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PROBES OF INITIAL-STATE INTERACTIONS IN DILEPTON ANGULAR DISTRIBUTIONS

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(To be published in Proceedings of the Workshop on Drell-Yan Processes, Fermilab (1982))

ABSTRACT

We discuss the angular distribution of dileptons $d\sigma/d^4Qd\Omega$, emphasizing phase sensitivity as a probe of initial-state interactions in QCD. The coherent nature of Sudakov effects is discussed, along with the presence of imaginary parts related by analyticity. Angular-distribution structure functions which describe interference between longitudinal and transverse virtual photons, e.g., can be used to probe phase differences that depend on large momenta. These evolve according to $\exp(ic\ \ell n\ \ell n(\tilde{Q}^2/\Lambda^2_{QCD}))$ where $\tilde{Q}^2$ is a large scale. We report on a complete calculation at $O(\alpha_s^2)$ of the $q\bar{q} \to \gamma^* +$ gluons channel which confirms the cancellation of small (cutoff) scales, and describe a complementary experiment involving spin. We discuss the limit $x \to 1$ of the distribution $d\sigma/dQ^2dxd\cos\theta$, and point out an unusual and interesting effect that a momentum-dependent phase can produce here.

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QCD is a theory employing interaction via the exchange of massless vector gluons. The 1/r character of the perturbative interaction generally produces similar logarithmic behavior for both very large and very small energy scales, which presuming factorization, can be separated into short and long distance dependence. Although it has been understood for some time that a coulomb-like phase shift must accompany the distorted long-distance propagation of color charges, the short distance, large momentum transfer implications of phase dependence has not been emphasized until recently.[1] By analogy with QED we call the leading, large momenta dependent component of the phase the Chromo-Coulomb Phase Shift (CCP). Since the momentum dependence at the large endpoint arises from gluons interacting from a short (renormalization scale) distance $x_i \sim 1/\sqrt{\mu^2}$ to asymptotically small distances $x_f \sim 1/\sqrt{s}$, one should be able to calculate the effect in perturbation theory. These observations lead to two questions:

1) Can the CCP be experimentally exploited to teach us about the application of QCD to hadron physics?
2) Is the theoretical status of QCD sufficiently developed to permit a reliable calculation of phase dependence?

The answers to both of these questions at this time is "yes" for certain experiments. Lepton pair production is an excellent prototype where phase differences occur in a nearly ideal situation.

Lepton pair production is a good laboratory to study effects of the CCP, in spite of canonical notions, based on partons, that the final state sum is totally incoherent. This is because the QCD description of production of a pair at measured Q^μ hinges on Sudakov-type resummation.[2] The striking cancellation pattern of Sudakov effects amounts to a high order of destructive interference of real amplitudes. Accompanying real Sudakov corrections are imaginary parts, demanded by analyticity, which are the key to the calculability of the CCP. Since analyticity represents the underlying causal time-ordering structure of the theory, such effects can be characterized as initial (or final) state interactions.

The fact that observables are real is no excuse for ignoring phases: indeed the pair angular distribution ($d\sigma/d^4Q d\Omega$) is directly sensitive to interference between different types of production.[3] For example, the azimuthal (ϕ) distribution, which we discuss in some detail below, always involves an interference between the very different longitudinal (L) and transverse (T) virtual photon polarizations. Another interesting example concerns the region $x \to 1$, whch as pointed out by Berger and Brodsky[4], depends on the coherent sum of L and T lowest and higher twist contributions. In both of these examples the new feature that enters is the momentum dependence of the CCP.

In the situations above, e.g., the real part of the exponentiated phase is projected out through interference. Since the real part oscillates, the signal of the CCP in action is an oscillatory scaling behavior in an appropriate physical variable. This is a rather unorthodox prediction, so we should lay out the ground rules and assumptions. We presume that the applicability of perturbative QCD to LPP will be established at, e.g., the leading twist level.[2,5] The CCP need not immediately violate the usual factorization beliefs, since the dependence on the IR cutoff versus the large momenta can be arranged to factor apart; the same phenomena occurs in QED. Closely related to this is the factorization question of active/spectator interactions raised by Bodwin, Brodsky, and Lepage, which is also discussed by Lindsay, Ross and

Sachrada.[6]. The question appears to be connected with the non-commuting matrix aspects of constant contributions and is still controversial.

