AIIM

Association for Information and Image Management

1100 Wayne Avenue, Suite 1100
Silver Spring, Maryland 20910

301/587-8202

Centimeter

Inches

1 of 1

Conf-930702--40

APPLICABILITY OF EQUIVALENT STATIC METHOD TO SEISMIC RESPONSE OF PIPING AND OTHER COMPONENTS*

B. J. Hsieh
Reactor Engineering Division
Argonne National Laboratory
Argonne, IL USA

For
1993 American Society of Mechanical Engineers Conference
Denver, Colorado
July 25–29, 1993

February 1993

*Work sponsored by the U.S. Department of Energy, under contract number W–31–109–Eng–38.

APPLICABILITY OF EQUIVALENT STATIC METHOD TO SEISMIC RESPONSE OF PIPING AND OTHER COMPONENTS

B J Hsieh
Reactor Engineering Division
Argonne National Laboratory
Argonne, IL

ABSTRACT

The Equivalent Static Method (ESM) is a simple and cost effective approach in the design of systems and components subjected to seismic loads. However, its applicability is restricted to systems which can be represented by a "simple model." In this paper the restriction to a simple model is examined using the example of a propped cantilever, for which some codes or standards explicitly state that ESM is not applicable. By comparing ESM results for the propped cantilever with those for a regular (un-propped) cantilever, it is found that ESM can conditionally be applied to the propped cantilever configuration.

INTRODUCTION

Key components, including piping of nuclear power plants, must be qualified for seismic loads that are likely to occur at the plant site. One way of qualification by analysis is the time-history analysis. This requires detailed input histories at the component location together with a compatibly refined mathematical model, e.g., a finite-element model, of the component or system. The method is relatively labor-intensive and time-consuming. A somewhat simpler and less work-intensive approach is to use the Response Specrtum Method, for which only the design response spectrum for the system must be specified. However, it is often desirable to have even simpler methods in order to obtain conservative response estimates of a system with relatively little effort.

Such a method is the so-called Equivalent Static Method (ESM). Its successful use depends on the characteristics of the component and the nature of the seismic input. The conditions for ESM applicability are described in many design and analysis codes and standards. In general the requirement is that the system can be represented by a simple model, i.e., has simple dynamic characteristics. This paper examines the ESM issue of simple structure and the corresponding amplification factors. Specifically the focus is on propped cantilevers with various lengths of overhang.

BACKGROUND

To understand the essence of the equivalent static method, a summary of the standard seismic analyses is given below:

Time history method -

For given input motion histories, the equations of motion can be numerically integrated in time in a step-by-step manner to yield directly the total response. This approach is known as the direct-integration time-history method. This method can also be used for nonlinear systems in which the normal modes are not clearly defined.

Alternatively for linear systems, an eigensystem analysis can be first performed on the system to obtain all the important vibration modes. Each mode is uncoupled from the rest. Dependent on the seismic input time history, each modal response is obtained by numerically or analytically integrating in time the corresponding modal equation of motion. Each modal response therefore is a function of time. The total response of the system is the sum of all the modal response histories, and is a function of time. This approach is known as the normal-mode time-history method.

Response spectrum method -

The vibration modes of the system are first obtained. The maximum response of each mode is then obtained at the corresponding modal frequency from the given seismic design response spectrum. This is so because the response spectrum is a representation of the maximum responses (acceleration, velocity, or displacement) of a family of idealized linear single-degree-of-freedom damped oscillators as a function of natural frequencies of the oscillators to a specified vibratory motion at their supports. The design response spectrum is a smoothed response spectrum obtained by analyzing, evaluating, and statistically combining a number of individual response spectra derived from earthquake records. Each modal response, therefore, is a constant. The total response of the system is the sum of all the modal responses. The summation can be done absolutely, or by SRSS or 10% grouping or other appropriate algorithms.

Equivalent Static Method-

Similar to the response spectrum method, this method directly uses the (design) response spectrum. The system is analyzed by subjecting it to an equivalent static load proportional to its mass distribution in the direction of the earthquake. The load magnitude is determined from the design response spectrum together with a load or static coefficient (amplification factor). The value of this coefficient is consequential in providing conservative estimates to the responses of the system.

