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THE LINK BETWEEN RESOLVED RESONANCES AND THE OPTICAL MODEL

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NEUTRON STRENGTH FUNCTIONS: THE LINK BETWEEN RESOLVED RESONANCES AND THE OPTICAL MODEL*

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ABSTRACT

Neutron strength functions and scattering radii are useful as energy and channel radius independent parameters that characterize neutron scattering resonances and provide a connection between R-matrix resonance analysis and the optical model. The choice of R-matrix channel radii is discussed, as are limitations on the accuracies of strength functions. New definitions of the p-wave strength function and scattering radius are proposed. For light nuclei, where strength functions display optical model energy variations over the resolved resonances, a doubly reduced partial neutron width is introduced for more meaningful statistical analyses of widths. The systematic behavior of strength functions and scattering radii is discussed.

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The neutron strength function provides a link between resonance scattering data and the optical model. Its usefulness in helping to determine optical potential parameters arises from the fact that a detailed analysis of s and p-wave scattering resonances can provide separate determinations of the real and imaginary parts of the optical model s-wave scattering amplitude η_0 and the imaginary parts of the p-wave amplitudes $\eta_{1,1/2}$ and $\eta_{1,3/2}$. These are four independent parameters as against the single parameter, a weighted sum of the real parts of the η's, which an average total cross section measurement over a comparable energy interval determines. And if, in special circumstances, d-wave resonances, or a p-wave background scattering phase can also be measured, the number of measured parameters becomes even greater. For this reason, strength functions have played an important role in determining optical potentials from the inception of the model to the present [1-3]. Beyond this they have been applied to more detailed discussions of nuclear dynamics, such as intermediate structure, [4-9] and are useful for the construction of statistical resonance cross section models from the optical potential.

There are also some disadvantages to the use of resonance parameters for the determination optical potential parameters. While average total cross sections can be measured over practically unlimited energy ranges with very high accuracies, resonance parameters can be determined only within a limited low neutron energy interval that contains a limited number of resonances. Since strength funcitons are averages over resonance partial widths and since partial widths fluctuate according to the Porter-Thomas distribution [10] which has a huge standard deviation of $\sqrt{2}$ relative to its average, the standard deviation of the average of a sample of N partial widths is $\sqrt{2/(N-1)}$ of its value. Therefore, a strength function derived from only 10 resonances has a 45% probable error, in addition to experimental uncertainties, and even 50 resonances can determine a strength function to within no better than 20%. In addition to this basic statistical error in strength function determination, there are also problems associated with energy and channel radius dependences of these parameters, which we shall discuss presently. Such energy variations may be large within the energy range of analyzable resonances, particularly in the case of lighter nuclei. Energy dependent strength functions may be useful in normalizing resonance widths for more reliable statistical analyses and intermediate structure determinations. I shall return to this point later also.

The only practical and generally applicable method for analyzing resonance data is the R-matrix method [11,12]. Such an analysis yields for each partial wave ℓj a set of resonance energies $E_\mu^{J(\ell j)}$ and a corresponding set of resonance amplitudes $\gamma_\mu^{\ell j}$. These R-matrix resonance parameters are theoretical constructs which have no directly measurable physical significance and their values depend on the more or less arbitrary choices of channel radii $a_{\ell j}$ and boundary conditions $B_{\ell j}$. For resonance analysis purposes the most convenient choice of $B_{\ell j}$ are those values which will cancel the channel shift functions at zero neutron energy and I will use that choice throughout.

When the R-matrix is calculated by solving the Schrödinger equation with a nuclear interaction Hamiltonian as was done for example by Takeuchi and Moldauer [13], it is essential that all channel radii be chosen, so as to be greater than the range of the nuclear interaction between the neutron and the target nucleus. However we are dealing here with an empirical R-matrix whose only function is to parameterize the resonances. Therefore we need not be concerned about such a restriction on channel radii and may choose the $a_{\ell j}$ to be near the half falloff radius of the optical potential. That this is satisfactory is demonstrated in Fig. 1 where it is shown that the s and p wave strength functions and the R' parameter calculated for ^{60}Ni from an optical potential with half falloff at 4.74 fm are all independent of channel radii for $a_{\ell j}$ between 2 and 10 fm. Theoretically the use of small $a_{\ell j}$ is justified by imagining that there exists some Hamiltonian whose interaction is confined within the $a_{\ell j}$ and which yields exactly the E^J_μ and $\gamma^{\ell j}_\mu$ which lie within the analyzed energy interval. Since this is a finite number of parameters we should be able to construct the required equivalent Hamiltonian. The advantage of using channel radii of optical potential range is that this reduces the energy dependences of the strength functions and avoids the introduction of spurious optical model peaks, as demonstrated in Fig. 2. It would perhaps be useful to establish a standard channel radius for R-matrix fitting of resonance data. The standard adopted in the calculations reported here is $a = 1.25A^{1/3} + 0.5$ fm for all channels.

