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Exclusive Photon-Photon Processes *

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Abstract

Exclusive $\gamma\gamma \to$ hadron pairs are among the most fundamental processes in QCD, providing a detailed examination of Compton scattering in the crossed channel. In the high momentum transfer domain ($s, t,$ large, θ_{cm} for t/s fixed), these processes can be computed from first principles in QCD, yielding important information on the nature of the QCD coupling α_s and the form of hadron distribution amplitudes. Similarly, the transition form factors $\gamma^*\gamma, \gamma^*\gamma \to \pi^0, \eta^0, \eta', \eta_c \ldots$ provide rigorous tests of QCD and definitive determinations of the meson distribution amplitudes $\phi_H(x, Q)$. We show that the assumption of a frozen coupling at low momentum transfers can explain the observed scaling of two-photon exclusive processes.

1 Introduction

Exclusive two-photon processes provide highly valuable probes of coherent effects in quantum chromodynamics. For example, in the case of exclusive final states at high momentum transfer and fixed θ_{cm} such as $\gamma\gamma \to p\overline{p}$ or meson pairs, photon-photon collisions provide a timelike microscope for testing fundamental scaling laws of PQCD and for measuring distribution amplitudes, the fundamental wavefunctions of hadrons. [1] At very high energies $s >> -t$, diffractive processes such as $\gamma\gamma \to$ neutral vector (or pseudoscalar) meson pairs with real or virtual photons can test the QCD Pomeron (or the $C = -1$ exchange Odderon) in a detailed way utilizing the simplest possible initial state. [2] In the case of low momentum transfer processes, the comparison of the two-photon decay width for a given $C = +$ resonance with its inferred two-gluon width provides an indirect discovery tool for gluonium. As discussed at this conference by H. Paar, [3] CLEO has reported a very small upper limit for the coupling $\Gamma(\gamma\gamma \to f_J^0(1220)$ due to the absence of a signal for K_sK_s decays, whereas a large $gg \to f_J^0(1220)$ coupling is inferred from Mark III and BES observations of $J/\psi \to \gamma f_J^0$ decays. Using Chanowitz's "stickiness" criteria, [4] this points to a gluonium interpretation of the f_J^0.

Traditionally, $\gamma\gamma$ data has come from the annihilation of Weisäcker–Williams effective photons emitted in $e^-e^\pm$ collisions. Data for $\gamma\gamma \to$ hadrons from $ep \to e'p'R^0$

events at HERA has also now become available. The HERA diffractive events will allow studies of photon and pomeron interference effects in hadron-induced amplitudes. As emphasized by Klein, [5] nuclear-coherent $\gamma\gamma \to$ hadrons reactions can be observed in heavy-ion collisions at RHIC or the LHC, *e.g.* $Z_1 Z_2 \to Z_1 Z_2 \pi^+ \pi^-$. Eventually $\gamma\gamma$ collisions will be studied at TeV energies with back-scattered laser beams, allowing critical probes of Standard Model and supersymmetric processes with polarized photons in exclusive channels such as Higgs production $\gamma\gamma \to W^+W^-$, and $\gamma\gamma \to W^+W^-W^+W^-$. [6]

2 Hard Exclusive Two-Photon Reactions

Exclusive two-photon processes such as $\gamma\gamma \to$ hadron pairs and the transition form factor $\gamma^*\gamma \to$ neutral mesons play a unique role in testing quantum chromodynamics because of the simplicity of the initial state. [1] At large momentum transfer the direct point-like coupling of the photon dominates at leading twist, leading to highly specific predictions which depend on the shape and normalization of the hadron distribution amplitudes $\phi_H(x_i, Q)$ the basic valence bound state wavefunctions. The most recent exclusive two-photon process data from CLEO [7] provides stringent tests of these fundamental QCD predictions.

Exclusive processes are particularly challenging to compute in QCD because of their sensitivity to the unknown non-perturbative bound state dynamics of the hadrons. However, in some important cases, the leading power-law behavior of an exclusive amplitude at large momentum transfer can be computed rigorously via a factorization theorem which separates the soft and hard dynamics. The key ingredient is the factorization of the hadronic amplitude at leading twist. As in the case of inclusive reactions, factorization theorems for exclusive processes [1, 8, 9] allow the analytic separation of the perturbatively-calculable short-distance contributions from the long-distance non-perturbative dynamics associated with hadronic binding. For example, the amplitude $\gamma\gamma \to \pi^+\pi^-$ factorizes in the form

$$\mathcal{M}_{\gamma\gamma\to\pi^+\pi^-} = \int_0^1 dx \int_0^1 dy\, \phi_\pi(x,\tilde{Q})\, T_H(x,y,\tilde{Q})\, \phi_\pi(y,\tilde{Q}) \qquad (1)$$

where $\phi_\pi(x,\tilde{Q})$ is in the pion distribution amplitude and contains all of the soft, non-perturbative dynamics of the pion $q\bar{q}$ wavefunction integrated in relative transverse

momentum up to the separation scale $k_\perp^2 < \tilde{Q}^2$, and T_H is the quark/gluon hard scattering amplitude for $\gamma\gamma \to (q\bar{q})(q\bar{q})$ where the outgoing quarks are taken collinear with their respective pion parent. To lowest order in α_s, the hard scattering amplitude is linear in α_s. The most convenient definition of the coupling is the effective charge $\alpha_V(Q^2)$, defined from the potential for the scattering of two infinitely heavy test charges, in analogy to the definition of the QED running coupling. Another possible choice is the effective charge $\alpha_R(s)$, defined from the QCD correction to the annihilation cross section: $R_{e^+e^-\to\text{hadrons}}(s) \equiv R_0(1 + \alpha_R(s)/\pi)$. One can relate α_V and α_R to $\alpha_{\overline{MS}}$ to NNLO using commensurate scale relations [10].