Evidently the time-history method may be preferred, when the goal is to examine a system's state under a particular seismic event when the recorded time histories are input to the analysis. For design and qualification purposes, because the seismic input may only be specified through a smoothed design response spectrum, a spectrum-consistent time history is generally not available without major effort. Under these circumstances, the equivalent static method may be preferred.

EQUIVALENT STATIC METHOD

The seismic load is applied as an equivalent static load (magnitude determined from the given response spectrum) proportional to the mass distribution of the structure. Because the mass distribution of the structure is used, the ESM is not entirely a static method and could just as well be called the equivalent dynamic method.

The major parameter that must be selected in this approach is the factor multiplying the mass distribution to define the equivalent static load. This factor has the unit of acceleration and must reflect the dynamic characteristics of both the seismic load and the structure. Therefore, the factor is dependent on the design acceleration response spectrum and on the shape and boundary condition of the structure. The factor generally is taken to be the product of the maximum spectral acceleration of the design response spectrum and a (static) coefficient to include the "higher mode effect."

The essence of ESM is that the dynamic behavior of the structure can be adequately captured by the fundamental mode alone (within a scalar multiplier) and that the fundamental mode can be adequately described by the static deflection of the mass-proportional load. Natural frequencies of the structure are not required; however, when available, they may be used to refine the approach. Thus instead of the maximum spectral acceleration, the spectral acceleration at the fundamental frequency or the maximum spectral acceleration on the higher-frequency end can be used. Any ESM will yield a conservative estimate of the response, if a sufficiently large acceleration or coefficient is chosen. The success of ESM lies in providing a conservative estimate without undue penalty by judicially choosing the spectral acceleration and static coefficient and their combination.

To further illustrate the essence of the ESM approach and application, guidelines from some of the important codes and standards are summarized below:

IEEE Recommended Practice for Seismic Qualification of Class 1E Equipment for Nuclear Power Generating Stations[1]

...

6.3 Static Coefficient Analysis. This is an alternate method of analysis that allows a simpler technique in return for added conservatism. ... A static coefficient of 1.5 has been established from experience to take into account the effects of multifrequency excitation and multimode response for linear frame-type structures, such as members physically similar to beams and columns, which can be represented by a simple model. ... In a static coefficient analysis, the seismic forces on each component of the equipment are obtained by multiplying the values of the mass times the maximum peak of the RRS times the static coefficient. The resultant force should be distributed over the component in a manner proportional to its mass distribution. The stress *at any point in the equipment can then be determined by combining the stress at that point due to the earthquake loading in each direction using the SRSS method.*

USNRC Standard Review Plan[2]

...

An equivalent static load method is acceptable if:

(1) Justification is provided that the system can be realistically represented by a simple model and the method produces conservative results in terms of responses. Typical examples or published results for similar structures may be submitted in support of the use of the simplified method.

(2) The design and associated simplified analysis account for the relative motion between all points of support.

(3) To obtain an equivalent static load of a structure, equipment, or component which can be represented by a simple model, a factor of 1.5 is applied to the peak acceleration of the applicable floor response spectrum. A factor of less than 1.5 may be used if adequate justification is provided.

ASCE Standard[3]

...

3.2.5 Equivalent-Static Method

3.2.5.3 Simple Multi-Degree-of-Freedom Models

(a) For cantilevers with nonuniform mass distribution and other simple models in which the maximum response results from loads in the same direction, the equivalent-static load shall be determined by multiplying the structure, equipment, or component masses by an acceleration equal to 1.5 times the peak acceleration of the applicable response spectrum. Smaller values may be used, if justified, or the floor ZPA value may be used if it is shown that the fundamental frequency is so high that no dynamic amplification will occur.

(b) The equivalent static method does not apply to propped cantilevers.

SIMPLE MODEL

When a structure, equipment, or component is classified as simple, and hence can be represented and analyzed by a simple model, then the ESM estimates conservatively the response. Generally, the acceleration used is the maximum spectral acceleration and the static coefficient is 1.5. The above cited codes and standards allow lower value of either the acceleration or the static coefficient, if justified.

Unfortunately, there is no clear definition of "simple model." One definition could be that those whose dynamic responses can be conservatively estimated by ESM are simple models. This definition, however, begs the question. For familiar structures the simplicity probably can be established from experience or from reported results in the open literature: Such as IEEE's recognition of linear frame-like structures (beams and columns), and ASCE's of cantilever. For general unknown and complex structures, the simplicity can only be resolved by complete dynamic analyses if it can not be established a priori by "engineering judgment."

