CONF-790602--65

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AN EVALUATION OF STOCHASTIC VOID MODELS FOR BUBBLE WORTH CALCULATIONS*

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*Research sponsored by the Reactor Research and Technology Division, U. S. Department of Energy under contract W-7405 eng 26 with the Union Carbide Corporation.

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The reactivity due to the collapse of a large number of small bubbles in a reactor core has been analyzed with Behrens' method,[1,2] Benoist's method,[3] and the probability table method [4] Each analysis has used a different stochastic model for the bubble distribution within the reactor core: the Nicholson-Goldsmith calculations have employed a "closely-spaced holes" model; the Gelbard-Lell calculations have employed a "random cube" model; the Hoffman-Petrie calculations have employed a "nuclear model". However, any of the models can be used with any of the methods. In this paper, the three void models are evaluated. Based on this analysis, the closely-spaced holes model is recommended as the most suitable for analysis of bubble collapse in a molten, LMFBR core.

To study the void distribution, the moment-generating function is used:

$$G(q) = \sum_{n=0}^{\infty} \frac{(-q)^n}{n!} \langle s^n \rangle , \qquad (1)$$

where $\langle s^n \rangle$ is the n-th moment of the next collision site distribution in the bubbly system.

In the Hoffman-Petrie calculations with the nuclear model, an explicit expression for the distribution of next collision sites was obtained. With the substitution of this expression into Eq. 1, the following moment-generating function is obtained for this model:

$$G_{HP}(q) = \frac{1}{1 + q/\Sigma_t + (\Sigma/\Sigma_t)[1 - \exp(-qd)]} , \qquad (2)$$

where Σ_t is the total macroscopic cross section of the compressed material surrounding the bubbles, Σ is $\alpha/d/(1-\alpha)$, α is the void fraction, and d is the mean chord length of the bubbles.

For the closely-spaced holes model, usually only the second moment of the next collision distribution is calculated. However, the procedures used to evaluate the second moment can be extended to higher moments. In this fashion we obtain an expression for the moment-generating function implied in the Nicholson-Goldsmith calculations:

$$G_{NG}(q) = \frac{1}{1 + q/\Sigma_t - (\Sigma/\Sigma_t) \sum_{n=1}^{\infty} \frac{(-q)^n}{n!} \langle L^n \rangle} , \qquad (3)$$

where $\langle L^n \rangle$ is the n-th moment of the chord-length distribution for a single bubble.

Equation (3) reduces to Eq. (2) if

$$\langle L^n \rangle = \langle L \rangle^n , \qquad (4)$$

since $\langle L \rangle = d$. The meaning of Eq. (4) is that the variance of the chord-length distribution for the nuclear model is zero. Therefore, in the nuclear model all chord-lengths through the bubbles are equal; whereas for the closely-spaced holes model, as applied by Nicholson and Goldsmith, all bubbles are spheres of the same size. Since there is probably, in actuality, a distribution of bubble radii, neither model can be considered fully comprehensive. The difference between bubble worths calculated with these models is probably less than the uncertainty due to the analytic representation of the actual problem. On the other hand, the assumption of constant chord-length is far less natural than that of constant bubble size and seems to give no advantage in implementation. Therefore, the nuclear model appears less attractive than the closely-spaced holes model.

No analytic expression for the moment-generating function corresponding to the random cube model of Gelbard and Lell was obtained. However, these authors have shown from Monte Carlo estimates that the second moment for the random cube model and that for the closely-spaced holes model are not very different. Furthermore, as will be shown, the difference in calculated bubble worth between these two models, for a representative problem, is similar to that between the closely-spaced holes and nuclear models. As previously indicated, the difference is probably less than the uncertainty due to the analytic representation of the physical situation. Hence, repetition of the relatively costly Monte Carlo calculations for the random cube model is not justified.

To determine the effect of these models on bubble worth, the change in the multiplication constant due to bubble collapse in a molten LMFBR core is estimated with Behrens' formula:[5]

$$\Delta k \sim \frac{k_\infty - 1}{k_\infty} \left[\frac{\langle s^2\rangle_i}{\langle s^2\rangle_f} - 1\right] , \qquad (5)$$

where $\langle s^2\rangle_i$ is the second moment of the next flight distribution for the system prior to bubble collapse and $\langle s^2\rangle_f$ is the corresponding quantity after bubble collapse. Nicholson[2] estimated that k_∞ for a large LMFBR would be about 1.3. Table 1 contains the bubble worth as calculated by Eq. (5) with $k_\infty = 1.3$ and a void fraction of 0.2 using each of the three models for $\langle s^2\rangle_i/\langle s^2\rangle_f$. The worth is shown as a function of the neutron mean free path in the molten fuel mixture. Based on Lillie's cross sections[6] for the inner core of an LMFBR, the neutron mean free path in the fuel is about 4 cm. For this value, the random cube model estimate of Δk is about 11% greater and the nuclear model is about 11% smaller than that of the closely-spaced holes model.

In this paper, the three models used to obtain bubble worth in a molten LMFBR core are studied and are shown, for the problem of interest, to give comparable results. Since the closely-spaced holes model is more reasonable from a physical standpoint than the nuclear model and is easier to implement than the random cube model, we recommend its use for bubble collapse calculations of a molten, LMFBR core.

Table 1. Bubble Worth for Void Fraction of 20% and k_∞ of 1.3

Model	Neutron Mean Free Path (cm)			
	2	3	4	5
Random Cube	0.0182	0.0120	0.0090	0.0074
Closely-Spaced Holes	0.0162	0.0108	0.0081	0.0065
Nuclear	0.0144	0.0096	0.0072	0.0058

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