In our calculations we employ the usual QCD improved parton model. Standard perturbation theory at one-loop (or in some cases two-loop) order is necessary to determine the CCP coefficients which involve hard gluon momenta; the IR cutoff vanishes in momentum dependence involving the evolution of a phase difference. We begin by studying the obviously interference-sensitive terms in $d\sigma/d^4Q d\Omega$.

I. AMPLITUDE DESCRIPTION

Let us review the description of the production of a virtual photon (γ^*) in the collision between hadrons A and B. We begin at the level of the amplitude

$$\langle N|J^\mu|P_A\lambda_A P_B\lambda_B\rangle = \varepsilon^\mu_{T1}\rho_{T1}e^{i\omega_{T1}} + \varepsilon^\mu_{T2}\rho_{T2}e^{i\omega_{T2}} + \varepsilon_L\rho_L e^{i\omega_L} \qquad (1)$$

where $\langle N|$ is the final state minus the γ^*, ρ,ω are real functions of momenta, α_s, etc. and the hadron helicities $\lambda_{A,B}$ have been displayed. The symbols ε^μ_{T1}, etc. stand for the polarizations of the γ^*, which are spanned by two transverse and one longitudinal direction in a particular frame. We choose the Collins-Soper frame[7] for definiteness; here ε^μ_L, e.g., $\propto Z^\mu/\sqrt{-Z^2}$, where

$$Z^\mu = P^\mu_A Q\cdot P_B - P^\mu_B Q\cdot P_A \qquad (2)$$

The amplitudes in Eq.(1) are not themselves measurable. What is measurable is a density matrix-like object for the virtual photon:

$$W^{\mu\nu} = \sum_N \langle P_A\lambda_A P_B\lambda_B | J^\mu | N\rangle\langle N | J^\nu P_A\lambda_A P_B\lambda_B\rangle (2\pi)^4 \delta^4 (P_A + P_B - Q - P_N) \ . \qquad (3)$$

If one were trying to eliminate phases explicitly, one could argue that the probability to create a γ^* depends on diagonal elements summed so that phases cancel:

$$\sum_\mu \langle J^\mu\rangle\langle J_\mu\rangle^* \propto W^\mu_\mu \propto d\sigma/d^4Q \ .$$

For a simple enough model this might be plausible. The creation probability, however, is clearly re-distributed in dilepton phase space $\Omega = (\theta,\phi)$ acording to important off-diagonal elements of $W^{\mu\nu}$ in which phase differences occur, since

$$\frac{d\sigma}{d^4Q d\Omega} = \frac{\alpha^2}{2(2\pi)^4}\frac{1}{Q^2 s}(\delta^{ij} - \hat{\ell}^i\hat{\ell}^j)W^{ij} \ , \qquad (4)$$

where $\hat{\ell}^i$ is the direction of either lepton in the pair rest frame. We are interested in having different γ^*'s interfere, since this should involve significant phase differences. For example, the leading real $(T^*L + L^*T)$ interference term can be written

$$W^{\mu\nu}_{TL} \propto \sum_{\substack{N \\ \lambda_A\lambda_B}} \{\varepsilon^\mu_{T1}\varepsilon^{\nu*}_L \rho_{T1}\rho_L e^{i(\omega_{T1} - \omega_L)} + c.c.\} \ ; \qquad (5)$$

this term enters the general form of the angular distribution[3] via

$$\sum_{\substack{N \\ \lambda_A \lambda_B}} d\sigma/d^4Q d\Omega \propto W_{TT}(1 + \cos^2\theta) + W_{LL}\sin^2\theta$$
$$+ W_{TL}\sin 2\theta \cos \phi + W_{T-T}\sin^2\theta \cos 2\phi \ . \tag{6}$$

From Eq.(5,6) we see that the sin 2θ cos ϕ coefficient $W_{TL}(Q^2 s, \ldots)$ depends sensitively on

$$\mathrm{Re}\ e^{i(\omega_{Tl} - \omega_L)} = \cos(\omega_{Tl} - \omega_L) \tag{7}$$

and should oscillate if $\omega_{Tl} - \omega_L$ is momentum dependent. (Of course W_{T-T}, another interference term, is also phase sensitive in the same sense). The remarks based on one photon exchange (i.e. QED) are model independent; QCD enters when one ponders whether $\omega_T(Q^2, s, \ldots; \alpha_s)$ should equal $\omega_L(Q^2, s, \ldots; \alpha_s)$. Perturbation theory can answer this question, as follows.

