SAND--89-0376C

DE89 007428

AN ANALYTICAL STUDY OF A VIBRATION TEST METHOD USING EXTREMAL CONTROL OF ACCELERATION AND FORCE

David O. Smallwood
Sandia National Laboratories
Albuquerque NM

Biography

David Smallwood received his BSME degree from New Mexico State University in 1962 and his MSME degree from New York University in 1964. He has worked for Sandia National Laboratories since 1967 and is currently the Division Supervisor of the Experimental Mechanics Division IV. He is a fellow of the IES.

Abstract

A vibration test method has been proposed where control is accomplished using extremal control of the force and acceleration at the input to a test item. This proposal is examined with several examples. The method does limit the acceleration input at frequencies where the test item responses tend to be unrealistically large. However the method's application is not straightforward and care must be taken in the application of the method.

Nomenclature

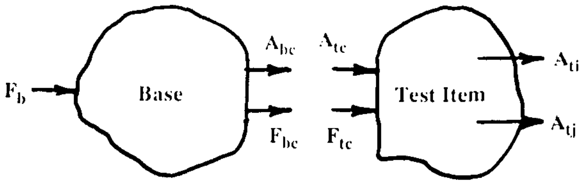
$(A_{bc})_{fr}$	the free acceleration of the base at the connection point
A_i	acceleration at location of force input to a shaker
A_c	acceleration at the interface between the fixtures and the test item
$\mathbf{A}$	accelerance, a frequency response function, the ratio of acceleration response and force input
$\mathbf{A}_{ii}$	driving point accelerance of a shaker
$\mathbf{A}_{ic}$	transfer accelerance, motion at force location, force at test item interface
$\mathbf{A}_{ci}$	transfer accelerance, motion at test item interface, force into shaker
$\mathbf{A}_{cc}$	driving point accelerance at interface to test item
b	as a subscript, a base variable
bl	as a subscript, a blocked force
C	a control spectral density, usually an envelope of an auto spectral density
c	as a subscript, an interconnection variable
F_i	the input force to a shaker
F_c	the force into a test item mounted on a shaker
$(F_{bc})_{bl}$	the blocked force of the base
fr	as a subscript, a free acceleration
m_r	modal mass
S	an auto (power) spectral density
t	as a subscript, a test item variable
ψ	mode shape matrix
ζ	fraction of critical damping
ω	frequency
ω_r	natural frequency of mode r
$'$	as a superscript, a matrix transpose

Introduction

It has long been recognized that the practice of establishing acceleration control spectra at the input to a test item by enveloping either field measurements or predictions leads to very conservative vibration tests [1-6]. Many proposals have been made in the past to introduce force control or combinations of force and acceleration control to reduce this conservatism [1-3]. Otts [1] proposed force control. Murfin and Witte [2,3] proposed acceleration control with a force limit. The force limit was based on the accelerance of the test item. Witte [4] proposed control on the product of the magnitudes of force and acceleration as a compromise between infinite impedance testing (acceleration control) and zero impedance testing (force control).

In spite of their advantages the methods were not widely used for several reasons. First impedance measurements of test items and base structures were not generally available. Second measurements of interface forces in both the field and laboratory were difficult. Third it was difficult to implement the methods using the control systems at the time. Some of these objections still hold today, but impedance measurements are much more common today, although still difficult, and digital control methods have increased the options for control.

The recent paper by Scarton and Kern [5] proposes extremal control of force and acceleration. This is essentially an acceleration controlled test with a force limit, or force control with an acceleration limit. They propose to establish both a force control spectrum, C_{Fb}, and an acceleration control spectrum, C_{Ab}. The acceleration control is determined from the free motion of the structure on which the test item will be mounted (the base). The force control spectrum is determined from the blocked force of the base. The blocked force is the interface force required to fix the base, i.e. keep the base interface from moving. The test would be controlled by which ever parameter, force or



$$A_{bc} = A_{tc}$$
$$F_{bc} = -F_{tc}$$

Figure 1. Subsystems Showing External and Connection Forces.

acceleration, reached the control spectrum first. This is

$$S_{At} = \text{but not} > C_{Ab} \quad \text{or} \quad S_{Ft} = \text{but not} > C_{Fb} \tag{1}$$

This is known as an extremal control method.

Theory

For simplicity, this paper will assume the test item is mounted on a structure for use at a single point. Motion is restricted to a single degree of freedom (single axis of input). For testing the test item is removed from the use structure and mounted on a shaker with fixtures. Again it will be assumed that the input to the test item during the test can be described by a single point in one axis. All variables relating to motion or force are given in the frequency domain. The ideas expressed in this manner as scalar functions of frequency are easier to understand, but can be easily extended to multiple inputs and axes using the matrix notation of Newbert [7]. Motion measurements will be assumed to be acceleration, and frequency response functions (FRF's) will be accelerance, $\mathbf{A}$, the ratio of acceleration and force. Velocity or displacement could be used, but acceleration is by far the most common measurement actually used. The general case is illustrated in Fig. 1. It is assumed that all the excitation forces are in the base, and the test item is passive (no forces applied to the test item except through the connection point).

The extremal test method depends on two spectra. An acceleration spectra is established which is equal to or larger than the interface motion in the use environment at all frequencies. Ideally this envelope would be established from use measurements with the test item attached to the use structure (the base). It is shown later that if these measurements are not available the free motion of the base can be used. A force spectra is established which is equal to or larger than the use interface force at all frequencies. If this force is not available it is shown that the base blocked force can be used. The test is then controlled by which ever quantity reaches its limit value first.

The use of the free base motion and the blocked force will now be examined.

