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Stability Study of High β Flux Conserving Equilibria

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OAK RIDGE NATIONAL LABORATORY

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ABSTRACT

The Flux Conserving Tokamak (FCT) model suggested that rapid heating would yield equilibria with high relative energy density ($\beta = 2p/B^2$) while nonetheless allowing control over q, the so-called safety factor for instability within the ideal magnetohydrodynamic (MHD) plasma model. In this study, we show that this is adequate to provide stability to β values of 10%, if there is a conducting metal shell in the vicinity of the plasma.

High values of β (>5%) are required for an economically credible tokamak fusion reactor. Recently, Sykes, Wesson, and Cox[1] have reported stability calculations which predicted stable plasmas with a total average $\beta(\bar{\beta} = 2 \int p d\tau / \int B^2 d\tau)$ $B = |\vec{B}|$ of 12% for a D-shaped plasma with an aspect ratio (A) of 2.4. Toroidal wave numbers (n) of 1, 2, and 3 were considered. Further investigation of their high β equilibria using a model which treats high instability mode number showed that as $n \rightarrow \infty$ the critical $\bar{\beta}(\bar{\beta}_c)$ was only 6%, even when a stabilizing, perfectly conducting shell was assumed to be exactly at the surface of the plasma, so that kink instabilities would not have been seen.[2]

We report stable equilibria with $\bar{\beta}_c$ = 10%. In our case, the perfectly conducting shell assumption was relaxed; we assumed a shell with a radius 20% larger than the plasma radius. A shell at this distance could represent the effect of appropriately chosen first wall material around the plasma. The shell inhibits but does not always prevent kink instabilities.

Flux conserving tokamak[3,4] equilibria with quite high β values[5] have now been analyzed using the computer code ERATO,[6] which treats the plasma stability within a linear ideal MHD model, using a finite element energy principle approach.

The equilibria studied represent D-shaped plasmas with aspect ratio of 4 and elongation of ~1.65. Reference equilibria were generated with specified values of q_0 (safety factor at the magnetic axis), q_s (safety factor at the surface), and β_p ($=2 \int p d\tau / \int B_p^2 d\tau$, B_p is the poloidal component of $\vec{B}$). The reference equilibria were then continuously scaled[7] to reduce $\bar{\beta}$ until stability was found at a critical value $\bar{\beta}_c$. The values of β_p and q_s/q_o are approximately constant during the scaling, so they can be used as labels for the equilibria. We studied nine equilibria with β_p = 1, 2.5, and 3.5, and q_s/q_o = 2, 3, and 4.

Figure 1 shows the flux contours for the equilibria with β_p = 2.5 and q_s/q_o = 2.0, which gave the highest $\overline{\beta}_c$. A conducting shell was assumed to lie at the location of the dashed line. Figure 2 shows the current density and pressure profiles for the same equilibrium. The pressure profile is bell shaped as expected, while the current density profile is peaked toward the outside, as is typical of high β FCT equilibria.[5] There is a tendency toward a "hollow" current density when β is high and the q profile is flattened. Increasing β_p beyond 2.5 gives a pronounced hollow and lower $\overline{\beta}_c$; while the pressure profile retains its character but is more strongly peaked. A smaller β_p gives a current profile with no hollow and the usual pressure profile. Increasing q_s/q_o (with β_p fixed at 2.5) reduces the outside peaking of the current density and smooths out the inside maximum, while broadening the pressure profile with no significant change in shape. Figure 3 shows the q profiles used and emphasizes the very flat character of the case q_s/q_o = 2.0 which gave the highest β_c.

Figure 4 shows regions of stability for the n = 1 mode as a function of q_s. The curves define constant β_p contours with values 1.0, 2.5, and 3.5. On each curve, q_s/q_o varies from 4 at the bottom to 2 at the top. Smaller q values correspond with higher $\overline{\beta}_c$. Diagrams for n = 2, 3, and 4 are similar. The wide current profile (small q_s/q_o) is most stable to internal modes, localized away from the plasma surface while the wall (at a radius 20% greater than the plasma radius) is sufficient to stabilize external kink modes which have a significant displacement of the plasma surface. An optimum β_p is observed to occur near a value $\beta_p \sim A/2$. The value of q_0 is approximately the same along the constant β_p lines with larger values at higher β_p.

The highest value found for $\overline{\beta}_c$ occurs at $q_s/q_0 = 2.0$ and $\beta_p = 2.5$ for n = 1, 2, 3, and 4. Figure 5 shows the beta dependence of the instability growth rate. There is a region of stability for n = 1 at very high β (>30%); this was not studied further because higher n modes were very unstable for the cases treated and in the high β region $q_0 < 1$ which would lead us to expect resistive instabilities[8] which are not included in the present analysis. Tearing modes are found experimentally to limit discharges to $q_0 \gtrsim 1$.

All the results shown are based on convergence studies and extrapolation to infinitesimal numerical grid spacing. Figure 6 shows the growth rates obtained using 30, 35, 40, and 45 grid points in both the radial and poloidal directions. A polynomial expansion in the grid spacing was used to fit the curve; the 50-grid point data served as a check on the fitting procedure, and the growth rates were then extrapolated to zero grid spacing values. Figure 7 shows the dependence of the squared growth rate on $\overline{\beta}_c$ when $N_G = 50$ grid points is used, when quadratic convergence is assumed ($\gamma^2 = a + b/N_G{}^2$, N_G is the number of grid points), and when a quartic convergence expansion is used. As is seen, it is essential that a complete convergence study be used.

