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AUTHOR(S): Takaharu Otsuka

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Los Alamos National Laboratory
Los Alamos, New Mexico 87545

MICROSCOPIC CALCULATION FOR DEFORMED NUCLEI

Takaharu Otsuka

Theoretical Division, Los Alamos National Laboratory

Los Alamos, New Mexico 87545

and

Japan Atomic Energy Research Institute, Tokai, Ibaraki, Japan

ABSTRACT

The microscopic basis of the Interacting Boson Model for deformed nuclei is discussed. The IBM Hamiltonian is constructed microscopically in the following two steps. In the first step, the collective nucleon pairs of $J=0^+$ (S), 2^+ (D), etc. are mapped onto the corresponding bosons. Nucleon-nucleon interactions are also mapped onto boson-boson inter actions. This mapping method for deformed nuclei was proposed recently, and it turned out that this method is consistent with the Hartree-Fock-Bogoliubov + angular momentum projection calculation. Low-lying collective states primarily consist of S and D pairs. Consequently, the corresponding boson states mainly consist of s and d bosons, while there are some admixture of g-bosons. In the second step, effects of these g-bosons are included within the s-d boson space by a unitary transformation which transforms a combination of d and g bosons into a new d-boson. By minimizing the coupling between "new" d and g bosons with an appropriate mixing angle, one can neglect the coupling and obtain the IBM Hamiltonian with s and d bosons. It is demonstrated that the s-d Hamiltonian thus derived indeed reproduces spectra of the original s-d-g Hamiltonian.

1. INTRODUCTION

I am going to talk about a microscopic study of the Interacting Boson Model (IBM)[1] for deformed nuclei. This work has been carried out in collaboration partly with Joe Ginocchio, and also partly with Naotaka Yoshinaga.

I shall begin with a brief review of the IBM from the microscopic point of view. There are two major microscopic assumptions for the IBM. Assumption I is the following[2-11] : There are two collective nucleon pairs of $J=0^+$ (S) and $J=2^+$ (D). The S and D pairs are coherent nucleon pairs. Since there are neutrons and protons, there are neutron S and D pairs, and proton S and D pairs. It is assumed that these pairs play dominant roles in low-lying collective states. This may be regarded as a generalization of the BCS theory so that one can treat the quadrupole deformation in terms of coherent pairs.

Assumption II is that the S and D fermion pairs can be approximated by the s and d bosons. The Hamiltonian of the s and d bosons is assumed to consist of the single boson energy and the boson-boson interaction. This assumption reduces the tremendous complexity of multifermion problems. In this talk, we test the validity of these assumptions in deformed nuclei, and show how one can construct the IBM Hamiltonian from a microscopic Hamiltonian.

2. S-D PAIR DOMINANCE IN THE INTRINSIC STATE OF DEFORMED NUCLEI

The ground state band of the deformed nucleus is described by the intrinsic state,[12] which is well approximated by a condensate state of the Cooper pair in the deformed single-particle orbits.[13] This Cooper pair is denoted hereafter as the Λ pair created by the $\Lambda^\dagger$ operator. The amplitudes in $\Lambda^\dagger$ are determined, for instance, by the BCS calculations in the deformed orbits. The intrinsic state ϕ of an N-pair system is written as $\phi \propto (\Lambda^\dagger)^N|0\rangle$ (Ref. 6).

The $\Lambda^\dagger$ operator is rewritten as $\Lambda^\dagger = \sum_J x_J A^{\dagger(J)}$, where x_J denotes amplitudes, and $A^{\dagger(J)}$ is obtained by projecting from $\Lambda^\dagger$ onto an angular momentum J (see Ref. 6). The operators $A^{\dagger(J)}$ are denoted by $S^\dagger$, $D^\dagger$ and $G^\dagger$ for J=0, 2 and 4, respectively[6];

$$\Lambda^\dagger = x_0 S^\dagger + x_2 D_0^\dagger + x_4 G_0^\dagger + \dots . \quad (1)$$

It has been shown[6-9] that the S-D probability, $x_0^2 + x_2^2$, is more than 85% in deformed nuclei with the deformation parameter $\delta \sim 0.30$ and the pairing gap $\Delta \sim 1.0$ MeV. For an N-pair system (2N = the number of valence nucleons), however, one has to consider the N-th power of $\Lambda^\dagger$;

$$(\Lambda^\dagger)^N = (x_0 S^\dagger + x_2 D_0^\dagger)^N + N\, x_4 G_0^\dagger (x_0 S^\dagger + x_2 D_0^\dagger)^{N-1} + \dots . \quad (2)$$

The pure S-D component in eq. (2) becomes less and less dominant for larger N and fixed x_J's. This could be a serious problem for large N. It is then of great interest to see whether this fact is relevant to physical observables or not. As an example, I would like to consider the intrinsic quadrupole moment Q_{in} (of the ground band), which is written as $Q_{in} = \langle\Lambda^N|Q_0|\Lambda^N\rangle$ with $|\Lambda^N\rangle$ being the normalized intrinsic state, and Q_0 being the m=0 component of the one-body quadrupole operator Q_m.

