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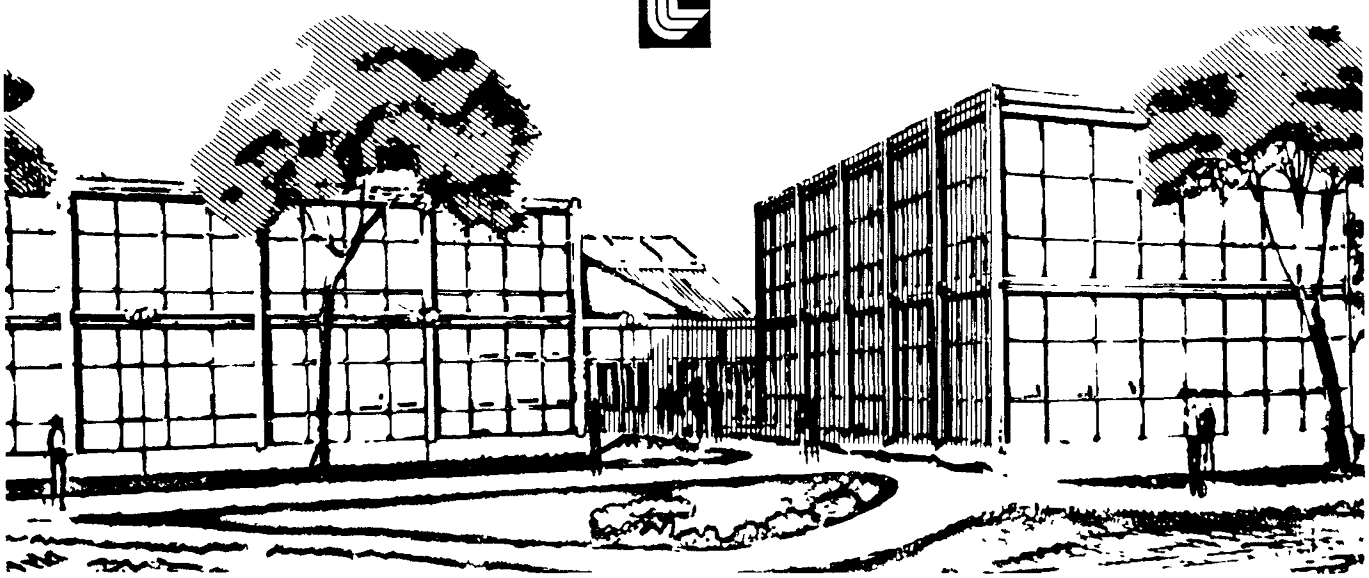

DISCLAIMER

SYNTHESIS OF HYDROCODE AND FINITE ELEMENT TECHNOLOGY
FOR LARGE DEFORMATION LAGRANGIAN COMPUTATION*

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ABSTRACT

Large deformation engineering analysis at Lawrence Livermore Laboratory has benefited from a synthesis of computational technology from the finite difference hydrocodes of the scientific weapons community and the structural finite element methodology of engineering. Two- and three-dimensional explicit and implicit Lagrangian continuum codes have been developed exploiting the strengths of each. The explicit methodology primarily exploits the primitive constant stress (or one point integration) brick element. Similarity and differences with the integral finite difference method are discussed. Choice of stress and finite strain measures, and selection of hour glass viscosity are also considered. The implicit codes also employ a Cauchy formulation, with Newton iteration and a symmetric tangent matrix. A library of finite strain material routines includes hypoelastic/plastic, hyperelastic, viscoelastic, as well as hydrodynamic behavior. Arbitrary finite element topology and a general slide-line treatment significantly extends Lagrangian hydrocode application. Computational experience spans weapons and non-weapons applications.