Figure 8 shows the dependence of $\overline{\beta}_c$ on toroidal wave number. The point near $n = \infty$ was obtained from ballooning theory.[9,10] Direct evaluation at $n = \infty$ gives instability but inclusion of 1/n corrections[10] suggests that the unstable region lies only at very high n values where validity of the MHD theory is suspect because of finite gyroradius and kinetic effects. Applying the correction,

$$\gamma^2 = \gamma_0^2 + \frac{1}{2n|\nu'(\chi_0)|}\left(\frac{\partial^2\gamma^2}{\partial\chi_0^2}\ \frac{\partial^2\gamma^2}{\partial\psi^2}\right)^{1/2} , \qquad (1)$$

where γ_0 is the $n = \infty$ growth rate, ν' is the local shear, and χ_0 is the lower limit of integration in computing the global shear (see Ref. 10), we find a critical n of 150. This large 1/n correction (which cancels the first order term) occurs because ν' is nearly zero at the most unstable ψ surface. We have applied corrections only to order 1/n, and these are sufficiently large to cause concern over the convergence of this expansion for the cases treated. As a result, this usual expansion probably provides only a rough estimate of the finite n corrections. This point is under further investigation.

The results for all the equilibria and all the toroidal wave numbers have roughly the same quantitative behavior as shown in Fig. 4 for n = 1 except for a progressively lower $\overline{\beta}_c$ as n is increased. Thus, we can use the data to suggest a scaling law — at least for the regime in the present study. The results in Fig. 4 suggest a relation

$$\overline{\beta}_c = U\left(\frac{\beta_p}{A}\right)^{3/2} / q_s^2 A , \qquad (2)$$

where U (for $n \to \infty$) is ~8.6 in this study, but must be regarded as a function of all plasma parameters not varied in the present results. The functional form in Eq. (2), however, does not include the optimum β_p

behavior exhibited in Fig. 4. The rough constancy of q_0^c (the critical safety factor on axis) along each line in Fig. 4 suggests

$$q_0^{c\ 2} = (1 - \frac{\beta_p}{A})^{-1} . \tag{3}$$

Shafranov and Yurchenko[11] found similar q_0^c dependence from a Mercier criterion (localized mode) study. Including Eq. (3) in Eq. (2) gives

$$\overline{\beta}_c = \left\{ 11 q_0^2 \left(\frac{\beta_p}{A}\right)^{3/2} (1. - \frac{\beta_p}{A}) \right\} / q_s^2 A \quad . \tag{4}$$

Equation (4) is an empirical representation of a large set of results for equilibria with A = 4.0. Additional equilibria with different aspect ratios are now being studied to ascertain the A-dependence shown in Eq. (4).

It is our conclusion that stability requirements for a D-shaped plasma with elongation of 1.65 allow stable average β's as high as 10%, even at a high aspect ratio of 4, with $q_s/q_0 = 2.0$ and $\beta_p = \frac{1}{2} A$. Further studies of the dependence of U on the plasma shape, the elongation, and the proximity of the conducting shell may lead to higher beta values, while future analysis of resistive and kinetic instabilities may reduce performance somewhat. In any event, the present results lead us to be significantly more encouraged than was the case earlier.

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Figure Captions

Fig. 1. Flux contours for the equilibria with the highest $\overline{\beta}_c$ (=10%). The dashed line gives the location of the conducting shell.

Fig. 2. The current density and pressure profiles for the equilibria with the highest $\overline{\beta}_c$. Both are shown at z = 0.

Fig. 3. Safety factor profiles.

