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AUTHOR(S): Thomas P. Wangler

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EMITTANCE GROWTH FROM SPACE-CHARGE FORCES

by

THOMAS P. WANGLER

Accelerator Technology Division, MS-H817
Los Alamos, NM 87545

Space-charge-induced emittance growth has become a topic of much recent interest for designing the low-velocity sections of high-intensity, high-brightness accelerators and beam-transport channels. In this paper we review the properties of the space-charge force, and discuss the concepts of matching, space-charge and emittance-dominated beams, and equilibrium beams and their characteristics. This is followed by a survey of some of the work over the past 25 years to identify the mechanisms of this emittance growth in both ion and electron accelerators. We summarize the overall results in terms of four distinct mechanisms whose characteristics we describe. Finally, we show numerical simulation results for the evolution of initial rms-mismatched laminar beams. The examples show that for space-charge dominated beams, the nonlinear space-charge forces produce a highly choatic filamentation pattern, which in projection to the 2-D phase spaces results in a 2-component beam consisting of an inner core and a diffuse outer halo. In the examples we have studied the halo contains only a few percent of the particles, but contributes about half of the emittance growth.

1. INTRODUCTION

Many accelerator applications require output beams with high phase-space density or high brightness. To achieve this goal it is necessary to control all sources that cause dilution of the phase-space density. In practice, it is difficult to measure the full six-dimensional phase-space density. Instead, the projected two-dimensional phase-space distributions are measured, and the effective areas occupied by the beam in those projections are characterized by rms emittances, which can be calculated for any arbitrary distribution. The evolution of the rms beam size is expressed in terms of rms emittance through the envelope equation.

In the presence of nonlinear forces or coupling between planes, arising either from external focusing or from self fields, the rms emittances can increase even when Liouville's theorem is satisfied. In general, for those applications that require an output beam capable of being focused to a very small spot, such emittance growth effects must be avoided. Examples include intense heavy ion beams for heavy-ion fusion and high-brightness photocathode electron guns for electron linear accelerators. These phenomena are studied using computer simulation of the multiparticle dynamics.

For beams with high average intensity, one may be concerned not with the rms or average phase-space areas, but with the outer part of the distribution, which determines particle losses on the accelerator structure. Relatively small losses in a high-energy accelerator may produce enough radioactivation of the accelerator structure or radiation damage of components to create practical difficulties in maintenance and operation of a facility.[1] For this case even if the focusing of an intense output beam does not impose difficult requirements on the final emittances, rms-emittance growth is still to be avoided because such growth generally means that the population of the outer regions of phase space has increased, an effect known as beam-halo formation. When this is a concern, the use of a single number (such as rms emittance) to characterize the distribution has limited usefulness, and one must look in more detail at the distribution.

A major cause of emittance growth in low-velocity intense beams is the Coulomb self force. In most accelerator beams this is predominantly a collective force, and small-impact-parameter binary collisions are usually believed to have little effect on the dynamics. This smoothed or average Coulomb force is called the space-charge force and is described by a repulsive self-electric field and an attractive self-magnetic field. The magnetic term is only important for relativistic beams and its contribution reduces the total space-charge force. In recent years emittance growth mechanisms from space-charge forces have been studied by computer simulation of intense low-velocity beams, especially for ion linear accelerators and beam transport lines. This has resulted in increased understanding of the mechanisms of this rms-emittance growth. In circular accelerators the space-charge force causes a spread of the betatron frequencies, which, in the presence of nonlinear resonances caused by magnetic field errors, also leads to emittance growth.[2] In this paper, however, we will restrict the discussion to emittance growth caused directly from the space-charge forces.

The concept of emittance and the definition of rms-emittance have been reviewed by Lawson.[3] It should be noted that the definition in this paper differs from his by not using the factor of 4 for the rms emittance. The K-V envelope equation is also discussed in Lawson's article, in which it is pointed out that the emittance term corresponds to a negative radial pressure gradient, which when added to the space-charge term gives the total effective repulsive force that affects the rms beam size. Comparison of the space-charge and emittance terms establishes the general criterion for determining the conditions under which space-charge effects are large enough to be of concern. Thus, when the ratio of the space-charge to emittance term approaches or exceeds unity, the space-charge force will generally be important.

The space-charge force is complicated because the field depends upon the time-varying charge density of the beam. In general, it is nonlinear and time dependent, and one observes coupling between the three planes. In the presence of external focusing forces, one observes phenomena that are common in plasma physics, such as plasma oscillations and Debye shielding. The plasma period determines a basic time scale for these phenomena, and the Debye length determines a basic length scale for the particle distribution. The net force, consisting of the external focusing plus the time-dependent space-charge force, may be either attractive or repulsive, and the sign of the net force may even vary across the beam. These conditions can lead to very chaotic behavior, as will be discussed later, and one must rely on numerical simulation codes to study the detailed dynamics.

2. EMITTANCE GROWTH MECHANISMS

2.1. Matched Beams

We distinguish between what we will call internal matching and rms matching. Internal matching constrains the six-dimensional phase-space distribution to make the isodensity contours coincide with the particle phase-space trajectories. For an internally matched beam, the distribution will be in equilibrium in the accelerator channel, and no emittance growth will occur, even though nonlinear forces may act on the beam. Such an equilibrium distribution is independent of time (stationary) if the focusing is uniform along the accelerator, or is periodic for a periodic-focusing channel. Examples of equilibrium distributions have been studied for two-dimensional transport channels.[4] The most frequently studied is the Kapchinskij-Vladmiriskij (K-V) distribution.[5] Unfortunately this distribution is physically unrealistic because the beam is distributed on the surface of a hyperellipsoid in four-dimensional phase space, resulting in no particles in the central core of this four-dimensional space. This distribution results in uniform ellipses for all two-dimensional projections.

