E_\gamma = E_\gamma [\text{odd-Z (or N)}] - E_\gamma [\text{even-even}] = 0$ . This requirement is equivalent to that discussed in refs. 12,13].

An equally interesting situation occurs for perfect negative decoupling,  $a = -1$ . The transition energies for the two signatures of the band in the odd-A nucleus equal the average transition energy of the neighboring transitions in the even-even nucleus, that is,  $E_{\gamma}(I-1/2 \rightarrow I-5/2) = E_{\gamma}(I-3/2 \rightarrow I-7/2) = 0.5 [E_{\gamma}(I \rightarrow I-2) + E_{\gamma}(I-2 \rightarrow I-4)]$ , see Figure 1 and equation (4). This condition, which remains untested for superdeformed nuclei, will be demonstrated for normal deformations in subsection 3.2.

A third condition,  $a = 0$ , valid for the numerous configurations not dominated by  $\Omega = 1/2$  components, also is depicted in Figure 1. Here the average transition energy of the two signatures in the odd-A nucleus is equal to the even-even transition, i.e.,  $E_{\gamma}(I \rightarrow I-2) = 0.5 [E_{\gamma}(I+1/2 \rightarrow I-3/2) + E_{\gamma}(I-1/2 \rightarrow I-5/2)]$ . Even though  $\Omega$  is not conserved in rotating systems, it remains a good quantum number for high-j configurations up to reasonably large rotational frequencies (see Figure 2 and the discussion in reference <sup>14</sup>). That is, the energies are independent of signature. In this regime the quadrupole deformation term in the single-particle hamiltonian dominates the Coriolis term. In contrast to the unique pattern of gamma-ray transitions for  $a = \pm 1$ , the pattern described for  $a = 0$  is valid for all values of  $a$ , providing  $a$  is equal for the two signatures of the odd-A nucleus (see equation 4).

Since the present paper is concerned with low-spin states of normally-deformed nuclei, it is appropriate to note the greatly-increased sensitivity of gamma-ray transition energies for "identical bands" in neighboring isotopes and isotones at small values of angular momentum to the decoupling parameter,  $a$ , see equation 4.

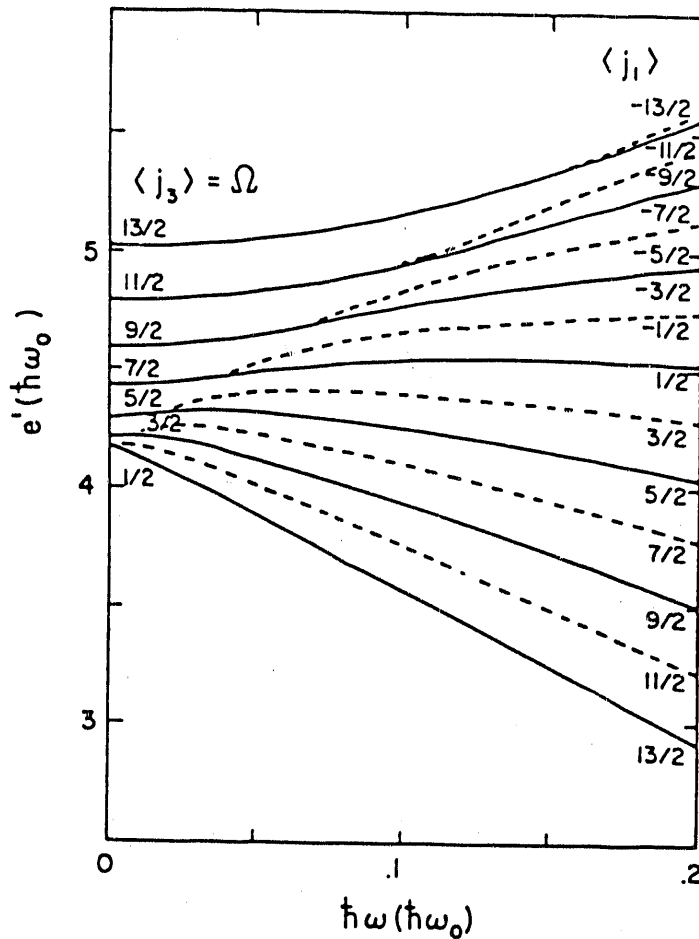


Figure 2: Calculated spectrum of single-particle states for the  $i_{13/2}$  intruder states in a rotating axially-deformed nucleus ( $\epsilon_2 = 0.25$ ) in the absence of pair correlations. Solid and dashed curves correspond to states ( $\alpha = 1/2$  and  $-1/2$ , respectively). Values of  $\langle j_3 \rangle = \Omega$  and  $\langle j_1 \rangle$  are given in the limit of no rotation and infinite rotation where they are conserved quantities.