A single channel R-function resonance analysis of a total or elastic neutron cross section is obtained by fitting the measured cross section to the appropriate following formula

$$\sigma = \sum_{J,\pi,\ell,j} \frac{(2J+1)}{2(I+1)} \sigma_{J,\pi,\ell,j} \quad , \tag{1}$$

$$\sigma^{\text{total}}_{J,\pi,\ell,j} = \frac{2\pi}{k^2} (1 - \text{Re}\, U_{J,\pi,\ell,j}) \quad , \tag{2}$$

$$\sigma^{\text{elastic}}_{J,\pi,\ell,j} = \frac{\pi}{k^2} \left|1 - U_{J,\pi,\ell,j}\right|^2 \quad , \tag{3}$$

where

$$U_{J,\pi,\ell,j} = e^{-2i\chi_{\ell j}} \frac{1 - L^{*}_{\ell j} R_{J,\pi,\ell,j}}{1 - L_{\ell j} R_{J,\pi,\ell,j}} \quad , \tag{4}$$

$$R_{J,\pi,\ell,j} = R^{\infty}_{J,\pi,\ell,j} + \frac{(\gamma_{\mu}^{J,\pi,\ell,j})^2}{E_{\mu}^{J,\pi} - E} \quad , \tag{5}$$

and for s and p waves and for our choice of boundary conditions

$$\chi_o = \rho_o \qquad\qquad L_o = i\rho_o$$
$$\chi_{1j} = \rho_{1j} - \tan^{-1}\rho_{1j} \qquad\qquad L_{1j} = \frac{\rho_{1j}^2}{1 + \rho_{1j}^2}(1 + i\rho_{1j}) \tag{6}$$

where $\rho_{\ell j} = ka_{\ell j}$ and k is the channel wave number and $a_{\ell j}$ is the arbitrarily chosen channel radius. The object of such a fit is to obtain the background R-functions R^{∞}, the R-matrix poles E_{μ} and the pole channel amplitudes γ_{μ}. Where appropriate these formulas must be generalized to multichannel R-matrix formulas, but the final set of resonance parameters is of the same type.

At low enough energies $R_{1j}\times \mathrm{Re}L_{1j}$ is small enough compared to unity so that $\mathrm{Re}L_{1j}$ can be ignored. However, this is not always true for the whole range of analyzable resonances in lighter nuclei as is demonstrated in Fig. 3.

The connection between the R-matrix resonance parameters and the optical model amplitudes $\eta_{\ell j}$ is given by

$$\eta_{\ell j} = e^{-2i\chi_{\ell j}} \frac{1 - L^{*}_{\ell j} \tilde{R}_{\ell j}}{1 - L_{\ell j} \tilde{R}_{\ell j}} \tag{7}$$

where

$$\tilde{R}_{\ell j} = R^{\infty}_{\ell j} + i\pi s_{\ell j} \tag{8}$$

$$s_{\ell j} = \frac{(2j + 1)}{2} \sum_{J} \frac{\langle (\gamma_{\mu}^{J,\pi,\ell,j})^2 \rangle}{D_{J,\pi,(\ell,j)}} \tag{9}$$

where $\langle \;\; \rangle$ indicates an average over resonance μ and D_J is the mean spacing of R-function poles E_{μ} of appropriate total angular momentum and parity. The coefficient and sum in Eq. (9) is required when the optical model is independent of target spin I and total angular momentum J, as is assumed here. The same sum and coefficient defines $R^{\infty}_{\ell j}$ in terms of the $R^{\infty}_{J,\pi,\ell,j}$. The optical model η's are obtained from an integration of the Schrödinger equation with the optical potential interaction as is performed in all optical model computer programs.

Equations (7)-(9) define the background R-functions R^{∞} and the R-strength functions s in terms of the complex optical model scattering amplitudes η and the channel parameters χ and L. R^{∞} and s vary slowly with energy as a result of the optical model giant resonance energy variation. But these parameters depend strongly upon the choice of channel radius by virtue of the channel radius dependences of χ and L. To obtain a useful parameter independent of arbitrary radii, it is necessary to remove this channel radius dependence, without at the same time introducing the strong kinematical energy dependences of partial neutron widths. This is the purpose of the strength function definitions which are given below.

Solving Eq. (7) for $s_{\ell j}$ we get for low neutron energies

$$4\pi\, P_{\ell j}\, s_{\ell j} = T_{\ell j} \quad , \tag{10}$$

where $P_{\ell j} = \mathrm{Im}\, L_{\ell j}$ and the transmission coefficient

$$T_{\ell j} = 1 - |\eta_{\ell j}|^2 \quad . \tag{11}$$

At these low neutron energies the transmission coefficients can be related to the characteristics of cross section resonances by

$$T_{\ell j} = 2\pi\, \bar{\Gamma}_{\ell j}/D \quad , \tag{12}$$

where $\bar{\Gamma}_{\ell j}$ is the average partial neutron resonance width and D the resonance spacing. In contrast to the R-function parameters, these are real physical quantities that can be measured and that are independent of any channel radius. Moreover, their kinematical energy dependencies at low neutron energy are known. In particular $\bar{\Gamma}_{\ell j}/D$ behaves there as $E^{(\ell + 1/2)}$. Therefore the desired channel radius and energy independent definition of the strength function is obtained by dividing the left hand side of Eq. (10) by $E^{(\ell + 1/2)}$.