The ASCE's guidelines explicitly exclude the application of ESM to propped cantilevers, without clearly defining what they are. This exclusion is counter-intuitive and also appears to contradict the IEEE guidelines.

In the following, it is demonstrated that for purpose of ESM application at least some propped cantilevers are just as simple as the regular unpropped cantilever.

PROPPED CANTILEVER

A (regular) cantilever is well-defined: It is a beam structure with one end fixed and the other free; i.e., an overhanging beam. A propped cantilever can mean a cantilever whose overhanging free end is propped, resulting in a structure without overhang; or a beam with one end fixed and the other simply supported or hinged. In such case, it is not a cantilever at all. Alternatively, a propped cantilever is still a cantilever with a characteristic overhang. The prop then must be a simple support somewhere between the free and fixed ends. To cover all the possible cases, the second interpretation is used in this study. When the support is at the free end, the first interpretation results.

The dynamic mode shapes and the static deflection due to the mass-proportional load are essential for ascertaining the applicability of ESM to propped cantilevers without actually performing complete dynamic analyses. The ESM requires closeness between the fundamental mode and static deflection shapes; and the particular static coefficient of 1.5 requires that the fundamental mode constitutes more than two-thirds of the total response. Therefore it is necessary to have measures for

(1) the closeness between the two shapes, (2) the modal participation.

To identify when ESM applies, the prop location is varied to cover the case of uniform regular cantilever, the fixed-hinged beam, and many intermediate propped cantilevers. No actual time-history or response-spectral analysisis is performed; the applicability of ESM is deduced by comparing these measures to those of the regular fixed-free cantilever, for which it is known that ESM is applicable.

(1) Closeness of shapes between fundamental mode and static deflection:

The mode and static deflection of a propped uniform cantilever are obtained by solving the regular beam equations.

For the static deflection under uniform mass-proportional loading, the solution is obtained by combining the solution of a regular fixed-free cantilever under uniform load and a solution of the same cantilever subjected to a concentrated force at the prop location. The magnitude and direction of the force are such that the displacement at the prop location will cancel out that produced by the uniform load. The respective solutions for uniform and concentrated loads can be obtained from handbooks, such as Roark's[4].

For the dynamic modes, the propped cantilever is divided into two segments, one from the fixed end to the prop and the other from the prop to the free end. A regular eigensystem is formulated for each segment. Through a match of the segmental mode shapes at the prop location, a characteristic or frequency equation emerges. The natural frequency of the propped cantilever is then obtained by a numerical iterative scheme from an initial estimate. The complete mode shape is then the union of the segmental mode shapes.

The closeness between the mode shape and the deflection shape is measured by the normality N as follows:

$$\begin{aligned} N &= \int \phi(x) y_s(x) dx \\ 1 &= \int \phi^2(x) dx \qquad (1) \\ 1 &= \int y_s^2(x) dx \end{aligned}$$

where y_s is the normalized static deflection, and ϕ is the normalized mode shape.

The Mathematica[5] software is used in both the static and the eigensystem processes.

(2) Modal participation:

The static coefficient multiplying the spectral acceleration in the ESM reflects the relative contributions of the dynamic modes. Each modal participation typically depends on the dynamic nature of the structure and of the seismic load. The larger the contribution of the fundamental mode (first mode in this study), the closer the static coefficient can be to unity.

Without explicitly solving the dynamic system with specific input, the participation can only be estimated. It is related to the participation factor (of the input) defined by

$$PFD = \int \phi(x) p(x) dx \qquad (2)$$

Here $\phi(x)$ is the (fundamental) mode shape, $p(x)$ is the mass-proportional load which is constant in x for uniform cantilevers, and PFD stands for dynamic participation to distinguish it from the static participation defined below.

The ESM approximates $\phi(x)$ by $y_s(x)$, the static deflection due to $p(x)$; therefore, a static participation factor, PFS, can be similarly defined by

$$PFS = \int y_s(x) p(x) dx \qquad (3)$$

The comparison between PFD and PFS together with N gives an indication of the closeness between $\phi(x)$ and $y_s(x)$.

Table 1 summaries the results for the propped cantilever. The PFD corresponds to the first or fundamental dynamic mode shape, the nondimensional prop location, a, is the ratio of the length between the fixed end and prop to the total length. The results are obtained by using Mathematica software to derive the above integrands and to integrate.