Norton's Theorem shows that ([7], Eq. 9.33)

$$(A_{bc})_{fr} = -\mathbf{A}_{bc}\,(F_{bc})_{bl} \tag{2}$$

The motion of the base at the interface with the test item removed (the free motion) is equal to the driving point accelerance of the base looking into the interface multiplied by the blocked force of the base at the interface. The blocked force is the force required to keep the interface motion zero. The minus sign is needed to keep the direction of the forces correct, but will not be important for this discussion.

It can also be shown that the following two relationships hold when the test item is connected to the base (Newbert [7], Eqs. 9.32 and 9.33).

$$A_{tc} = A_{bc} = [\mathbf{A}_{tc}/(\mathbf{A}_{bc}+\mathbf{A}_{tc})]\,(A_{bc})_{fr} \tag{3}$$

$$F_{tc} = -F_{bc} = [\mathbf{A}_{bc}/(\mathbf{A}_{bc}+\mathbf{A}_{tc})]\,(F_{bc})_{bl} \tag{4}$$

The interface motion of the test item and the base are the same, hence, $A_{tc}=A_{bc}$. The force into the test item is the negative of the force into the base. We have to be careful and not interpret the forces, F, and accelerations, A, as auto or power spectra. They are complex functions of frequency.

If we take the ratio of the test item interface acceleration, A_{tc}, and base free interface motion, $(A_{bc})_{fr}$, and the ratio of the force into the test item at the interface, F_{tc}, and the blocked force of the base into the interface we get,

$$A_{tc}/(A_{bc})_{fr} = \mathbf{A}_{tc}/(\mathbf{A}_{bc}+\mathbf{A}_{tc}) \tag{5}$$

$$F_{tc}/(F_{bc})_{bl} = \mathbf{A}_{bc}/(\mathbf{A}_{bc}+\mathbf{A}_{tc}) \tag{6}$$

Adding Eqs. (5) and (6) gives

$$A_{tc}/(A_{bc})_{fr} + F_{tc}/(F_{bc})_{bl} = 1 \tag{7}$$

This is Eq. (5) in Scharton [5].

At first glance Eq. (3) appears to indicate that the interface acceleration with the test item attached could not be larger than the free interface acceleration, but this is not the case. The interface acceleration can larger than the free acceleration at particular frequencies. The accelerances are complex numbers and the magnitude of a sum can be smaller than the magnitude of either of the parts. The same is true for the interface forces, they can larger than the blocked force at a particular frequency. Motion and force at the interface larger than the free motion and blocked force is just a reflection of the fact that the poles and zeros (the resonances and anti-resonances) of the system move when the test item is attached.

Similarly, Eq. (7) does not insure a conservative test when the terms on the left side of the equation are controlled such that the magnitude of at least one of the terms is greater than or equal to one (an extremal test). Further justification for using the blocked force and free motion of the interface as test envelopes is needed.

If it is assumed that the peaks in the accelerance are dominated by a single mode the peaks are approximately [8]

$$|\mathbf{A}_{jk}(\omega_r)| \approx (\psi_{rj}\ \psi_{rk})/(2m_r\zeta_r) \quad (8)$$

where j is the response location, k is the forcing location, ω_r is frequency of the rth mode, ψ is the mode shape, m_r is the modal mass, and ζ_r is the fraction of critical damping.

It is reasonable to expect the peaks in the interface acceleration spectra of the combined system to be smaller than the peaks in the free motion since the modal mass for the combined system will be larger than the modal mass for the base alone. This is the basis for using an envelope of the peaks of the free motion in place of an envelope of the interface motion when establishing the acceleration test envelope.

Similar arguments can be made for assuming the peaks in the interface force for the combined system will be smaller than the peaks in the blocked force. The force peaks will be at the antiresonances of the interface. The depth of the antiresonances and hence the amplitudes of the force peaks are also dependent on the modal mass and the damping values.

However, the above is not always true. Consider the very simple case where the base is a mass driven by a force. The blocked force and the free acceleration will be constants independent of frequency. The test item is a spring mass. Clearly when the spring-mass is connected to the base mass a resonant frequency will exist. At that frequency the interface force between the masses will be larger than an envelope of the blocked force, and the interface motion between the base mass and the spring will be larger than an envelope the free motion.

The form of the above equations will now be considered for random inputs. If the coherence between the free motion and the motion with the test item attached is unity Eq. (3) gives

$$S_{Atc} = |[\mathbf{A}_{tc}/(\mathbf{A}_{bc}+\mathbf{A}_{tc})]|^2\ (S_{Abc})_{fr} \quad (9)$$

Similarly, if the blocked force and the force with the test item attached has unity coherence Eq. (4) gives

$$S_{Ftc} = |[\mathbf{A}_{bc}/(\mathbf{A}_{bc}+\mathbf{A}_{tc})]|^2\ (S_{Fbc})_{bl} \quad (10)$$

The auto (power) spectra can only be estimated where the free and attached motion measurements are made using the same stationary random input. The phase information for random inputs in Eqs. (5) and (6) can be measured in principle only for very special cases. One example would be three identical bases all excited by the same random force. One base would be free, one base blocked, and one base attached to the test item. The cross spectra between the responses could then be used to determine the complex ratio of accelerances on the right sides of Eqs. (5) and (6).

The above equations will now be used to establish test envelopes. The acceleration test envelope, C_{Ab}, is established by enveloping the peaks of the interface motion from use measurements or from the free acceleration of the base. The force envelope, C_{Fb}, is established from interface force measurements in the use environment or more commonly from base blocked force estimates. The test will then be controlled such that the test spectra are never larger than these envelopes,

$$S_{At} = \text{but not} > C_{Ab}$$

or

$$S_{Ft} = \text{but not} > C_{Fb}$$

which are Eqs. (1) and (2) and the basis for the proposed test method. The method has the advantage that details of the interface motion and force are not required.