### 3. THE ROLE OF "IDENTICAL BANDS" IN NORMALLY-DEFORMED NUCLEI

#### 3.1 Analogues of the Positively-Decoupled Superdeformed $^{151}\text{Tb}$ in Light Nuclei

The first example of a low-spin "identical band" that comes to mind is the "ancient" case<sup>15]</sup> of  $^{20}\text{Ne} - ^{19}\text{F}$ , shown in Figure 3. This example is the analogue of superdeformed  $^{152}\text{Dy} - ^{151}\text{Tb}$ . The unpaired proton in  $^{19}\text{F}$  occupies the  $1/2^- [101]$  Nilsson state, which has the same position *vis-a-vis* the p-shell as the  $1/2^- [301]$  with

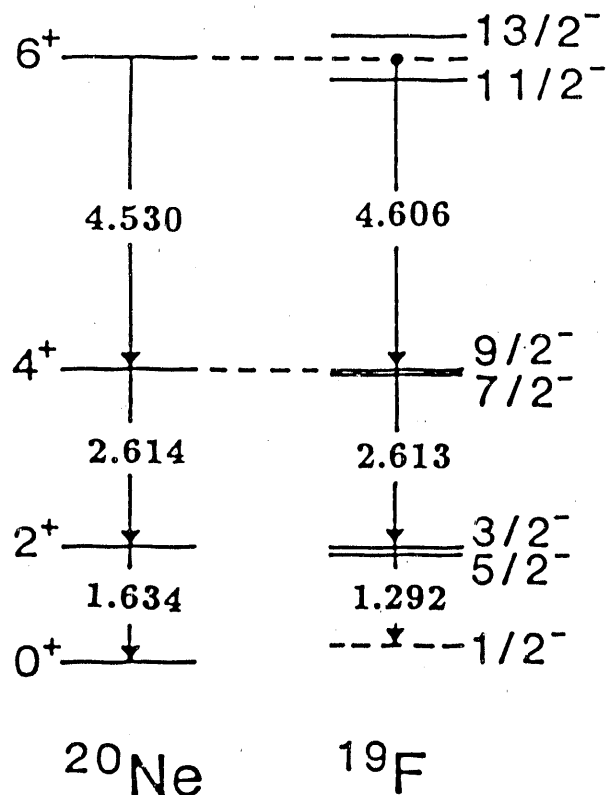


Figure 3: Comparison of the low-lying spectrum of states in  $^{20}\text{Ne}$  (left) with those states in  $^{19}\text{F}$  associated [15] with an unpaired  $1/2^- [101]$  proton. These data are taken from reference [17].

respect to the fp-shell. Neon-20 -  $^{19}\text{Ne}$  with a valence  $1/2^- [101]$  neutron provides a similar example<sup>16,17</sup>]; however, the data are less complete than for  $^{19}\text{F}$ . These light examples of "identical bands" are remarkable in light of the eight percent variation in the moment of inertia expected for macroscopic rotors of  $A = 19$  and  $20 - \mathcal{I} \propto A^{-5/3}$  and the oft expected unstable shapes of light nuclei. On the other hand, these light nuclei have many features in common with superdeformed nuclei, e.g., large deformations and essentially, no pair correlations. For example, the quadrupole deformation of  $^{20}\text{Ne}$  is predicted<sup>18</sup>] to be  $\epsilon_2 = 0.35$ .

Analogs of the positively-decoupled bands in superdeformed  $^{151}\text{Tb}$  based on the  $[301]$ ,  $[501]$ , ... orbitals are rare in heavy normally-deformed nuclei. For normal deformation such orbitals occur just below shell closures, hence the nuclei are not stably-deformed when these configurations are near the Fermi level. Other examples of positively-decoupled bands ( $a = 1$ , see Figure 1) should occur for single-nucleon occupation of the  $1/2^- [321]$ ,  $1/2^- [541]$ , and  $1/2^- [521]$

orbitals ( $1/2^-[\tilde{2}20]$ ,  $1/2^-[\tilde{4}40]$ , and  $1/2^-[\tilde{4}20]$  respectively, in the pseudospin formalism). The  $1/2^-[\tilde{3}21]$  and  $1/2^-[\tilde{5}41]$  configurations lie low in their respective shells; therefore, they usually are not observed in deformed nuclei<sup>19]</sup> except as intruders in the next lower shell. Likewise, the Coriolis term in the single-particle hamiltonian ( $= -\omega j_1$ ) mixes other components into the wave function of these configurations even at small rotational frequencies,  $\omega$ , because of the large values of  $j_1$  (the component of the intrinsic angular momentum along the rotational axis), see Table 1. This mixing for high- $j$ , low- $\Omega$  configurations is shown in Figure 2. Indeed, the observed decoupling parameter for the  $1/2^-[\tilde{5}41]$  quasiproton configuration (and other configurations with large values of  $j_1$ ) do not agree with the pseudo SU(3) limiting value, see Table 1. Thus, the  $1/2^-[\tilde{5}21]$  orbital is the best candidate for additional examples of positively-decoupled "identical bands" in normally-deformed nuclei. Positive decoupling is known for quasineutron bands based on this configuration, see Table 1 and ref. 19. A preliminary analysis of this configuration in the ytterbium isotopes indicates the presence of nearly "identical bands" when this orbit is far from the Fermi level. These observations presuppose the findings of the next subsection. Nearly "identical bands" also have been observed<sup>26]</sup> in neighboring odd- $Z$  and odd-odd lutetium isotopes differing by a  $1/2^-[\tilde{5}21]$  neutron.

