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TITLE: NUMERICAL MODELS OF CTX EQUILIBRIUM

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The open mesh flux conserver (MFC) of the Los Alamos spheromak (CTX) is equipped with Rogowski loops that measure the current in many of its individual segments. The distribution of induced current in the toroidal hoops of the MFC can be used to construct numerical models of the CTX equilibirum.

Assuming the plasma is in an axisymmetric zero-beta ideal MHD equilibrium, the magnetic field can be represented in cylindrical coordinates as

$$\underset{\sim}{B} = \frac{\hat{e}_\varphi}{r} \int_0^\psi \lambda(\psi')d\psi' + \nabla\psi \times \frac{\hat{e}_\varphi}{r} \quad , \tag{1}$$

with

$$\underset{\sim}{j} = \nabla\times\underset{\sim}{B} = \lambda(\psi)\underset{\sim}{B} + \underset{\sim}{j}_I \quad . \tag{2}$$

where $\underset{\sim}{j}_I$ is the induced current in the MFC and $\lambda = j_\parallel/B$. The poloidal flux function ψ satisfies the Grad-Shafranov equation

$$-\Delta^*\psi = \lambda(\psi) \int_0^\psi \lambda(\psi')d\psi' + \sum_j I_j r_j \delta(r-r_j)\delta(z-z_j) \quad . \tag{3}$$

The sum is over the toroidal hoops of the MFC, which have coordinates (r_j, z_j) and carry current I_j. The operator $\Delta^* = r^2\nabla\cdot r^{-2}\nabla$, and $\lambda(\psi)$ is taken to be a polynomial ,

$$\lambda(\psi) = \lambda_0 + \lambda_1\psi + \lambda_2\psi^2 + \dots \quad , \tag{4}$$

with adjustable parameters λ_k. Defining the flux due to the plasma current

$$- \Delta^{*}\psi_{P}(r,z) = \lambda(\psi) \int_{0}^{\psi} \lambda(\psi')d\psi' \quad , \tag{5}$$

and the Greens' function of the j^{th} toroidal hoop

$$-\Delta^{*}G_{j}(r,z) = r_{j}\delta(r-r_{j})\delta(z-z_{j}) \quad , \tag{6}$$

the solution to Eq. (3) can be written.

$$\psi(r,z) = \psi_{P}(r,z) + \sum_{j} I_{j}G_{j}(r,z) \quad . \tag{7}$$

Applying the flux conserving boundary condition $\psi(r_k,z_k) = 0$ at each hoop, Eq. 7 becomes

$$\sum_{j} I_{j}G_{j}(r_{k},z_{k}) = - \psi_{P}(r_{k},z_{k}) \quad . \tag{8}$$

The matrix $G_j(r_k,z_k) \equiv M_{jk}$ is the mutual inductance between hoops j and k. A finite aspect ratio correction is added to the diagonal elements to give the correct self-inductance. The current is then determined by

$$I_{j} = - \sum_{k}(M^{-1})_{jk} \psi_{P}(r_{k},z_{k}) \quad . \tag{9}$$

The algorithm for solving Eq. (3) is now evident: start with a guess for ψ and solve Eqs. (5), (9), and (7) in that order, and repeat until the solution converges. (10-20 interations is typical.) This procedure determines the hoop currents as a function of the λ parameters $I_j = I_j(\lambda_0\lambda_1\lambda_2 \ldots)$. Combining this with the measured values of the hoop currents I_j^* and their uncertainties ΔI_j^*, a mean square error is compute

$$E(\lambda_0\lambda_1\lambda_2 \ldots) = \sum_j \left[\frac{I_j^* - I_j(\lambda_0\lambda_1\lambda_2 \ldots)}{\Delta I_j^*}\right]^2 , \tag{10}$$

and the set of parameters that gives the best fit to the data can be determined.

If only one parameter is used in Eq. (4), the λ profile is flat and there is only one solution; the minimum energy state shown in Fig. 1. With two parameters the slope of the λ profile can be varied, generating a family of equilibria that can model the time evolution of CTX during a shot. Figure 2 shows two parameter fits to the Rogowski loop data at four different times in a typical shot. The first row of figures shows the data along with the computed hoop current distributions that give the best fit. The second and third rows show the λ and q profiles. The first column is at the beginning of the shot when the spheromak is being sustained and the n = 1 oscillation is observed. The second column is just after the sustainment ends and the plasma is close to the minimum energy state with no oscillations. Resistive decay causes current peaking near the magnetic axis in column three where an n = 2 oscillation is seen, and continues to column four where the n = 3 oscillation takes over. Notice that the q on axis is close to one when the n = 1 oscillation is observed, and close to 1/2 and 1/3 when the n = 2 and n = 3 oscillations are seen.