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Introduction

Large deformation engineering analysis at Lawrence Livermore Laboratory has benefited from a synthesis of computational technology from the hydrocode/finite difference codes on the scientific side of the house, and the structural/finite element codes available to the engineering community. Since our engineering efforts span the nuclear explosives, conventional munition, and nuclear reactor fields, the similarities and differences, strengths and weaknesses of each methodology became apparent. Given the broad applications base of a large laboratory, in-house methods development activities have concentrated on providing a flexible architecture to exploit the two technologies for 1) spatial discretization, 2) explicit and implicit time integration, 3) slide-line/void treatment 4) and "equation of state" or "constitutive" material libraries.

The prototypes which spawned the multitude of hydrocodes are the well known HEMP of Wilkins [1], and the lesser known LLL contemporary TENSOR [2]. Non-linear, finite element technology is characterized by the explicit HONDO [3], the implicit MARC [4] and NONSAP/ADINA [5]. Neither the overall field nor these codes will be surveyed in detail, but our developments will be discussed in the context of the strengths and weakness of this base. A valuable contributor arriving during the early part of our work was HONDO, by Key. This explicit finite element code helped bridge the finite difference/finite element, explicit/implicit gap. A survey paper of Belytschko helps focus this discussion [6].

Lagrangian Formulation

The key to our treatment, which is characteristic of Lagrangian hydrocodes, is the simple statement of local momentum balance in the current configuration.

$$\underset{\sim}{\nabla} \cdot \underset{\sim}{\sigma} + \underset{\sim}{b} = \rho \dot{\underset{\sim}{v}} \tag{1}$$

with Cauchy stress, divergence derivatives per current configuration, and body force per unit current volume. If the current configuration $\underset{\sim}{x}$ were considered the independent variable, then the total derivative of particle velocity would involve the advective chain rule derivative, and we would have an Eulerian formulation. What establishes a Lagrangian formalism despite the current configuration is the expression of the current configuration $\underset{\sim}{x}$ in terms of the initial configuration through the spatial discretization into nodal motions which track material points through time.

$$\underset{\sim}{x}\,(\underset{\sim}{X},t) = \underset{\sim}{x}\left[\underset{\sim}{X}(\underset{\sim}{s}),t\right] = \sum_{m} \underset{\sim}{\phi}_m(\underset{\sim}{s})\bar{\underset{\sim}{x}}_m(t) \tag{2}$$

The motion can evolve through time in terms of the fixed basis $\underset{\sim}{\phi}$, which can be used to project eq. (1) for its approximate solution.

$$\int_V \underset{\sim}{\phi}^T \,(\underset{\sim}{\nabla} \cdot \underset{\sim}{\sigma} + \underset{\sim}{b} - \rho \dot{\underset{\sim}{v}})\, dV = 0 \tag{3}$$

or

$$[M]\{\dot{\bar{v}}\} = \{P\} - \{F\} \tag{4}$$

with the stress divergence vector

$$\{F\} = \int_V B^T \underset{\sim}{\sigma}\, dV \quad , \tag{5}$$

$B = \nabla\underset{\sim}{\phi}$ is the usual linearized gradient tensor, but at the current configuration.

Explicit Codes with Primitive Elements

Consider the right hand side of eq. (4) a function evaluation. That evaluation involves a strain/deformation computation, a material routine to evaluate or update the stress, and finally the stress divergence force calculation. For the explicit simple centered difference scheme and lumped mass

$$\begin{aligned} [M]\{\Delta v\}_{n-1/2} &= \Delta t\left[\{P\}_n - \{F\}_n\right] \\ \{\Delta d\}_n &= \Delta t\,\{v\}_{n+1/2}\ ,\ \underset{\sim}{x}_n = \underset{\sim}{X} + \underset{\sim}{d}_n \end{aligned} \tag{6}$$

and cost is proportional to function evaluation cost which increases with the order of the element. Choice of basis function between primitive and higher order isoparametric elements trades function evaluation cost against direct solution cost for implicit time integration schemes. The stability criterion for higher order elements also becomes more restrictive.