### 3.2 Negatively-Decoupled "Identical Band" Based on the $1/2^+[\tilde{4}11]$ Orbital — The Role of Pair Correlations

Negatively-decoupled "identical bands" ( $a = -1$ , see Figure 1) have not yet been observed in superdeformed nuclei. In the pseudo-SU(3) limit such bands should occur when the odd nucleon occupies a normal-parity state with an even number of oscillator quanta ( $N$  even and  $\tilde{N}$  odd) — see equation 5. The  $1/2^+[\tilde{4}31]$  and  $1/2^+[\tilde{4}11]$  Nilsson states (with pseudospin quantum numbers  $1/2^+[\tilde{3}30]$  and  $1/2^+[\tilde{3}10]$ , respectively) are candidates for this decoupling. Indeed, negative decoupling is observed for the  $1/2^+[\tilde{4}11]$  proton band (see Table 1),

Table 1 Decoupling Parameter Summary for  $\Omega = 1/2$  Bands  
in Rare Earth Nuclei<sup>a)</sup>

Nilsson Config.	Pseudo spin	$j_1$ <sup>b)</sup>	#Cases <sup>c)</sup>	Decoupling Parameter	
				Exp. Ave. <sup>d)</sup>	Pseudo <sup>e)</sup> SU(3)
<u>Proton States — Odd-Z Nuclei</u>					
[431]	$\widetilde{[330]}$	3.1	0	-	-1
[420]	$\widetilde{[321]}$	3.2	8	1.26 (.40)	0
[411]	$\widetilde{[310]}$	2.6	23	-0.79 (.18)	-1
[400]	$\widetilde{[301]}$	1.4	8	0.15 (.20)	0
[541]	$\widetilde{[440]}$	4.0	28	4.25 (1.73)	1
[550]	f	5.4	1	-1.11	f
<u>Neutron States — Odd-N Nuclei</u>					
[400]	$\widetilde{[301]}$	1.4	6	0.11 (.37)	0
[541]	$\widetilde{[440]}$	3.6	0	-	1
[530]	$\widetilde{[431]}$	4.1	7	0.06 (.70)	0
[521]	$\widetilde{[420]}$	3.3	37	0.61 (.23)	1
[510]	$\widetilde{[411]}$	2.9	28	0.07 (.18)	0
[501]	$\widetilde{[400]}$	1.4	0	-	1
[660]	f	6.5	2	3.86 (1.05)	f
[651]	[550]	5.0	2	2.22 (1.52)	-1

a) Data as tabulated in tables XXVI and XXVII of ref. 19.

b) Projection of intrinsic angular momentum on rotational axis, see eq. (10), as calculated from Nilsson wave functions<sup>25]</sup> at  $\delta = .30$ .

c) Number of experimental cases tabulated in ref. 19.

d) Average experimental decoupling parameter tabulated in ref. 19. The standard deviation is given in parenthesis.

e) Decoupling parameter in pseudo SU(3) limit, see ref. 11.

f) Unique- (nonnormal-) parity state. Not included in pseudospin formalism. In limit of pure configuration and no rotation

$a = (-1)^{j-1/2}(j+1/2) = -6$  and  $7$  for  $1/2[550]$  and  $1/2[660]$ , respectively.

which plays a similar role in the  $N = 4$  shell to the  $1/2^-[521]$  configuration *vis-a-vis* the  $N = 5$  shell. The  $1/2^+[411]$  configuration is negatively decoupled vice the positive decoupling of the  $1/2^-[521]$  configuration because the decoupling parameter,  $a$ , is proportional to  $(-1)^{\tilde{N}}$ , see equation 5. Again, the  $1/2^+[431]$  configuration lies near the shell gap and is not observed in stably deformed nuclei<sup>19</sup>].

The gamma-ray energy difference for transitions in neighboring odd-Z and even-even isotopes differing by a  $1/2^+[411]$  quasiproton is shown in Figure 4 as a function of  $E_\gamma$  for a series of lutetium isotopes. In this figure, the analogue of the figure used to demonstrate "identical" superdeformed bands<sup>5</sup>],

$$\begin{aligned} \Delta E_\gamma = & 0.5 [E_\gamma(Z-1; I-1/2 + I-5/2) + E_\gamma(Z-1; I-3/2 + I-7/2)] \\ & - 0.25[E_\gamma(Z; I + I-2) + E_\gamma(Z; I-2 + I-4) + E_\gamma(Z-2; I + I-2) \\ & + E_\gamma(Z-2; I-2 + I-4)]. \end{aligned} \quad (6)$$

Here  $I$  and  $Z$  are even integers. That is, not only are the neighboring transitions in the even-even system averaged to account for negative decoupling,  $a = -1$  (see Section 2 and Figure 1), but the transitions in the two signatures of the odd-Z system (which would be equal for perfect decoupling) also are averaged. Likewise, to minimize differences in deformations the transition energies of neighboring even-even isotones also are averaged.

The difference in  $E_\gamma$  associated with the occupation of a  $1/2^+[411]$  quasiproton vary from one part per thousand for  $^{175}\text{Lu}$  to greater than ten percent for  $^{165}\text{Lu}$ , see Figure 4. This demonstrates that in certain circumstances the moment of inertia apparently is not reduced in normally-deformed odd-A nuclei relative to their "fully-correlated" even-even neighbors. That is, *sequences in normally-deformed neighboring isotones also can be "identical."* In fact, the decrease in the magnitude of  $\Delta E_\gamma$  (associated with a reduced moment of inertia for the occupation of the  $1/2^+[411]$  quasiproton) is anti-correlated with the bandhead energy of this configuration. This correlation is demonstrated even more clearly in Figure 5, where the lowest

