Conf-901072--11

DOE/ER/40423--20

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STAGGERED WEAK MATRIX ELEMENT MISCELLANY

Stephen R. Sharpe

Institute for Theoretical Physics
University of California
Santa Barbara, CA 93106

and

Department of Physics, FM-15
University of Washington
Seattle, Washington 98195[1]

Abstract

I report on work, done with Rajan Gupta and Greg Kilcup, using staggered fermions to study weak matrix elements in quenched QCD. I give an update on the $\Delta I = 1/2$ rule and on matrix elements relevant for ϵ'. I show results of a study of the dependence of B_K on non-leading terms in the chiral expansion. I present our first results for B_K from a quenched calculation at $\beta = 6.4$ on $32^3 \times 48$ lattices, based on an ensemble of 12 configurations.

Talk presented at the International Symposium Lattice '90
Tallahassee, Florida, USA, October 8-12, 1990

November 1990

[1]Permanent address

1 INTRODUCTION

This talk gives a progress report on the work of the staggered part of the weak matrix element Grand Challenge collaboration. During its second year, this collaboration received nearly 7500 hours of Cray-2 time, to be shared between Wilson and staggered factions. The staggered faction (Rajan Gupta, Greg Kilcup and myself) have used their share to extend the results obtained in the first year on the $\Delta I = 1/2$ rule, ϵ'/ϵ and B_K in quenched QCD. The motivation and much of the methodology has been explained in previous talks,[1, 2] and in our recent paper on the spectrum.[3] We want to do calculations with reliable estimates of statistical and systematic errors. Our new results indicate that the assessment of these errors which I gave last year was too optimistic.[2] We also have new results on the spectrum and on the $I = 2$ pion scattering length; these are discussed in a separate talk [4].

2 UPDATE ON $\Delta I = 1/2$ RULE

Our aim is to understand the amplitudes for kaon decays into two pions in either an $I = 0$ or $I = 2$ state (A_0 and A_2 respectively). Particularly problematic is the large ratio of these amplitudes, $A_0/A_2 \approx 22$. It is difficult to calculate $K \to \pi\pi$ amplitudes directly, as clarified by Maiani and Testa.[5] Instead, we use lowest order current algebra to relate $A_{0,2}$ to the simpler $K \to \pi$ and $K \to 0$ amplitudes.[6] I label the amplitudes so obtained $A_{0,2}^{K\pi}$. Calculating $A_2^{K\pi}$ is equivalent, in the chiral limit, to calculating B_K, for which statistical and finite volume errors are small.[7] Calculating $A_0^{K\pi}$, however, is still difficult, for there are the notorious "eye" diagrams as well as the simpler "eights". We also need to know what numbers to expect for $A_0^{K\pi}$ and $A_2^{K\pi}$, since they differ from $A_{0,2}$ due to corrections to lowest order current algebra.

Last year we were encouraged by three facts. First, higher order corrections to current algebra were found to be large in a model calculation. The dominant correction is probably due to final state interactions (FSI). Isgur *et al.* [8] found, in a potential model, that FSI enhance A_0 by $1.5 - 2$ and reduce A_2 by a similar factor. This would bring our calculation of $A_2^{K\pi}$ (i.e. B_K) into reasonable agreement with the experimental A_2. In addition we would only need aim for $A_0^{K\pi}/A_2^{K\pi} = 6 - 9$ rather than 22. Second, we had results for the eight diagrams for $A_0^{K\pi}$ with small statistical errors. From these diagrams alone we found $A_0^{K\pi}/A_2^{K\pi}|_{eights} = 3.6(2)$ on $16^3 \times 40$ lattices at $\beta = 6$. Finally, we had preliminary results for the eye contribution to $A_0^{K\pi}$ on the same lattices. The errors were large, but the eyes did increase A_0, and it appeared possible that their contribution could be similar in magnitude to that from the eights. Putting this all together, an understanding of the $\Delta I = 1/2$ rule as an accumulation of factors of 1.5 and 2 appeared possible.