II. WHY PHASES DIFFER: THE HARD INTERACTION DEPENDENCE

To define angle ϕ one must have $Q_T \neq 0$; power counting in QCD shows that one quark must be far off-shell to produce ϵ_L^μ. Hence $W_{TL} \sim O(Q_T/\sqrt{\hat{s}})$, where $\hat{s}$ is the subprocess energy. We therefore consider a QCD Born term at $Q_T \neq 0$ (Fig.1a) and two kinds of loop corrections: "soft" (Fig.1b) and "hard" (Fig.1c) in Feynman gauge. Labeling momenta as shown, let us consider the region $Q_T^2 \sim k_T^2 \sim$ fixed as $k^+ \sim \eta\sqrt{s} \to \infty$ with η a small number, with $p^+, r^- \sim \sqrt{\hat{s}} \to \infty$ and $p^-, r^+ \sim$ fixed. Then the quark on the left in Fig.1 goes far off-shell $\left((r-k)^2 \equiv \tilde{p}^2 \sim \eta\sqrt{\hat{s}}\right)$, interfering with production on the right (complex conjugate) side that is nearly on mass shell $\left((p - k)^2 \sim \text{fixed}\right)$. For this configuration it is most efficient (to leading power of $Q_T/\sqrt{s}$) to attach the L photon to the left, off-shell side and the T photon to the right, on-shell side as shown. Now a key point: for the resummed cross section, logarithms from truly "soft" gluons (cutoff $\lesssim |\ell_j| \lesssim \sqrt{k_T^2}$) produce a phase which has no knowledge of the hard production point determining L or T: such phases should cancel. However, hard perturbative gluon loops with momenta $\sqrt{k_T^2} \lesssim |\ell_j| \lesssim \sqrt{s}$, e.g., have to be sensitive to the hard interaction, so here $\omega_T \neq \omega_L$. A somewhat heuristic representation of these ideas comes by attaching ordinary Sudakov form factors $\exp(-B_T)$, $\exp(-B_L)$ to the on- or off-shell vertices, respectively, and keeping track of imaginary parts. Letting cutoff dependence be regulated by quark mass (m) and gluon mass (λ), one finds[8]

$$B_T + i\omega_T = \frac{C_F\alpha_s}{2\pi}\left\{\frac{1}{2}\,\ell n^2(Q^2/m^2) - i\pi\,\ell n(Q^2/\lambda^2)\right\}$$

$$B_L + i\omega_L = \frac{C_F\alpha_s}{2\pi}\left\{\ell n(Q^2/m^2)\,\ell n(Q^2/\tilde{p}^2) + \frac{1}{2}\,\ell n^2(Q^2/\tilde{p}^2) - 2i\pi\,\ell n(Q^2/\tilde{p}^2) - i\pi\,\ell n(Q^2/m^2)\right\} \tag{8}$$

so that in this example

$$\exp\left(i(\omega_T - \omega_L)\right) = \exp\left(+\frac{C_F i\pi\alpha_s}{2\pi}\left(2\ell n(Q^2/\tilde{p}^2) - \ell n(m^2/\lambda^2)\right)\right) . \tag{9}$$

This suggests that the cutoff dependence in (m^2,λ^2) has isolated, or factored itself from the momentum dependence. The point to be made is that hard gluons for which perturbative QCD are applicable do imply $\omega_T \neq \omega_L$, and that a real calculation is called for.

III. CALCULATIONAL STRATEGY

Instead of calculating the real parts for Eq.(3) directly, we will proceed with a calculation for the imaginary parts at $O(\alpha_s^2)$. There are several reasons for this:

1) Real parts of CCP origin begin at $O(\alpha_s^3)$ in this process because $\alpha_s\cos(\omega_T - \omega_L) \sim \alpha_s - \alpha_s^3(\omega_T^{(1)} - \omega_L^{(1)})^2/2! + \ldots.$ One can see how interference develops at two-loop order; a contribution, with imaginary parts denoted by dotted lines, is shown in Fig.2. Needless to say, these are very difficult calculations.