I begin with considering a schematic example in order to make discussions as transparent as possible. This example is the purely aligned limit where the normal phase appears as the solution of the BCS calculation. In this limit, $\Lambda^\dagger$ is given by uniform v-factors below the Fermi level and vanishing v-factors above it.[6] In order to calculate Q_{in}, we first evaluate an unnormalized matrix element, $\langle 0|\Lambda^N Q_0 (\Lambda^\dagger)^N|0\rangle$, which is the overlap between the state, $(\Lambda^\dagger)^N|0\rangle$, and the state,

$$Q_0(\Lambda^\dagger)^N|0\rangle = N\langle\Lambda|Q_0|\Lambda\rangle\ (\Lambda^\dagger + \lambda^\dagger)\ (\Lambda^\dagger)^{N-1}|0\rangle \quad . \quad (3)$$

Here, the $\lambda^\dagger$ operator is introduced as, $[Q_0, \Lambda^\dagger] = \langle\Lambda|Q_0|\Lambda\rangle\ (\Lambda^\dagger + \lambda^\dagger)$, with orthogonality $\langle 0|\Lambda\lambda^\dagger|0\rangle = 0$. In the aligned limit, one can easily obtain $\langle 0|\Lambda^N \Sigma^\dagger (\Lambda^\dagger)^{N-1}|0\rangle = 0$, and then,

$$\langle 0|\Lambda^N Q_0 (\Lambda^\dagger)^N|0\rangle = N\langle\Lambda|Q_0|\Lambda\rangle \langle 0|\Lambda^N(\Lambda^\dagger)^N|0\rangle \ . \tag{4}$$

In other words, $\Sigma^\dagger$ appears in eq. (3) as a consequence of the commutator $[Q,\Lambda^\dagger]$, while it has no effect on Q_{in} due to the complete cancellation among various terms in $\Sigma^\dagger$. Since the quantity in eq. (4) has to be normalized by the norm $\langle 0|\Lambda^N(\Lambda^\dagger)^N|0\rangle$, one finally obtains a simple relation,

$$Q_{in} = \langle\Lambda^N|Q_0|\Lambda^N\rangle = N\langle\Lambda|Q_0|\Lambda\rangle \ . \tag{5}$$

It should be pointed out that eq. (5) is exact in the aligned limit, and that all effects of the Pauli principle are included. The N-pair matrix element is given by the product of the number of pairs, N, and the one-pair matrix element $\langle\Lambda|Q_0|\Lambda\rangle$, while the complicated N-pair norm is removed. In other words, as far as Q_{in} is concerned, the Λ pairs are "independent of each other" or behave as "spectators". This is referred to as the "independent-pair" property of condensed pairs. Eq. (5) implies that the fraction of the S-D contributions in $\langle\Lambda^N|Q_0|\Lambda^N\rangle$ is determined by the one-pair matrix elements $\langle\Lambda|Q_0|\Lambda\rangle$. Using eq. (1), the S-D contribution can be calculated. The total S-D fraction turned out to be more than 70% for reasonably large N, due to the S-D dominance in each Λ-pair (see Ref. 6). Thus, the S and D pairs play dominant roles in the quadrupole matrix element, and the amplitudes of spin-J components in each Λ pair are indeed crucial for this physical observable.

I now turn to more realistic cases where the super-phase takes place in the BCS calculation. Up to eq. (3) there is no difference from the purely aligned limit. The difference arises, when $\Sigma^\dagger$ is acted on $(\Lambda^\dagger)^{N-1}|0\rangle$. In the super-phase case, occupation probabilities are different among deformed single-particle orbits, and lower orbits are more occupied than higher orbits. Thus, lower orbit terms in $\Sigma^\dagger$ are more suppressed than higher orbit terms. Therefore, in spite of the orthogonality $\langle 0|\Lambda\Sigma^\dagger|0\rangle=0$, the state $\Sigma^\dagger(\Lambda^\dagger)^{N-1}|0\rangle$ is not orthogonal to $(\Lambda^\dagger)^N|0\rangle$. In fact, by examining the single-particle amplitudes of $\Sigma^\dagger$, it can be seen that $\Sigma^\dagger(\Lambda^\dagger)^{N-1}|0\rangle$ is comprised primarily of $(\Lambda^\dagger)^N|0\rangle$ usually. I then introduce a quantity representing this non-orthogonality,

$$\varepsilon = - \langle 0|\Lambda^N\Sigma^\dagger(\Lambda^\dagger)^{N-1}|0\rangle/\langle 0|\Lambda^N(\Lambda^\dagger)^N|0\rangle \ . \tag{6}$$