The plausibility of this scenario has diminished this year. On the theoretical front, a systematic analysis of higher order ($O(p^4)$) corrections using the chiral Lagrangian including weak interactions has been undertaken.[9] The analysis allows one, in principle, to use experimental information to extrapolate the physical amplitudes towards the chiral limit, in which limit $O(p^4)$ effects can be neglected. The extrapolated amplitudes can then be compared directly to $A^{K\pi}$, as long as the latter are also extrapolated to the chiral limit. The details of the analysis are not yet available, but it is clear that the effect of higher order

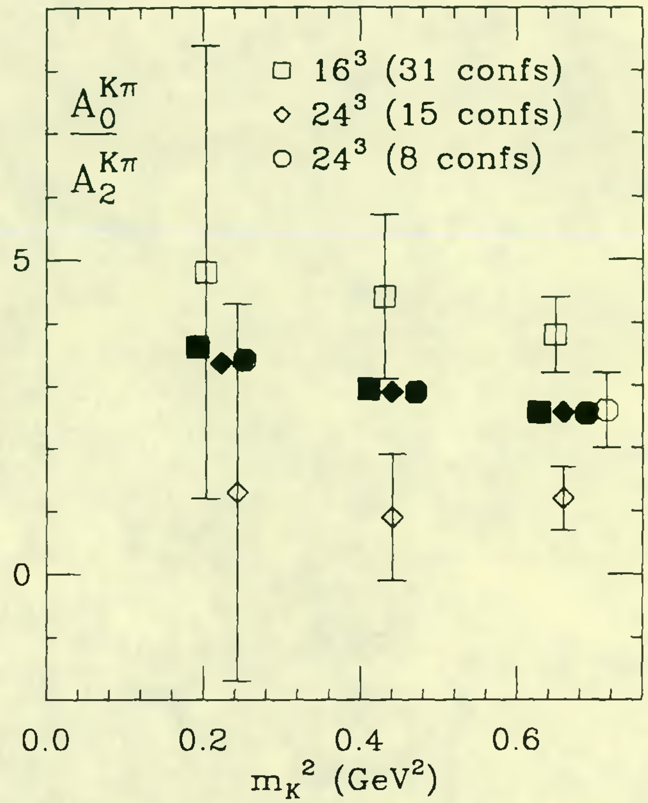

Figure 1: $A_0^{K\pi}/A_2^{K\pi}$ ($\beta = 6$, $1/a$=2GeV). Solid points are from eight diagrams only. Some points are offset horizontally for clarity.

terms is smaller than that of Isgur *et al.* For example, lattice calculations should aim for $A_0^{K\pi}/A_2^{K\pi} \approx 0.62 \times 22 \approx 14$.[10] The shortcoming of this analysis is that the $O(p^4)$ terms do not capture all of the effects of FSI. A partial resummation of higher order terms is underway.[10]

As for the computations, we repeated our calculation of $A_0^{K\pi}$ on two sets of $24^3 \times 40$ lattices at $\beta = 6$. The first set of 15 lattices is that used to calculate B_K.[7] The second set of 8 lattices is independent, and has been analyzed with the timeslices containing the wall sources in Landau rather than Coulomb gauge. The results for B_K (and thus $A_2^{K\pi}$) on these lattices agree with those on the first set. Results from all lattices for $A_0^{K\pi}/A_2^{K\pi}$ are shown in Fig. 1. Perturbative corrections are not included.

The results from eight diagrams are of similar quality to those for B_K: there is no volume dependence, and the two 24^3 samples give compatible results. (The errors are slightly smaller than the symbols.) By comparison, the contribution of the eye diagrams flips sign when going from the 16^3 to the first set of 24^3 lattices. At the heaviest mass ($m_q \sim 1.5 m_s$) this appears to be statistically significant. This prompted us to repeat the calculation at this mass on the second set of 24^3 lattices. The result lies between the other two, with the eye contribution being consistent with zero. We conclude that we do not have a credible signal for the eye diagrams, particularly in the chiral limit. Clearly we need much greater statistics and/or an improved method of calculating the eye loops.