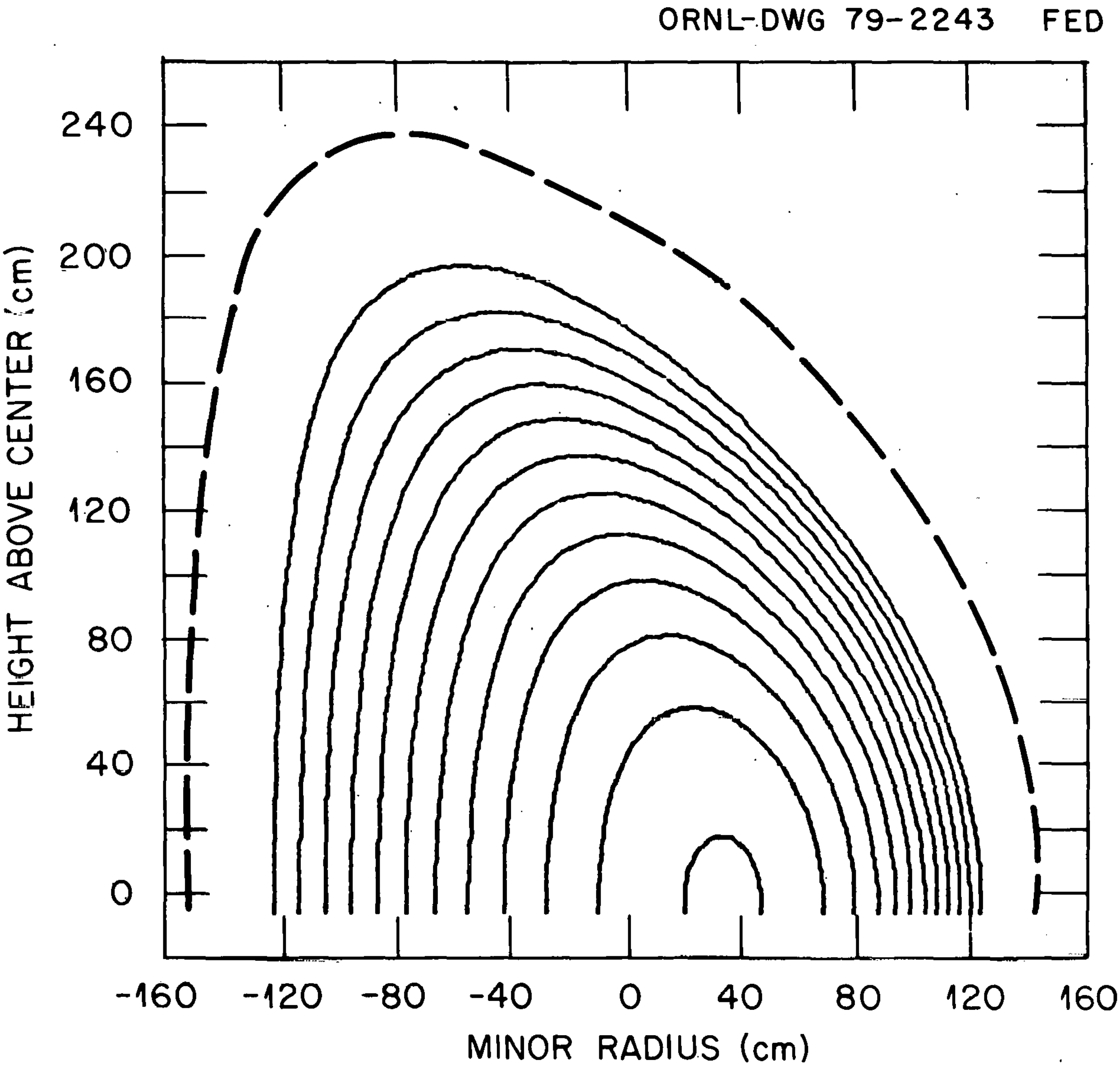
Fig. 4. $\overline{\beta}$ vs q_s for all n = 1. The shaded region on the left of each curve is stable.

Fig. 5. γ^2 vs $\overline{\beta}$ for n = 1, 2, 3, and 4 using the equilibria with the highest $\overline{\beta}_c$.

Fig. 6. The growth rate squared vs N_G^{-2} for n = 4 (N_G is the number of grid points). A polynomial fit was used to find the growth rate as $N_G \to \infty$.

Fig. 7. The growth rate squared vs $\overline{\beta}$ using: 1) 50 grid points (N_G = 50), 2) convergence study assuming N_G^{-2} convergence, and 3) convergence study using a polynomial fit (convergence expansion).

Fig. 8. $\overline{\beta}_c$ vs n^{-1}.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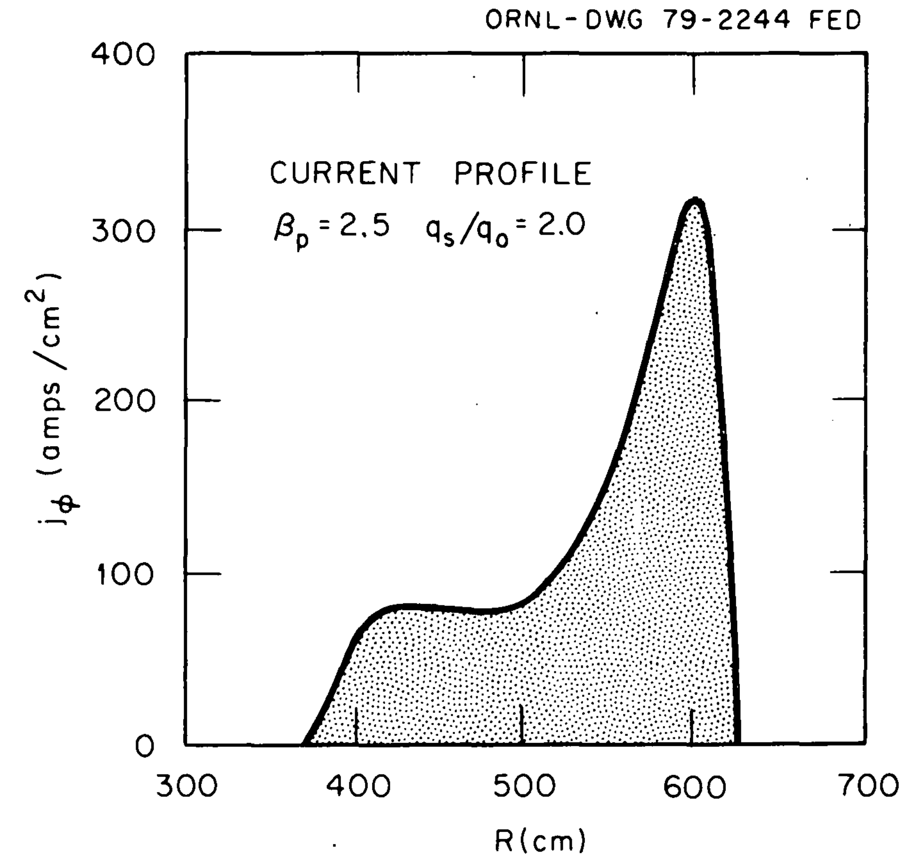
240
200
160
120
80
40
0
HEIGHT ABOVE CENTER (cm)
-160
-120
-80
-40
0
40
80
120
160
MINOR RADIUS (cm)