In keeping with our finite difference analog, we seek a primitive element with constant stress. Then at the element level, eq. (5) becomes

$$\{F\}_e = \left\{ \int_{V_e} B^T \, dV \right\} \underset{\sim}{\sigma}_e \tag{7}$$

For triangles, tetrahedra, and the 2D and 3D linear isoparametric, this integral is exactly computed with one-point quadrature (axisymmetric geometry is an exception and will be discussed later). It is interesting to see this for the linear isoparametric [7].

$$\int B^T dV = \int_{V_e} \frac{1}{|J(\underset{\sim}{s})|} \left(\hat{B}_o + \hat{B}_1 \underset{\sim}{s} \right) |J| \, d\underset{\sim}{s} = 4\hat{B}_o \tag{8}$$

For the two-dimensional Cartesian case, with $z_{ij}=z_i-z_j, z=x,y$ and a counterclockwise element nodal connectivity

$$\{F^x\}_e = \begin{Bmatrix} -\sigma_{xx} y_{24} + \sigma_{xy} x_{24} \\ -\sigma_{xx} y_{31} + \sigma_{xy} x_{31} \\ \sigma_{xx} y_{24} - \sigma_{xy} x_{24} \\ \sigma_{xx} y_{31} - \sigma_{xy} x_{31} \end{Bmatrix}, \quad \{F^y\}_e = \begin{Bmatrix} \sigma_{yy} x_{24} - \sigma_{xy} y_{24} \\ \sigma_{yy} x_{31} - \sigma_{xy} y_{31} \\ -\sigma_{yy} x_{24} + \sigma_{xy} y_{24} \\ -\sigma_{yy} x_{31} + \sigma_{xy} y_{31} \end{Bmatrix}$$

which is equivalent to the integral difference representation of Wilkins [1] for a stress derivative at node "k"

$$\underset{\sim}{F}_k = \frac{1}{A_k} \oint \underset{\sim}{n} \cdot \underset{\sim}{\sigma} \, d\ell \tag{9}$$

where contributing constant stress elements are accumulated. This contour is always expressed as a diamond through four quadrilaterals attached to a point (with degenerate special cases for free edge and corner points). It just as easily assembles an arbitrary finite element topology (Fig. 1).

Axisymmetry

It's been known that the one point brick is equivalent to the integral finite difference method in Cartesian coordinates, but a non-trivial difference occurs in the axisymmetric case.

$$\rho\ddot{r} = \frac{\partial\sigma_{rr}}{\partial r} + \frac{\partial\sigma_{rz}}{\partial z} + \frac{\sigma_{rr}-\sigma_{\theta\theta}}{r}$$
$$\rho\ddot{z} = \frac{\partial\sigma_{rz}}{\partial r} + \frac{\partial\sigma_{zz}}{\partial z} + \frac{\sigma_{rz}}{r} \qquad (10)$$

The typical hydrocode (HEMP,TENSOR) differences the derivative terms by the contour integrals discussed, but the final, axisymmetric term is simply arithmetically averaged at a node. In fact, since the equation is first divided by the density, a common mass vector is lost

$$\ddot{r}_k = \left(\frac{f_r}{m} + \beta\right)_k$$
$$\ddot{z}_k = \left(\frac{f_z}{m} + \alpha\right)_k \qquad (11)$$

where "f" is the force per unit circumference identical to the Cartesian term,

$$m = \oint \rho dA = \frac{1}{4} \sum_e \frac{\rho V}{r_c}$$
$$\beta = \frac{1}{e}\sum\left(\sigma_{rr}-\sigma_{\theta\theta}\right)\left(\frac{A}{\rho V}\right) \qquad (12)$$
$$\alpha = \frac{1}{e}\sum\sigma_{rz}\left(\frac{A}{\rho V}\right)$$

where ρV is the conserved element mass, but A is the time dependent area.