The envelope of the test motion can be estimated from models, from field measurements, or from free motion estimates. Unfortunately the interface force in a use environment or blocked force is seldom measured directly. Estimates of the base blocked force require knowledge of the base driving point accelerance at the connection point and an estimate of the free motion. The blocked force envelope is then estimated using Eq. (2). This has the advantage of being independent of the test item and does not require an estimate of the interface motion or force with the test item in place. The disadvantage is that the base driving point accelerance must be known.

Interface force estimates in the use environment require different information. If details of the interface motion in the use environment are known the interface force can be estimated from

$$S_{Ftc} = S_{Atc}\ /|\mathbf{A}_{tc}|^2 \quad (11)$$

The auto spectrum of the force at the interface into the test item in the use environment, S_{Ftc}, is given by the auto spectrum of the interface motion in the use environment, S_{Atc}, divided by the magnitude squared of the driving point accelerance of the test item, A_{tc}. This has the advantage of requiring no information about the base driving point accelerance. The disadvantage is the requirement for acceleration estimates in the use environment with the test item in place, the free acceleration of the base is not used. This was the approach of Murfin and Witte [2-3].

Examples

The examples will all assume the envelopes are formed from the free base acceleration and the blocked force.

The first example will be for the simple system with one rigid body mode and two dynamic modes shown in Fig. 2.

The natural frequencies, damping values, modal masses, and mode shapes for the base are given below.

$$\omega_{nb} = [0\ 50]$$
$$\zeta_b = [0\ .05]$$
$$m_i = [1500\ 500]$$
$$\psi_b = \begin{bmatrix} 1 & .5 \\ 1 & -1 \end{bmatrix}$$

The combined system will have resonances at 29 and 50 Hz.

The input force spectrum is white with a value of 1. The driving point accelerance at the connection point is shown in Fig. 3. The free acceleration and block force spectra are shown in Figs. 4 and 5. The acceleration and force limit spectra chosen are also shown on these figures.

The next two plots (Figs. 6 and 7) show the responses at two points (the interface and one response point, in this case the mass M on the right of Fig. 2) in the test system under three conditions. The first solid curve is the response spectra when the combined system is driven with the white force. This is a simulated use environment and is used for comparison with two simulated test environments. The dashed curve is the response for an acceleration control simulated test of the test item removed from the base. In this case the motion of the interface, Fig. 6, is equal to the control spectrum, which is established by enveloping the free motion of the base when the base alone was driven by the white force. The dotted line is the response for a simulated extremal test of the test item. In this case the interface motion, Fig. 6, is the acceleration control spectrum with the force limit imposed. The extremal test is essentially the same as an acceleration controlled test except the input acceleration is notched by the force limit near 29 Hz. As desired, the response of the second mass is always larger than combined system response for both test strategies. The extremal test is less conservative near the 29 Hz resonance as desired. In this case, using Eq. 11, the motion of the interface with the test item attached is always $(1/2)^2$ or 1/4 the free acceleration since the driving point accelerance of the base and the test item are identical.

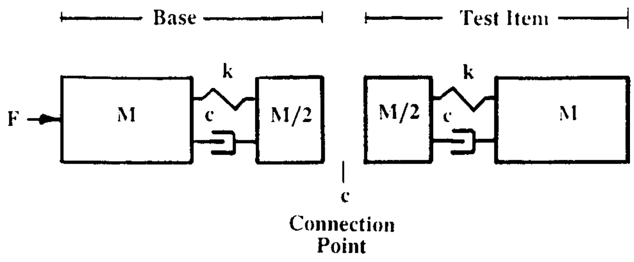


Figure 2. 3 Degree-of-Freedom System.

Legend for All the Figures Comparing Acceleration In the Examples

—— solid, response of the combined system.

- - - dash, response of the test item with an acceleration controlled test.

.... dots, response of the test item force-acceleration extremal test.

The base for the second example has 7 degrees of freedom and the test item has 6 degrees of freedom. The parameters are given below.

$$\omega_{nb} = [0\ 50\ 100\ 350\ 550\ 700\ 800]$$
$$\zeta_b = [0\ .01\ .01\ .02\ .05\ .03\ .01]$$
$$m_b = [150\ 100\ 50\ 40\ 30\ 10\ 20]$$
$$\psi_b = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -.9 & .7 & -.8 & .7 & -.6 & -.5 \end{bmatrix}$$
$$\omega_{nt} = [0\ 200\ 450\ 560\ 750\ 810]$$
$$\zeta_t = [0\ .02\ .03\ .02\ .05\ .03]$$
$$m_t = [40\ 30\ 17\ 10\ 10\ 8]$$
$$\psi_t = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & .5 & 0 & -.7 & -.9 & -.1 \\ 1 & .1 & -.5 & .5 & 1 & .8 \\ 1 & -.1 & -.4 & .5 & -.8 & -.7 \\ 1 & -.6 & .5 & -.2 & 0 & .9 \\ 1 & -1 & 1 & -.9 & .8 & .9 \end{bmatrix}$$

As before the force input has unity spectral density. The free acceleration and blocked force of the base to this input together with the limit envelopes are shown as Figs 8 and 9. A force limit at 1000 was originally used, but is lowered to 100 for more interesting results. The response of the interface is shown as Fig. 10. The response of the other 5 locations in the test item were calculated. They showed