2) Imaginary parts, on the other hand, begin at $O(\alpha_s^2)$ and are easily isolated. For example, among the many graphs at this order, comparatively few have non-vanishing imaginary parts. Furthermore, fewer still survive the T,L projection. [9]

3) Imaginary parts are directly measurable, in fact, because of a very nice experiment using a polarized proton and complementary to the measurement of W_{TL}.[9] The experiment is technically feasible since only one hadron needs to be polarized. One kind of aparatus that is available (and approved) takes the form of the CERN UA6 gas jet target. The complementarity enters because one can show[9]

$$\mathrm{Re}\left(e^{i(\omega_{T1}-\omega_L)}\right) \rightarrow (\text{coeff. of } \cos\phi \ \sin 2\theta) \tag{10a}$$

$$\mathrm{Im}\left(e^{i(\omega_{T2}-\omega_L)}\right) \rightarrow (\text{coeff. of } \lambda_A \sin\phi \ \sin 2\theta) \tag{10b}$$

where λ_A is one proton's helicity; the other particle (B) is an unpolarized hadron. The $\cos\phi$, $i\sin\phi$ dependences in Eq.(10) are just remnants of the spin 1 (T) photon projections. The extra phase i comes with a single helicity ($\gamma_5 = -\bar{\gamma}_5$), while parity conservation is maintained by the fact that transverse direction $\hat{T2}$ is a pseudovector combination of $\hat{T1}$ and $\hat{Z}$.

We estimate that $|\omega_{T1}| \sim |\omega_{T2}| + O(Q_T/\sqrt{\hat{s}})$, since deviations from azimuthal symmetry transform in this way. In that case, the phase difference of the spin coefficient (10b) and the unpolarized coefficient W_{TL} (10a) will have the same momentum dependence to leading order.

An important and non-trivial physical consistency check, both on the calculation and on the applicability of perturbative QCD, is whether the IR cutoff dependence actually vanishes in the sum over graphs as indicated by Eq.(9). This is because the direct calculation leads to the task of finding imaginary pats of terms of order $\alpha_s^2 \ell n^2(\pm \text{ various scales})$, including $\alpha_s^2 \ell n^2(\pm Q^2/\text{cutoff})$. In our calculation we continue these, obtaining $i\pi\alpha_s \ell n(|Q^2|/\text{cutoff})$, e.g., along with momentum dependent pieces such as $i\pi\alpha_s \ell n(|Q^2/\hat{s}|)$. Although we have not yet organized all the momentum dependent terms, we can report that cutoff dependence (imaginary terms of $O(1/\varepsilon)$ in $4 + \varepsilon$ dimensions) does indeed cancel. This in itself supports the applicability of QCD.

The phenomenology of the cutoff-free, momentum dependent results will be discussed elsewhere.[9] Let us only remark here that data from the NA3 collaboration[10] does exist for all of the structure functions of Eq.(6) including W_{TL}. The data is not of sufficient quality to determine if oscillations with respect to, e.g., Q_T^2/Q^2 exist, but neither are oscillations ruled out. Significant fluctuations may be present in this data, in fact, which become more apparent when the kinematic factor of $Q_T/\sqrt{s}$ is divided out.[9] We urge that upcoming precision, high statistics data be binned for the separate Q^2, Q_T^2 dependences of W_{TL} and W_{T-T} at fixed s.

IV. THE $x \to 1$ LIMIT

Some time ago Berger and Brodsky[4] pointed out that higher twist, longitudinal γ^*'s should dominate lower twist, transverse γ^*'s in the distribution $d\sigma/dQ^2dxd\cos\theta$ as $x \to 1$ for the case of $\pi p \to \mu^+\mu^- + X$. One can summarize this result at the amplitude level as follows. In the notation of Eq.(1) one finds

$$\lim_{x\to 1} \langle N|J^\mu|P_A P_B\rangle \propto (1-x)\varepsilon_T^\mu + (k_T/Q)\varepsilon_L^\mu \qquad (11)$$