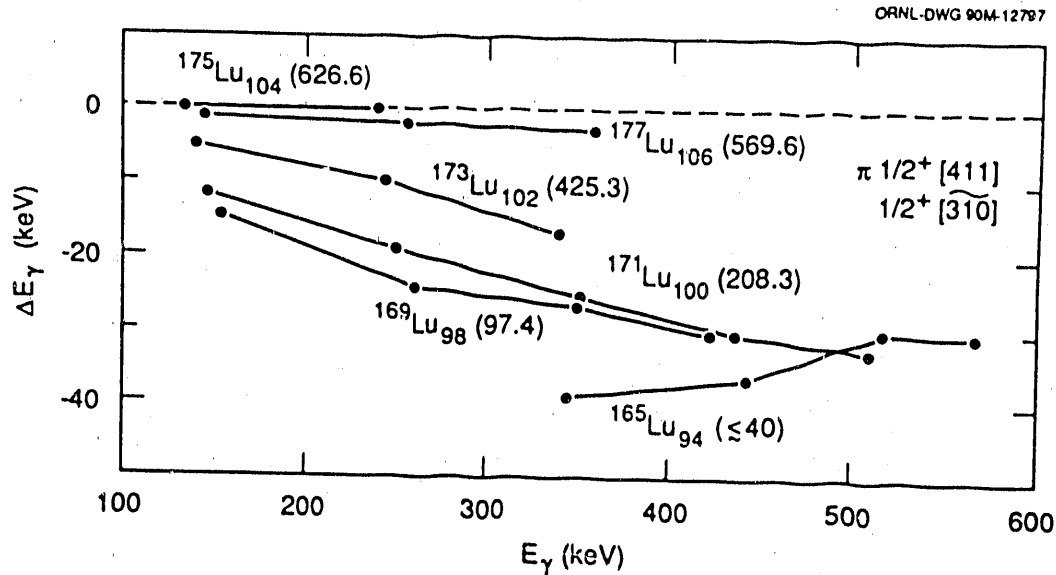


Figure 4: Differences in gamma-ray transition energies between states in the odd-mass lutetium isotopes, associated with the  $1/2^+[411]$  quasiproton configuration, and the yrast states in the neighboring ytterbium and hafnium isotones,  $\Delta E_\gamma$ , as a function of the gamma-ray transition energies in the lutetium isotopes. Neighboring transitions in the even-even systems (accounting for negative decoupling --see Figure 1 and equation (4)); transitions in the two signatures of the odd-Z system (equal in the fully decoupled limit --see Figure 1); and transitions in neighboring even-even isotones (to minimize variations in the mean field) are averaged as defined in equation (6). The energy of the  $1/2^+[411]$  bandhead is given in keV (in parenthesis) for each lutetium isotope. These data are taken from the most recent Nuclear Data Sheets.

spin point for  $\Delta E_\gamma$  (corresponding to the averaged  $5/2^+ + 1/2^+$  and  $7/2^+ + 3/2^+$  transitions for odd-Z and the averaged  $2^+ + 0^+$  and  $4^+ + 2^+$  transitions for even-even -- see equation 6) -- is shown as a function of the  $1/2^+[411]$  bandhead energy for a variety of holmium, lutetium, and tantalum isotopes.

Indeed, we have "rediscovered" the microscopic nature of pair correlations. When a configuration is far from the Fermi level, it does not contribute to the pair correlations so the pairing contributions to the moment of inertia is unaffected by its occupation. Mathematically, the product of the occupation and deoccupation parameters ( $V$  and  $U$

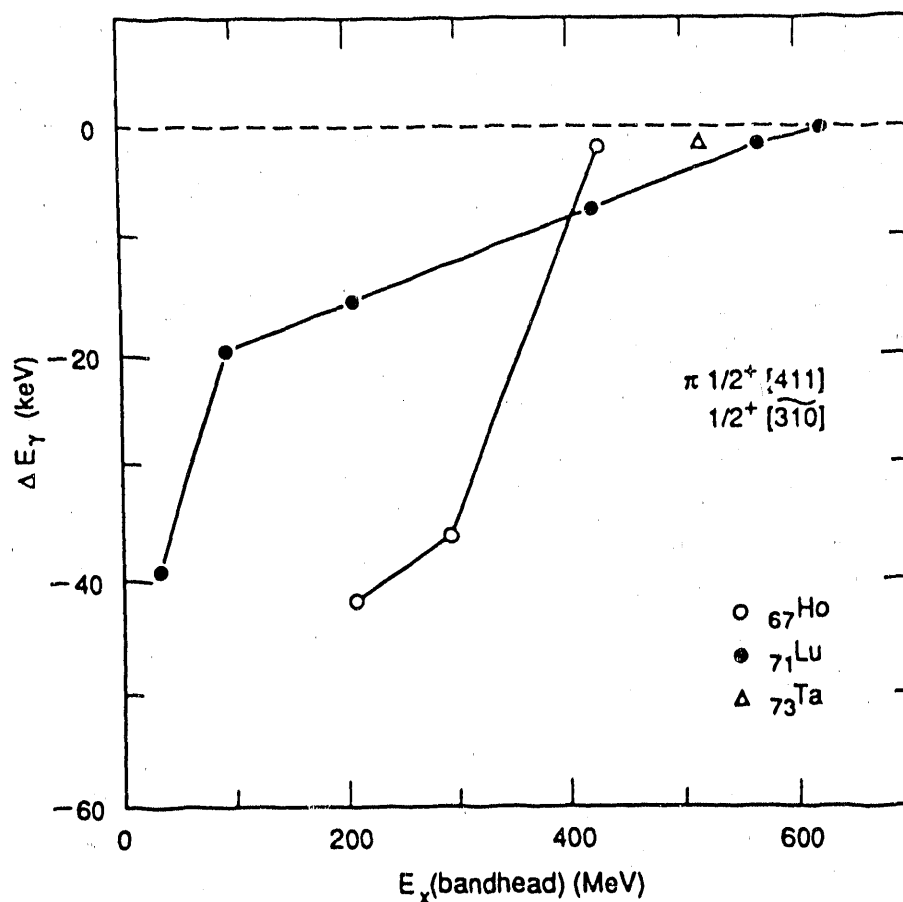


Figure 5: Correlation between,  $\Delta E_\gamma$ , the difference in gamma-ray transition energy between the lowest states in the odd-mass lutetium isotopes, associated with the  $1/2^+[411]$  quasiproton configurations in holmium, lutetium, and tantalum isotopes and that of the yrast states in the neighboring isotones and the excitation energy of the bandhead of this configuration in the odd-Z isotopes. The appropriate averages described in the caption to Fig. 4 and the text were also made for these data. These data are taken from the most recent Nuclear Data Sheets.