This parameter ε has a positive value $0.3 \sim 0.4$ for usual deformed nuclei, and is quite insensitive to δ and Δ. This negative non-orthogonality therefore yields a blocking effect on the Q operator, reducing $\langle 0|\Lambda^N Q_0(\Lambda^\dagger)^N|0\rangle$ from the r.h.s. on eq. (4) by a factor $(1-\varepsilon)$. Since Q_{in} is an average of quadrupole matrix elements of low-lying ground-band members, ε represents the mean non-orthogonality blocking effect in this region. Although there should be other blocking effects which are contained in the norm $\langle 0|\Lambda^N(\Lambda^\dagger)^N|0\rangle$, this norm does not appear in the normalized matrix element.

Similar to eq. (5), $Q_{in}=\langle\Lambda^N|Q_0|\Lambda^N\rangle=N(1-\varepsilon)\langle\Lambda|Q_0|\Lambda\rangle$ is obtained. This expression is regarded as a generalization of eq. (5) and is also refered to as the "independent-pair" property, since ε is very insensitive to δ and Δ and $\Sigma^\dagger(\Lambda^\dagger)^{N-1}|0\rangle$ is primarily $\propto(\Lambda^\dagger)^N|0\rangle$. Using eq. (1), Q_{in} is expanded as,

$$Q_{in} = N(1-\varepsilon)[2x_0x_2\langle S|Q_0|D_0\rangle + x_2^2\langle D_0|Q_0|D_0\rangle + \dots] \ , \tag{7}$$

where $\langle S|Q_0|D_0\rangle = \langle D_0|Q_0|S\rangle$ etc. are used. The fraction of the S-D pair contributions is now evaluated. The coefficient $(1-\varepsilon)$ is left in this evaluation, since it represents an overall reduction. As an example, I take a system of 16 neutrons in the N=82-126 major shell with realistic spherical single-particle energies.[12] For δ=0.3, a Nilsson+BCS calculation is carried out, and the Λ pair is obtained with Δ = 0.8 MeV. Resultant square of these amplitudes are x_0^2=33.8%, x_2^2=54.2% and x_4^2=11.5%. Using these amplitudes, fractions of contributions from various terms in eq. (7) are calculated. Fractions from $\langle S|Q_0|D_0\rangle$, $\langle D_0|Q_0|D_0\rangle$, $\langle D_0|Q_0|G_0\rangle$ and $\langle G_0|Q_0|G_0\rangle$ are, respectively, 52%, 18%, 25%, and 4%. The S-D pairs account for 70% of Q_{in}, while the total probability of the pure S-D components in $|\Lambda^N\rangle$ is much less (see eq. (2)). One thus finds contributions from the S-D pairs dominant in the N-pair matrix element. The same conclusion is obtained for other deformed nuclei.

3. FERMION-BOSON MAPPING FOR DEFORMED NUCLEI

The "independent-pair" property which means essentially that the complicated fermion N-pair norm can be eliminated in the normalized N-pair matrix element, leads us to a new fermion-boson mapping method as described in the following. The nucleon Λ pair is mapped onto a boson,

$$\Lambda^\dagger = x_0 S^\dagger + x_2 D_0^\dagger + x_4 G_0^\dagger \quad \to \quad \lambda = x_0 s^\dagger + x_2 d_0^\dagger + x_4 g_0^\dagger \ . \tag{8}$$

Consequently, the nucleon intrinsic state is mapped as,

$$|\Lambda^N\rangle \quad \to \quad |\lambda^N) . \tag{9}$$

The boson image of the nucleon quadrupole operator is given as,

$$Q \to Q^B = q_1(d^\dagger s + s^\dagger \tilde{d}) + q_2[d^\dagger \tilde{d}]^{(2)} + q_3\,[g^\dagger \tilde{d} + d^\dagger \tilde{g}]^{(2)} + \dots \tag{10}$$

with coefficients $q_1 = (1-\varepsilon)\langle S||Q||D\rangle/\sqrt{5}$, $q_2 = (1-\varepsilon)\langle D||Q||D\rangle/\sqrt{5}$, etc. In this mapping, the equality $\langle\Lambda^N|Q_0|\Lambda^N\rangle = (\lambda^N|Q_0^B|\lambda^N)$ holds (see eq. (7)). I note that the "independent-pair" property contains the bosonic structure characterized by the factorization $(\lambda^N|Q_0^B|\lambda^N) = N\cdot(\lambda|Q_0^B|\lambda)$.