$$S_{\ell j} = \frac{2}{2\ell + 1}\, \frac{P_{\ell j}}{E^{\ell + 1/2}}\, s_{\ell j} \quad , \tag{13}$$

which includes a traditional factor of $(2\ell + 1)^{-1}$, and where, again by tradition, E is measured in electron volts. Equations (10), (12), (13) yield the traditional definition of the S-wave neutron strength function

$$S_0 = \frac{1}{\sqrt{E}} \frac{\bar{\Gamma}_o}{D} \quad . \tag{14}$$

For p-waves, we obtain by the same method

$$S_{1j} = \frac{2j+1}{3E^{3/2}} \frac{\bar{\Gamma}_{1j}}{D} \quad . \tag{15}$$

In contrast, the traditional formulation of the p-wave strength function

$$S_{1j}^{\text{traditional}} = \frac{2j+1}{3\sqrt{E}} \frac{1+(ka)^2}{(ka)^2} \frac{\bar{\Gamma}_{1j}}{D} = \frac{2}{3} \frac{ka}{\sqrt{E}} S_{1j} \tag{16}$$

does have the desired energy independence at low energies, but it also has a very strong and undesirable channel radius dependence which is demonstrated in Fig. 1. Therefore, to be interpretable, any value of $S_1^{\text{traditional}}$ must be supplemented by a channel radius value. In contrast the definition (15) of S_1 is radius and energy independent up to energies where the value of $(ak)^2$ becomes appreciable compared to unity, that is, typically up to some tens of kilovolts. Above such energies other kinematical as well as dynamical effects such as optical model variability start coming into play, in any case. I would therefore recommend that Eq. (15) be adopted as the definition of the p-wave strength function.

If E is measured in electron volts, then S_0 is of order 10^{-4} and S_{1j} of Eq. (15) is of order 10^2.

In addition to the strength function, the background R-function R^∞ provides another channel parameter that can be connected with the optical model. Solving Eq. (7) for the s-wave R_o^∞, we obtain in the low energy limit the following connection with η_o, via the background scattering radius R'.

$$-\frac{\arg(\eta_o)}{2k} \equiv R' = a_o(1-R_o^\infty) \tag{17}$$

where the defining relationship is on the left and the connection with resonance analysis is on the right. R' is again an energy and radius independent parameter at low energies.

If we do the same for p-waves, we obtain the relation

$$R'_{1j} = \frac{P_{1j}}{k} \left(\frac{1}{3} - R_{1j}^\infty\right) \tag{18}$$

for the p-wave background scattering radii. These quantities are again independent of the channel radii a_{1j}, but they are obviously proportional to the neutron energy E at low energies. In order to obtain energy and radius independent p-wave potential scattering radii we must define the reduced p-wave scattering radii

$$R'^{o}_{1j} = R'_{1j}/E \quad , \tag{19}$$

which yield numbers in the vicinity of unity when E is measured in MeV. It should be noted that R'^{o}_{1j} may assume negative values in the vicinity of a p-wave strength function peak, as seen in Fig. 4. P-wave potential scattering radii have recently been measured [14].

In the lighter nuclei, A < 50, resonance cross sections can often be measured and analyzed by R-matrix parameters within energy regions where any or all of the corresponding strength functions $S_{\ell j}$ or $R'^{o}_{\ell j}$ display appreciable energy or radius dependences that cannot be removed. They are caused in part by the breakdown of low energy approximations and in part by optical model giant resonance effects. Moreover, the resonance spacings may be so large that no useful averages can be obtained within subintervals of small energy variability. In such circumstances it is useful to recognize these energy variabilities and build them explicitly into the definitions. For this purpose it is useful to consider a doubly reduced partial width parameter $\Gamma^{oo}_{\mu\ell j}$ from which both the kinematic channel energy dependence and the dynamical optical model energy dependence has been factored out:

$$\Gamma^{oo}_{\mu\ell j} = \frac{\Gamma_{\mu\ell j}\; S_{\ell j}(1\ \mathrm{eV})}{E_{\mu}^{\ell+1/2}\; S_{\ell j}(E_{\mu})} \quad , \tag{20}$$

Alternately one could use reduced R-matrix amplitudes $\gamma_{\mu}^{\ell jo} = \gamma_{\mu}^{\ell j}\sqrt{s_{\ell j}(1\ \mathrm{eV})/s_{\ell j}(E_{\mu})}$. Here $S_{\ell j}(E_{\mu})$ or $s_{\ell j}(E_{\mu})$ is determined by the optical potential and the latter can be adjusted in order to satisfy

$$\frac{2j+1}{2\ell+1}\,\frac{\bar{\Gamma}^{oo}_{\mu\ell j}}{D} = S_{\ell j}(1\ \mathrm{eV}) \quad . \tag{21}$$

This provides a systematic method of optical model fitting of resonance data in lighter nuclei.