respectively) is small when the configuration is far from the Fermi level. Thus, the pair gap,  $\Delta$ , given<sup>4]</sup> by

$$\Delta = G_0 \sum_i U_i V_i, \quad (7)$$

remains essentially unchanged when the occupied, or "blocked" odd orbital is removed from the summation. Hence, the kinematic moment of inertia, given<sup>20]</sup> microscopically by

$$\mathcal{J}^{(1)} = \sum (j_i / \omega) V_i \quad (8)$$

also remains unchanged, if  $j_1$ , (often denoted  $j_x$ ) the component of intrinsic angular momentum on the rotational axis, is small. ( $\omega$  is the angular frequency of rotation.) If pair correlations are reduced by the occupation of an odd single-particle, the occupation coefficients,  $V$ , also are modified leading to an increased occupation of the highly-aligned orbits (those with large values of  $j_1$ ). These orbits, lying low in the shell, contribute to larger moment of inertia, and thus an increased "blocking" effect. An analogous expression also can be written for the dynamical moment of inertia<sup>20]</sup> with a similar dependence on the pair correlations through the occupation coefficients.

### 3.3 The Configuration Dependence of "Identical Bands" — the Role of $j_1$

A correlation between  $\Delta E_\gamma$ , the shift in transition energy between odd-A nuclei and neighboring even-even isotones, and the bandhead energy also is observed for other configurations, e.g., the  $7/2^+[404]$  (see Figure 6) and the  $3/2^+[411]$  (see ref. 21]) quasiprotons. In this case the appropriate averages for  $a = 0$  (equal values of  $a$  in the two signatures of the odd-A nucleus) are used — see equation (4) and Figure 1. That is,

$$\begin{aligned} \Delta E_\gamma = & 0.5 [E_\gamma(Z-1; I+1/2 + I-3/2) + E_\gamma(Z-1; I-1/2 + I-5/2)] \\ & - 0.5 [E_\gamma(Z; I + I-2) + E_\gamma(Z-2; I + I-2)]. \end{aligned} \quad (9)$$

Again,  $I$  and  $Z$  are even integers. In the case of the  $7/2^+[404]$  band, sizable shifts in  $\Delta E_\gamma$  remain for the largest bandhead energies, indicating the importance of a degree of freedom other than pair correlations. Indeed,  $\Delta E_\gamma$  is expected to be anticorrelated with  $j_1$ , see equations (4) and (8). This correlation is demonstrated in Figure 7 where  $\Delta E_\gamma$  is shown as a function of  $E_\gamma$  for a variety of configurations<sup>22]</sup> in  $^{167}\text{Lu}$  having nearly the same bandhead energy, but with values of  $j_1$  varying between 1.9 and 4.9 (for the bandhead). The component of intrinsic angular momentum along the rotational axis,  $j_1$ , is given by