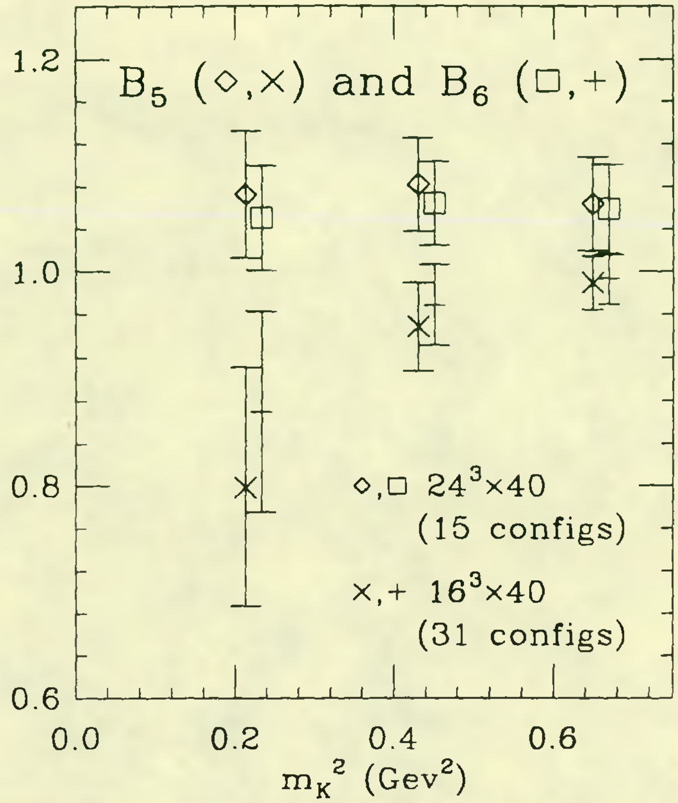

Figure 2: B_5 and B_6 at $\beta = 6$, with offsets for clarity.

3 UPDATE ON ϵ'

As explained in Greg Kilcup's talk last year,[1] we can calculate the matrix elements of the penguin operators which contribute to ϵ' ($\mathcal{O}_{5,6}$ in our notation[11]), despite the fact that they involve eye diagrams. We do, however, have to make the approximation of using $K \to \pi$ and $K \to 0$ matrix elements. We express the results in terms of B-parameters,[11] which are unity if the vacuum insertion approximation is valid. Last year we had results from the 16^3 lattice at $\beta = 6$; we now have results on the first set of 24^3 lattices. The results (without perturbative corrections) are shown in Fig. 2.

I am encouraged that we are able to calculate $B_{5,6}$ with small statistical errors: they result from a cancellation between eyes, eights and a subtraction term. Furthermore, the matrix element has the correct chiral behavior, namely a finite limit as $m_K \to 0$. There are, however, suggestions of finite volume effects at the smaller quark masses. It is possible that this is due to chiral logarithms,[2] but this has not been thought through. In any case, our experience with finite volume effects in matrix elements related to B_K suggests that results from 24^3 lattices will be close to those from infinite volume.[2]

The comparison with experiment must be done as for $A_{0,2}$, since we are calculating the imaginary parts of the $K \to \pi\pi$ amplitudes. Thus we must extrapolate our results to the chiral limit. Taking the 24^3 data as our best estimate, we see that the extrapolation to the chiral limit is smooth, giving values slightly greater than, but consistent with, unity, for both B_5 and B_6. I expect this conclusion to be unaffected by perturbative corrections, which largely cancel in the ratios defining the B parameters. Our result of a few years ago,

$B_6(m_K) = 0.4(3)$,[11] is lower than our new results by 2σ. If the difference is significant, it might be explained by a combination of the stronger coupling and smaller volume of the earlier calculation.