In principle, similar tables can be constructed for higher dynamic modes. However, selecting the initial estimates and checking the final convergence for the iterative solution of the nonlinear transcendal frequency equation is a tedious, manual task, particularly when many modes are examined. Thus the propped cantilever is also analyzed by the ANSYS software[6], which directly gives the participation factor and effective mass of each mode. The number of modes extracted is only limited by the degrees of freedom of the finite element model, the accuracy of the eigensystem solver, and the patience of the analyst. Eighteen modes are extracted in the current study.

Table 2 shows the relative modal participation factors in percentage of the total participation of all modes. Since the

TABLE 1
SUMMARY OF PARTICIPATION AND NORMALITY BETWEEN FIRST MODE SHAPE AND STATIC DEFLECTION

Nondimensional Prop Location	PFD	N	PFS
0/8	0.782962	0.9999	0.789323
1/8	0.743342	0.999905	0.74913
2/8	0.695408	0.999923	0.699631
3/8	0.630579	0.999957	0.634131
4/8	0.531135	0.99983	0.543177
5/8	0.332942	0.981584	0.468487
6/8	0.233876	0.793142	0.66661
7/8	0.735872	0.99946	0.746453
8/8	0.860001	0.999922	0.863743

TABLE 2
SUMMARY OF RELATIVE MODAL PARTICIPATION FACTORS

	Nondimensional Prop Location								
Mode	0/8	1/8	2/8	3/8	4/8	5/8	6/8	7/8	8/8
1	31%	31%	27%	22%	20%	12%	10%	32%	41%
2	17%	17%	14%	9%	9%	25%	33%	8%	4%
3	10%	10%	6%	12%	24%	9%	2%	1%	16%
4	7%	6%	1%	17%	4%	6%	12%	16%	2%
5	6%	5%	14%	6%	5%	10%	8%	3%	10%
6	5%	3%	12%	1%	7%	7%	5%	11%	1%
7	4%	2%	6%	3%	9%	2%	10%	1%	7%
8	3%	1%	3%	4%	0%	6%	0%	8%	1%
9	3%	1%	1%	8%	2%	5%	5%	1%	5%
10	3%	7%	3%	3%	6%	2%	3%	6%	1%
11	2%	7%	1%	1%	2%	4%	2%	1%	4%
12	2%	4%	4%	1%	1%	3%	5%	5%	1%
13	1%	2%	5%	4%	2%	1%	0%	1%	2%
14	2%	2%	2%	3%	4%	4%	3%	4%	1%
15	1%	1%	1%	1%	1%	0%	0%	1%	2%
16	1%	1%	0%	1%	0%	2%	1%	2%	1%
17	0%	1%	1%	1%	2%	2%	0%	1%	1%
18	1%	0%	0%	3%	2%	2%	0%	1%	0%

For each propped cantilever, the entry in this table is

$(PFD)_i = \int \phi_i(x)p(x)dx$; ϕ_i = ith mode shape

$$\text{Entry} = |(PFD)_i| \Big/ \sum_{i=1}^{18} |(PFD)_i|$$

factors are signed, the total participation is defined as the sum of the absolute contribution of each of the modes. This tends to reduce the portion of the first mode compared to an algebraically summed total. Also because the participation factors relate directly to the composition of the input load (or the mass distribution) rather than directly to the response, they can not be directly used to judge the ESM applicability of any particular propped cantilever in a definitive sense. For example, it is not clear from this table why ESM is only applicable to the regular cantilever (a = 0/8) per ASCE's guidelines. However, given that it is and all other conditions being equal, then it is reasonable to deduce that the ESM