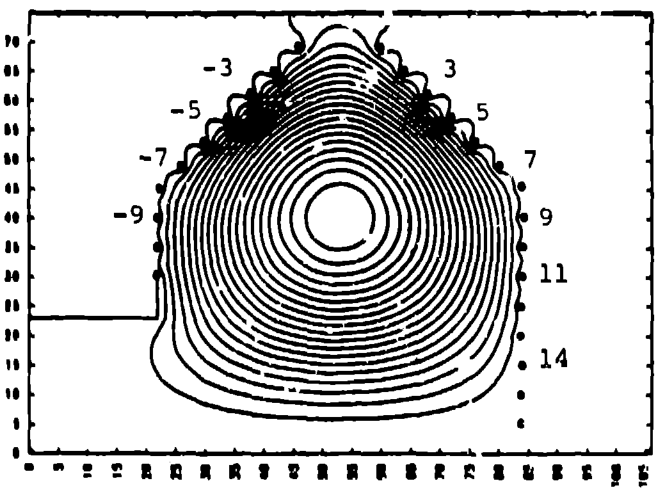

Figure 1. The minimum energy state of CTX

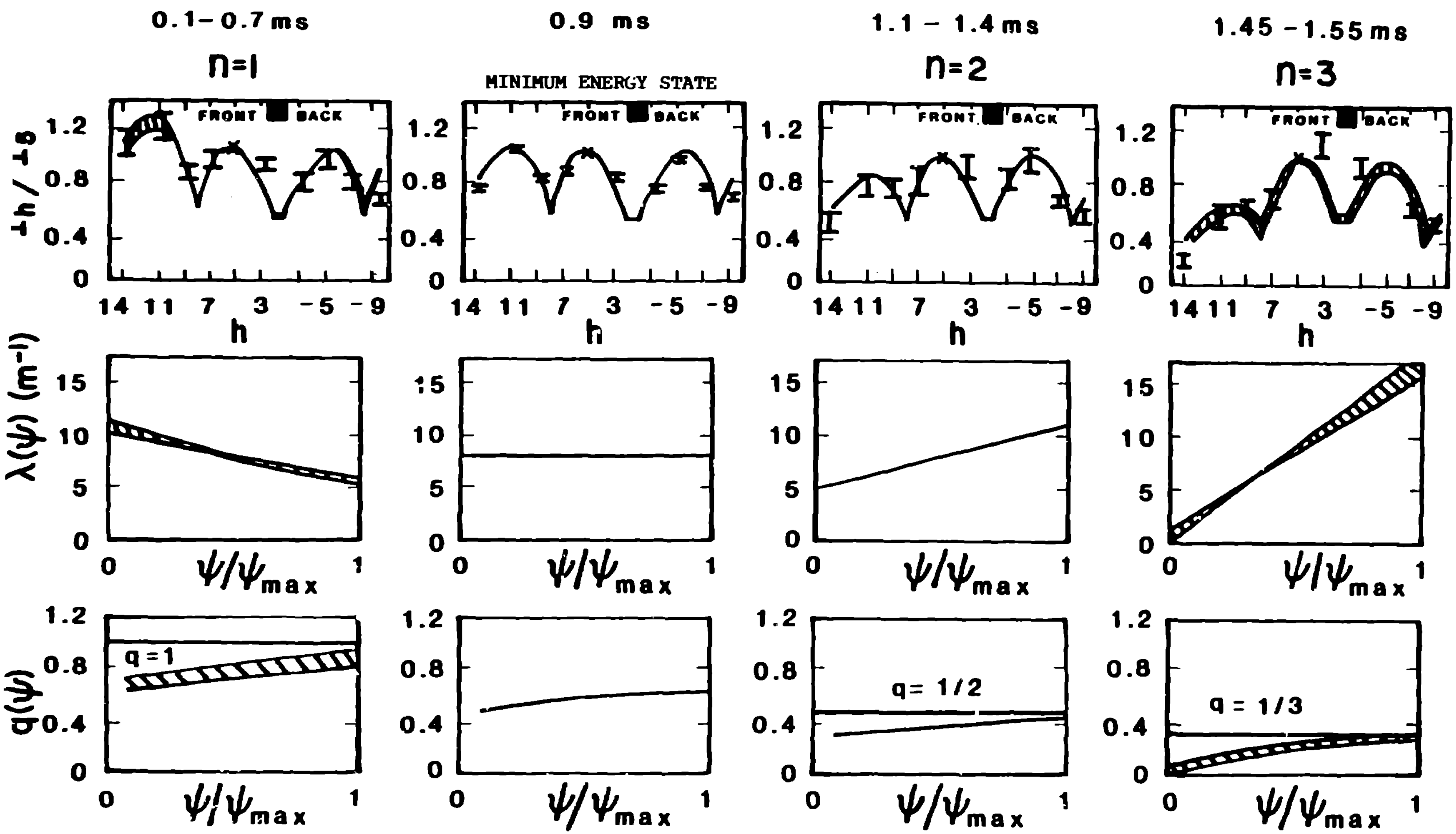

Figure 2. Two parameter models of CTX equilibrium at four different times during a typical shot in which n = 1 oscillations are observed (Column 1), followed by a quiescent period (Column 2), then by n = 2 oscillations (Column 3) and n = 3 oscillations (Column 4).