Fig. 1.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CURRENT PROFILE
$\beta_p = 2.5 \quad q_s/q_o = 2.0$
j_ϕ (amps/cm^2)
400
300
200
100
0
300
400
500
600
700
R(cm)

Fig. 2(a)

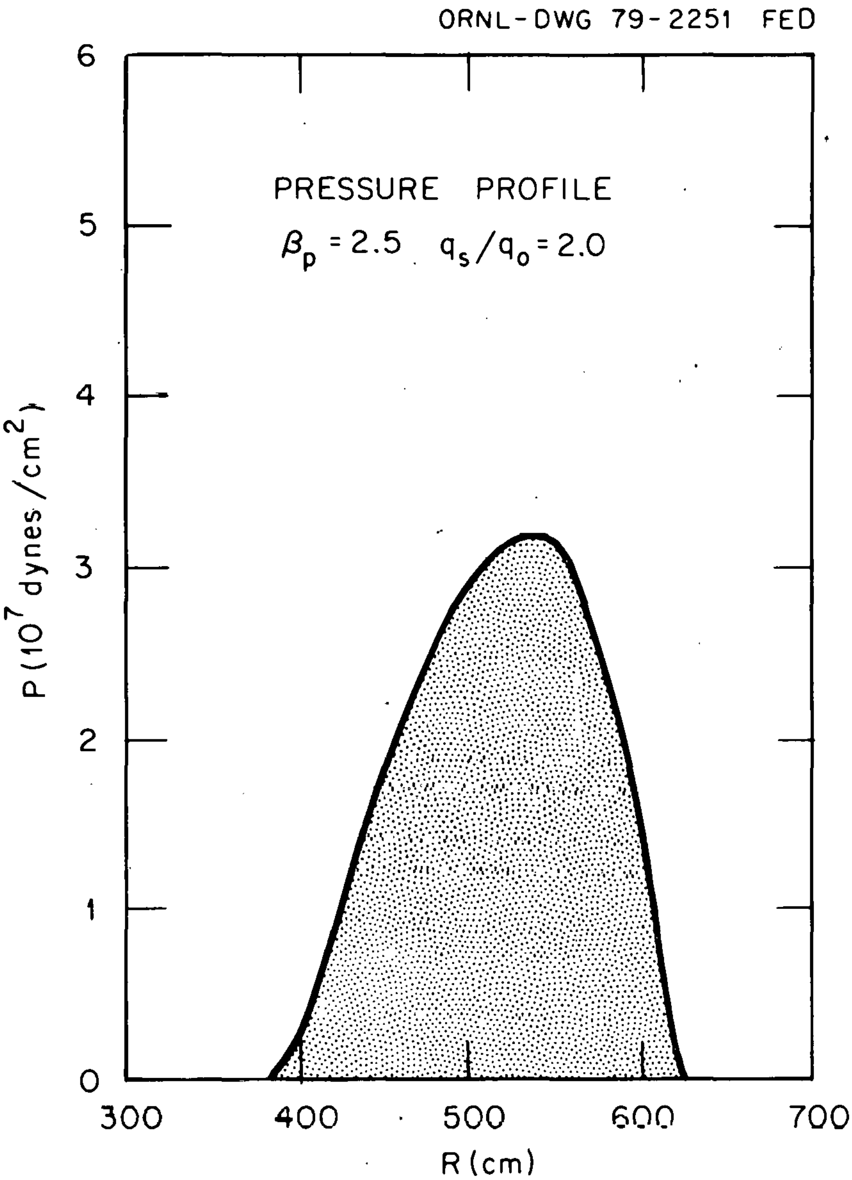
ORNL-DWG 79-2251 FED
PRESSURE PROFILE
$\beta_p = 2.5$ $q_s/q_o = 2.0$
P (10^7 dynes/cm^2)
0
1
2
3
4
5
6
300
400
500
600
700
R (cm)

Fig. 2(b)