In addition, the doubly reduced widths are convenient for statistical analyses in lighter nuclei. The $\Gamma^{oo}_{\mu\ell j}$ should be distributed according to the Porter-Thomas distribution, while the distributions of singly reduced widths $\Gamma^{o}_{\mu\ell j} = \Gamma_{\mu\ell j}/E_{\mu}^{\ell+1/2}$ may be distorted by optical model effects. Also the cumulative values of doubly reduced widths $\Sigma_{E_\mu<E}\ \Gamma^{oo}_{\mu\ell j}$ give a more reliable indication of intermediate structure than do the customary stairway plots of unreduced or singly reduced widths. A case in point is the recent analysis of γ^2_μ for s and p-wave neutron resonances in ^{29}Si by Newson et al. [6], over a neutron energy range of up to 4.5 MeV. As indicated in Fig. 2, there is much optical model variation in the R-strength functions over this energy range. Moreover, this variation depends strongly upon the choice of channel radii. Thus the rapid rise in the s-wave strength above 3 MeV in Ref. (6) could possibly arise from the rapid rise of the s-wave R-strength function in the same region as shown in Fig. 2B. Similarly the apparent peak in the p-wave strength in Ref. [6] seems to be correlated with the optical model peak in the p1/2 strength in Fig. 2B. The appearance of spurious channel radius dependent peaks can be avoided by first obtaining an optical model fit to the resonance and other data as indicated in Eq. (21), and then constructing stairway plots with the doubly reduced widths of Eq. (20). Energy variations in the latter can then be interpreted as true intermediate resonances.

Finally, Fig. 4 displays the mass number dependence of the s and p-wave strength functions and (reduced) background scattering radii, as defined here, and as calculated from the overall optical potential of Ref. [2] with a fixed spin orbit potential of 7 MeV.

AFTERTHOUGHT: An alternative to the definition (13) might be the following

$$S'_{\ell j} = \frac{2}{2\ell+1}\,\frac{(ka)^{2\ell+1}}{E^{\ell+1/2}}\,s_{\ell j} \quad . \tag{13'}$$

This removes some of the kinematical energy dependence from the strength function at the expense of introducing an explicit channel radius dependence. The result of the definition (13') would be to introduce an additional factor of $1+(ka)^2$ into the definition (15) of the p-wave strength function. For the case of ^{28}Si, $S'_{1,1/2}$ and $S'_{1,3/2}$ are shown dotted in Fig. 2a.

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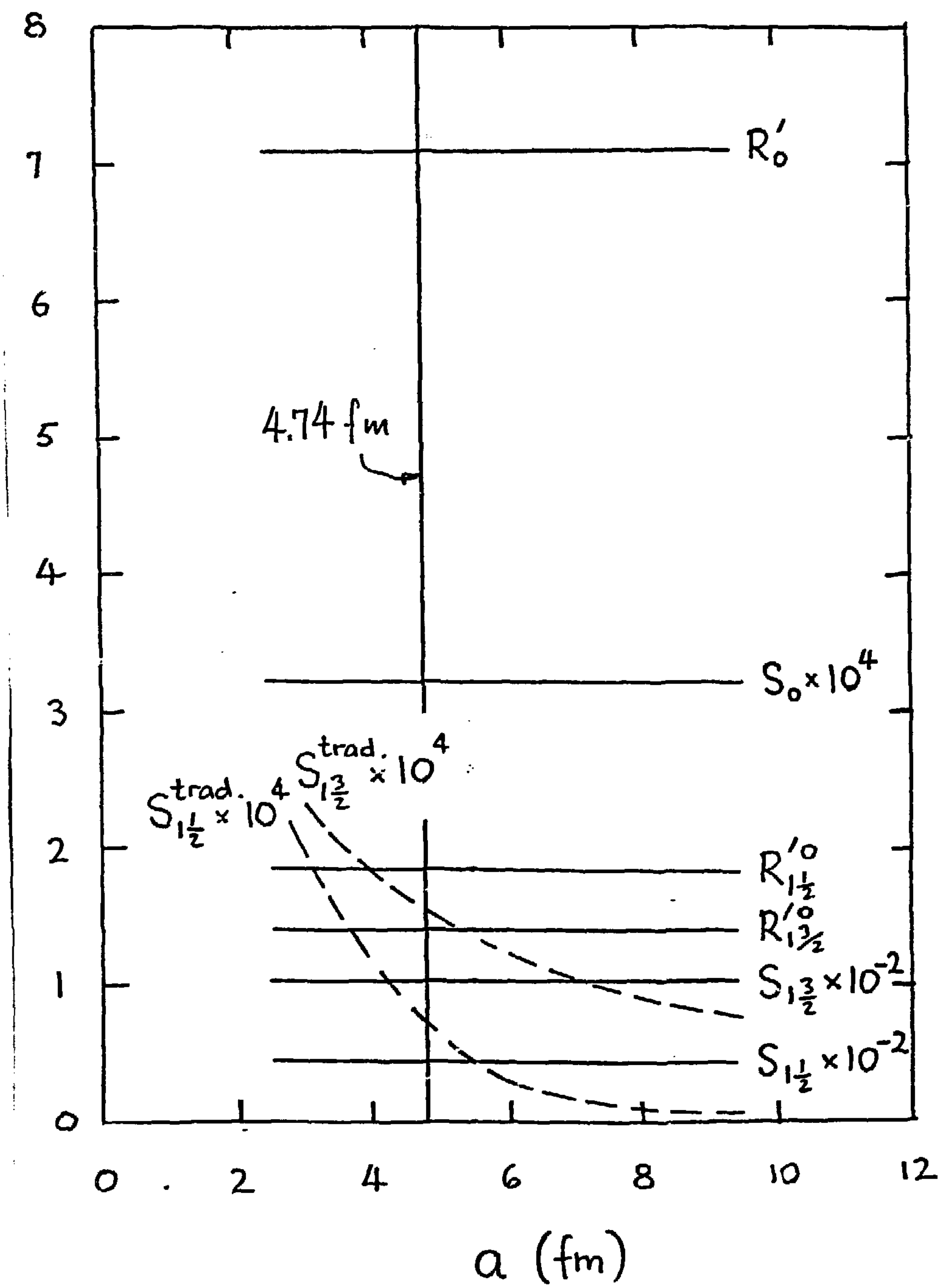