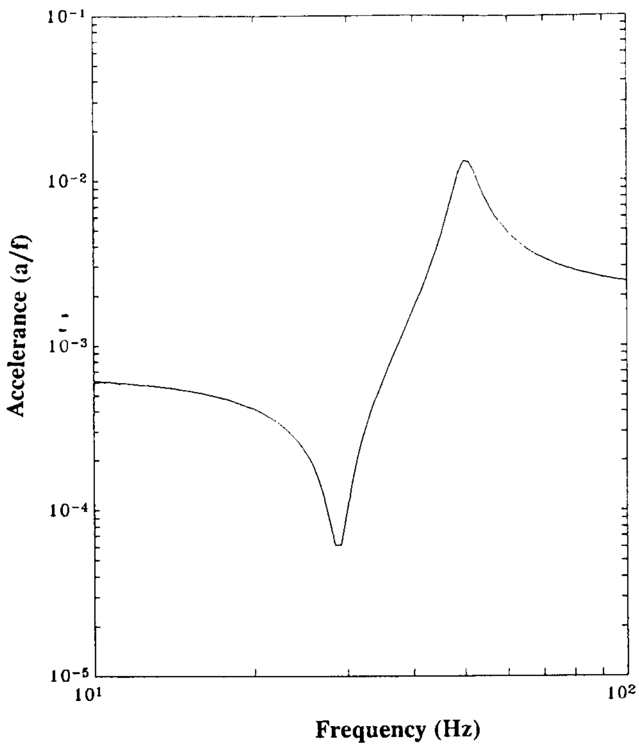


Figure 3. 3 Degree-of-Freedom System, Connection Point Driving Point Accelerance.

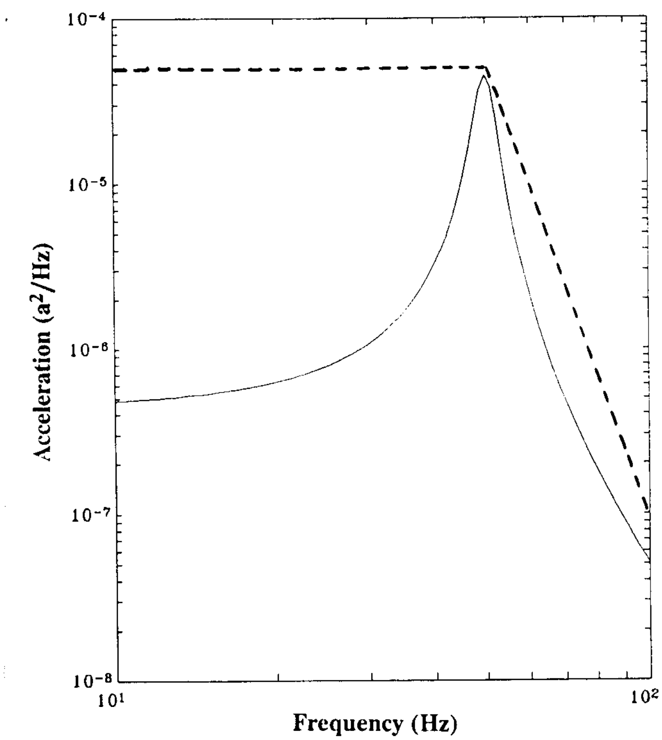


Figure 4. 3 Degree-of-Freedom System, Base Free Acceleration.

similar results. The response of location 4 is shown in Fig. 11. The interpretations of the curves is the same as for the first example. The solid curve is for a

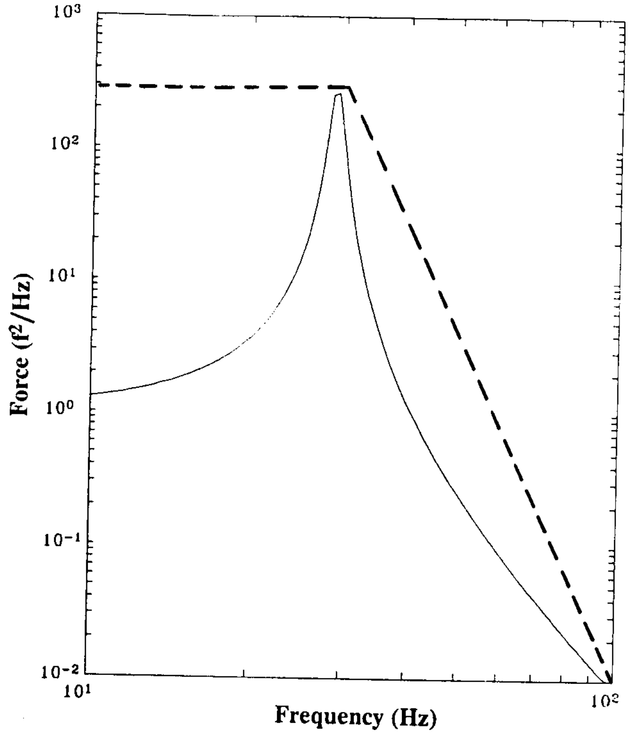


Figure 5. 3 Degree-of-Freedom System, Base Blocked Force.

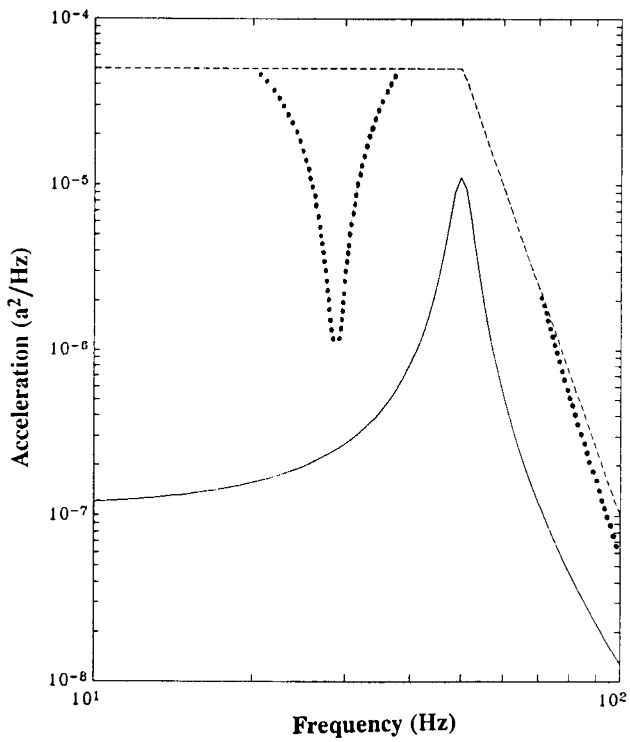


Figure 6. Acceleration at Input to Test Item, Example 1.