Given a beam that is not internally matched one would like to be able to transform it into an internally matched equilibrium distribution, without increasing the rms emittance in the process. We do not know whether it is possible in principle to perform such a transformation without accompanying emittance growth. Nevertheless, it is feasible to match the rms beam sizes for each degree of freedom. This is accomplished by providing a beam-optics transformation to make the rms sizes constant. In a periodic channel the rms sizes will undergo a periodic flutter about their average values. An rms-matched beam is not generally internally matched, and beam distribution is not in equilibrium; therefore, the beam will evolve with the possibility of irreversible emittance growth. Nevertheless, rms matching is an important characteristic of an internally matched equilibrium distribution, and injection of an rms-matched beam can be considered a first approximation of the desired internally matched beam.

Numerical simulations of nonequilibrium beams in uniform focusing channels show that such beams often evolve to quasi equilibrium distributions, which change only slowly as the beam is accelerated. The evolution of the beams is usually accompanied by rms-emittance change as a result of both nonlinearity and coupling between degrees of freedom. Experience has shown that the velocity distributions of the final beams are Maxwellian-like. When focusing is linear, the spatial distribution of a space-charge-dominated beam consists of an approximately uniform charge-density core of density n. The density increases to zero over a finite distance approximately equal to the Debye length λ_D given nonrelativistically by

$$\lambda_D = \sqrt{\varepsilon_0 kT/nq^2} \quad , \tag{1}$$

where q is the charge per particle and ε_0 is the free space permittivity. In Eq. 1 the thermal energy is given by $kT = mc^2\varepsilon^2/a^2$, where mc^2 is the particle-rest energy, a is the rms emittance, and ε is the rms-normalized emittance, defined in the Sacherer[6] convention, with no factor of 4 included (but the relativistic $\beta\gamma$ factor).

For space-charge dominated beams $\lambda_D \ll a$, the equilibrium spatial distribution is approximately uniform with a sharp falloff at the edges. For emittance-dominated beams $\lambda_D \gg a$, the Debye tail occupies essentially the entire spatial extent of the beam, resulting in a peaked Gaussian-like charge density. Among two-dimensional continuous equilibrium beams, the K-V distribution is anomalous because it always has uniform charge density, regardless of the relative importance of emittance and space charge. However, this distribution does not correspond to the final equilibrium state of beams observed in numerical simulation studies.

It is further observed in numerical simulation that the emittance growth of beams that evolve to a final equilibrium distribution is associated mostly with a halo of low-density particles in phase space. This halo is especially undesirable for high-duty-factor linacs because it results in particle losses on the accelerator walls and in radioactivation of the accelerator.

2.2. Historical View of Space-Charge-Induced Emittance Growth

In early emittance growth studies[7,8] at Brookhaven of bunched beams in high-current proton drift-tube linear accelerators, space-charge forces associated with longitudinal to transverse coupling were identified as the primary source of observed transverse emittance growth. It was also found that this emittance-growth mechanism leads to a lower limit for the output emittance as input emittance is decreased at fixed-beam current. Later studies[9] showed that emittance growth could be physically correlated with the dependence of the transverse oscillation frequency on the longitudinal position of the particles in the bunch. At least in the early stages of the emittance growth, the ellipse orientations in transverse phase space depended on the axial position along the bunch, and the overall phase-space area was enlarged.

Early work was also carried out by P. Lapostolle,[10] who combined the numerical simulation studies with analytical work leading to the rms envelope equation and also first described some of the mechanisms discussed in this paper. An observation of longitudinal emittance decrease associated with the transverse emittance increase led Lapostolle to the suggestion of equipartioning.[11] In this picture, the emittance changes are the result of the evolution of a high-current beam towards a thermal equilibrium distribution in which approximate thermal-energy balance or equipartioning, would be established. We refer to this process as the thermal-energy-transfer mechanism. This suggests that if the input beam could be equipartioned in the accelerator in addition to being rms matched, a better approximation to the ideal internally matched beam would result, and therefore space-charge-induced emittance growth would be minimized.

Exact equipartitioning would mean that the mean-square, center-of-momentum velocities in each degree of freedom would be equal. This corresponds to the condition that

$$\sigma_x \varepsilon_x = \sigma_y \varepsilon_y = \sigma_z \varepsilon_z \tag{2}$$

where σ_x, σ_y, and σ_z are phase advances per focusing period, associated with both the external focusing and the space-charge forces. For a given set of input rms-normalized emittances ε_x, ε_y, and ε_z, and a given beam intensity, the condition given by Eq. 2 imposes a constraint on the relative focusing forces. Thus, while rms matching is achieved by providing a suitable beam-optical matching section before the beam is injected into the accelerator, the equipartitioning condition depends both on the input beam, through the beam current and the emittances, and on the focusing in the accelerator.

Additional understanding of the equipartitioning dynamics was obtained from the work of Hofmann,[12] who identified the longitudinal-to-transverse space-charge effects with coherent instabilities associated with anisotropy in the beam. Underlying this approach is a particular mechanism for the emittance growth;[13] the growth is the result of the excitation of unstable collective modes of oscillation of the beam. Some modes involve radial and azimuthal (quadrupole, sextupole, etc.) density oscillations of the beam. The first reported study of such modes for a two-dimensional, round K-V distribution in a uniform focusing channel was made by Gluckstern,[14] who identified many modes and derived their stability characteristics. Studies for the K-V beam in a quadrupole channel were made by Hofmann et al.[15] and some modes were found to be unstable although not all cases lead to emittance growth.