The phenomenological consequences of these numbers is rather obscure. There are many contributions to ϵ', and that of $\mathcal{O}_6$ is important but not dominant in the presently allowed range of m_t. Most analyses actually use $B_5 = B_6 = 1$, which is consistent with what we find.[12] An important point is that vacuum insertion (including the 1/3 from the color Fierz) works well for all the components (eights, eyes and subtraction) of the matrix elements. This is unlike B_K, for which vacuum saturation fails badly at intermediate steps.

4 CHIRAL BEHAVIOR OF B_K

The remainder of the talk concerns B_K, which is

$$B_K = \frac{\langle \bar{K}'|\bar{s}'\gamma_\mu(1+\gamma_5 d')\ \bar{s}\gamma_\mu(1+\gamma_5 d)|K\rangle}{\frac{4}{3}\langle \bar{K}'|\bar{s}'\gamma_\mu\gamma_5 d'|0\rangle\langle 0|\bar{s}\gamma_\mu\gamma_5 d|K\rangle}\,, \tag{1}$$

where the primes restrict contractions.[13] In this section we examine the dependence of quenched B_K on the quark masses. Our results are at $\beta = 6$ using $m_q = 0.01, 0.02, 0.03$, which we refer to as 1, 2, and 3, respectively. The lowest mass is $\approx m_s/2$. Previously we studied combinations such that $m_{s'} = m_s$ and $m_{d'} = m_d$, e.g. $(s'd', s, d) = (11, 11)$, $(12, 12)$, etc. We expect [9, 14] $(y = m_K^2/(4\pi f_\pi)^2)$

$$B_K = B_0(1 - (3 + a\delta)y\ln y + by + cy\delta)\,, \tag{2}$$

$$\delta = (m_s - m_d)^2/(m_s + m_d)^2\,, \tag{3}$$

up to terms of $O(y\delta^2)$. I have expanded in δ to save space, and since $\delta \le$ [redacted] 0.25 in our calculations. B_0, b and c are the coefficients to be calculated, while a is known: $a = 1/3$ and 1, in full and quenched QCD respectively. In full QCD there is also a correction term $\propto m_u + m_d + m_s$.

Equation 2 predicts that quenched B_K will depend on δ, and not just on m_K^2. This is tested in Fig. 3, which shows our updated results on the $24^3 \times 40$ lattices. The issue is whether (12,12), (13,13) and (23,23) lie on the same curve as (11,11), (22,22) and (33,33). There is a small difference of marginal significance, indicating that the coefficient c is small. The same pattern holds on $16^3 \times 40$ lattices, within larger errors.

We have also looked at $m_{s'} + m_{d'} \neq m_s + m_d$ (specifically (11,13) and (11,33)). Here the weak operator inserts energy, allowing new higher order terms, e.g. $m_{K'}^2 - m_K^2$. These points are also shown in Fig. 3, plotted against $m_{K'}m_K$, which is the natural generalization of m_K^2. Again there is no significant deviation from the single curve.

In conclusion, the slope in Fig. 3 shows that there is a significant higher order contribution to B_K, but it comes dominantly from the chiral logarithm and b term in Eq. 2. Why the coefficient c is small deserves further investigation.