In the usual finite element method, the volume weighted integral of (3) leads to a conserved mass vector, and the stress divergence (7) leads to

$$\left\{F\right\}_e = \left\{\int_A B^T \, r \, dA\right\} \underset{\sim}{\sigma}_e \tag{13}$$

This constant stress form Key uses in HONDO, integrating the element by numerical quadrature. Inspection of (8) shows that cancellation of the Jacobian permits exact integration. We found one point integration equally accurate.

$$\left\{F\right\}_e = 4\hat{B}_o \, r_c \, \underset{\sim}{\sigma}_c \tag{14}$$

We do not find any difference in accuracy for problems of small deformation, but in large spherical deformation, the radial bias of the volume weighted finite element method introduces significant asymmetries, which was in fact the reason for the historical choice for the area integral method.

We pay the small price of a time dependent "mass" vector, but avoid the problem of the hoop term by a straight area Galerkin form.

Hourglass Modes

The nemesis of the primitive brick element is the hourglass zero energy mode, which is a singularity at equilibrium and a growing error in transient problems, especially for contact problems where surface smoothness is important. Proper stabilization should be directed at global rather than local modes, but constraint counts are difficult to automate in general, and so far, all have attempted element wise artificial hourglass forces. Explicit hydrocodes use a viscous restoring force based on angular velocities of zone edges. All seem equivalent for rectangular zones, but generalize in ad hoc manner. We have found both the "triangle Q" of Wilkins and the isoparametric form of Key [3] to be both computationally expensive and unreliable. We have been using the "rotational Q" of TENSOR [2], with good results. Koslov and Frazier [8] present a stiffness treatment, seeking an exact quadratic hourglass amplitude. Both the idea of a stiffness (perhaps critically damped), and that of creating a proper energy mode (rather than a trace stabilization), have merit. However, we find their generalization to the non-rectangular case to be incorrect. Their modes, while orthogonal in the sense defined, do not lead to zero gradients at the strain computation point (see Appendix). We have successfully experimented with the mode orthogonal to the rigid body and zone centered strain modes. The key result is that the nodal weights of the mode are independent of element shape, i.e., the same as for the rectangle. Hourglass computations can be expensive; about ten percent of the total cost in 2D and forty percent in 3D. The new method is cheap, and we await more experience in 3D.

Finite Strain Measures and Constitutive Modeling

We offer nothing new in constitutive theory or stress point algorithmic implementation. The radial return method goes back to Wilkins [1] for the case of isotropic hardening. Finite deformation plasticity is treated by the Jaumann form of the Cauchy stress increment. Krieg and Key made a significant extension of kinematic hardening to finite rotation [9].

Deformation gradients are computed for Green-St. Venant or Almansi strain measures. Where constititive models are given in terms of Piola stresses, they are transformed to Cauchy stress for stress divergence calculations. In addition, we include hydrodynamic and crush models, as well as high explosive burn.

Explicit Experience

Three years ago the challenge of large deformation inelastic dynamic response problems taxed the capabilities of HEMP with its limited logical mesh topology, and primitive one way slide treatment. The complex layered structures, with slip and contact impact, left HONDO also of limited use to us. We launched an algorithm development for contact/impact in 2D 10 which dramatically improved our capability (Fig. 1B). The requirement for 3D response led to a developmental 3D explicit code, DYNA3D 11,12 . We battled the primitive element where hourglass problem unsuccessfully, and resorted to a 20 node brick with 8 point quadrature. Figure 2 shows a successfull large deformation elastic/ plastic impact. Such brute force explicit dynamics is expensive,ten hours of CDC 7600 time. A 2D version, with higher order elements, the HEMP difference scheme and a synthesis of the material routines of HEMP and HONDO led to our own DYNA2D [13]. Experience found that our nodal constraint contact method [9] which worked well for solid structures, exacerbated the primitive element hourglass problems in hydrodynamics and problems of large plastic flow. So a new theory, based on hydrocode techniques of surface stress averaging, was extended to the general slide-line finite element topology [14].