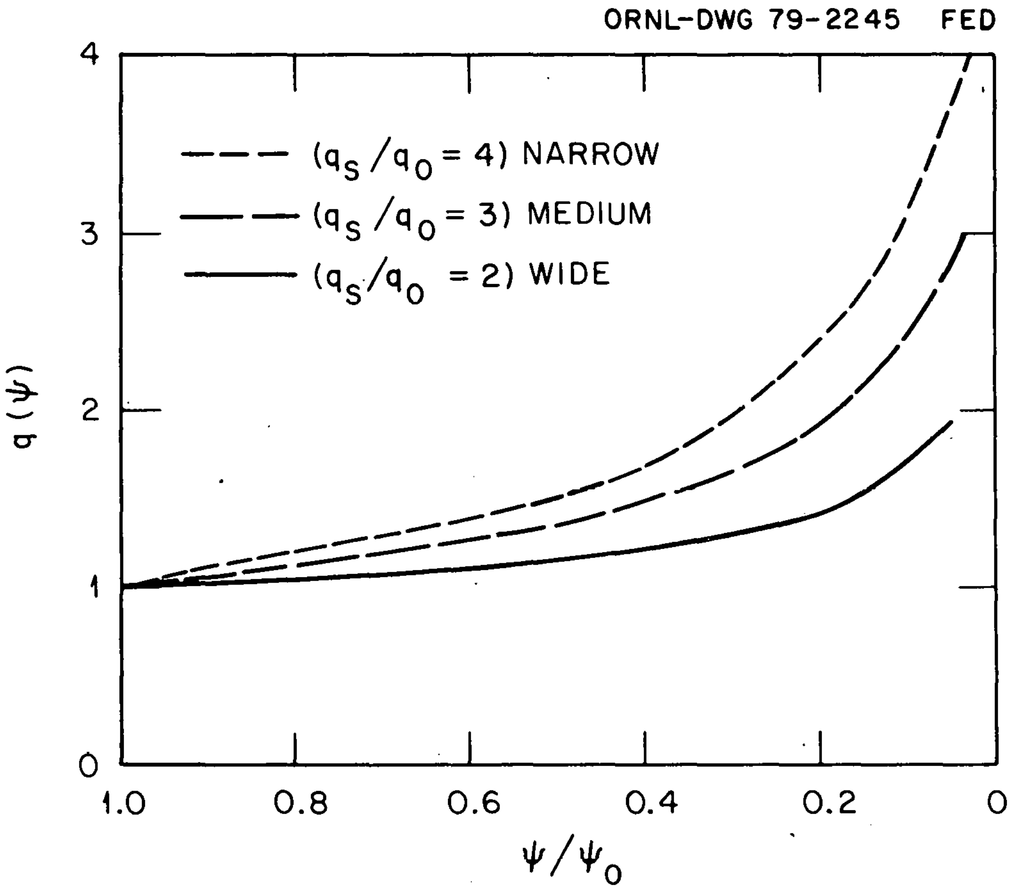
ORNL-DWG 79-2245 FED
$(q_s/q_0 = 4)$ NARROW
$(q_s/q_0 = 3)$ MEDIUM
$(q_s/q_0 = 2)$ WIDE
$q(\psi)$
0
1
2
3
4
1.0
0.8
0.6
0.4
0.2
0
ψ/ψ_0

Fig. 3

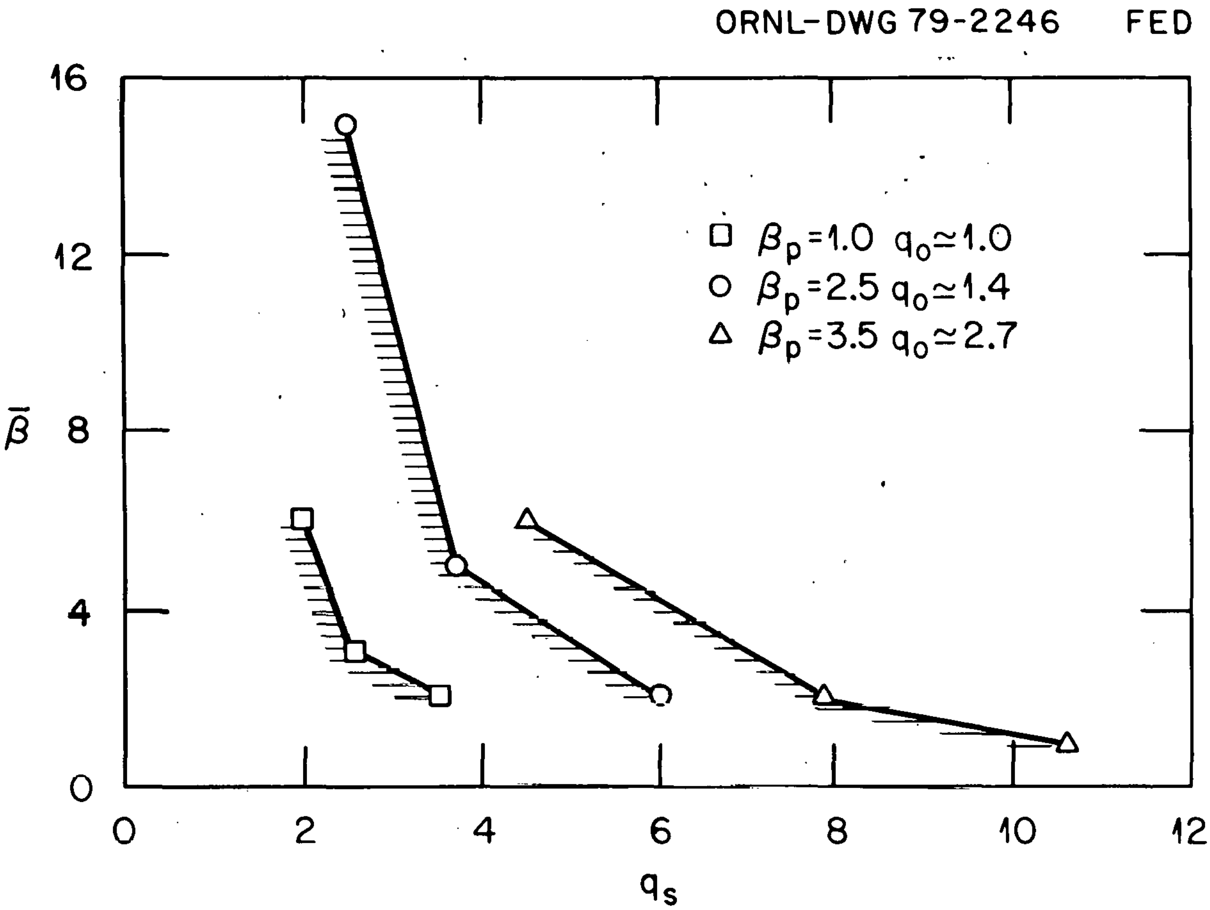
ORNL-DWG 79-2246 FED
16
12
8
4
0
$\bar{\beta}$
0
2
4
6
8
10
12
q_s
β_p=1.0 q_o≃1.0
β_p=2.5 q_o≃1.4
β_p=3.5 q_o≃2.7

