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NONPERTURBATIVE RENORMALIZATION OF QED IN LIGHT-CONE QUANTIZATION*

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ABSTRACT

As a precursor to work on QCD, we study the dressed electron in QED non-perturbatively. The calculational scheme uses an invariant mass cutoff, discretized light-cone quantization, a Tamm-Dancoff truncation of the Fock space, and a small photon mass. Nonperturbative renormalization of the coupling and electron mass is developed.

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1 Introduction

We are in the process of studying dressed fermion states in a gauge theory. To give the work specific focus, we concentrate on the nonperturbative calculation of the anomalous moment of the electron. [1] This is not intended to be competitive with perturbative calculations. [2] Instead it is an exploration of nonperturbative methods that might be applied to QCD and that might provide a response to the challenge by Feynman [3] to find a better understanding of the anomalous moment.

The methods used are based on light-cone quantization [4] and on a number of approximations. Light-cone coordinates provide for a well-defined Fock state expansion. We then approximate the expansion with a Tamm–Dancoff [5] truncation to no more than two photons and one electron. The Fock-state expansion can be written schematically as $\Psi = \psi_0|e\rangle + \psi_1|e\gamma\rangle + \psi_2|e\gamma\gamma\rangle$. The eigenvalue problem for the wave functions ψ_i and the bound-state mass M becomes a coupled set of three integral equations. To construct these equations we use the Hamiltonian H_{LC} of Tang *et al.* [6] The anomalous moment is then calculated from the spin-flip matrix element of the plus component of the current. [7] The regulator is an invariant-mass cutoff $\sum_i (P^+/p_i^+)(m_i^2 + p_{\perp i}^2) \leq \Lambda^2$. Additional approximations and assumptions are a nonzero photon mass of $m_e/10$, a large coupling of $\alpha = 1/10$, and use of numerical methods based on discretized light-cone quantization (DLCQ). [4]

2 Renormalization

We renormalize the electron mass and couplings differently in each Fock sector, as a consequence of the Tamm–Dancoff truncation. [8] The bare electron mass in the one-photon sector is computed from the one-loop correction allowed by the two-photon states. We then require that the bare mass in the no-photon sector be such that $M^2 = m_e^2$ is an eigenvalue.

The three-point bare coupling e_0 is related to the physical coupling e_R by $e_0(\underline{k}_i, \underline{k}_f) = Z_1(\underline{k}_f)e_R/\sqrt{Z_{2i}(\underline{k}_i)Z_{2f}(\underline{k}_f)}$, where $\underline{k}_i = (k_i^+, \mathbf{k}_{\perp i})$ is the initial electron momentum and \underline{k}_f the final momentum. The renormalization functions $Z_1(\underline{k})$ and $Z_2(\underline{k}) = |\psi_0|^2$ are generalizations of the usual constants. The amplitude ψ_0 must be computed in a basis where only allowed particles appear.

The function Z_1 can be fixed by considering the proper part of the transition amplitude T_{fi} for photon absorption by an electron at zero photon momentum ($\underline{q} = \underline{k}_f - \underline{k}_i \rightarrow 0$): $T_{fi}^{\text{proper}} = V_{fi}/Z_1(\underline{k}_f)$, where V_{fi} is the elementary three-point vertex. The transition amplitude can be computed from $T_{fi} = \psi_0\langle\Psi|V|i\rangle$, in which $|\Psi\rangle$ is the dressed electron state and $\psi_0 = \sqrt{Z_{2f}(\underline{k}_f)}$. The proper amplitude is then obtained from $T_{fi}^{\text{proper}} = T_{fi}/(Z_{2i}Z_{2f})$, where the Z_2 's remove the disconnected dressing of the electron lines.

Thus the solution of the eigenvalue problem for only one state can be used to compute Z_1 . Full diagonalization of H_{LC} is not needed. Because Z_1 is needed in the

construction of H_{LC} , the eigenvalue problem and the renormalization conditions must be solved simultaneously.

Most four-point graphs that arise in the bound-state problem are log divergent. To any order the divergences cancel if all graphs are included, but the Tamm-Dancoff truncation spoils this. For a nonperturbative calculation we need a counterterm $\sim \lambda(p_i^+, p_f^+) \log \Lambda$ that includes infinite chains of interconnected loops. The function λ might be fit to Compton amplitudes. [9] Thus we need to be able to handle scattering processes.

3 Preliminary Results and Future Work

Some preliminary results are given in Fig. 1. In the two-photon case there remain divergences associated with four-point graphs.

The next step to be taken in this calculation is renormalization of the four-point couplings, followed by numerical verification that all logs have been removed. Construction of finite counterterms that restore symmetries will then be considered. We can also consider photon zero modes, Z graphs, and pair states.

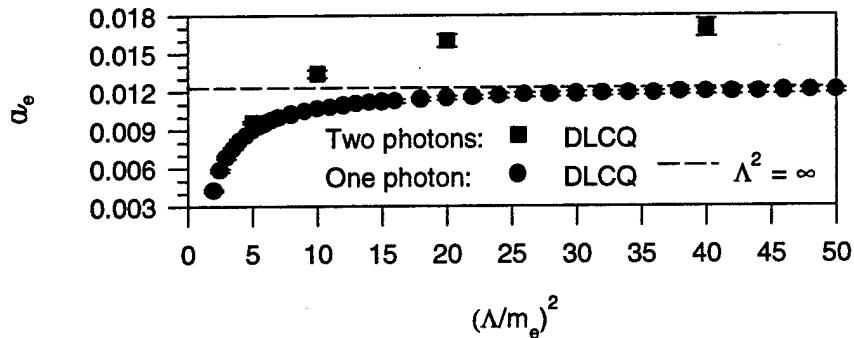


Figure 1: Electron anomalous moment as a function of the cutoff Λ^2 , extrapolated from DLCQ calculations. The photon mass is $m_e/10$, and the coupling is 1/10.

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