Implicit Method

Our recent 2D implicit large deformation code, NIKE2D, is also built around the Cauchy formulation, but will not be reviewed here. The rotational terms of the Jaumann rate are treated explicitly, leading to a symmetric tangent matrix. Newton iteration, with accurate right hand side evaluation has lead to excellent results [15,16]. A penalty function slide-line treatment and vectorized large capacity equation solver provides efficient 7600 and CRAY-1 computation.

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Appendix

That the hourglass weights are independent of element shape is seen by examining the velocity gradient (or strain) derivative for the one point element.

$$\begin{Bmatrix} \frac{\partial z}{\partial x} \\ \frac{\partial z}{\partial y} \end{Bmatrix} = \frac{1}{2A} \begin{bmatrix} y_{31} & -y_{24} \\ -x_{31} & x_{24} \end{bmatrix} \begin{Bmatrix} z_{24} \\ z_{31} \end{Bmatrix} = 0$$

where

$$y_{31}x_{24} - y_{24}x_{31} = 2A \neq 0,$$
$$z = \dot{x},\dot{y} \text{ or } x,y,\ z_{y} = z_i - z_j$$

Thus,

$$z_{24} = z_{31} = 0 \quad , \quad z_2 = z_4,\ z_3 = z_1$$

and the rigid body constraint $\sum z_i = 0$ yields the normalized shape (-1, 1, -1, 1), independent of element shape.