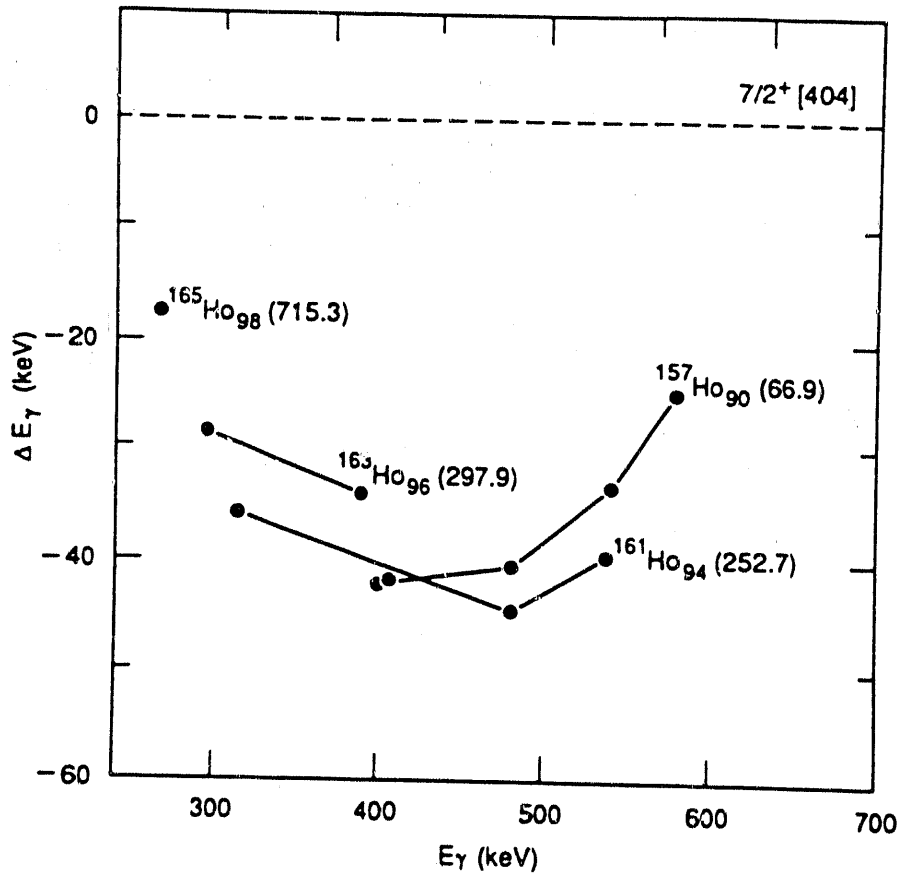


Figure 6: Differences in gamma-ray transition energies between states in the odd-mass holmium isotopes, associated with the  $7/2^+[404]$  quasiproton configuration, and the yrast states in the neighboring dysprosium and erbium isotones,  $\Delta E_\gamma$ , as a function of the gamma-ray transitions energies in the holmium isotopes. The two signatures of the odd-Z system (see equation (4) and Figure 1) and the transitions in neighboring even-even isotones (to minimize variations in the mean field) are averaged as defined in equation (9). The energy of the  $7/2^+[404]$  bandhead is given in keV (in parenthesis) for each holmium isotope. These data are taken from the most recent Nuclear Data Sheets.

$$j_1 = \sqrt{j(j+1) - K^2} . \quad (10)$$

In terms of the cranking model<sup>23]</sup>  $j_1 = -de'/d\omega$ . That is,  $j_1$  is the negative of slope on a routhian plot of  $e'(\omega)$ .

Thus to complete the story it is necessary to consider the aligned angular momentum associated with the "identical bands," since  $j_1$  will contribute to the moment of inertia. In the absence of pair correlations equation (8) reduces to  $\mathcal{J}(1) = \sum j_1/\omega$ , where the summation is over occupied orbitals. Indeed, the  $j_1$  for the  $1/2^- [301]$

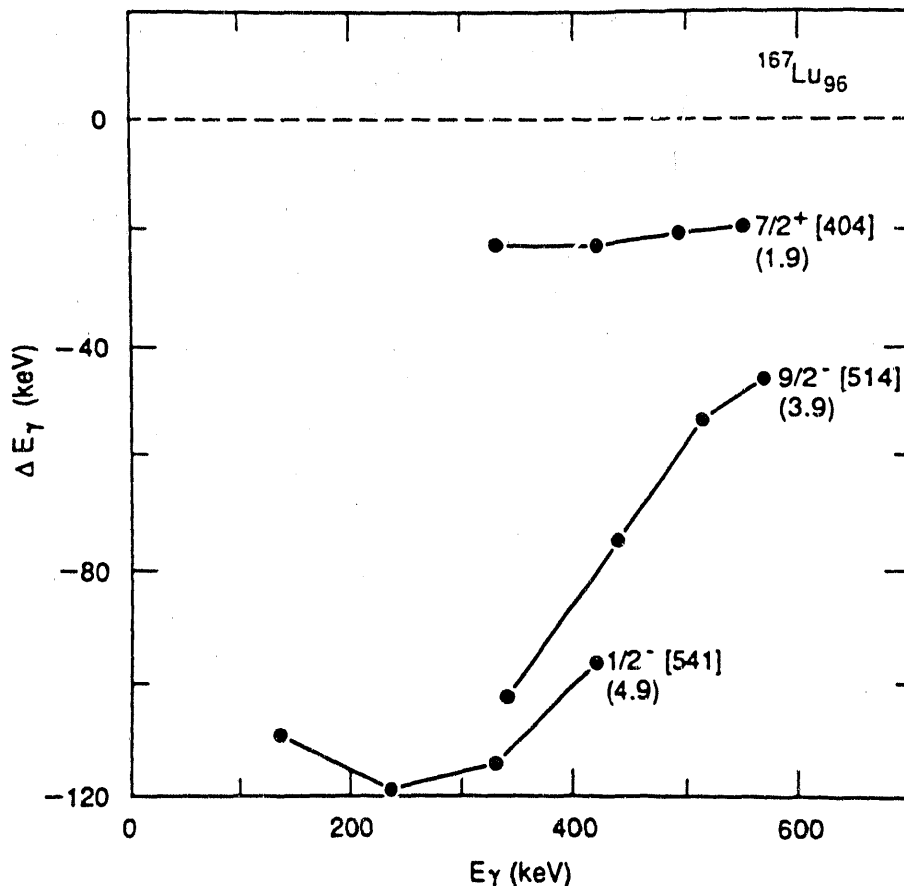


Figure 7: Differences in gamma-ray transition energies between states [22] in  $^{167}\text{Lu}$ , associated with three different quasiproton configurations, and the yrast states in the neighboring isotones,  $^{166}\text{Yb}$  and  $^{168}\text{Hf}$ , are shown as a function of the gamma-ray transition energies in  $^{167}\text{Lu}$ . The two signatures of the odd-Z system (see equation (4) and Figure 1) and the transitions in neighboring even-even isotones (to minimize the variations in the mean field) are averaged as defined in equation (9). The value of  $j_1$ , the component of the intrinsic angular momentum aligned with the rotational axis, — see equation (10) — is given in parenthesis for each isotope.

configuration associated with the superdeformed  $^{151}\text{Tb}$  "identical band" is small: ( $= 1.3$  for  $\delta = 0.3$ ). Though  $\Omega = 1/2$  for this configuration, its wave function is dominated by shell-model components having small values of  $j$ . For large deformations the small  $j$ -values associated with the  $1/2^-$ [301] configuration remain rather constant even at a sizable rotational frequency due to the lack of nearby orbitals that mix with this configuration<sup>8</sup>].