TABLE 3
SUMMARY OF RELATIVE MODAL EFFECTIVE MASS

	Nondimensional Prop Location								
Mode	0/8	1/8	2/8	3/8	4/8	5/8	6/8	7/8	8/8
1	63%	62%	53%	42%	30%	12%	6%	63%	77%
2	19%	19%	14%	7%	7%	56%	68%	4%	1%
3	7%	6%	3%	12%	44%	7%	0%	0%	12%
4	3%	3%	0%	24%	1%	3%	9%	17%	0%
5	2%	1%	14%	3%	2%	8%	4%	0%	5%
6	1%	1%	10%	0%	4%	4%	2%	7%	0%
7	1%	0%	2%	1%	6%	0%	6%	0%	2%
8	1%	0%	1%	2%	0%	3%	0%	4%	0%
9	1%	0%	0%	5%	0%	2%	2%	0%	1%
10	0%	3%	0%	1%	3%	0%	0%	2%	0%
11	0%	3%	0%	0%	0%	2%	0%	0%	1%
12	0%	1%	1%	0%	0%	1%	1%	1%	0%
13	0%	0%	2%	2%	0%	0%	0%	0%	0%
14	0%	0%	0%	1%	1%	1%	0%	1%	0%
15	0%	0%	0%	0%	0%	0%	0%	0%	0%
16	0%	0%	0%	0%	0%	0%	0%	0%	0%
17	0%	0%	0%	0%	0%	0%	0%	0%	0%
18	0%	0%	0%	1%	0%	0%	0%	0%	0%
MM	97%	89%	91%	94%	91%	94%	90%	90%	97%
1st	61%	55%	48%	39%	27%	11%	5%	57%	74%

For each propped cantilever, the entry in this table is

$(PFD)_i = \int \phi_i(x)p(x)dx$; ϕ_i = ith mode shape

$$\text{Entry} = (PFD)_i^2 \Big/ \sum_{i=1}^{18} (PFD)_i^2$$

$$MM = \sum_{i=1}^{18} (PFD)_i^2 \Big/ \text{Total Mass of Cantilever}$$

TABLE 4
SUMMARY OF FIRST MODAL FREQUENCY AND PARTICIPATION RELATIVE TO THAT OF REGULAR CANTILEVER

Nondimensional Prop Location	Natural Frequency	Participation Factor	Effective Mass	Modified Mass
0/8	1.00	1.00	1.00	1.00
1/8	1.22	0.98	0.98	0.90
2/8	1.53	0.87	0.84	0.79
3/8	2.01	0.71	0.67	0.64
4/8	2.80	0.63	0.48	0.45
5/8	4.20	0.37	0.19	0.18
6/8	6.11	0.32	0.10	0.09
7/8	5.62	1.02	1.00	0.93
8/8	4.38	1.31	1.22	1.22

The participation factor is relative to the absolute sum.
The effective mass is relative to the total modal mass of 18 modes.
The modified mass is relative to the total physical mass.

should apply also to propped cantilevers with nondimensional prop location of 1/8, 7/8, and 8/8.

Table 3 is similar to Table 2 but shows the relative modal masses in percentage of the total modal mass of the 18 modes. Since each modal mass is the square of the corresponding modal participation factor, the modal mass is unsigned. It can be shown that the total physical mass of the structure is the total modal mass, if an infinite number of modes is included and if the mode shapes are normalized in a certain way.

Because the seismic effect is transmitted to a structure as an inertia load, the percentage of total modal mass relative to the total physical mass is an important indicator of the adequacy of the finite element model. The **MM** row in Table 3 lists this percentage. Each entry when referred to the total physical mass must be modified accordingly. Row **1st** is the modified percentage of the first or fundamental modal mass. It is evident that ESM is at least as applicable for the case a = 8/8 as for the case a = 0/8, and the cases a = 1/8 and a = 7/8 are also tolerable.

Table 4 summarizes the first modal properties relative to the case of a = 0/8. The natural frequency clearly shows nonlinearity in terms of prop location. The cantilever is stiffest at prop location of 6/8. In this case, clearly the ESM does not apply, unless a large static coefficient is used.

DISCUSSION AND CONCLUSIONS

The reference point is that the ESM applies to the regular, unpropped cantilever with a static coefficient of 1.5. The applicability of ESM to propped cantilevers is first examined by analyzing the fundamental dynamic mode shape and static deflection shape, both related to the mass distribution. The central requirement is that the two shapes must be close to each other. However, the shapes need not be infinitesimally close to each other. The closeness needs only be compatible to that for the regular cantilever. The closeness measure, Normality N in Table 1, indicates that the these shapes are amazingly close for all cases except the case of a = 6/8 and maybe a = 5/8. A different sense of closeness is provided by PFD and PFS. These are all plotted in Figure 1. Note the two participations differ significantly in the range 5/8 ≤ a ≤ 7/8. Here the ESM is not applicable. However, the static deflection may actually pick up more mass than the first mode.