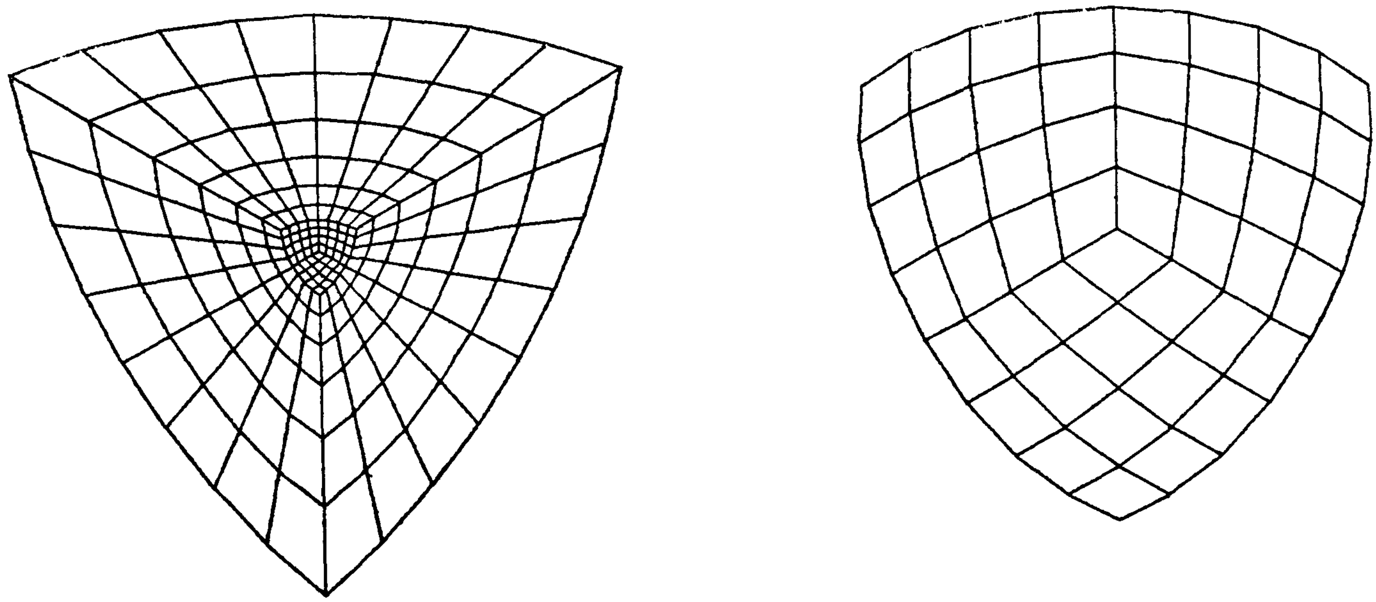

Figure 1A

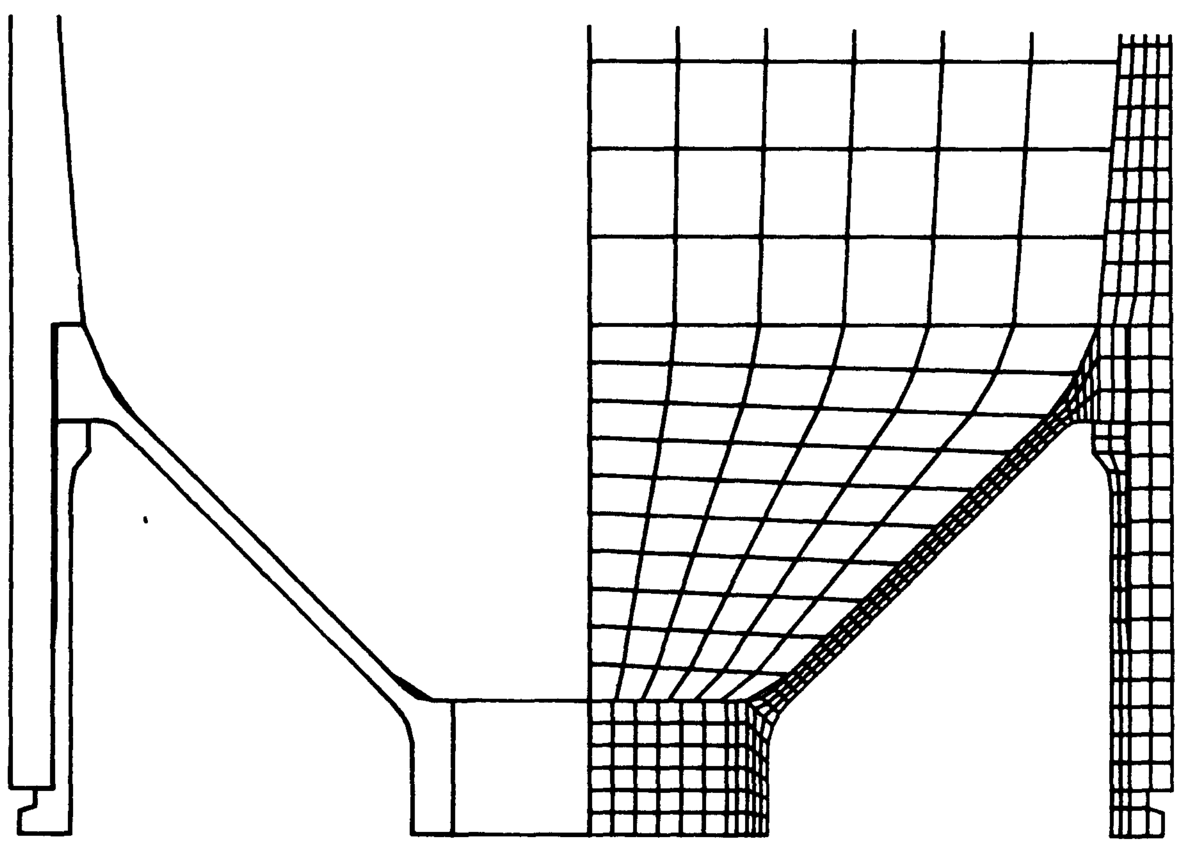

Figure 1B

INITIAL *DYNA3D* DEVELOPMENT EXPLOITED 20 NODE BRICK ISOPARAMETRIC ELEMENTS