The validity of this mapping is examined by looking at other matrix elements. As an example, I consider matrix elements related to a state $|D_2\Lambda^{N-1}\rangle$, where D_2 denotes the m=2 component of D_m. This state is considered to be one of the major components of the γ-band intrinsic state. Matrix elements $\langle D_2\Lambda^{N-1}|Q_0|D_2\Lambda^{N-1}\rangle$ and $\langle D_2\Lambda^{N-1}|Q_2|\Lambda^N\rangle$ are compared in table 1 with the corresponding boson predictions $(d_2\lambda^{N-1}|Q_0^B|d_2\lambda^{N-1})$ and $(d_2\lambda^{N-1}|Q_2^B|\lambda^N)$. Note that the diagonal matrix element above is related to the γ-band intrinsic quadrupole moment which is the same order of magnitude as Q_{in}. The off-diagonal one is related to the ground-gamma E2 transition which is one order of magnitude less than Q_{in}. Table 1 clearly demonstrates that these fermion matrix elements are reproduced very well by the one-body boson operator Q^B.

The agreement exhibited in table 1 suggests that the non-orthogonality blocking effect for $|D_2\Lambda^{N-1}\rangle$ over deformed single-particle orbits is not very different from that for $|\Lambda^N\rangle$. In fact, only one pair is different between the two states; it is either the D_2 component of the D-pair or the Λ pair in which D_0 is one of the major components. I emphasize that the agreement in table 1 is not exceptional and a similar agreement can be seen generally in deformed nuclei. Further investigation on the validity of the present mapping will be reported in a forthcoming paper.

Parameters of boson quadrupole operators are evaluated for ^{158}Gd. Here, δ=0.25 was taken, and the pairing strength was chosen so that Δ~0.9 MeV. The Λ pair was calculated from the Nilsson+BCS. Following the above mapping procedure, parameters of Q^B in eq. (10) are calculated.

In order to construct an s-d boson (or IBM) system, the g-boson is eliminated in the next step, and its effects are taken into account by renormalization of s-d boson terms by the method of Ref. 5. Although the renormalization method has been improved considerably since Ref. 5 as discussed in a subsequent section, essential properties in this table remain unchanged. The quadrupole field Q is responsible for the admixture of the g-boson.[5] Since Q is mapped onto the one-body boson field Q^B, the renormalization can be done for each boson separately. In other words, when a d-boson is coupled to a g-boson by Q^B, the other bosons behave as spectators. This independence property simplifies appreciably the renormalization procedure.[5)]

Parameters of the s-d boson quadrupole operator are thus calculated microscopically with the renormalization. The result of this calculation should be compared to that obtained by phenomenological fitting IBM calculations. In table 2, these two results are shown in a convention in which the boson quadrupole operator is defined as

$$Q^B_\tau = d^\dagger_\tau s_\tau + s^\dagger_\tau \tilde{d}_\tau + \chi_\tau [d^\dagger_\tau \tilde{d}_\tau]^{(2)} \qquad (11)$$

with $\tau=\pi$ (proton) or ν (neutron) and χ_τ being a parameter, and the boson proton-neutron QQ interaction is defined as $-\kappa Q^B_\pi \cdot Q^B_\nu$ with the strength κ. The strength of the nucleon QQ interaction is taken from Ref. 14. The boson E2 operator is written as $T^{(E2)} = e^B_\pi Q^B_\pi + e^B_\nu Q^B_\nu$ with the boson effective charge e^B_τ. The nucleon effective charges are assumed to be 1.7e for protons and 0.7e for neutrons. In table 2, a reasonable agreement is s[illegible] between the microscopically calculated parameters and those obtained by fitting calculations. In table 2, unrenormalized values of the parameters are also indicated to show changes due to the renormalization.

These parameters are calculated also by the method of Ref. 10 (OAI), where the IBM parameters are determined by states $|S^N\rangle$ and $|S^{N-1}D\rangle$. This method is useful in and near spherical nuclei, while it yields, in deformed regions, too large a value of κ and too small a value of $|\chi|$ compared to the results of phenomenological fitting. Large $|\chi|$ is essential to obtain rotational spectra. In fact, $|\chi|=\sqrt{7}/2$ is the SU(3) limit.[15] This discrepancy is now solved, by introducing the new mapping where the blocking from a coherent linear combination of S, D and G is included properly.