where k_T is of the order of a hadron mass. Eq.(11) implies that $d\sigma/dQ^2dxd\cos\theta \propto (1-x)^2(1+\cos^2\theta) + (M^2/Q^2)\sin^2\theta$; this is the well known turnover from transverse to longitudinal polarization as $x \to 1$. However, the transverse amplitude has, in addition, a factor from Sudakov suppression as $x \to 1$.[11] This is a signal that effects of the CCP may occur. Briefly, an off-shell form factor,

$$\exp\left(-C_F \frac{\alpha_s(Q_0^2)}{2\pi} \ln(-Q^2/-p^2)\ln(-Q^2/-p'^2)\right) \qquad (12)$$

where p^2, p'^2 are spacelike quark legs, will multiply (1−x) in Eq.(11) to leading double-logarithmic accuracy. Continuing Eq.(12), including running coupling effects, and observing that $p^2 \cdot p'^2 \gtrsim$ const. Q^2 in the region of interest, one should multiply again by a phase associated with Eq.(12),

$$\exp(i\omega) = \exp\left[-i\pi c \ \ln \ \ln(Q^2/\Lambda^2_{QCD}) + i\delta\right] \tag{13}$$

with $c = C_F/(11 - 2/3\ N_f)$ and δ is a constant.

In addition to these Sudakov-related effects, there are recently discovered non-traditional higher twist effects that might produce transverse polarizations. One example is the violation of the Bloch-Nordsiek prescription discovered by Doria, Frenkel and Taylor,[12] who considered $q\bar{q} \to \gamma^* +$ gluons. Since the result is both higher twist (of order $\alpha_s^2 \ln(p^2/Q^2) \ln(Q^2/\text{cutoff}) p^2/Q^2$) and a remnant of truly soft gluons, it is not easy to estimate the implications of this effect.[13] For this discussion we will simply point out the interesting consequences of interference of the leading twist term with such anomalous contributions. The normalization would presumably be dependent on bound state, i.e. non-perturbative details, while the sensitivity of experiments to the $\cos[\omega(Q^2)]$ oscillation is determined by the x dependence. This latter problem is non-trivial because the limits $k_T \to 0$ and $x \to 1$ do not commute.[13] This underscores the need for further theoretical development of higher twist calculations.

IV. CONCLUSIONS

We have emphasized the application of the exponential pattern of logarthmic resummation in QCD to phase-sensitive observables in $d\sigma/d^4Qd\Omega$. If the usual understanding of factorization and Sudakov dominance is correct, then an oscillatory momentum dependence for W_{TL}, e.g., is a prediction of QCD. The momentum dependence of the argument of the oscillation is const. $\ell n\, \ell n(\tilde{Q}^2/\Lambda^2_{QCD})$, which varies rapidly enough in the moderate Q^2 region ($\ell n\, \tilde{Q}^2 \lesssim \ell n$ 100 GeV^2) to produce detectable oscillations. We believe that the usual smooth high twist and non-perturbative backgrounds can be clearly separated from the coherent effect of oscillations at large enough momentum transfers, since such backgrounds should be free from large momentum dependent logarithmic phases.

Observation of such oscillations would provide a superb test of QCD: the signal for present-day experiments is a wave of geometrically increasing wavelength.

REFERENCES

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* On leave from Centre de Physique Theorique, Ecole Polytechnique, Palaiseau (France)

1. J. P. Ralston and B. Pire, Phys. Rev. Lett. 49, 1605 (1982); B. Pire and J. P. Ralston, Phys. Lett. B117, 233 (1982).

2. Yu. Dokshitzer, et al., Phys. Rep. 53C, 269 (1980); G. Parisi and R. Petronzio, Nucl. Phys. B116, 427 (1979). For a review see A. H. Mueller, Phys. Rep. 73C, 237 (1981).

3. See, e.g., C. S. Lam and W. K. Tung, Phys. Rev. D18 2447 (1978); J. C. Collins, Phys. Rev. Lett. 42, 291 (1979).

4. E. Berger and S. J. Brodsky, Phys. Rev. Lett. 42, 940 (1979); E. Berger, Zeit. Phys. C4, 289 (1980).

5. See, e.g., J. C. Collins and D. E. Soper, Proc. Moriond Workshop on Lepton Pair Production, J. Tran Than Van, ed., Les Arcs (1981); J. Collins (these proceedings); D. Soper (these proceedings).