Fig. 4.

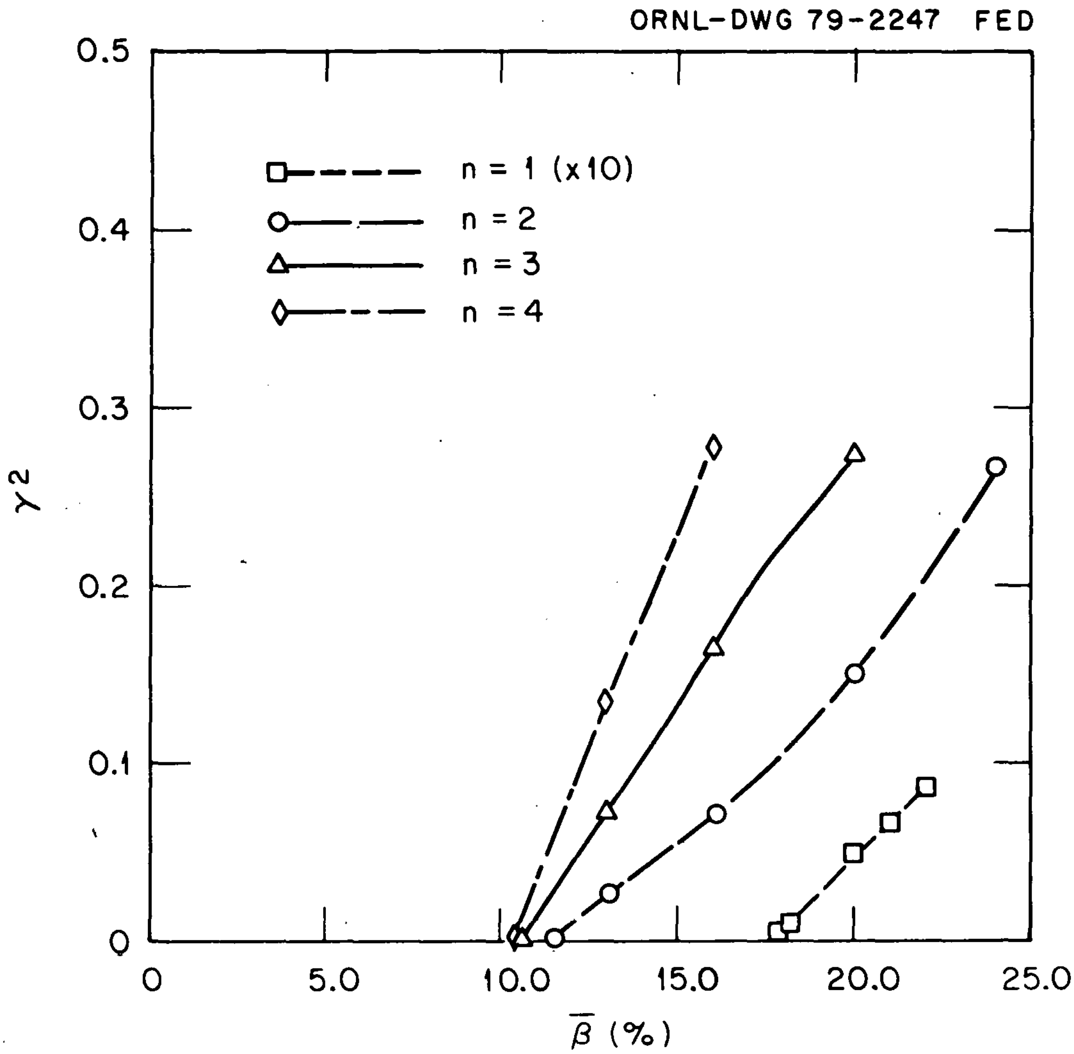
ORNL-DWG 79-2247 FED
n = 1 (x10)
n = 2
n = 3
n = 4
γ^2
0.5
0.4
0.3
0.2
0.1
0
0
5.0
10.0
15.0
20.0
25.0
$\overline{\beta}$ (%)

Fig. 5.