Hofmann[12] also studied the K-V distribution with an asymmetry between the x- and y-planes and derived the instability thresholds for the different modes. Although the studies correspond to continuous beams in an x-y geometry, Hofmann found that the same instability thresholds were approximately valid for the r-z geometry of a two-dimensional, bunched beam. It was found that equipartitioned beams were stable with respect to these instabilities and that generally the requirement for avoiding emittance growth even allowed some relaxation of exact equipartitioning. The predictions of Hofmann's model were further tested for high-current beams in drift-tube linear accelerators by Jameson[16] who confirmed that equipartitioned input beams produce the minimum emittance growth. Non equipartitioned beams could produce a significant transfer of energy and emittance between the longitudinal and transverse planes. Jameson showed from simulation how the parameters of the nonequipartitioned accelerator beam can change in the space defined by the mode-stability plots derived by Hofmann.[12] This behavior can be complicated and makes it difficult to derive simple design guidelines for avoiding this emittance growth. Perhaps the simplest design approach is to require exact equipartitioning as defined by Eq. 2. A less restrictive guideline is suggested by Hofmann,[12] whose criterion is that energy anisotropy is generally tolerable when the phase advance ratio $\sigma_l/\sigma_t < 1$ where σ_l and σ_t are the phase advances for longitudinal and transverse motion. The growth times from numerical simulation were typically about one to two transverse oscillation periods.

Additional studies of thermal-energy transfer and equipartitioning have been carried out for two-dimensional beams with different initial charge distributions in uniform transport channels.[17,18] Formulas for emittance growth were derived from the relationship between field energy and rms emittance described below, and the formulas were compared with numerical simulations.

A different space-charge-induced emittance-growth mechanism was discovered[19,20] that even affects bunched and continuous beams that are both rms matched and equipartitioned, but are internally mismatched. This mechanism has been called charge redistribution. When a beam is injected into a transport or accelerator channel, the charged beam particles, behaving like a plasma, adjust their positions to shield the external fields from the interior of the beam. For linear external fields in the extreme space-charge (zero-emittance) limit, this implies a charge rearrangement to a uniform density for producing a linear space-charge field with exact shielding. Finite-emittance rms-matched beams in numerical simulation evolve towards an internally matched charge density with a central uniform core and a finite thickness boundary, whose width is about equal to the Debye length. The rms-emittance growth results from the nonlinear space-charge fields, while the beam has nonuniform density and is undergoing internal plasma oscillations. The emittance growth can also be described as the result of the decoherence of the plasma oscillation phases for particles with different amplitudes (phase mixing), resulting in a distortion of the phase-space area. This mechanism of emittance growth has the smallest-known growth time; the full emittance growth occurs during only one-quarter of a plasma period, followed by damped emittance oscillations for typically ten or so additional plasma periods. In a high-current accelerator the full growth can occur within a single cell. This mechanism can become important when beams that have been internally matched to a strong focusing channel are injected after rms matching into a weaker focusing channel. In the strong focusing channel, where the matched beam size is small compared to the Debye length, the equilibrium spatial distribution is a strongly peaked, Gaussian-like distribution. In the weak focusing channel the rms beam size is large, and the corresponding equilibrium distribution is nearly uniform.

If the rms-matched input beam has the peaked spatial profile the beam density will change from peaked to nearly uniform in the weak focusing channel, and the corresponding change in the space-charge field energy of the distribution can be used to calculate the emittance growth. This results from the fact that for a fixed rms beam size, the space-charge field energy is minimum for a uniform beam and increases as beams become more nonuniform. The evolution of the beam from peaked to uniform is accompanied by conversion of space-charge field energy to thermal energy, which causes an increase in emittance. The emittance growth for a spherical bunch containing N particles, each with charge q, is obtained from the expression[21]

$$\frac{\varepsilon_f}{\varepsilon_i} = \left[1 + q^2 N a U_{ni}/60\sqrt{5}\pi\varepsilon_0\gamma^3 mc^2\varepsilon_i^2\right]^{1/2} \tag{3}$$

where a is the rms beam size, U_{ni} is the initial, dimensionless, nonlinear field-energy parameter, a function only of the shape of the initial distribution, and ε_i and ε_f are the initial and final rms-normalized emittances. Emittance growth from charge redistribution is sensitive to the initial spatial charge density. For an initial Gaussian profile of a spherical bunch, $U_{ni} = 0.308$, whereas for a uniform density,$U_{ni} = 0.0$. The equation shows that for a given available field energy, determined by U_{ni}, the emittance growth increases with increasing rms beam size a. This is because emittance is a measure of area occupied in the beam phase space, and for larger beams the increased velocity or divergence spread to be distributed over a larger area.

To avoid emittance growth from charge redistribution, it is necessary either to avoid transitions to accelerator channels with weaker focusing or to always provide input beams with spatial profiles that are as uniform as possible. Other guidelines for minimizing emittance growth from charge redistribution can be inferred from Eq. 3. For a given beam current I, defined as the average value over an rf period, the number N of particles per bunch is given by $N = I\lambda/qc$, where λ is the rf wavelength. Equation 3 predicts that the emittance growth of a bunch is less at high frequencies, a result that appears because a high-frequency linear accelerator has less charge per bunch for a given current. This condition was also reported[9] in studies of emittance growth that included equipartitioning effects and so is more generally valid than for the charge-redistribution effect alone. The charge redistribution mechanism has also been studied by Anderson,[22] who has derived formulas for the dynamics in the extreme space-charge limit.

Numerical simulation studies of transverse emittance growth in a radiofrequency quadrupole (RFQ) linac have also been reported.[23] The main features are:

1) the emittance growth is predominantly caused by space-charge forces, 2) most of the growth occurs while bunching the beam and so is a strong function of the longitudinal beam size, 3) above a certain current, the growth is weakly dependent on the beam current, 4) the growth is almost independent of the initial distribution, and 5) the final emittance approaches a lower limit as the initial emittance approaches zero at fixed-beam current. The emittance growth in the RFQ bunching section may be a combination of the equipartitioning effect and charge redistribution as the bunching forces increase the peak value of the beam current and drive the beam into a more space-charge dominated regime. A semiempirical emittance growth formula was obtained [23] based on Equation 3, which is in good agreement with the numerical simulation results. This formula shows the advantage of high frequency and strong focusing for control of space-charge-induced emittance growth in the RFQ.