4. RELATION TO THE ANGULAR MOMENTUM PROJECTION

I shall in this section talk about the consistency between the fermion-boson mapping introduced above and the Hartree-Fock-Bogoliubov (HFB) + angular momentum projection calculation. The fermion intrinsic wave can be written in the HFB scheme as,

$$\phi^F \propto (\Lambda^\dagger)^N |0\rangle \quad . \tag{12}$$

The boson analogue of ϕ^F is written, in the mean field approximation, as,

$$\phi^B \propto (\lambda^\dagger)^N |0) \ . \tag{13}$$

where $\Lambda^\dagger$ is defined in eq. (1) and $\lambda^\dagger$ is defined in eq. (8). The intrinsic wave function contains 0^+, 2^+, 4^+, etc. states of the ground state rotational band. The probability to find spin I member in the intrinsic state is calculated as,

$$P_I^X = \frac{[(2I+1)/2] \int d(\cos \vartheta)\, d_{00}^{\,I}(\vartheta) \langle \phi^X \hat{R}(\theta)\, \phi^X \rangle}{\langle \phi^X\ \phi^X \rangle} \tag{14}$$

where $d_{00}^{\,I}(\theta)$ denotes the d function, $\hat{R}(\theta)$ is $\exp(i\theta\, R_y)$, X stands for F (fermion) or B (boson), and the axially symmetric deformation is assumed.

The rotated wave function $\hat{R}(\theta)\phi$ is written as

$$\hat{R}(\theta)\, \phi^F \propto (K^\dagger)^N\ |0\rangle \tag{15}$$

with

$$K^\dagger = e^{\,i\theta_y \hat{R}} \Lambda^\dagger e^{-i\theta \hat{R}_y} \ , \tag{16}$$

and

$$\hat{R}(\theta)\, \phi^B \propto (\kappa^\dagger)^N\ |0\rangle \tag{17}$$

with

$$\kappa^{\dagger} = e^{i\theta \hat{R}_y} \lambda^{\dagger} e^{i\hat{R}_y} \tag{18}$$

Using eqs. (15) and (17), P_I^F and P_I^B are expressed as,

$$P_I^F = \frac{[(2I+1)/2] \int d(\cos\theta)\ d_{00}^{I}(\theta) \langle\Lambda^N K^{\dagger N}\rangle}{\langle\Lambda^N \Lambda^{\dagger N}\rangle} \tag{19}$$

and

$$P_I^B = \frac{[(2I+1)/2] \int d(\cos\theta)\ d_{00}^{I}(\theta)\ (\lambda^N \kappa^{\dagger N})}{(\lambda^N \lambda^{\dagger N})} \tag{20}$$

It is observed that fermion overlap in eq. (18) is approximated as

$$\langle\Lambda^N K^{\dagger}\rangle \sim \langle\Lambda K^{\dagger}\rangle^N \langle\Lambda^N \Lambda^{\dagger N}\rangle \tag{21}$$

for $\theta \ll 1$. Because of the equality,

$$\langle\Lambda K^{\dagger}\rangle = (\lambda\kappa^{\dagger}), \tag{22}$$

eq. (21) becomes

$$\langle\Lambda^N K^{\dagger N}\rangle \sim (\lambda\kappa^{\dagger})^N \langle\Lambda^N \Lambda^{\dagger N}\rangle \tag{23}$$

It is thus shown that P_I^F is equal to P_I^B in the approximation of eq. (21). I note that all θ dependence of $\langle\Lambda^N K^{\dagger N}\rangle$ are included in $(\lambda\kappa)^N$ in eq. (22), since $\langle\Lambda^N \Lambda^{\dagger N}\rangle$ is θ independent. In other words, the fermion intrinsic state and the boson intrinsic state have the same structure. Although the fermion norm $\langle\Lambda^N \Lambda^{\dagger N}\rangle$ is quite different from the boson norm $(\lambda^N \lambda^{+N})$, this difference is not relevant to normalized quantities as P_I^F and P_I^B. The approximation in eq. (21)

indicates that orthogonality in the one pair space persists in multi-pair space. This is a generalization of the independent pair property discussed in the previous section.

The approximation of eq. (21) is examined numerically for a deformed nucleus, as shown in table 3. The fermion wave function in eqs. (1) and (12) was obtained by the HFB calculation with particle number conservation. The intrinsic system has deformation parameter $\sim$ 0.25 and pairing gap $\sim$ 0.7 MeV. The boson intrinsic wave function was obtained by eqs. (9) and (13). The probabilties P_I^F in eq. (19) and P_I^B in eq. (20) are compared in table 3. One finds an excellent agreement, which suggests that the mapping in eq. (9) is indeed appropriate for deformed nuclei.