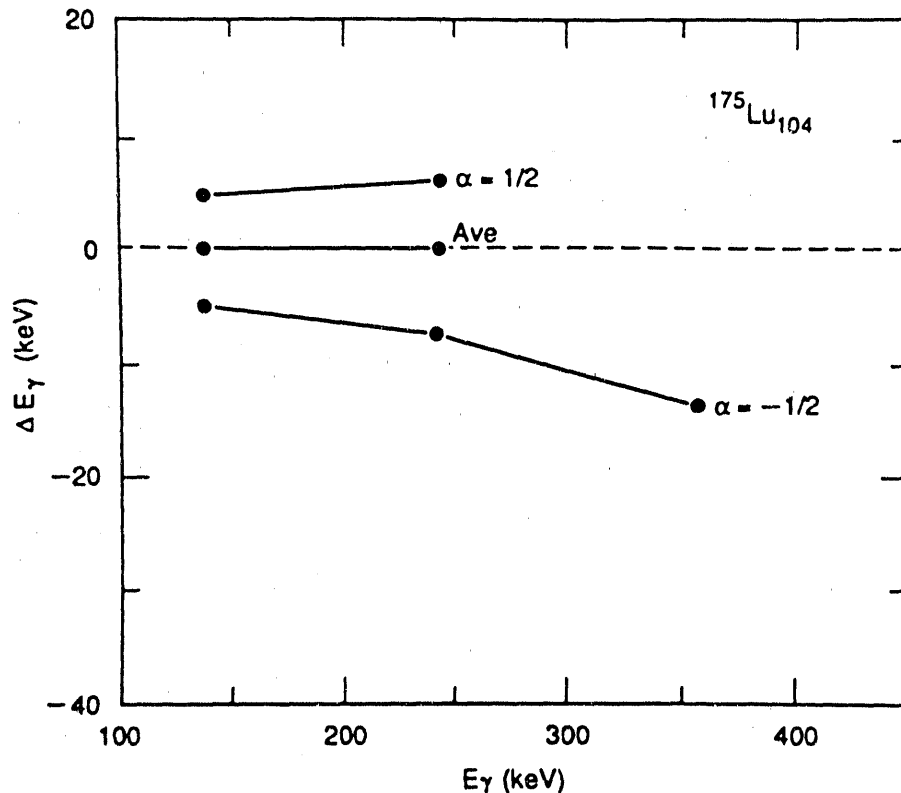


Figure 8: Illustration of the signature dependence of  $\Delta E_\gamma$  for the  $1/2^+[411]$  proton configuration in  $^{175}\text{Lu}$ , the best normally-deformed "identical band." The values of  $\Delta E_\gamma$  are defined by equation (6), except those values shown for the separate signatures (labeled  $\alpha = \pm 1/2$ ) do not include the signature average in the odd-Z system. The signature-averaged values (labeled Ave) are identical to those shown for this nucleus in Figure 4.

In contrast, the  $j_1$  associated with the  $1/2^+[411]$  configuration is somewhat larger (see Table 1), due to shell-model components with larger  $j$  values. However, we have masked the blocking effect for this configuration by averaging the two signatures. The  $\Delta E_\gamma$  values for the two signatures of this configuration in  $^{175}\text{Lu}$ , the best normally-deformed "identical band," are compared in Figure 8 with the average value of this quantity. Though the variation in  $E_\gamma$  between odd- and even-mass isotones is of the order of a few parts per thousand for the average, it increases to about three percent for each signature individually. Indeed, unpaired cranking-model calculations indicate a sizable  $j_1$  nearly equal in magnitude, but of opposite sign,