When a charged particle beam that is not internally matched has total transverse energy larger than that of a matched beam, excess or free energy can be transformed to thermal energy, resulting in emittance growth, provided nonlinear forces act on the beam. An example is the case of an rms-mismatched beam, where excess potential energy associated with the mismatch is available for emittance growth. Emittance growth is expected when the beam under the influence of the nonlinear space-charge force relaxes toward an equilibrium, or internally matched state. For a uniform, continuous, linear focusing channel, where transverse energy is conserved, Reiser[24] has recently derived an equation for the emittance growth of an initially rms-mismatched beam, assuming that all the excess energy from the initial state is converted into the thermal energy of a final matched beam. Numerical simulation studies[25] confirmed the formula and showed that for rms beam-size mismatches of 50% or more, the emittance growth is the result of a large, well-populated halo surrounding the core of the beam. The studies suggest that rms mismatch may be the source of most of the halo observed in high-current beams. Consequently, we identify this cause of emittance growth as the rms-mismatch mechanism.

In general, the change of the rms emittances can also be related to changes in the field energy associated with the self fields through a differential equation,[20,21,26] which shows that nonlinear space-charge fields are associated with the emittance growth. The application of these ideas to several different problems has been described by Hofmann.[27]

In a periodic focusing channel an additional emittance growth is caused by the envelope instability,[15,28] which occurs when the periodic focusing structure excites coherent modes of the beam. Not only does the envelope grow, but the modes with nonuniform density are excited, and the nonlinear fields cause emittance growth. This emittance-growth effect may be called the structure-resonance mechanism, and it can generally be avoided by designing the transport or accelerator channel at a zero-current phase advance per focusing period no larger than $\sigma_o = \pi/2$.

Space-charge-induced emittance growth also can be important for intense, low-emittance injectors for electron linear accelerators. Much recent work has been motivated by the development of high-brightness photocathode injectors for free electron laser (FEL) applications. For this application, a short laser pulse irradiates a photocathode inside an rf cavity to produce an intense short-bunch that is rapidly accelerated to relativistic energies. Before the electrons reach sufficiently high velocities, for the self-magnetic fields to cancel the self-electric forces, significant emittance growth can occur. As was found in studies of high-current proton linear accelerators[7–9] (see discussion earlier in this paper), most of the initial emittance growth was found to be caused by variation of the transverse space-charge force along the axial position in the bunch as the beam expands in the cavity.[29] It has been found from numerical simulation that the correlation of transverse ellipse orientation with axial position, which is the cause of the rms-emittance growth, can be removed before longitudinal mixing and thermalization occurs by refocusing the beam with a solenoid lens.[30–33] Although it is well known that rms emittance can decrease[20] as well as increase, the example shows that even for space-charge forces, the rms emittance growth is not necessarily irreversible. Also, because the electron beam is being accelerated in the injector to relativistic energies, the system can be designed to effectively freeze the low emittance configuration at the waist before acceleration to the final energy.

In general, the low mass of the electron means that electron beams can be accelerated very rapidly out of the low-velocity regime where space-charge forces have their most serious consequences. This leads to some differences in the space-charge problem for electrons and ions. Though the electron-emittance growth occurs while the beam is expanding and can be reversed by refocusing, low-velocity ions are generally transported and accelerated in a long periodic or quasi-periodic focusing structure. In both cases focusing can be used to control the emittance-growth effects.

2.3 Summary of Space-Charge-Induced Emittance Growth

The characteristics of the four distinct mechanisms: charge redistribution, rms mismatch, thermal-energy transfer, and structure resonances are given in Table I. All four mechanisms share the same fundamental source of emittance growth, which is the nonlinear part of the space-charge force, including coupling effects such as the dependence of the transverse space-charge fields on the longitudinal coordinate. The mechanisms differ in their source of free energy for emittance growth. The structure-resonance mechanism is the only one that is restricted to periodic focusing channels, but the restriction is mostly a theoretical one because practical focusing channels use discrete lenses and are often periodic or quasi-periodic. (In practice uniform focusing channels are thought of as a smoothed representation of a periodic channel.) The time scales listed for the emittance growth are typical effective values, based on the results of numerical simulation studies. Sometimes these studies show a very complex time dependence,[25] and it is clear that a single time constant is not always adequate to describe the physics. Furthermore, one must take care that binary collision phenomena, which can easily occur in the simulation codes, do not produce emittance growth that masks the collective physics we are trying to model. This topic has been treated in two excellent articles by Haber[34] and Haber and Rudd.[35]

Table I shows a strong sensitivity of emittance growth on distribution for both the charge-redistribution and the structure-resonance mechanisms. This conclusion for the former mechanism is well established, but for the latter case it is based on a study showing that emittance growth from structure resonances could be greatly reduced if the transverse velocity distribution was Gaussian-like with a halo.[36] Finally, the suggested cures for each mechanism are given in the last row. To minimize the effect of the structure resonance, we should also add the possibility of injecting with a Gaussian-like velocity distribution.

TABLE I.
MECHANISMS OF EMITTANCE GROWTH
INDUCED BY SPACE-CHARGE FORCES

	Charge Redistribution	RMS Mismatch	Thermal-Energy Transfer	Structure Resonance
Free-energy source	Nonuniform field energy	Potential energy	Thermal energy in other plane	Directed energy
Focusing system	Uniform and periodic	Uniform and periodic	Uniform and periodic	Periodic
Time scale	$\sim \tau_{\text{plasma}}/4$	$\sim 10\tau_{\text{plasma}}$	$\sim 10\tau_{\text{plasma}}$	$\sim 2\tau_{\text{betatron}}$
Distribution function sensitivity	Strong	Weak	Weak	Strong
Emittance growth formulas	Yes	Yes	Yes	No
For minimum growth	Uniform density or internal match	rms match	Equipartition	$\sigma_0 < \frac{\pi}{2}$