Fig. 1. Channel radius dependences of s and p-wave strength functions and (reduced) channel radii, Eqs. (14), (15), (17), (19), as well as traditional p-wave strength functions, Eq. (16), dashed lines, for an optical potential (^{60}Ni) with half falloff radius of 4.74 fm. (V = 53.1-0.3E, W = 7.9 + 0.25E, VSO = 8.0 MeV; R = $1.2A^{1/3}$, A = 0.6 fm).

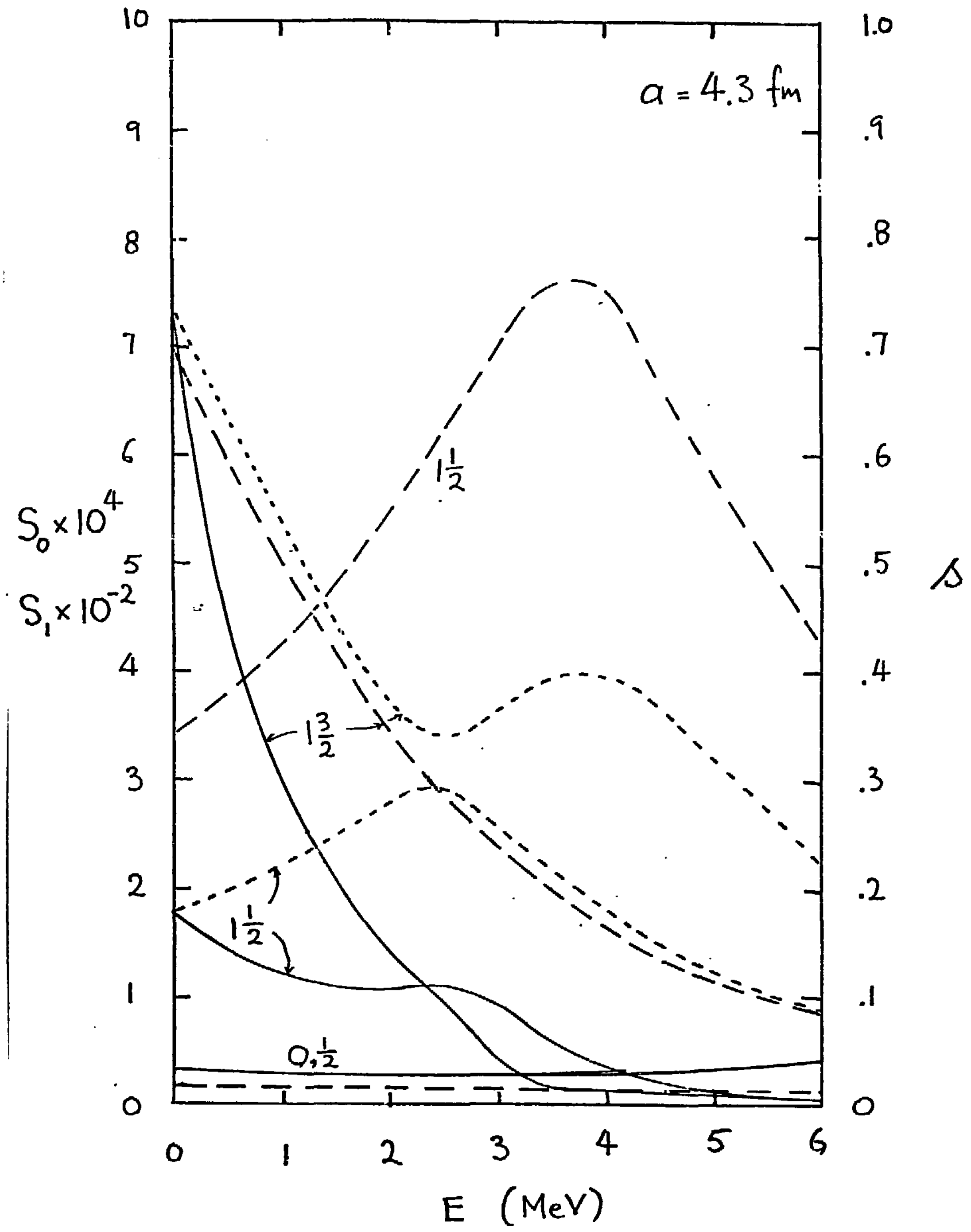

Fig. 2A. Energy dependences of s and p-wave R-strength functions Eq. (9), dashed lines, strength functions Eq. (13), full lines, and S_1, dotted lines, for a ^{28}Si optical model (V = 53.8, W = 3.0, VSO = 7.0 MeV; $R = 1.21A^{1/3}$, $A_V = 0.66$, $A_W = 0.48$ fm) Channel radii a = 4.3 fm.