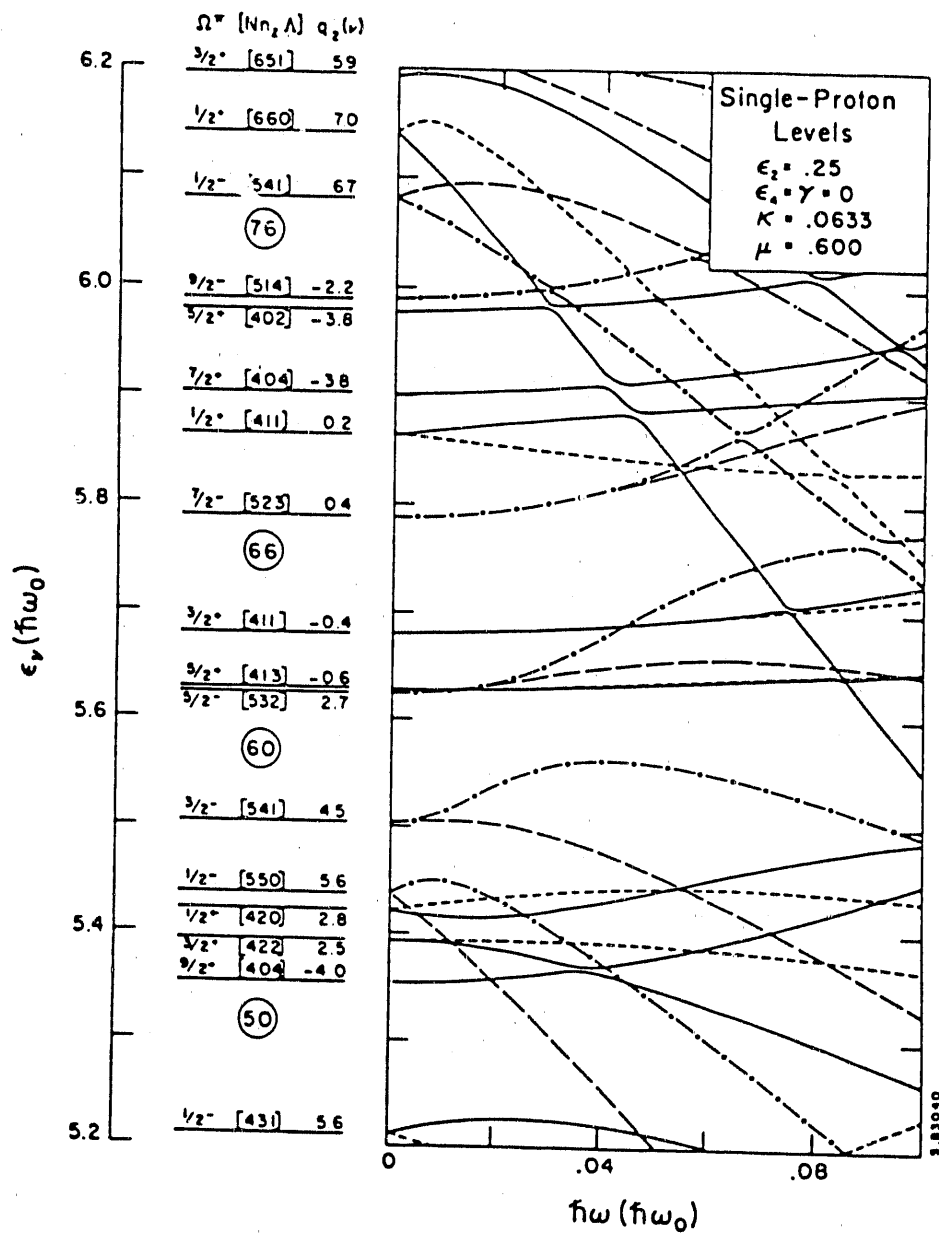


Figure 9: Spectrum of single-proton states calculated using the modified oscillator potential [23] for a rotating quadrupole-deformed nucleus ( $\epsilon_2 = 0.25$ ) in the absence of pair correlations.  $(\pi, \alpha) = (+, 1/2), (-, -1/2), (-, 1/2),$  and  $(+, -1/2)$  states are denoted by solid, short-dashed, dot-dashed, and long-dashed curves, respectively. To the left the asymptotic Nilsson quantum numbers,  $\Omega^\pi [N, n_z, \Lambda]$ , are given for each orbit. These quantum numbers, of course, are only valid for  $\hbar\omega = 0$ . Note that  $j_1$ , the contribution of each orbital to the aligned angular momentum, is the negative of the slope of  $\epsilon(\omega)$ .

for the two signatures of the  $1/2^+[411]$  proton configuration (see Figure 9).

The occupation of both signatures of configurations with equal-magnitude, opposite-signed values of  $j_1$  also will tend to cancel the  $j_1$  contributions to the moment of inertia in a manner similar to averaging the two signatures. Therefore, the existence of "identical bands" in neighboring isotope and isotones differing by one nucleon, is a more sensitive probe of the microscopic composition of the moment of inertia than the occurrence of such bands in neighboring even-even isotopes and isotones. This may be an explanation of the existence of nearly "identical bands" in e.g., superdeformed  $^{192}\text{Hg}^{24}$ ] and  $^{194}\text{Hg}^6$ ], where the Fermi level is near a group of high- $\Omega$  states that have not yet been strongly perturbed by the Coriolis interaction. Though these states are associated with sizable aligned angular momenta,  $j_1$ , in lowest order the contributions of the two signatures cancel.

#### 4. SUMMARY

The gamma-ray transition energies in neighboring odd- and even-mass isotones and isotopes for normally-deformed nuclear configurations are analyzed in a manner similar to that recently used for superdeformed configurations. Many of the uncertainties of the superdeformed data, such as spin, parity, and configuration assignments and relative excitation energies for the various decay sequences, are resolved for the normally-deformed cases. These data indicate that the "identical bands" observed for superdeformed nuclei can also occur in normally-deformed nuclei when: (i) the "blocked" orbital lies outside the pair gap; (ii) the aligned angular momentum,  $j_1$ , of the "blocked" orbital is very small; and (iii) the two signatures of the same configuration are averaged or summed. Such findings imply that the existence of "identical bands" in neighboring odd- and even-mass nuclei are associated with weak pair correlations. (See ref. 10 for a more comprehensive discussion of pair correlations in superdeformed nuclei.) Likewise, these data show that the existence of "identical bands" in neighboring nuclei differing by a single nucleon is a more sensitive probe of the microscopic composition

of the moments of inertia than the occurrence of such bands in neighboring isotopes and isotones differing by two nucleons.

Though a detailed comparison of the same configuration for super- and normal-deformation (outside of the pair gap) remains to be made, it appears that the best examples of "identical bands" are the superdeformed cases. The larger deformations "usually" separate configurations with large Coriolis matrix elements thereby reducing the configuration mixing induced by rotation. Likewise, the larger angular momentum of the superdeformed cases reduces the effects of nonperfectly-decoupled bands, see equation (4).

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