simulated use environment, the dashed curve is for a simulated acceleration controlled test and the dotted curve is for a simulated extremal test. As can be seen

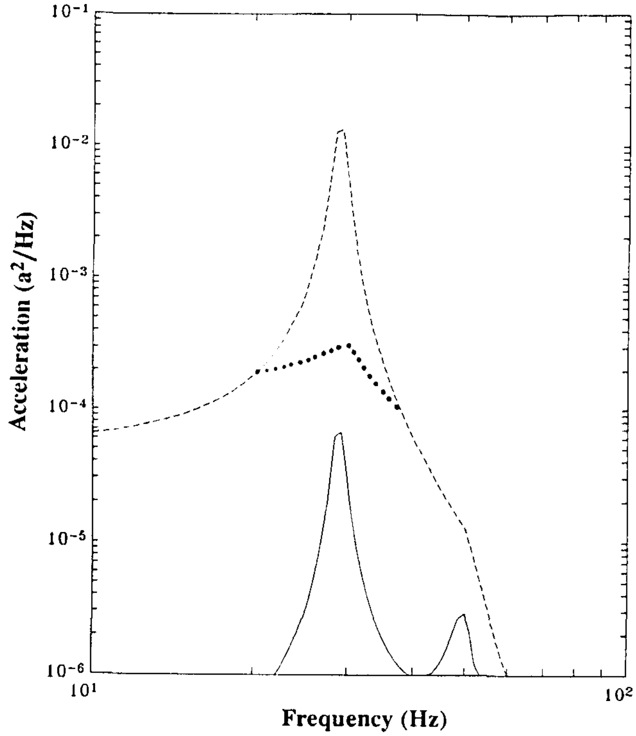


Figure 7. Acceleration of Mass 2, Example 1.

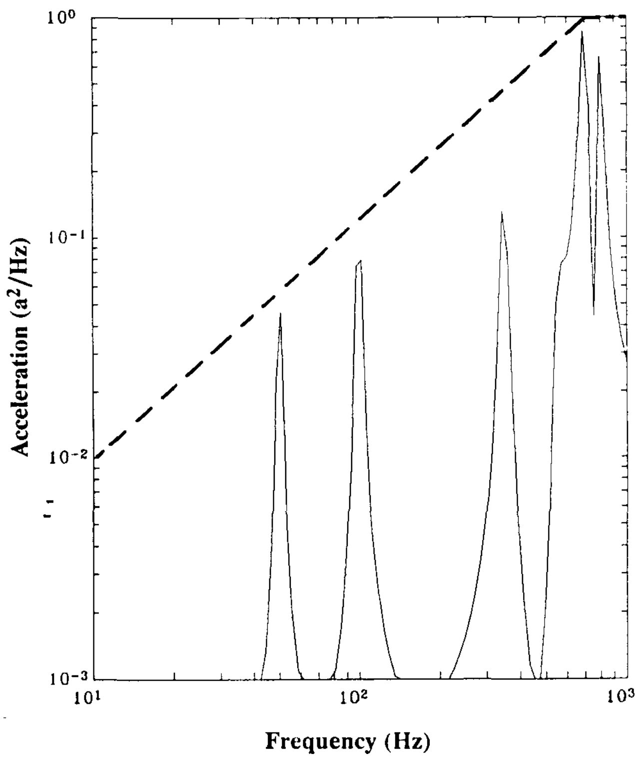


Figure 8. Free Acceleration of the Base, Example 2.

from Fig. 10 the extremal test was controlled by acceleration except in a few frequency bands. Figure 11 illustrates the force limit did reduce the response away

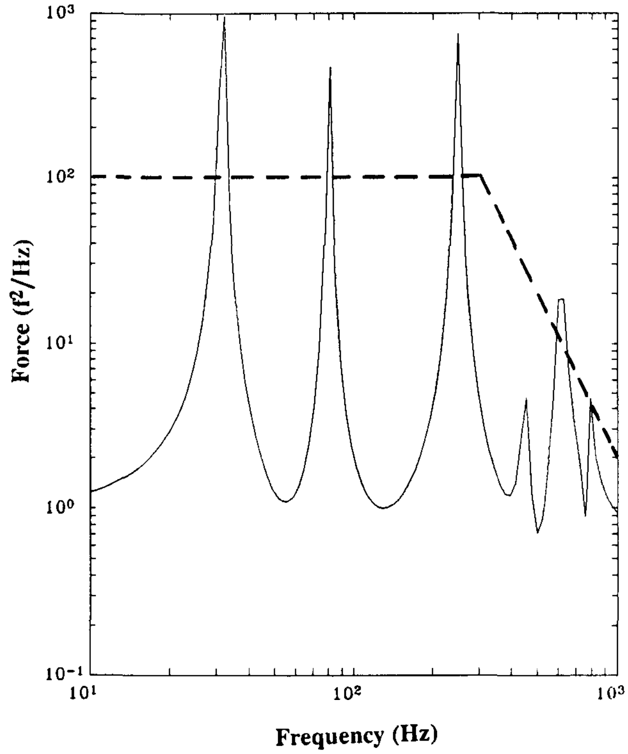


Figure 9. Block Force of the Base, Example 2.