We have similarly calculated matrix elements of the interaction $Q_\pi Q_\nu$. The corresponding boson matrix elements are calculated for the boson operator in eq. (10). Again, one finds good agreement in table 4 between fermion and boson calculations. We thus conclude that the mapping method introduced in the previous section reproduces the HFB + angular momentum projection calculation.

5. RENORMALIZATION OF g-BOSON EFFECTS

I have so far discussed the fermion-boson mapping, in which nucleon pairs of S, D, G, etc. are mapped onto bosons s, d, g, etc. One can simply neglect effects of bosons of 6^+, 8^+, and higher. Effects of g-bosons, however, have to be included. The g-boson is mixed through the following term in the Q Q interaction,

$$f\ q_3\ \ Q_\pi \cdot (g_\nu^\dagger \tilde{d}_\nu + d_\nu^\dagger \tilde{g}_\nu)^{(2)} \tag{24}$$

where f is a coupling constant, and q_3 is introduced in eq. (10). This interaction can be rewritten as

$$f\, q_3 \;\; \left((Q_\pi g_\nu^{\dagger})^{(2)} \cdot \tilde{d}_\nu \;+\; d_\nu^{\dagger} \cdot (Q_\pi \tilde{g}_\nu)^{(2)} \right). \tag{25}$$

Therefore, if $(Q_\pi g_\nu^{+})^{(2)}$ behaves like a quadrupole boson, effects of the g boson can be treated by including $(Q_\pi g_\nu^{+})^{(2)}$ as a part of the "d" boson. For this purpose, we introduce a unitary transformation,

$$U = e^{\phi_\nu Q_\pi \cdot (g_\nu^{\dagger} \tilde{d}_\nu \,-\, d_\nu^{\dagger} g_\nu)^{(2)} \,+\, \phi_\pi Q_\nu \cdot (g_\pi^{\dagger} d_\pi \,-\, d_\pi^{\dagger} \tilde{g}_\pi)^{(2)}} \tag{26}$$

A unitary transformation of the form $U = e^{Z}$ (Z is an operator) can be written in general as

$$U\, X\, U^{-1} \;=\; X + [\, Z, X \,] + \tfrac{1}{2}[\, Z, [\, Z, X \,]] + \;\ldots.. \tag{27}$$

If one has relations,

$$[\, Z, [\, Z, X \,]] = -\alpha^{2} \;\; X \tag{28}$$

and

$$[\, Z, [\, Z, [\, Z, X \,]]] = -\beta^{2} \;\; [\, Z, X \,] \tag{29}$$

one obtains,

$$U\, X\, U^{-1} \;=\; X \;\; \cos\alpha \;\; + [\, Z, X \,] \;\; \frac{\sin\beta}{\beta} \;\; . \tag{30}$$

I shall apply eq. (27) to X = d with the transformation in eq. (26). One can obtain easily,

$$[\, Z, d_\nu^{\dagger} \,] \;\; = \;\; \phi_\nu \, [\, Q_\pi \, g_\nu^{\dagger}]^{(2)} \tag{31}$$

where Z stands for ln U for U in eq. (26). In deriving eq. (31) and also in the following, we assume that Q_π and Q_ν in U commute with any operator. In other words, Q_π and Q_ν in eq. (26) are treated as if they are c-numbers. Matrix elements of [Q, X] (X = arbitrary operator) should be much smaller than those of Q itself for low-lying collective states, since the coherent property of Q should be lost in [Q, X]. This assumption is clearly related to the collectivity of states, and also of Q_π and Q_ν.

The double commutator is written as

$$[\ Z,\ [\ Z,\ d_\nu^\dagger]] = -\phi_\nu^2 \sum_K \sqrt{5}\ \sqrt{2K+1}\ \ W(2\ 4\ K\ 2\ .\ 2\ 2) \times [\ [Q_\pi\ Q_\pi\ ]^{(K)}\ d_\nu^\dagger]^{(2)} \tag{32}$$

Although K in eq. (32) runs from 0 to 4, the K=0 coefficient is largest, and the scalar product $[Q\ Q]^{(K=0)}$ is enhanced in quadrupole collective states. It is, hence, a reasonable approximation to retain only the K=0 term in eq. (32). Secondly, we replace the operator $[Q\ Q]^{(0)}$ with its expectation value $<[Q\ Q]^{(0)}>$ with respect to an appropriate state, for instance, the intrinsic state of the rotational band or the ground state. Eq. (32) then becomes

$$-\phi_\nu^2 <[Q\ Q]^{(0)}> \frac{1}{\sqrt{5}}\ \ d_\nu^\dagger \tag{33}$$

This equation has the same form as eq. (28) with $\alpha = \sqrt{\phi_\nu [Q\ Q]^{(0)}\ \ /\sqrt{5}}$. We can thus evaluate all terms in eq. (27), and finally obtain the transformed form of the d operator as in eq. (30).