The closeness between fundamental mode shape and the static deflection shape indicates that the static solution is close to the one-mode modal solution, which in turn is hoped to be a good approximation to the actual response. For those whose closeness is compatible to that of the regular cantilever, one must also ensure that the fundamental mode alone has enough modal mass to sufficiently represent the inertia property. Again, the percentage of the first modal mass needs only be compatible to that of the regular cantilever, about 61% of the total physical mass; note that the static coefficient of 1.5 is expected to somehow compensate for the missing 39%. The summarized first-mode modal properties are plotted in Figure 2. Unequivocally, there is a substantial range of prop location where the first mode catches less than 90% of the mass relative to that of the regular cantilever. This is not admissible for ESM application where the static coefficient is 1.5. However, this under-representation of mass can be remedied by using a larger static coefficient for those propped cantilevers classified as having "close shapes."

In seismic analysis, the participation factor of a mode is the coefficient before the mode shape when the mass-proportional load is decomposed into the mode-shape or modal spectrum. It is a measure of the input load in terms of each mode. When the response is decomposed into the modal spectrum, the coefficient for each mode is termed mode coefficient in ANSYS. The participation factors can be calculated within a scalar multiplier without specific seismic input. But the mode coefficient depends on the response and hence can only be computed for a given seismic input. Actually, it is the dominance of the fundamental mode coefficient that ensures the success of ESM. Since generally the seismic input is unknown, the mode coefficient is unknown. The mode coefficients are related to the modal participation factors as a whole. This is so because the load and the response are linked by the equations that govern the behavior of the structure. Generally speaking, the structure works like a narrow-band filter with the passing frequency range around the first modal frequency. Therefore, for random-like input or uniform design response spectrum, the mode coefficient decreases rapidly for higher modes. The decrease rate is much faster than that of the modal participation factor. (This is also verified by ANSYS results for the current study.) Consequently, the participation factor or its square, the modal effective mass, underestimates the first-mode contribution to the response. Therefore, it is conservarive to use the participation factor to examine the ESM applicability.

Figures 1 and 2 lead to the following conclusion. For ESM applicability, a propped cantilever is cataloged into one of the following depending on the nondimensional distance, a, between the fixed end and the prop location:

(1) a = 0
This is the regular cantilever and ESM is applicable.

(2) 0 < a ≤ 1/2
ESM is conditionally applicable. The static coefficient should be modified by the factor, f = 1 + 22a/9.
This linear modification equation is obtained by drawing a straight line between the points a = 0 and a = 1/2 and realizing the mass at the latter is only 45% of the former. The static coefficient is then 1.5f. At a = 0.5, the coefficient can be as high as 10/3.

(3) 1/2 < a < 7/8
ESM is not applicable because the mode shape and static deflection are not close.