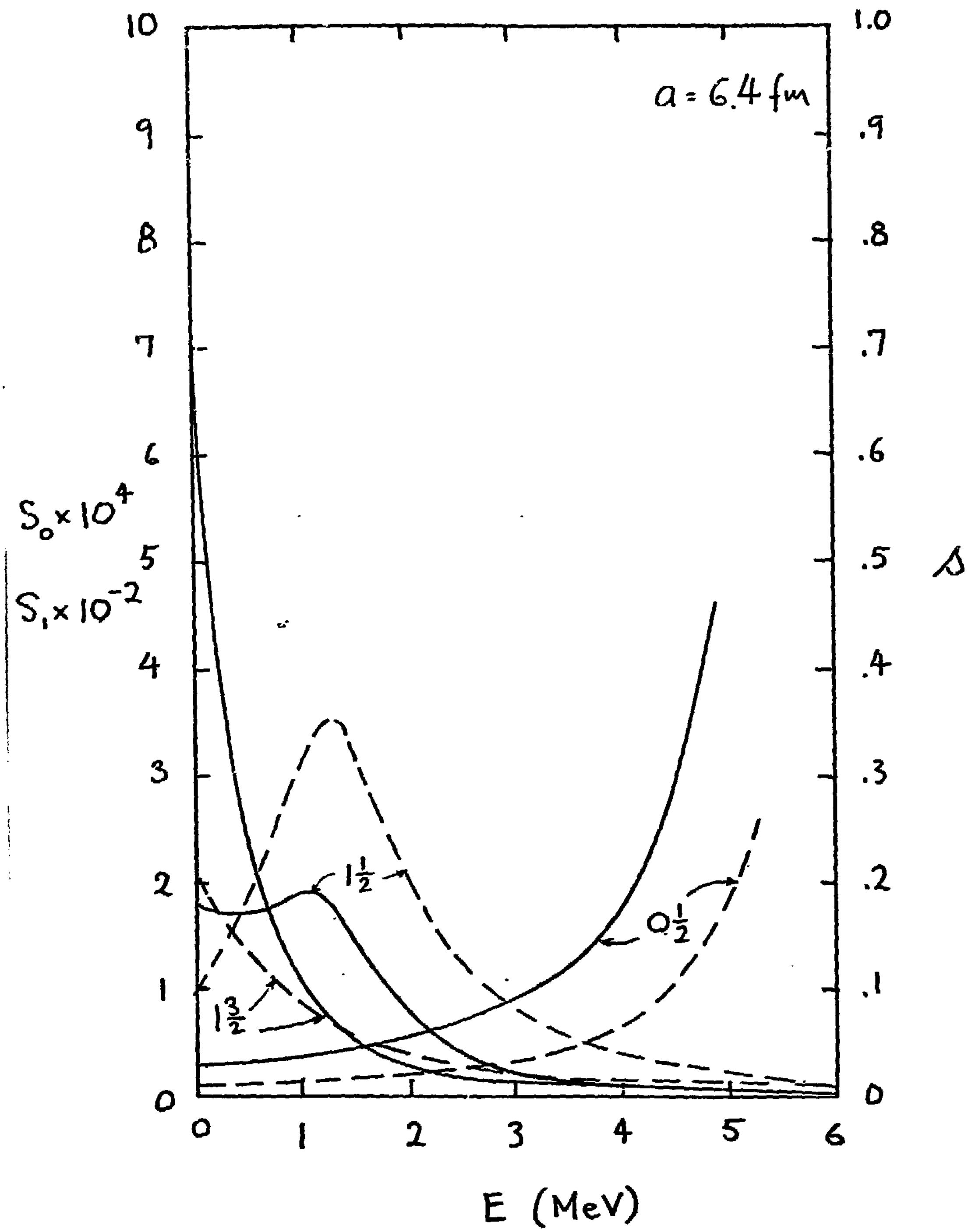

Fig. 2B. Same as 2A with a = 6.4 fm.