In practical calculation, terms of K≠0 in eq. (32) are also included effectively, by replacing these terms with

$$\frac{<[Q\ Q]^{(K)} d^\dagger>}{<[Q\ Q]^{(0)} d^\dagger\ >}\qquad [\ [Q\ Q^{(0)}\ d^\dagger\ \ ] \tag{34}$$

where $\langle \rangle$ stands for the expectation value of an appropriate state as discussed just above. In other words, effects of the K≠0 terms are included in some averaged way. Deviation or fluctuation from the approximation in eq. (34) can be included order by order in principle.

Based on the above prescription, all terms in the original boson Hamiltonian are transformed. The mixing angle ϕ_ν and ϕ_π in U can be determined from amplitudes in Eq. (8) for deformed nuclei. The mixing angle will be determined in other regions by utilizing a boson analogue of the RPA approximation. It is expected that the angle stays rather constant. The major effect of the unitary transformation is found in the single d boson energy. As already mentined at Eq. (25), the unitary transformation treats $(Qg^\dagger)^{(2)}$ as a part of the new d boson. Thus, the interaction in Eq. (25) is transformed as a part of the single d boson energy. This actually means an enormous reduction of the single d boson energy. The reduction is expressed as

$$Q_\pi \cdot (g_\nu^\dagger \tilde{d}_\nu + d_\nu g_\nu)^{(2)} \rightarrow (Q_\pi Q_\pi)\, \hat{n}_{d_\nu} \qquad (35)$$

where some numerical factors are omitted. Since the ground-state expectation value of (Q Q) becomes largest in the middle of the valence shell as a function of proton and neutron numbers, the reduction of the single d boson energy is largest there. Note that this trend is consistent with the conclusion of phenomenological studies.

We applied the unitary transformation to an s-d-g boson Hamiltonian

$$H_{s-d-g} = \epsilon_{d_\pi} n_{d_\pi} + \epsilon_{g_\pi} n_{g_\pi} + \epsilon_{d_\nu} n_{d_\nu} + \epsilon_{g_\nu} n_{g_\nu} - f\, Q_\pi Q_\nu \quad , \qquad (36)$$

where Q_π and Q_ν are given by Eq. (10). The strength parameter f is adjusted so that this Hamiltonian reproduces the experimental spectrum of ^{168}Gd.

The calculated spectrum is shown in Fig. 1. The spectrum of an sd Hamiltonian obtained by the above unitary transformation is also shown in Fig. 1. These two spectra are in an excellent agreement. The unitary transformation indeed works well. In Fig. 1, there is the spectrum of another s-d boson Hamiltonian which was obtained by dropping all g-boson terms in eq. (35). This spectrum has a scale three times larger than the other two, although the rotational spectrum is already seen. The comparison among these three spectra demonstrates the importance and usefulness of the unitary transformation. I note that the present method is not a perturbation, and that one can treat considerably large admixture of g bosons. After the unitary transformation, coupling between the new d boson and the new g boson becomes weak, and may be treated by perturbation, if one wished.

5. SUMMARY

I summarize this talk by mentioning the following two points.
(i) Matrix elements of operators in many nucleon collective states can be related to matrix elements of these operators in few nucleon states with an overall reduction due to blocking effects. This leads us to a simple fermion-boson mapping, even for deformed nuclei with many valence nucleons.
(ii) The g boson, which arises as a result of the fermion-boson mapping, can be eliminated by a unitary transformation from a combination of d and g bosons into a new d boson.

With this mapping principle and renormalization technique, a IBM Hamiltonian can be derived from a fermion Hamiltonian.