(4) 7/8 ≤ a ≤ 0.9118
ESM is conditionally applicable. The static coefficient

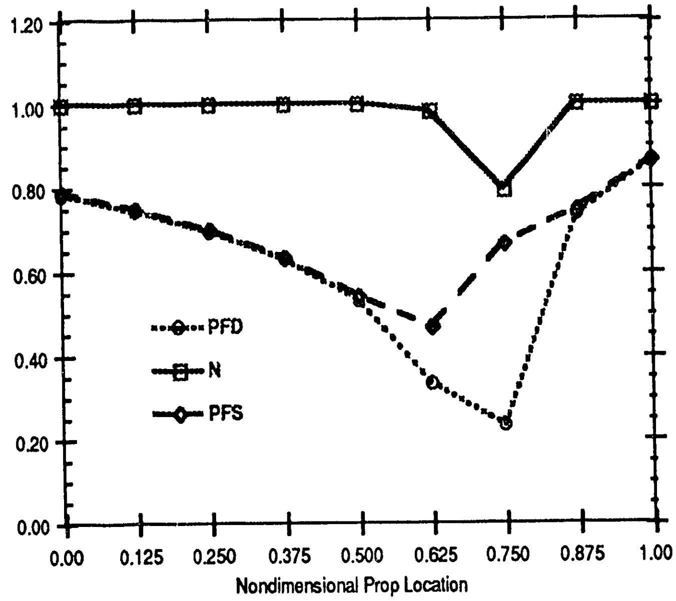

FIGURE 1
CLOSENESS BETWEEN 1ST MODE AND STATIC DEFLECTION

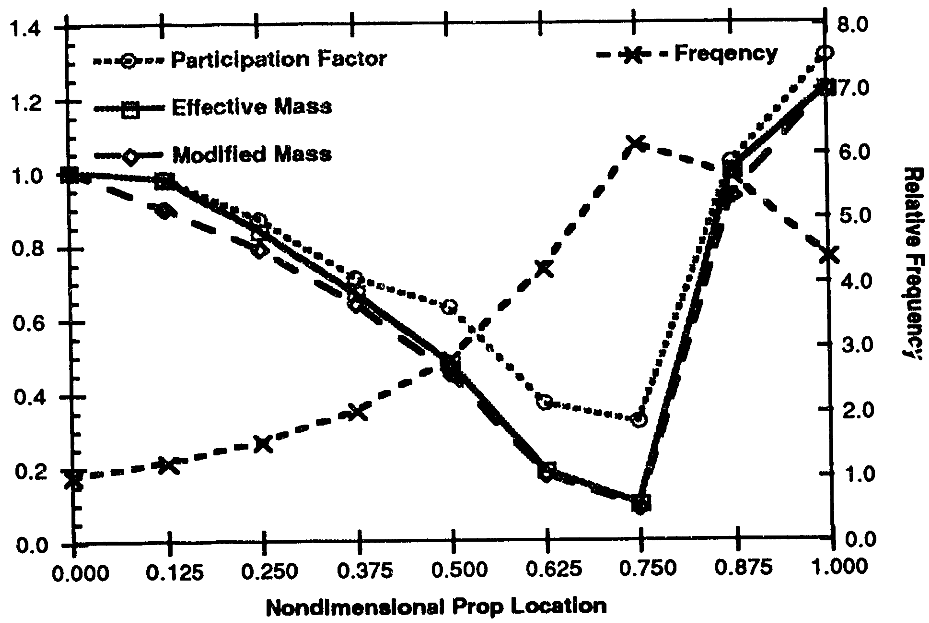

FIGURE 2
RELATIVE FIRST MODE PROPERTIES

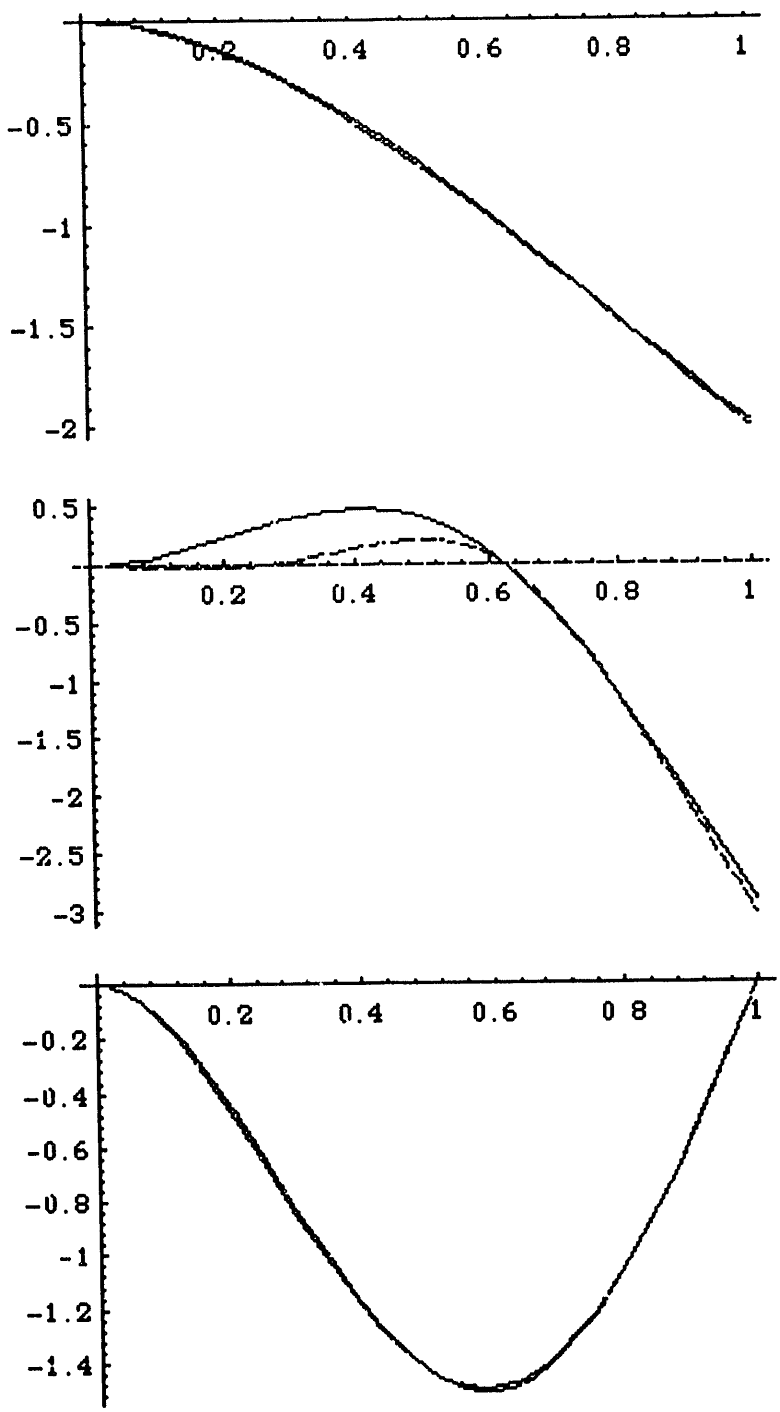

FIGURE 3
NORMALIZED FIRST-MODE AND STATIC-DEFLECTION SHAPES
FOR a = 0, a= 6/8, AND a = 1

should be modified by the factor, f = 2.8644 - 2.0448a. This linear modification equation is obtained by drawing a straight line between the points a = 7/8 and a = 1 and realizing the masses are respectively 93% and 122% of that at a = 0. The static coefficient is then 1.5f. At a = 0.9118, f = 1.

(5) $0.9118 \le a \le 1$
ESM is applicable. If desired, the static coefficient can actually be reduced by the same factor as derived above, f = 2.8644 - 2.0448a.
The static coefficient is then 1.5f. At a = 1, f = 0.8196, and the static coefficient becomes 1.23.

The worst case when ESM is not applicable, a = 6/8, has the highest frequency. Since the total physical mass is the same for all cases, this propped cantilever must have the highest stiffness. It is not immediately apparent why high stiffness has a detrimental effect on the EMS applicability. Figure 3 contains normalized 1st-mode shapes and static-deflection shapes for the cases of a = 0, a = 6/8, and a = 1. The violation for the a = 6/8 case of the closeness requirement is apparent (the dashed line is the static deflection). Note that the physical frequency for the a = 6/8 case may be way