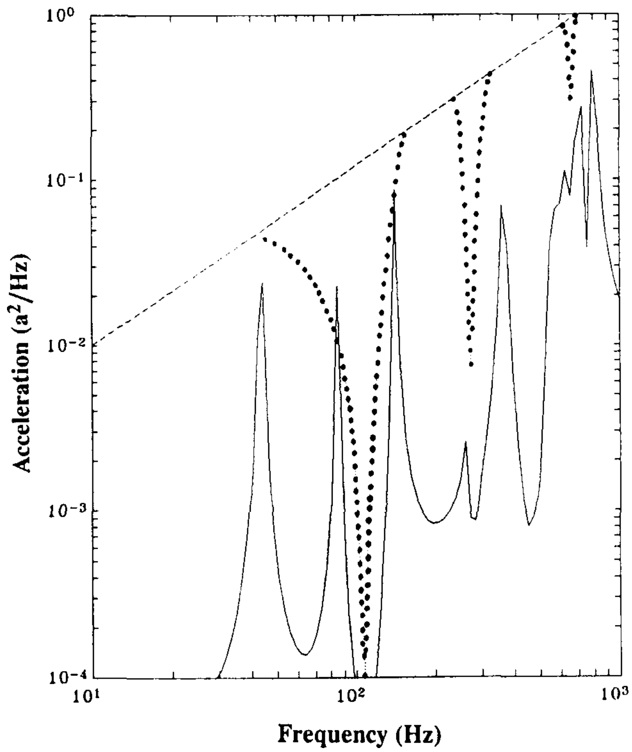


Figure 10. Acceleration at Input to the Test Item, Example 2.

from the interface at three frequencies where the response was much greater than the simulated use environment.

The third example is the same as the second except the last mode of the base was moved to 1800. The modal mass of this mode is changed to .0261, and the mode shape vector for the last mode is changed to [1 1]'. This is done to give the base a spring like behavior at high frequencies. This behavior is typical of many base structures which are dominated by local stiffness. An example would be a thin panel. The driving point accelerance is shown as Fig. 12. The free acceleration, blocked force, and the envelopes are shown as Figs. 13 and 14. The acceleration of the input (the interface) and location 4 are shown in Figs. 15 and 16. Notice that above 30 Hz the simulated extremal test, dots on the Figs., was force controlled. In this case the extremal test did an outstanding job of reproducing the simulated field results. This example suggests that force control for items mounted on light structures has merit.

Implementation

Implementation of the proposed method can be accomplished approximately as suggested by Scharton [5]. A more exact implementation generates a wish list for a vibration control system. Extremal control is available on current systems. However, the user should be aware of bias problems in any extremal control strategy [9].

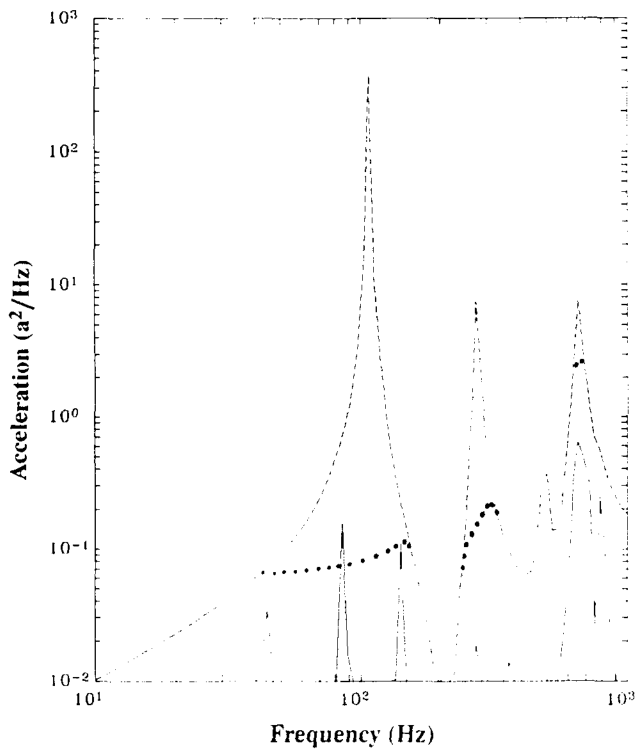


Figure 11. Acceleration of Location 4, Example 2.

These problems can be minimized by simultaneous sampling and calculating good estimates (many degrees of freedom) of the spectra before forming the extremal control spectrum. A needed feature, not currently available, is a different reference spectrum for each control channel.

Measurement of the input force into the test item during the vibration test can also present a challenge. Force gages are available which can measure tension-compression forces, and some gages are available which can measure moment and shear forces. These gages must be used with care and many of them generate outputs in response to forces in directions other than the sensitive axis. It is also difficult to incorporate force gages into many test setups. A simpler method, from the viewpoint of test setup, is desirable.

If the force, F_i, can be measured anywhere between the armature coils of the shaker and the interface to the test item a method exists to estimate the force at the test item interface, F_c. A matrix equation is written relating the forces and accelerations,

$$\begin{Bmatrix} A_i \\ A_c \end{Bmatrix} = \begin{bmatrix} \mathbf{A}_{ii} & \mathbf{A}_{ic} \\ \mathbf{A}_{ci} & \mathbf{A}_{cc} \end{bmatrix} \begin{Bmatrix} F_i \\ F_c \end{Bmatrix} \tag{12}$$

The subscript i refers to the location where the force is measured and the subscript c refers to the interface between the test item and its fixtures. First the