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Table 1. Comparison between exact and boson quadrupole matrix elements (fm^2) related to the D_2 pair for (a) the 16 neutron system in the N=82-126 shell with δ=0.3 and Δ=0.8 MeV and for (b) the 16 proton system in the Z= 50-82 shell with δ=0.25 and Δ=0.9 MeV. $Q_{in} = \langle\Lambda^N|Q_0|\Lambda^N\rangle$ $(=(\lambda^N|Q_0^B|\lambda^N))$ is also shown.

case	Q_{in}	$\langle D_2\Lambda^{N-1}\|Q_0\|D_2\Lambda^{N-1}\rangle$ exact	boson	$\langle D_2\Lambda^{N-1}\|Q_2\|\Lambda^N\rangle$ exact	boson
(a)	179	150	150	12	9
(b)	108	88	91	14	13

Table 2. Parameters in the boson QQ interaction and in the boson E2 calculated microscopically by the present mapping method with the renormalization due to elimination of g-boson. In the column "(unr.)", the unrenormalized result is shown in parentheses. Parameters obtained by a fitting calculation and those by the OAI method are also shown.

parameter	micro.	(unr.)	fitting	OAI
κ (Mev)	0.094	(0.12)	0.08 ~ 0.09	0.19
χ_π	-0.86	(-0.80)	-0.8 ~ -0.9	0.04
χ_ν	-1.18	(-1.03)	-1.1 ~ -1.2	-0.55
e_π^B(e fm^2)	12.0	(10.6)	12 ~ 14	14.3
e_ν^B(e fm^2)	10.0	(6.7)	12 ~ 14	8.4

Table 3 Probabilities of spin-I members in the intrinsic state

I	fermion (P_I^F)	boson (P_I^B)
0^+	3.44 %	3.46 %
2^+	15.44	15.58
4^+	21.63	21.93
6^+	21.20	21.57
8^+	16.51	16.74
10^+	10.75	10.71
12^+	6.01	5.78

Table 4 Matrix elements of the interaction f ($Q_\pi Q_\nu$)

I	fermion	boson
0^+	14.99 MeV	15.05 MeV
2^+	14.99	15.05
4^+	14.99	15.05
6^+	14.98	15.04
8^+	14.96	14.98
10^+	14.92	14.91

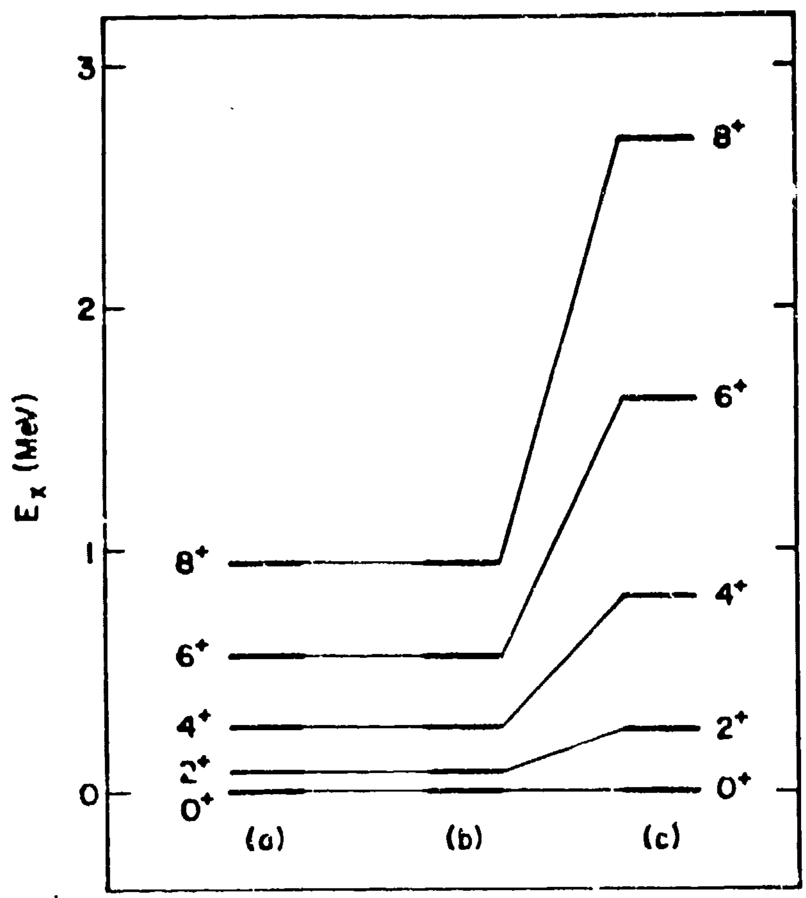

Fig. 1. Spectra obtained (a) from a Hamiltonian containing s, d, and g bosons, (b) from the s-d boson (IBM) Hamiltonian calculated from the above s-d-g boson Hamiltonian by the unitary transformation, and (c) from the s-d boson Hamiltonian obtained by just dropping all g-boson terms in the above s-d-g boson Hamiltonian.