3. PHASE-SPACE DYNAMICS

3.1. Numerical Simulation

So far we have discussed the phenomenon of rms-emittance growth and have identified four fundamental mechanisms, based upon the sources of free energy. Now we will use numerical simulation to look at the multiparticle dynamics and see what changes occur in phase space when the emittance grows. We will examine what may appear to be a relatively simple case of a round continuous beam in a uniform linear focusing channel with purely radial focusing. This system represents a smooth approximation for beams in quadrupole focusing channels and we expect that phenomena observed in the uniform channel will also be observed in the quadrupole channel. We use a numerical simulation code[37] for these studies in which the radial space-charge forces are calculated from Gauss's Law. Consequently, we are studying the effect of the collective forces acting on each particle and ignoring the small-impact-parameter binary Coulomb collisions. Our computer code has been run with 2000 simulation particles through 56 steps per plasma period, choices that we believe are adequate to represent the main features of the space-charge forces. We have chosen to study the dynamics of an initial rms-mismatched laminar (zero emittance) beam. Laminar beams are idealizations because all real beams have finite emittance. Nevertheless, the laminar beam represents the extreme space-charge limit and allows us to isolate the effects of the space charge.

In each of the following figures we show the distributions of a) the radial or $r - r'$ phase space, b) the projected or $x - x'$ (and $y - y'$) phase space, and c) the $x - y$ beam cross section. We show the $r - r'$ phase space because we expect the dynamics to appear simpler in $r - r'$ space when only radial forces act on a laminar beam. We assign an initial positive radius to all particles, but if during the simulation a particle crosses the axis, we change the sign of the radius.

3.2 Mismatched Uniform Density Laminar Beam

We begin by studying the dynamics of an initial uniform-density laminar beam with zero-velocity spread, which is rms mismatched so that the initial rms beam size is larger than the matched value by a factor of 1.5. Figures 1a through 1d show the beam characteristics for four different times, 0, 0.25, 0.50, and 0.75, measured in beam-plasma periods. The beam-plasma period for a uniform beam of density n_0 is defined in the usual way as $T_p = 2\pi/\omega_p$, and $\omega_p^2 = q^2 n_0/\varepsilon_0 m$ is the beam-plasma frequency. The phase-space plots show density (plasma) oscillations that are excited by the unbalanced external focusing and internal space-charge forces. The total force alternates at the beam-plasma frequency between focusing and defocusing. The charge distribution always remains uniform so that only linear

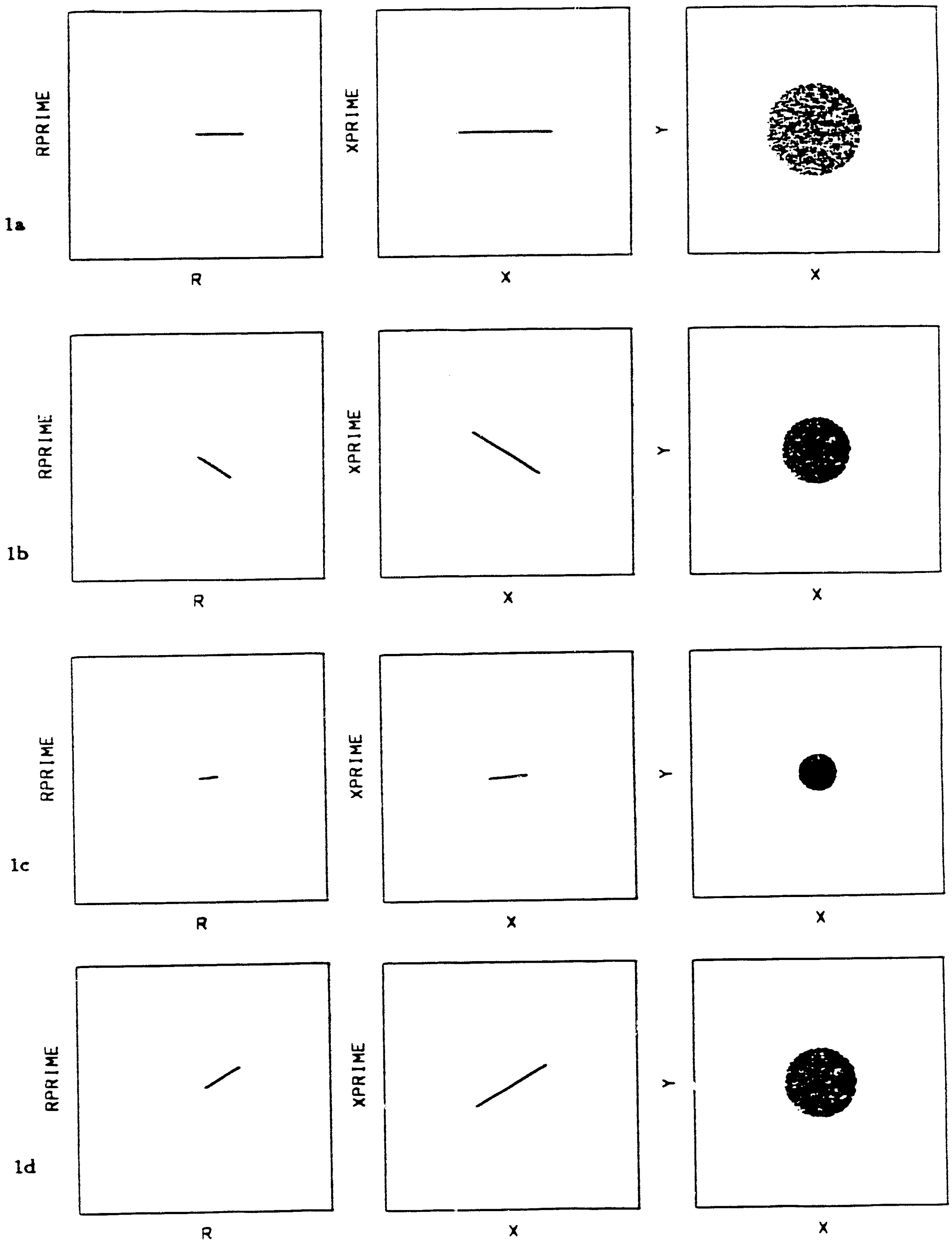


Fig. 1. Radial ($r - r'$) phase space, transverse ($x - x'$ and $y - y'$) phase space, and cross section ($x - y$) from simulation of an initial uniform-density laminar beam in a uniform linear focusing channel for a) $t = 0$, b) $t = 0.25$, c) $t = 0.50$, and d) $t = 0.75$ in units of beam-plasma periods. The initial rms beam sizes in x and y are 50% larger than the matched size.

forces act on the beam, and the emittance remains zero. In the absence of space-charge forces particles are focused by the external fields and cross the axis as they execute betatron oscillations. But because of the repulsive space-charge forces, they do not cross the axis.