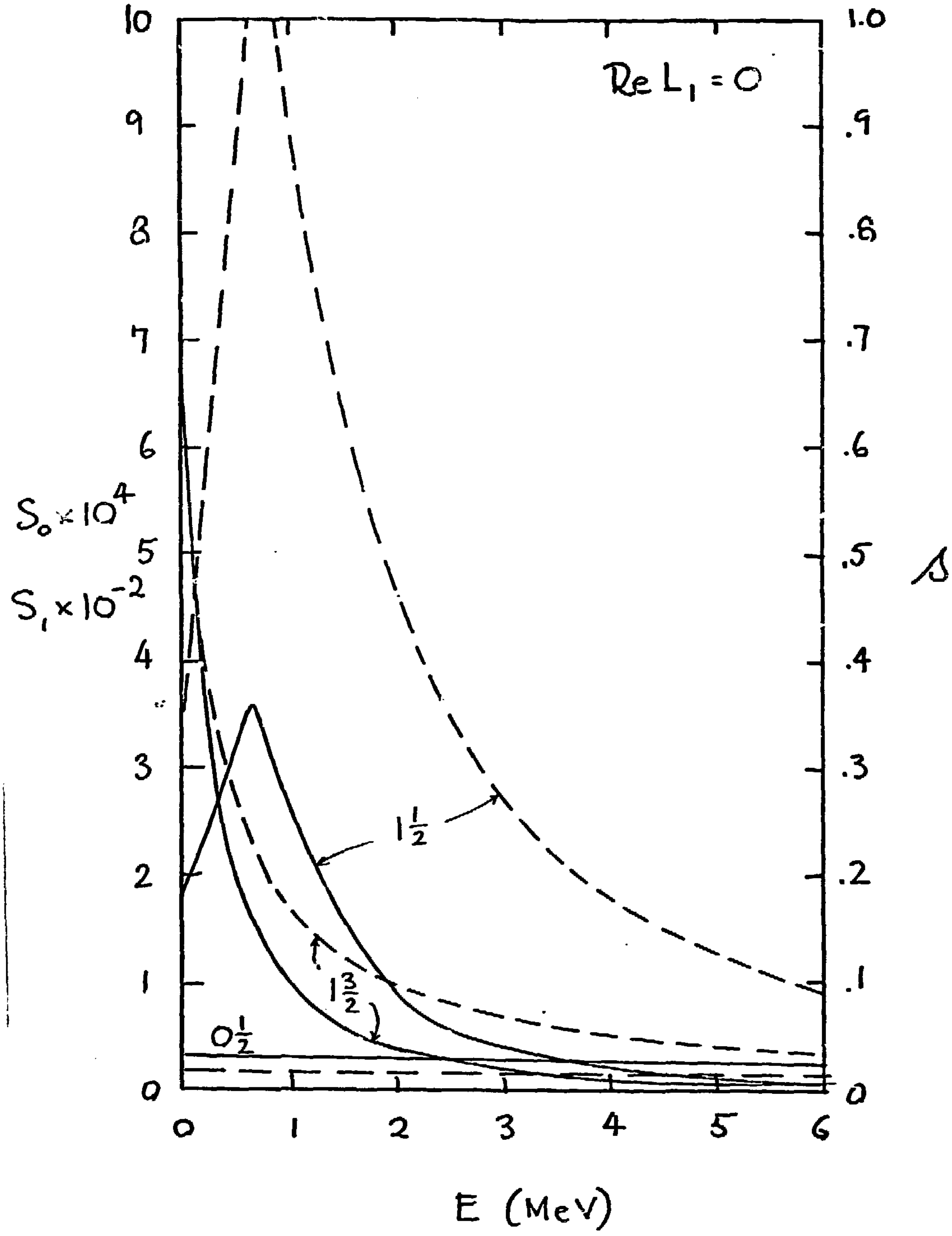

Fig. 2C. Same as 2A, but ignoring p-wave level shift factor ReL_{1j} in Eq. (7).