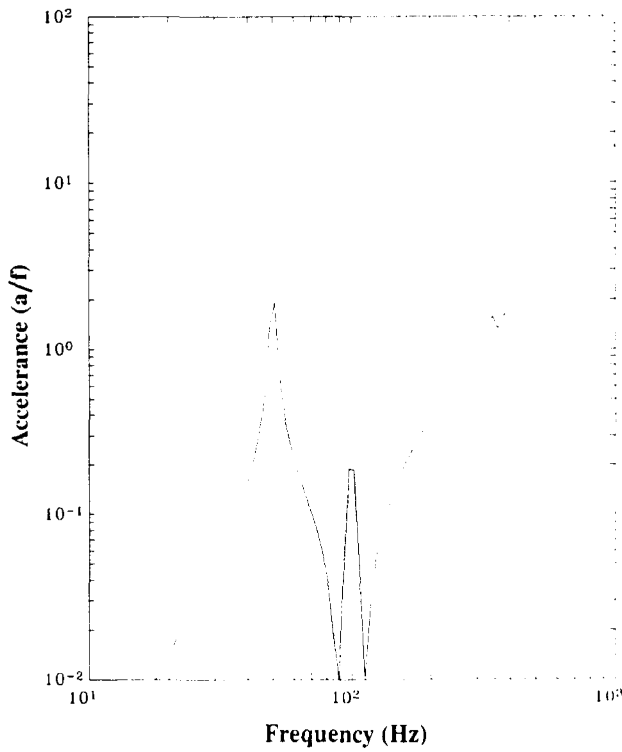


Figure 12. Driving Point Accelerance of the Base, Example 3.

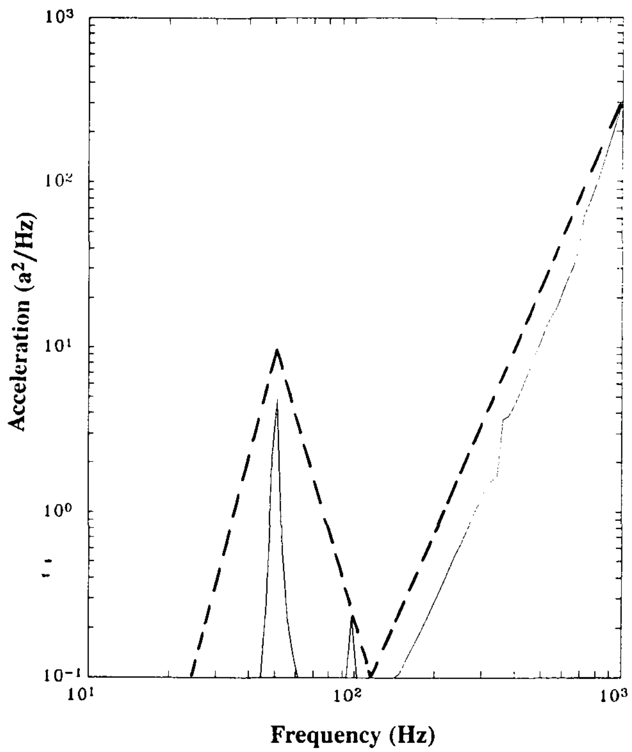


Figure 13. Free Acceleration of the Base, Example 3.

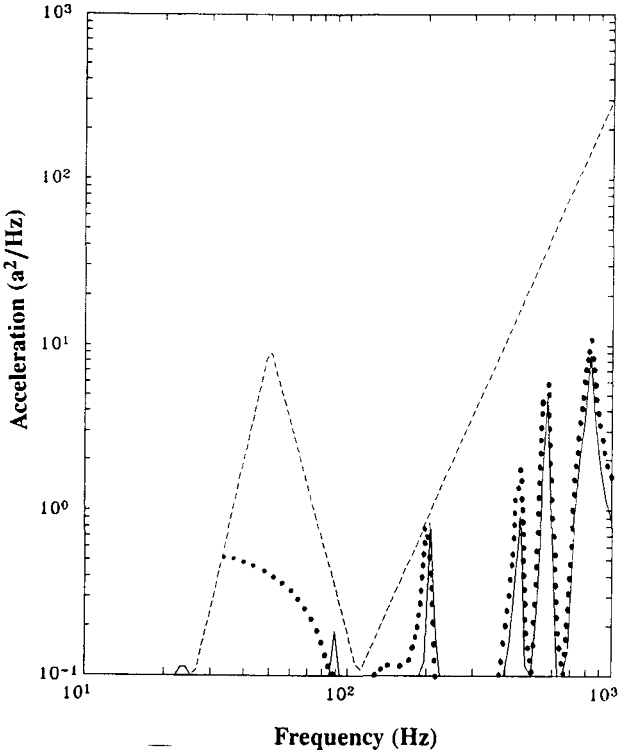


Figure 15. Acceleration at the Input to the Test Item, Example 3.

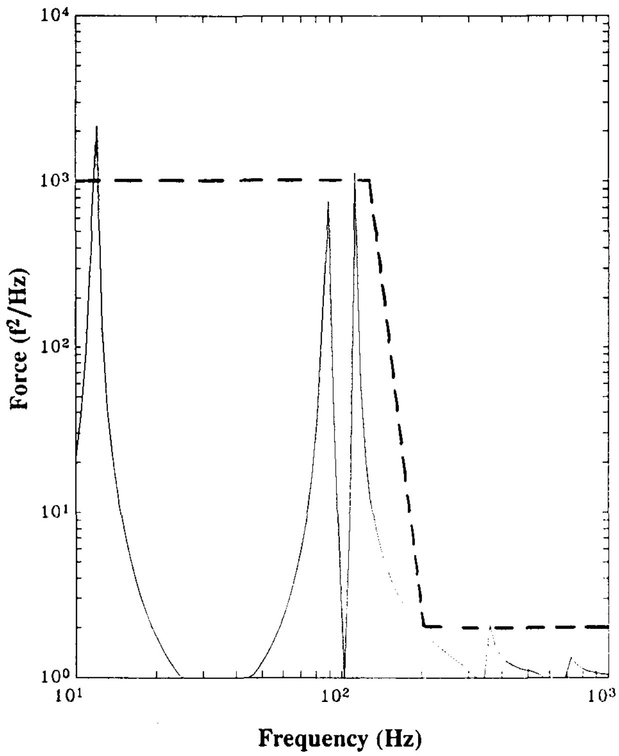


Figure 14. Blocked Force of the Base, Example 3.