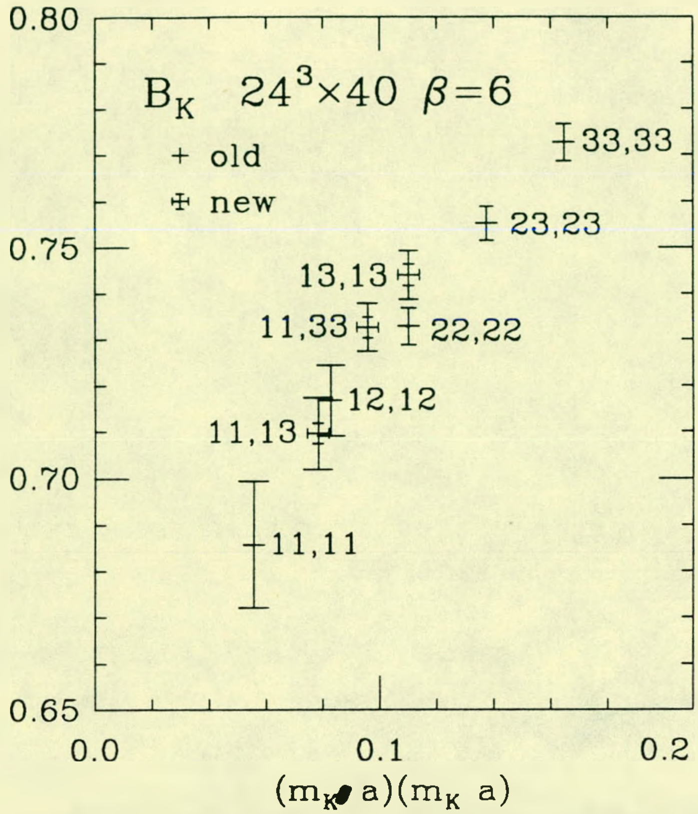

Figure 3: B_K, with quark masses indicated.

5 TOWARDS THE CONTINUUM LIMIT FOR B_K

Last year our results for $B_K(m_K)$ were very encouraging. We found 0.70(2) and 0.70(1) at $\beta = 6$ on $16^3 \times 40$ and $24^3 \times 40$ lattices, indicating very small finite volume effects. [7] In addition, there were indications that the finite lattice spacing errors were small at $\beta = 6$. Figure 4 shows the B_K at $\beta = 5.7$, 6 and 6.2. There appears to be a large decrease from $\beta = 5.7$ to 6, but the results at $\beta = 6$ and 6.2 are consistent. A linear fit to all points at $\beta = 6.2$ gives $B_K(m_K) = 0.68(4)$. We actually expect a drop of ≈ 0.01 between $\beta = 6$ and 6.2 due to the small anomalous dimension of the four-fermion operator.

To further test the lattice spacing dependence we have begun calculations at $\beta = 6.4$ on $32^3 \times 48$ lattices. The lattice spacing is roughly half that at $\beta = 6$, so the physical spatial volume is similar to 16^3 lattices. Cray-2 memory constraints restrict us to $N_t = 48$. We have generated 12 lattices so far, separated by 2000 overrelaxed/Metropolis sweeps in a 4:1 ratio.[3] To save on memory, we pack the configurations as 16 bit integers. We do not pack the propagators, however, since doing so introduces unacceptably large errors.

We use wall sources as before,[3] but make several changes in our methods,[1] some dictated by the small physical time extent of the lattice, others designed to simplify the calculation of f_π. Instead of Dirichlet boundary conditions (BC) in time, we use periodic and antiperiodic BC, and take the sum and difference to obtain propagators moving forward and backwards through the lattice. This is equivalent to doubling the lattice. Rather than fix source timeslices to Coulomb gauge, we keep them in Landau gauge. Finally, we use an additional set of sources half way along the lattices, at $t = 24$.