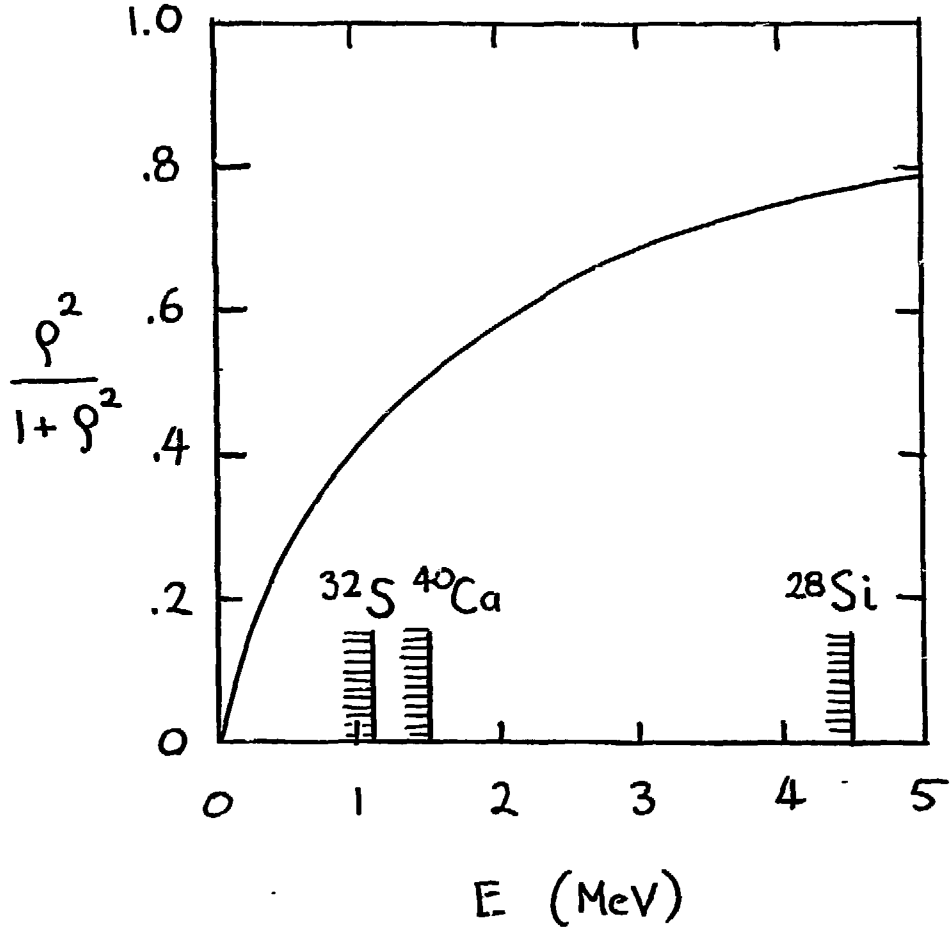

Fig. 3. Energy dependence of p-wave level shift factor ReL_{lj} for light nuclei compared to the limits of analyzable resolved p-wave resonances in ^{32}S, ^{40}Ca, and ^{28}Si.

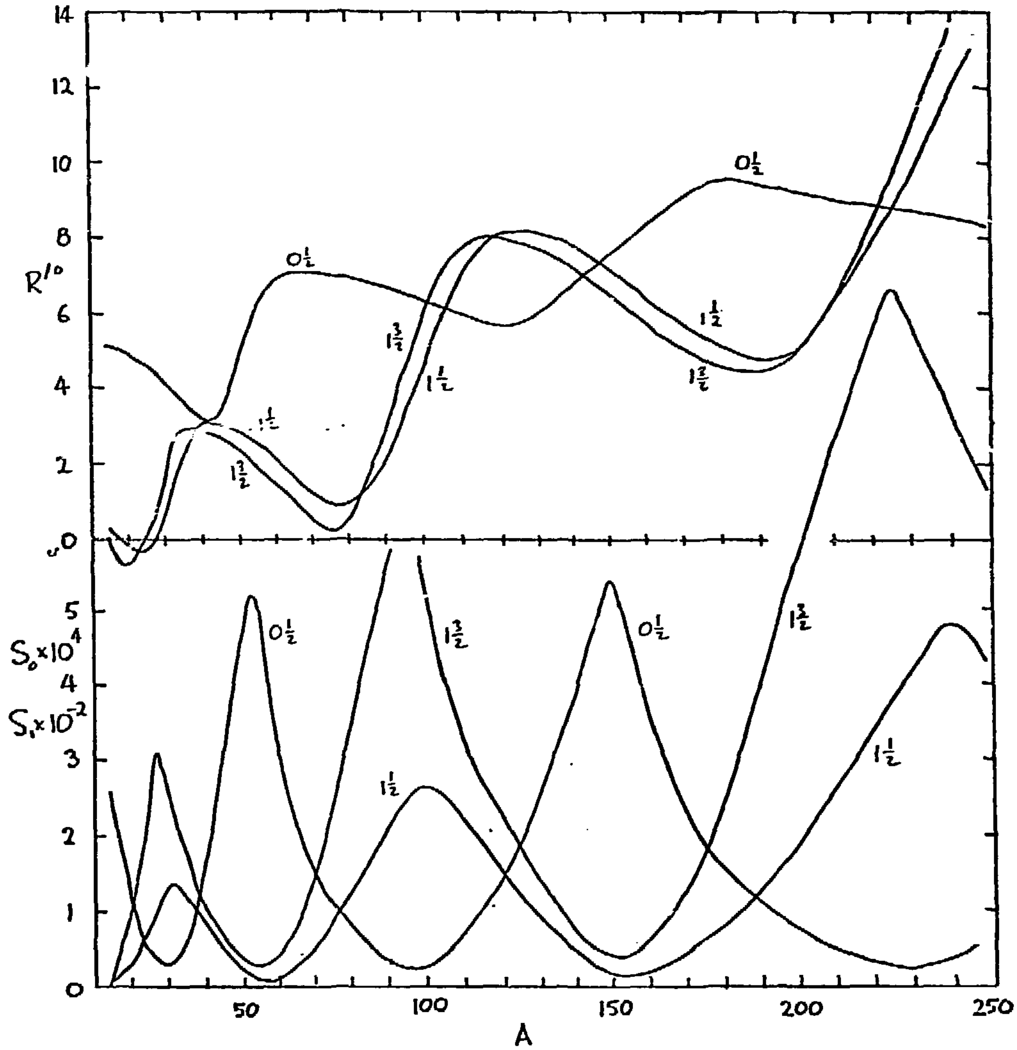

Fig. 4. Systematic mass number dependences of s and p-wave strength functions and (reduced) channel radii for an optical model with V = 46.0, W = 14.0, VSO = 7.0 MeV; $R_V = 1.37A^{1/3}$, $R_W = 1.447A^{1/3}$, $A_V = 0.62$, $A_W = 0.25$ fm.