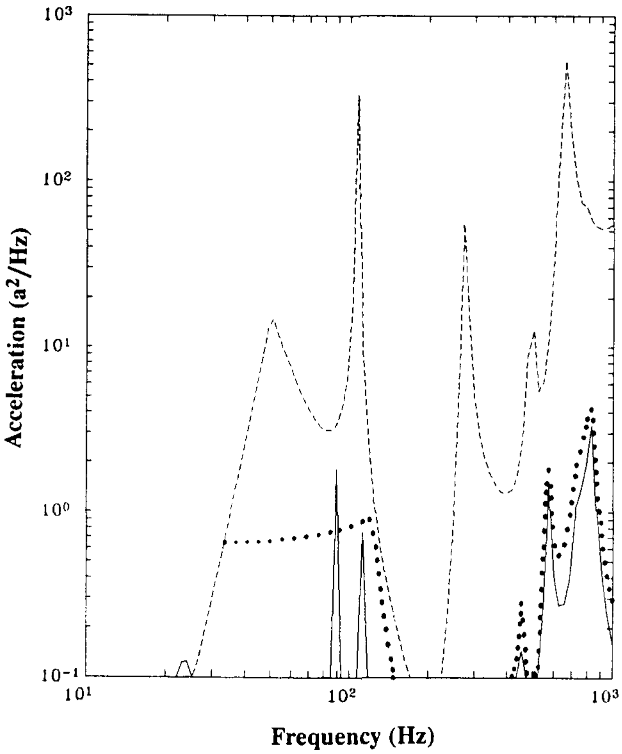


Figure 16. Acceleration of Location 4, Example 3.

system is driven with the test item removed, $F_c = 0$, and acceleration is measured at the test item interface, A_c, and at the location where the force was measured, A_i. These measurements allow the determination of the accelerances, $\mathbf{A}_{ii}$ and $\mathbf{A}_{ci}$. Using reciprocity

$$\mathbf{A}_{ic} = \mathbf{A}_{ci}. \qquad (13)$$

Solving the first equation in the matrix for the force into the interface, F_c, which is the negative of the force into the test item, F_t, gives,

$$F_t = (\mathbf{A}_{ic})^{-1} (\mathbf{A}_{ii} F_i - A_i) \qquad (14)$$

This equation works well except near the resonances of low damped systems. If the shaker and test fixtures are rigid then the accelerances are the inverse of the mass of the shaker armature and fixtures

$$\mathbf{A}_{ic} = \mathbf{A}_{ii} = 1/M \qquad (15)$$

Equation (14) reduces to

$$F_t = F_i - MA_i \qquad (16)$$

the usual form for mass subtraction. Even for low damped systems Eq. (14) can usually be used over a wider frequency band than Eq. (16).

The digital implementation of Eqs. (14) or (16) will require the insertion of a processor module between the calculation of the Fourier transforms of the input channels and the calculation of the input spectra. Ideally this would be a user defined module such that tomorrows method can also be implemented.

The force can be measured at the interface between the shaker and any fixtures with force gages. This will require the measurement of the motion at this same location, easy to do. The force could also be measured at the armature coil by measuring the current needed to drive the shaker. This requires a voltage proportional to the instantaneous drive current (calibrated in force units) and an acceleration measurement on the armature near the drive coil to implement Eq. (14).

Mass subtraction, Eq. (16), can be accomplished with a measurement of the shaker force (a direct measurement or a measurement of the drive current) and a measurement of the interface motion.

Conclusions

The revival of test control methods using force is appropriate considering the advances in testing technology in the last 15 years. The method reviewed in the paper shows real merit and should be investigated further. Other methods should be revisited.

References

1. J. V. Otts, "Force Controlled Vibration Tests: A Step Toward Practical Application of Mechanical Impedance," The Shock and Vibration Bulletin, No. 34, Part 5, pp 45-53, Feb. 1965.

2. W. B. Murfin, "Dual Specifications in Vibration Testing," The Shock and Vibration Bulletin, No. 38, Part 1, Aug. 1968.

3. A. F. Witte, " Dual Specifications in Random Vibration Testing, an Application of Mechanical Impedance." The Shock and Vibration Bulletin, No. 41, part 4, pp 109-118, Dec, 1970.

4. A. F. Witte, "A Force-Acceleration Control Technique for Vibration Testing," Instrument Society of America, 1969 Annual Conference Proceedings, Paper 69-560.

5. T. D. Scharton and D. L. Kern, "Using the VAPERS Program to Support the TOPEX Spacecraft Design Effort," 59th Shock and Vibration Symposium, Vol. IV, pp 21-36 Sandia National Laboratories Report SAND88-2473C, Dec. 1988. Available from NTIS, U.S. Dept. of Commerce, 5285 Port Royal Road, Springfield, VA 22161.

6. D. O. Smallwood, "The Application of Unloaded (Free) Motion Measurements and Mechanical Impedance to Vibration Testing," Proc. of the Institute of Environmental Sciences, pp 71-82, April 1976.

7. V. H. Neubert, Mechanical Impedance: Modelling/Analysis of Structures, 1987, Jostens Printing and Publishing Co., Science Park Road, State College, PA 16801.

8. D. J. Ewins, Modal Testing: Theory and Practice, Research Studies Press Ltd., Letchworth, Hertfordshire, England, 1986.

9. D. O. Smallwood and D. L. Gregory, "Bias Errors in a Random Vibration Extremal Control Strategy," The Shock and Vibration Bulletin, No. 50, Part 2, pp 199-205, Sept. 1980.