above the conventional cut-off frequency for rigid range of 33 Hz, where some references permit the application of ESM; e.g., *Piping and pipe support systems*[7]. This observation suggests that frequency and stiffness alone is not sufficient to deduce the applicability of the ESM.

The ESM can conditionally be applied to propped cantilevers. The blanket statement in ASCE's guidelines appears to be unnecessarily restrictive. The relaxation of this rule is beneficial in piping analysis where many propped cantilevers exist.

ACKNOWLEDGMENT

This work is sponsored by the US Department of Energy, under contract number W-31-109-Eng-38.

REFERENCES

[1] *IEEE Recommended Practice for Seismic Qualification of Class 1E Equipment for Nuclear Power Generating Stations*, IEEE Std 344-1987, Institute of Electrical and Electronic Engineers, pp.6-16

[2] USNRC Standard Review Plan, NUREG-0800, Rev.1-July 1981, US Nuclear Regulatory Commission, pp.3.7.2-5

[3] ASCE Standard, *Seismic Analysis of Safety-Related Nuclear Structures and Commentary on Seismic Analysis of Safety Related Nuclear Structures*, ASCE 4-86, September 1986, American Society of Civil Engineers

[4] *Roark's Formulas for Stress & Strain*, 6th ed., Warren C. Young, McGraw Hill, Inc.

[5] *Mathematica, A System for Doing Mathematics by Computer*, 2nd ed., v2.1.0, Wolfram Research

[6] *ANSYS Engineering Analysis System*, v4.4, Swanson Analysis Systems, Inc.,

[7] *Piping and pipe support systems*, P.R. Smith and T. J. Van Lann, §5.3.3.5, pp.125-127

DATE
FILMED
8 / 18 / 93

END