3.3 Gaussian Density Laminar Beam

Next we examine the dynamics of an initial Gaussian-density laminar beam with zero-initial-velocity spread, which is rms mismatched by the same factor 1.5. Figures 2a through 2l show a sequence of plots for different times in units of the plasma period (defined for the equivalent uniform beam with the same rms size). For this case, the external force is linear, but the space-charge force is nonlinear. Several new features are obvious. Most of the small amplitude trajectories undergo plasma oscillations (they do not cross the axis) and form an inner core. The large amplitude trajectories correspond to betatron oscillations (they cross the axis) and form an outer halo. In $r-r'$ space the halo looks like a ring-shaped filament. In $x-x'$ space the ring projects into a low-density disk. The projection effect is the result of the fact that any arbitrary point in $r-r'$ space projects to a straight line in $x-x'$ space that passes through the origin and ranges between $-r \leq x \leq r$. Although emittance growth has often been identified with a process of filamentation, we see that the filamentary halo in this problem is observed in the $r-r'$ phase space. In the usual $x-x'$ projected phase space this becomes a diffuse disk-like halo.

Even within a few plasma periods the nonlinear space charge force randomizes the distribution of points within the core. This randomization or thermalization is the result of a process in which the inner part of the filament in $r-r'$ space is stretched and folded many times. The stretching and folding is associated with local variations of the magnitude and sign of the space-charge force caused by local density variations.

The halo produced after several plasma periods is a common feature of all the space-charge mechanisms of emittance growth. We find that the outer filaments seen in $r-r'$ space contain mostly the particles with large initial amplitudes but also contain some particles with small initial amplitudes that were launched during the initial stages of randomization of the core. For our example, the halo is a very ordered structure in $r-r'$ space even after 20 plasma periods; unlike the core, the halo is not yet thermalized.

At present there is no established criterion for defining the halo. For the present example of an rms-mismatched Gaussian laminar beam, we find that an ellipse with the same Courant-Snyder parameters as the rms ellipse and with an emittance five times larger than the rms ellipse appears to enclose the core and exclude most of the halo. If we define the core particles to be all those contained within this ellipse, and define halo particles as those outside, we find that after