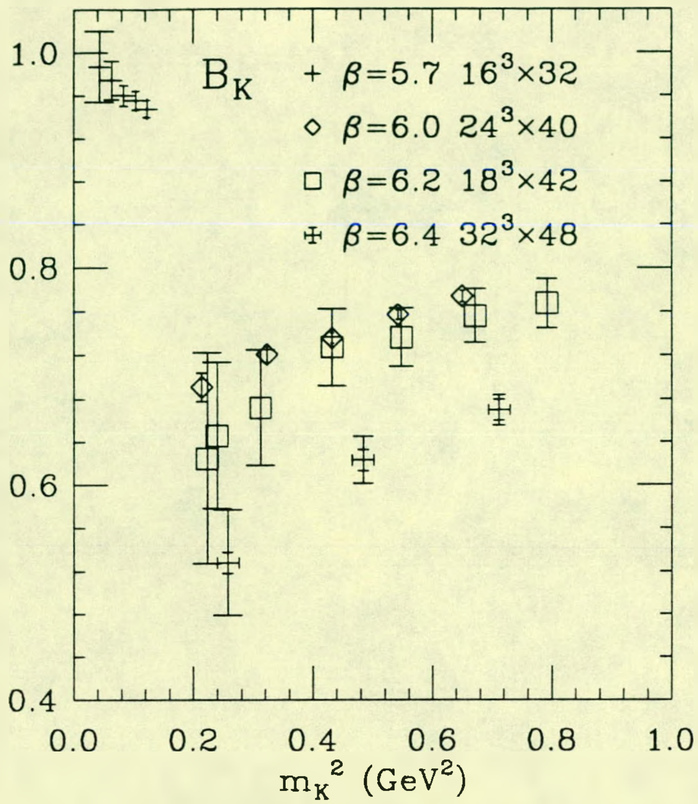

Figure 4: B_K for β =5.7, 6, 6.2 and 6.4. The scales used are $1/a$ =1, 2, 2.5 and 4 GeV, respectively.

Figure 5 shows how we combine propagators to calculate B_K. The method closest to our previous calculations uses the forward and backward propagators from the $t = 0$ source, and is illustrated in Fig 5a. The problem here is that the wall sources produce both pions and rhos, and the rho contamination does not decay until about half way across the lattice, so we cannot be sure we are calculating purely pionic matrix elements. This is the price we pay for using physically shorter lattices than at $\beta = 6$. To combat this we use a second method in which propagators from the source at $t = -24$ (equivalently $t = 72$ since we have effectively doubled the lattice) are combined with those from $t = 0$. For t in the first half of the lattice we are calculating the off-shell matrix element (Fig. 5b), while for $t = 32 - 40$ the dominant contribution comes from the desired on-shell matrix element (Fig. 5c). The advantage of this method is that rho contamination is suppressed by an additional 10 or so timeslices of propagation.

It turns out that the two methods give consistent results, which bolsters our confidence in the calculation. We use the second method to quote numbers. An added bonus is results for the off-shell matrix element. This should be equal to the on-shell matrix element in the chiral limit, and should also show the effects of final state interactions. We do see small final state interactions, and we find close agreement between the on- and off-shell matrix elements. We hope to use this to shed light on the connection between A_2 and $A_2^{K\pi}$.