6. G. T. Bodwin, S. J. Brodsky and G. P. Lepage, Phys. Rev. Lett. 47, 1799 (1981) and SLAC-PUB-2927 (1982); W. W. Lindsay, D. A. Ross and C. T. Sachrada, Southampton preprint SHEP-81/82-6. See also G. T. Bodwin (these proceedings) and W. J. Stirling (these proceedings).

7. J. C. Collins and D. E. Soper, Phys. Rev. D16, 2219 (1977).

8. F. Khalafi, Zeit. Phys. C11, 251 (1981).

9. J. P. Ralston and B. Pire, Argonne preprint ANL-HEP-CP-82-60, to be published in Proceedings of the 5th International Symposium on High Energy Spin Physics, Brookhaven (1982); J. P. Ralston and B. Pire (in preparation).

10. J. Badier, et al., Zeit Phys. C11, 195 (1981).

11. G. Parisi, Phys. Lett. 84B, 225 (1979); G. P. Lepage and S. J. Brodsky, Phys. Rev. D22, 2157 (1980). See also A. H. Mueller (Ref.2).

12. R. Doria, J. Frenkel and J. C. Taylor, Nucl. Phys. B168, 93 (1980); C. diLieto, S. Gendron, I. G. Halliday and C. T. Sachrada, Nucl. Phys. B183, 223 (1981).

13. One must carefully distinguish the $x \to 1$ infrared behavior of lepton pair production from the analogous $x \to 1$ deep-inelastic scattering. In the latter case, of course, the π structure function must become completely longitudinal at $x \cong 1$. New aspects of this are discussed in M. Soldate, SLAC-PUB-2998 (1982).

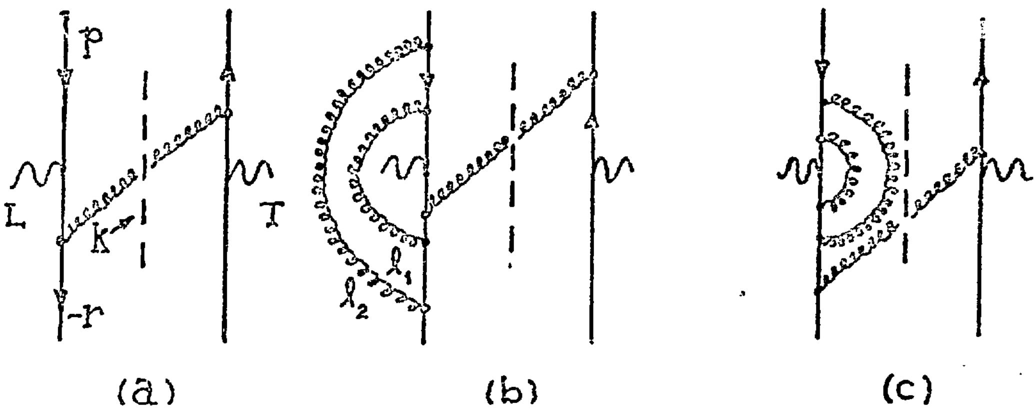

Fig.1. Diagrams for $W^{\mu\nu}$ describing interference of longitudinal (L) and transverse (T) photons, as discussed in Section II.

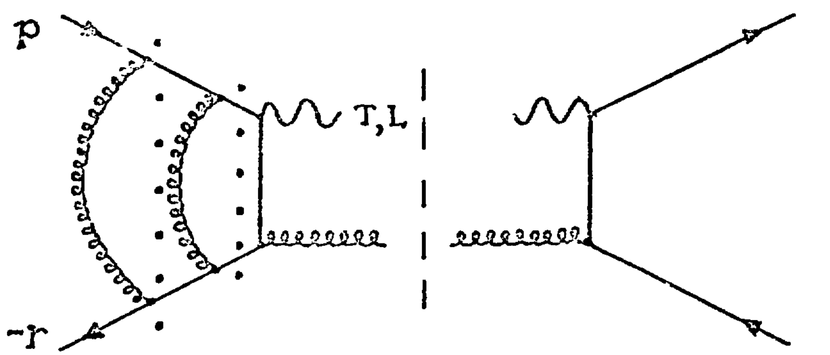

Fig.2. Real terms produced from interference of CCP terms contributing at $O(\alpha_s^3)$. The dotted line denotes the Cutkosky rule for imaginary parts.

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