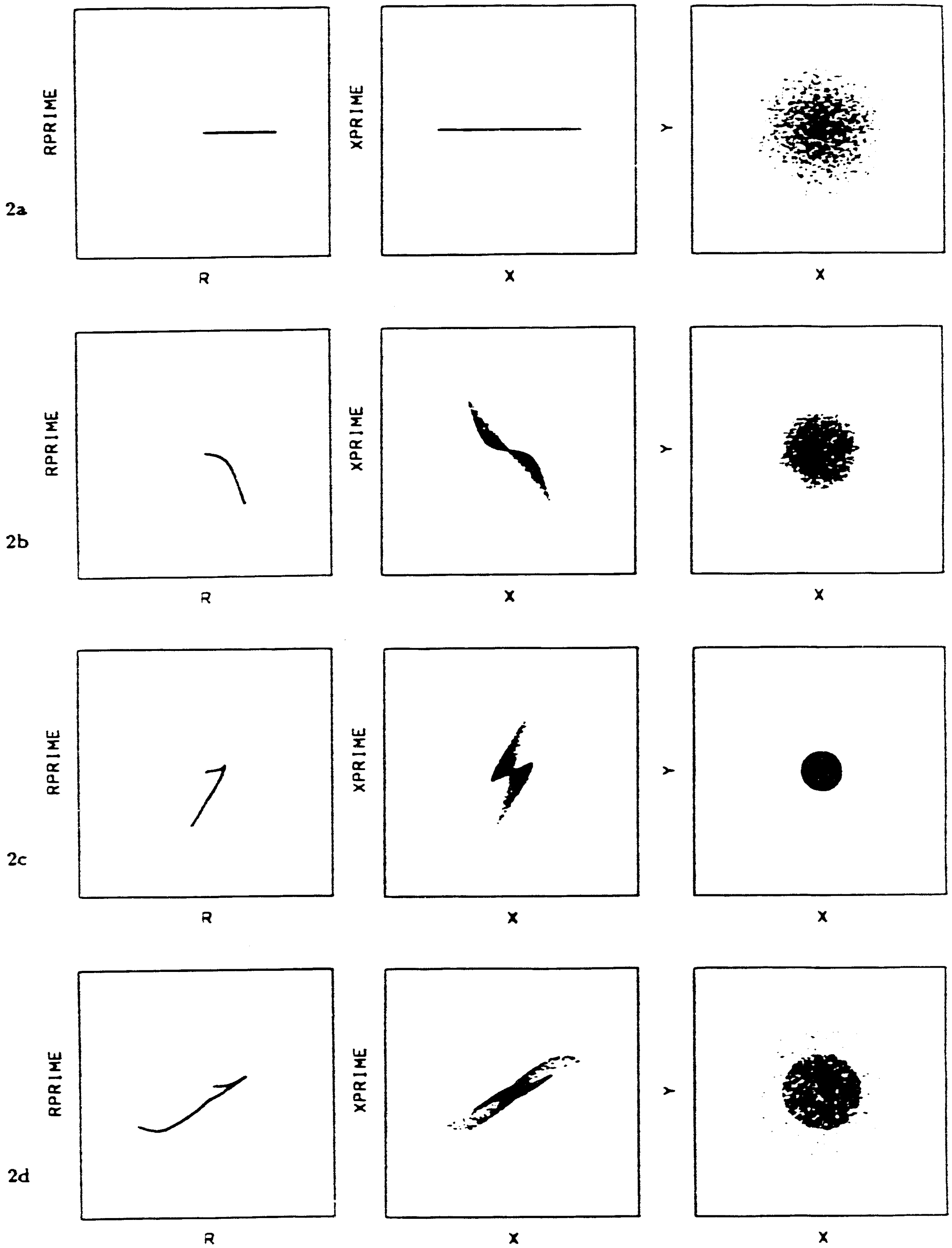
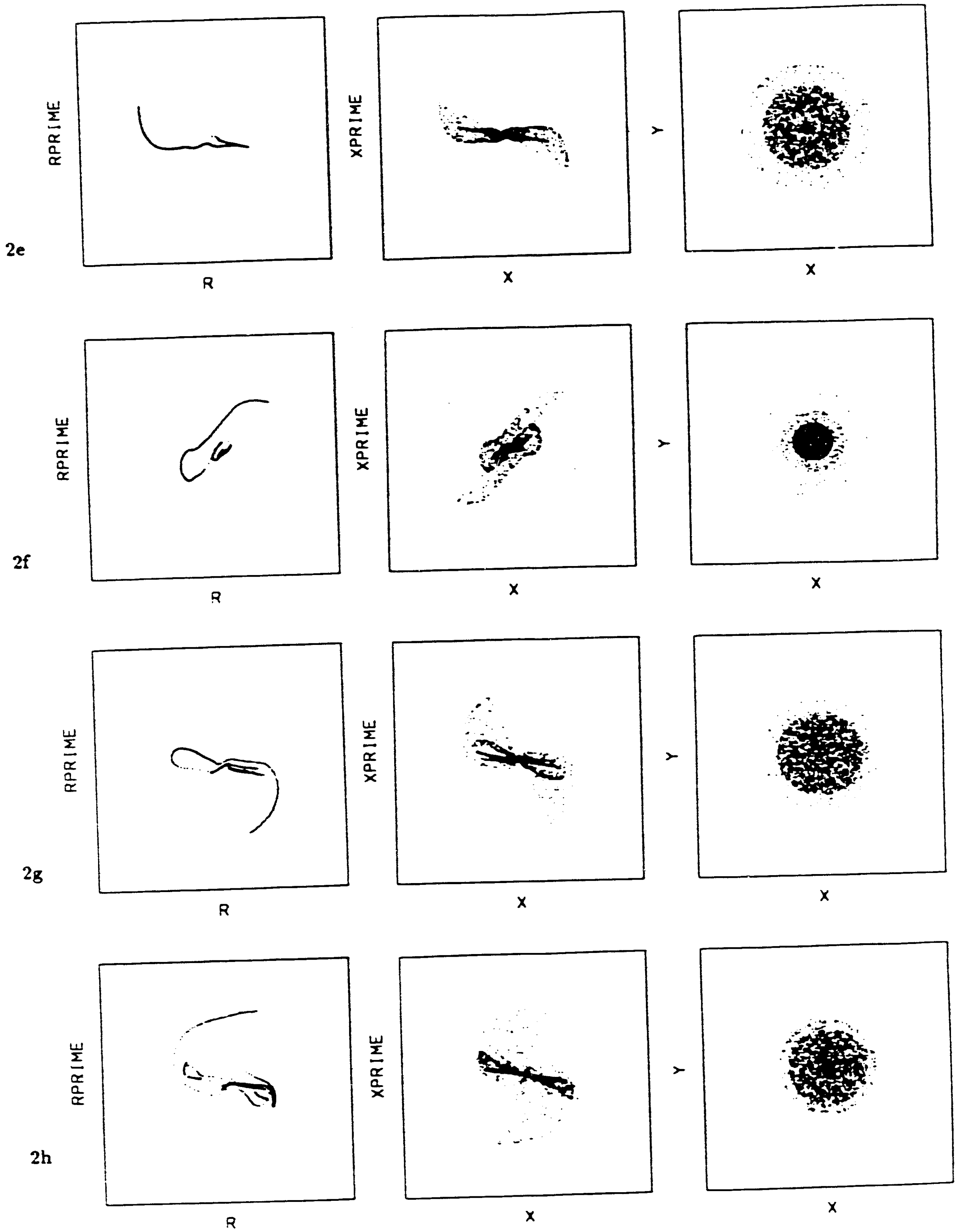


Fig. 2. Radial ($r - r'$) phase space, transverse ($x - x'$ and $y - y'$) phase space, and cross section ($x - y$) from simulation of an initial Gaussian-density laminar beam in a uniform linear focusing channel for a) $t = 0$, b) $t = 0.25$, c) $t = 0.50$, d) $t = 0.75$, e) $t = 1.00$, f) $t = 1.50$, g) $t = 2.00$, h) $t = 3.00$, i) $t = 4.00$, j) $t = 5.00$, k) $t = 10.00$, and l) $t = 20.00$ in units of beam-plasma periods. The initial rms beam sizes in x and y are 50% larger than the matched size.