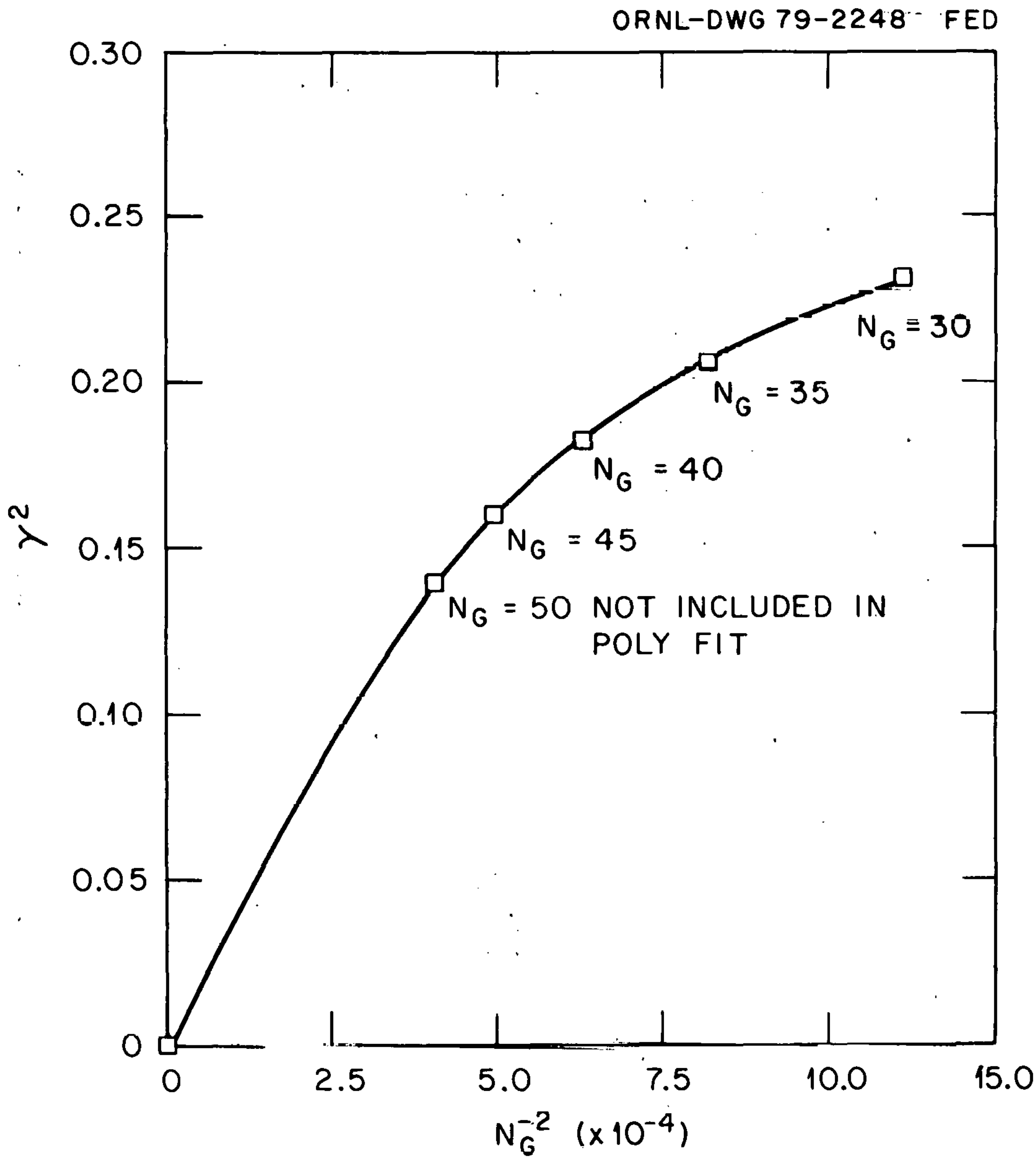
ORNL-DWG 79-2248 FED
0.30
0.25
0.20
0.15
0.10
0.05
0
γ^2
$N_G = 30$
$N_G = 35$
$N_G = 40$
$N_G = 45$
$N_G = 50$ NOT INCLUDED IN POLY FIT
0
2.5
5.0
7.5
10.0
15.0
N_G^{-2} (x 10^{-4})

Fig. 6.