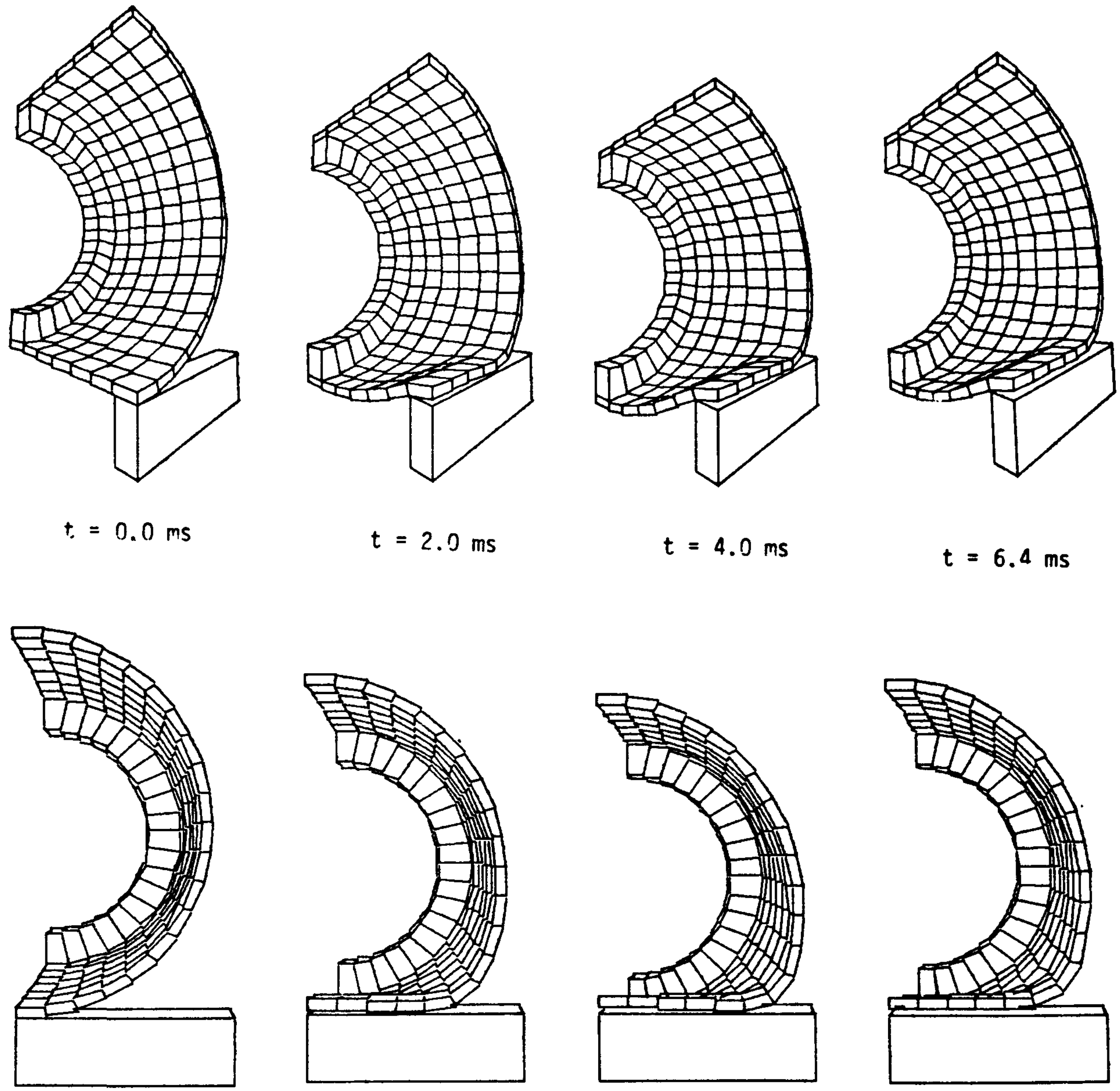

Figure 2

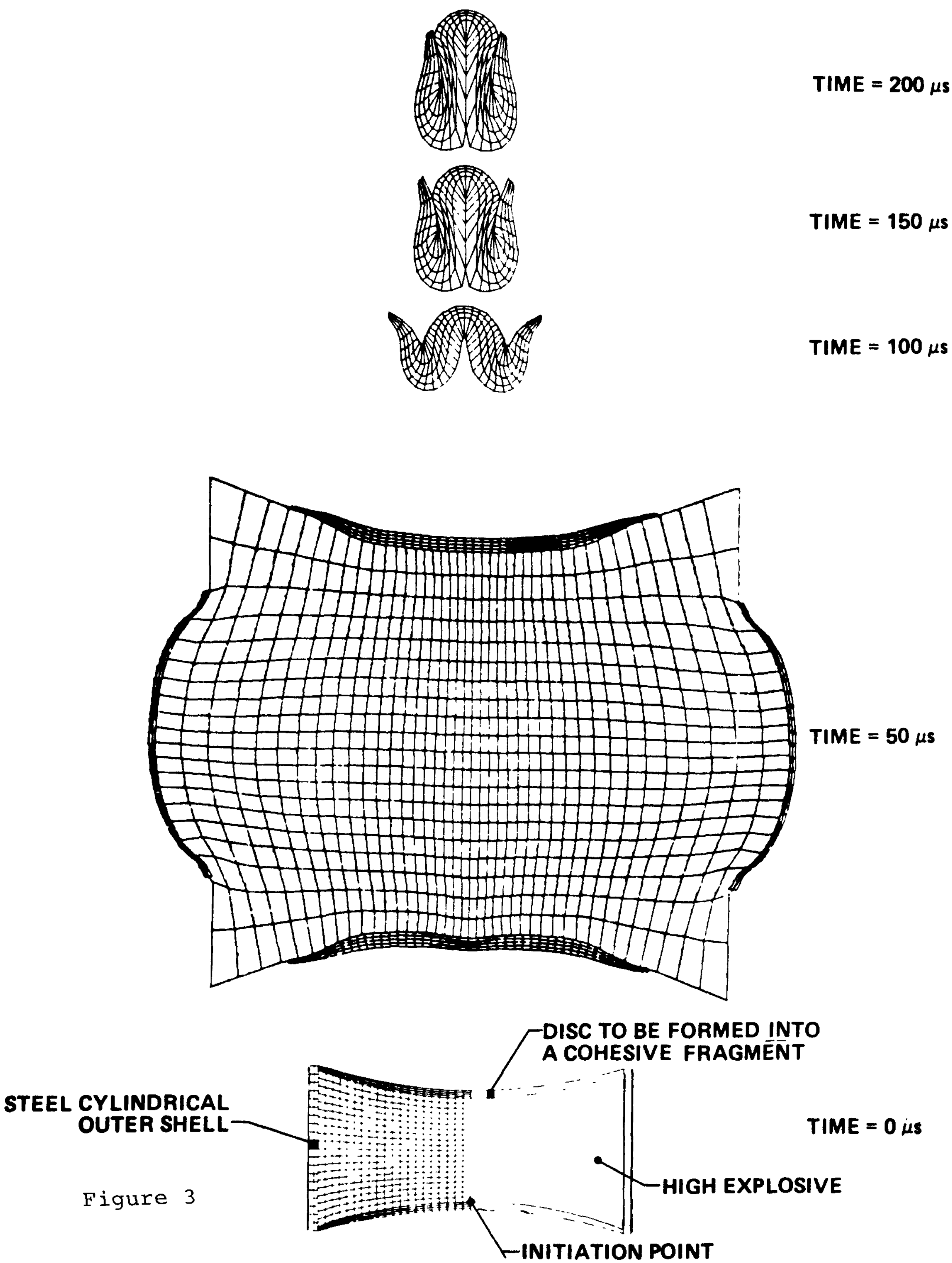

Figure 3

DYNA2D COMPUTES THE SELF FORGING FRAGMENT WARHEAD. SYNTHESIS OF THE BEST OF HEMP AND FINITE ELEMENTS ALLOWS MULTIPLE SLIDE-LINE HE EXPANSION AND CLOSURE CONTACT.

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