2e
RPRIME
R
XPRIME
X
Y
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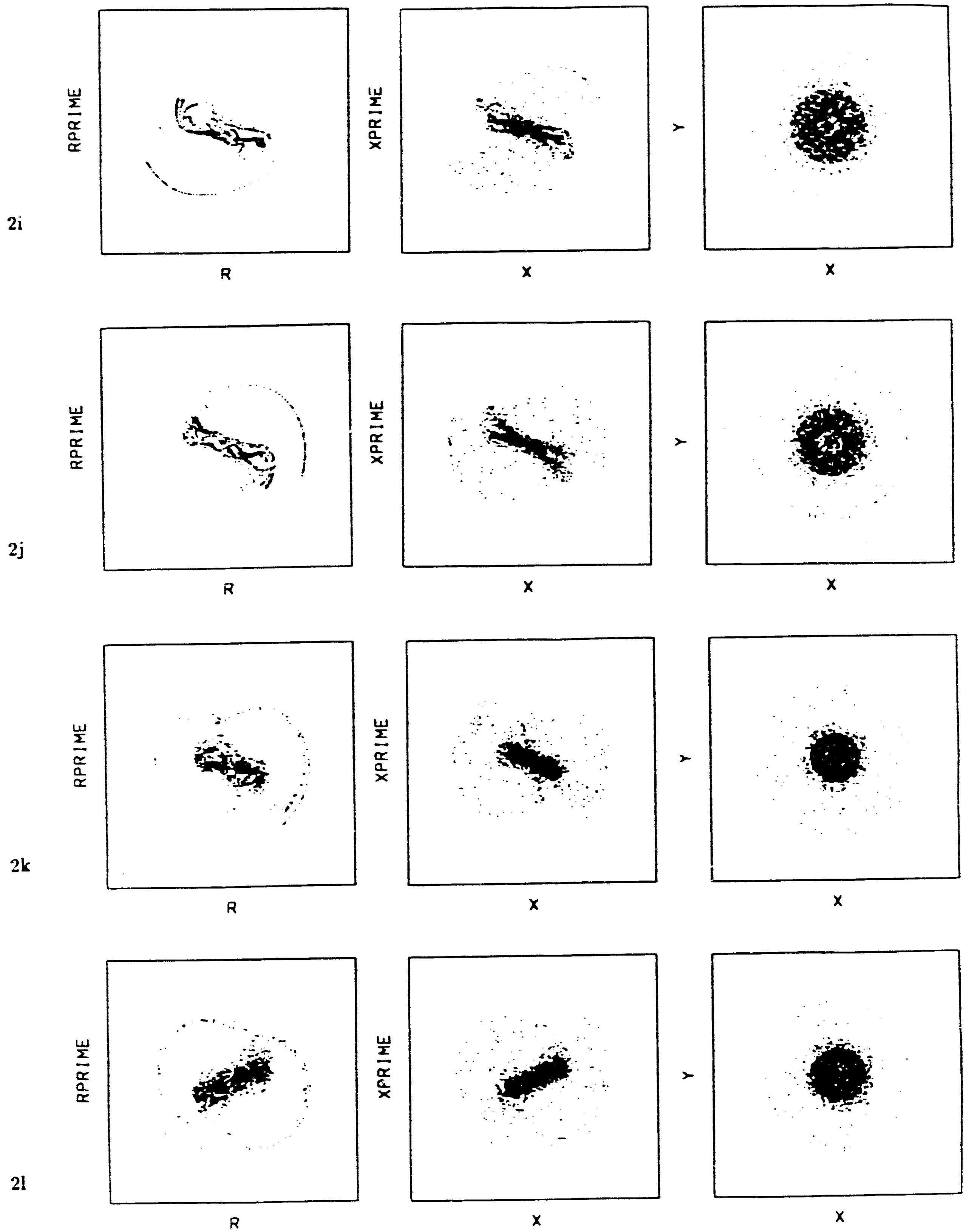
2f
RPRIME
R
XPRIME
X
Y
X
2g
RPRIME
R
XPRIME
X
Y
X
2h
RPRIME
R
XPRIME
X
Y
X

Fig. 2. (cont.)


2i
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XPRIME
X
Y
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2j
RPRIME
R
XPRIME
X
Y
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2k
RPRIME
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XPRIME
X
Y
X
2l
RPRIME
R
XPRIME
X
Y
X

Fig. 2. (cont.)

about 10 plasma periods, 6% of the particles are contained within the halo. For this example the core and the halo contribute about equally to the final rms emittance. Furthermore, the rms emittance of the core grows to its final value in about one-quarter of a plasma period, like that of an rms-matched beam through the charge-redistribution mechanism.[20] The emittance growth of the halo occurs over about 10 plasma periods. We need more studies to determine how these results vary with the amount of mismatch and to determine what happens when using more realistic beams with nonzero initial emittance.

4. CONCLUSIONS

We have discussed the rms-emittance growth caused by nonlinear collective space-charge forces, which is an important cause of loss of brightness for intense low-velocity beams. We have seen that four important emittance-growth mechanisms can be identified on the basis of different sources of free energy available for the growth. Numerical simulation studies have been of great value in giving us a better understanding of these complex phenomena. We have presented an example of the dynamics of a mismatched beam in the extreme space-charge limit, and we have seen that the rms-emittance growth is associated with the formation of both a core and a halo.

Even after more than 20 years we find that there are many questions about space-charge-induced emittance growth that have not been resolved. Many such questions are posed by Lawson[3] in these proceedings. Perhaps an important general question concerns the nature of the state of the beam after a few tens to a few hundred plasma periods, which represent a time scale of practical interest for many linear accelerators and transport systems. Is the beam or at least the core of the beam in some approximate equilibrium state? Is a Maxwell-Boltzmann distribution a good description? If equipartitioning is a characteristic of the beam, why is it so? One interesting and plausible hypothesis is that enough chaos is provided by the nonlinear space-charge forces for the beam to approach a state of maximum entropy. This may explain why beams in numerical simulations do not evolve towards highly ordered equilibrium states like the K-V distribution, and why we observe a tendency for beams to equipartition their kinetic energies. The entropy concept was first applied to beams, and its relationship to rms emittance was explored by Lawson, Lapostolle, and Gluckstern.[38] Recently, the maximum-entropy hypothesis was used to calculate the characteristics of the final distribution for a high-intensity expanding beam in free space.[39] It is clear that a lot of work still remains before we have a complete understanding of this interesting and important area of charged-particle-beam physics.

5. ACKNOWLEDGEMENTS

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