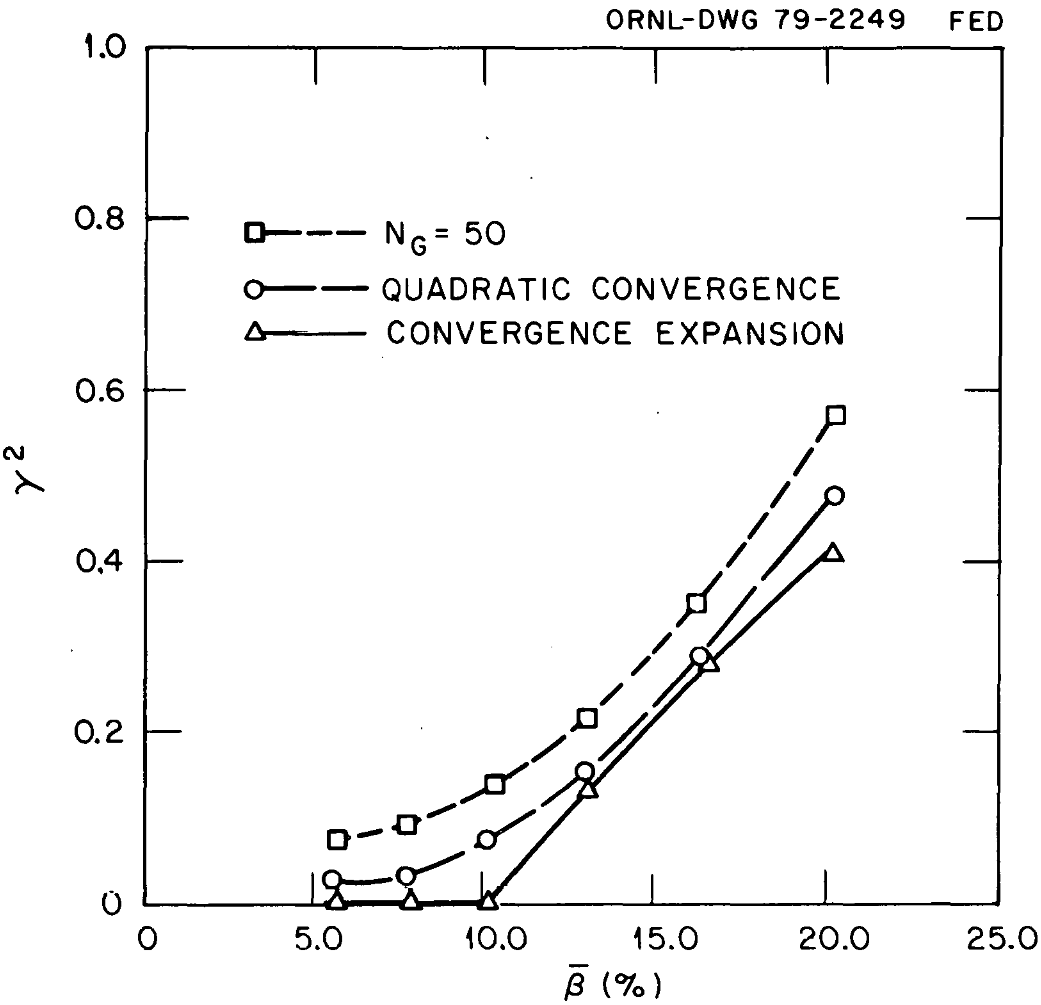
ORNL-DWG 79-2249 FED
1.0
0.8
0.6
0.4
0.2
0
γ^2
N_G = 50
QUADRATIC CONVERGENCE
CONVERGENCE EXPANSION
0
5.0
10.0
15.0
20.0
25.0
$\bar{\beta}$ (%)

Fig. 8.

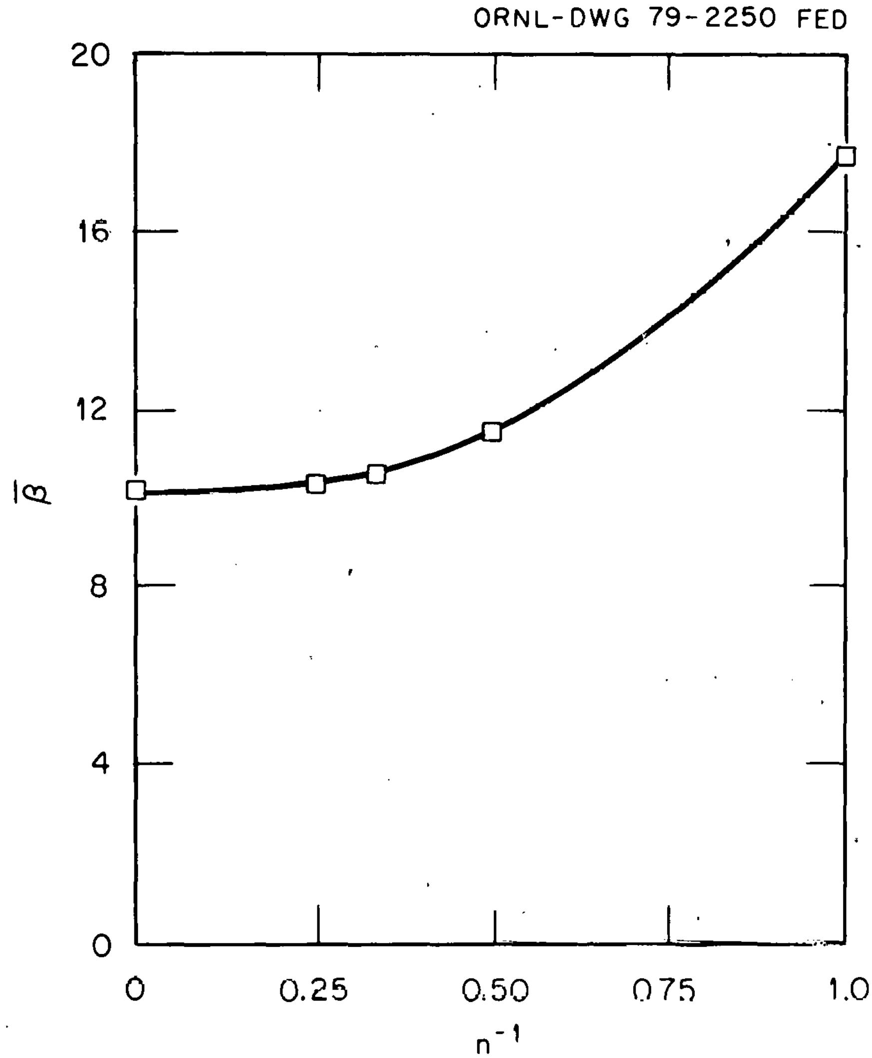

Fig. 7.

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