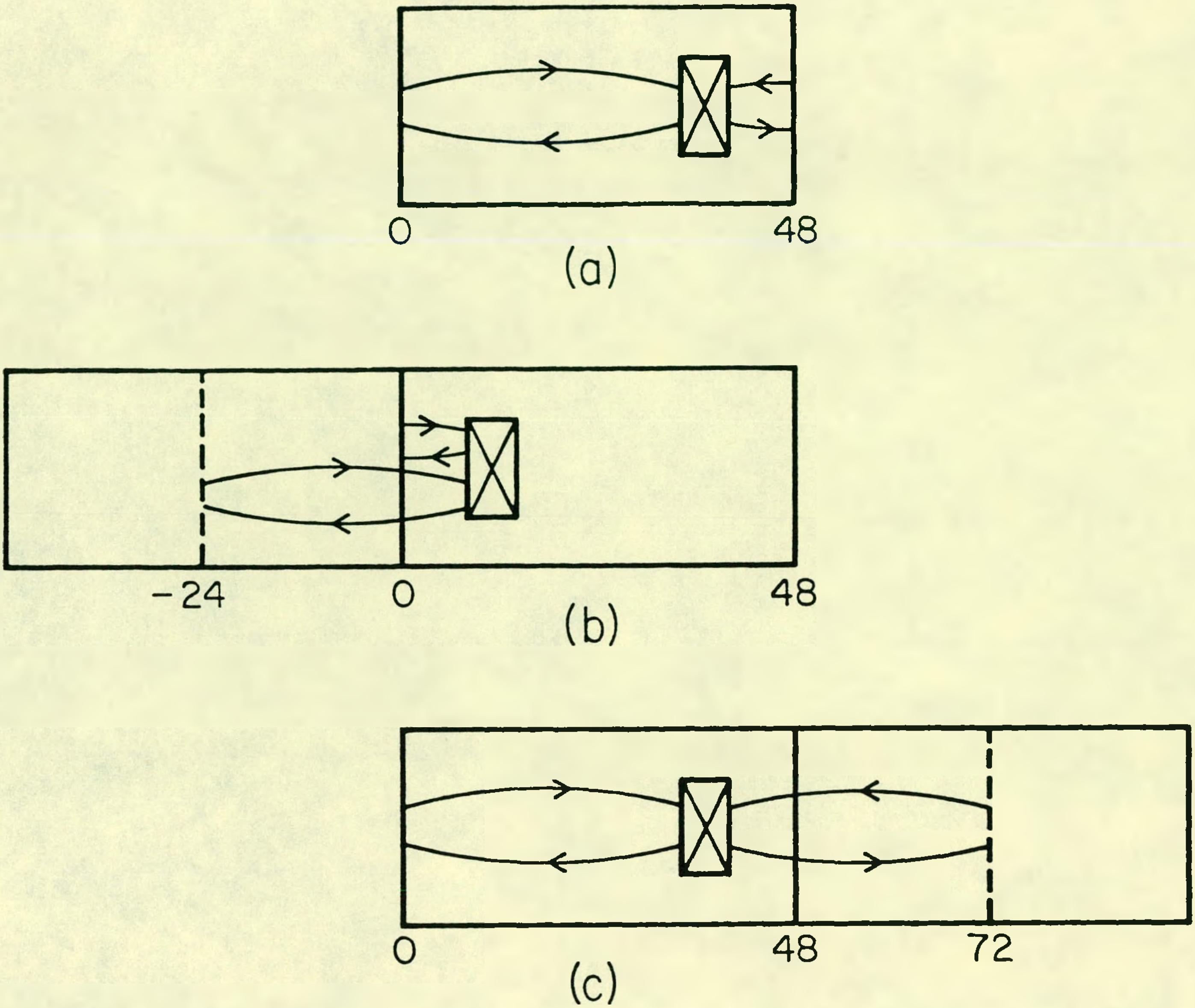

Figure 5: Methods for calculating B_K. Lines with arrows are propagators, boxes are weak operators.

The resulting bare B_K values are shown in Fig. 4. There is clearly a substantial drop from $\beta = 6$ to $\beta = 6.4$, as is particularly clear at the higher quark masses. Perturbative scaling would have the difference be tiny, ~ 0.02. Our results suggest, therefore, large scaling violations. Indeed, if one takes a slice through the data at the physical kaon mass, and plots the data versus lattice spacing, it is consistent with a linear slope. We do expect there to be $O(a)$ terms, and the magnitude we are finding is not unexpected. [15] Extrapolating to $a = 0$ will, if our results persist, lead to a smaller value of B_K than before. This would imply larger phases in the KM matrix, and consequently increased predictions for CP violating B-meson decays. Furthermore the discrepancy between $A_2^{K\pi}$ and A_2 would be reduced. Thus it is very important to understand what it happening.

To attempt to do this we plan to use much of the third year of the Grand Challenge to (a) accumulate more statistics at $\beta = 6.4$, (b) repeat the calculation at $\beta = 6.2$ on a $32^3 \times 48$ lattice (the 18^3 lattice used previously is small in physical units), (c) extend the $\beta = 5.7$ calculation to higher quark masses, (d) repeat the $\beta = 6$ calculation using the new methods on $16^3 \times 24$ lattices, and (e) improve the operators to remove $O(a)$ corrections.

ACKNOWLEDGEMENTS

Thanks are due to Claude Bernard, Joachim Kambor, Paul Mackenzie, Yosef Nir and Daniel Wyler for helpful conversations. This work is supported in part by an OJI award through contract DE-AT06-88ER40423, an Alfred P. Sloan Fellowship, and by NSF Grant No. PHY89-04035 (supplemented by funds from NASA).

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