The contributions from non-valence Fock states and the correction from neglecting the transverse momentum in the subprocess amplitude from the non-perturbative region are higher twist, *i.e.*, power-law suppressed. The transverse momenta in the perturbative domain lead to the evolution of the distribution amplitude and to next-to-leading-order (NLO) corrections in α_s. The contribution from the endpoint regions of integration, $x \sim 1$ and $y \sim 1$, are power-law and Sudakov suppressed and thus can only contribute corrections at higher order in $1/Q$. [1]

The distribution amplitude $\phi(x, \tilde{Q})$ is boost and gauge invariant and evolves in $\ln \tilde{Q}$ through an evolution equation [1]. It can be computed from the integral over transverse momenta of the renormalized hadron valence wavefunction in the light-cone gauge at fixed light-cone time [1]:

$$\phi(x, \tilde{Q}) = \int d^2\vec{k}_\perp \, \theta\left(\tilde{Q}^2 - \frac{\vec{k}_\perp^{\,2}}{x(1-x)}\right) \psi^{(\tilde{Q})}(x, \vec{k}_\perp). \tag{2}$$

A physical amplitude must be independent of the separation scale $\tilde{Q}$. The natural variable in which to make this separation is the light-cone energy, or equivalently the invariant mass $\mathcal{M}^2 = \vec{k}_\perp^{\,2}/x(1-x)$, of the off-shell partonic system [11, 1]. Any residual dependence on the choice of $\tilde{Q}$ for the distribution amplitude will be compensated by a corresponding dependence of the NLO correction in T_H. In general, the NLO prediction for exclusive amplitude depends strongly on the form of the pion distribution amplitude as well as the choice of renormalization scale μ and scheme.

The QCD coupling is typically evaluated at quite low scales in exclusive processes since the momentum transfers has to be divided among several constituents. In the BLM procedure, the scale of the coupling is evaluated by absorbing all vacuum

polarization corrections with the scale of the coupling or by taking the experimental value integrating over the gluon virtuality. Thus, in the case of the (timelike) pion form factor the relevant scale is of order $Q^{*2} \sim e^{-3}\mathcal{M}^2_{\pi\pi^-} \cong \frac{1}{20}\mathcal{M}^2_{\pi^+\pi^-}$ assuming the asymptotic form of the pion distribution amplitude $\phi_\pi^{\rm asympt} = \sqrt{3}\, f_\pi\, x(1-x)$. At such low scales, it is likely that the coupling is frozen or relatively slow varying.

In the BLM procedure, the renormalization scales are chosen such that all vacuum polarization effects from the QCD β function are re-summed into the running couplings. The coefficients of the perturbative series are thus identical to the perturbative coefficients of the corresponding conformally invariant theory with $\beta = 0$. The BLM method has the important advantage of "pre-summing" the large and strongly divergent terms in the PQCD series which grow as $n!(\alpha_s\beta_0)^n$, *i.e.*, the infrared renormalons associated with coupling constant renormalization [12, 13]. Furthermore, the renormalization scales Q^* in the BLM method are physical in the sense that they reflect the mean virtuality of the gluon propagators [13, 14, 15, 16]. In fact, in the $\alpha_V(Q)$ scheme, where the QCD coupling is defined from the heavy quark potential, the renormalization scale is by definition the momentum transfer caused by the gluon. Because the renormalization scale is small in the exclusive $\gamma\gamma$ processes discussed here, we will argue that the effective coupling is nearly constant, thus accounting for the nominal scaling behavior of the data [17, 18].

The heavy-quark potential $V(Q^2)$ can be identified via the two-particle-irreducible scattering amplitude of test charges, *i.e.*, the scattering of an infinitely heavy quark and antiquark at momentum transfer $t = -Q^2$. The relation

$$V(Q^2) = -\frac{4\pi C_F \alpha_V(Q^2)}{Q^2}, \tag{3}$$

with $C_F = (N_C^2 - 1)/2N_C = 4/3$, then defines the effective charge $\alpha_V(Q)$. This coupling provides a physically-based alternative to the usual $\overline{MS}$ scheme. As in the corresponding case of Abelian QED, the scale Q of the coupling $\alpha_V(Q)$ is identified with the exchanged momentum. The scale-fixed relation between α_V and the conventional $\overline{MS}$ coupling is

$$\alpha_V(Q) = \alpha_{\overline{MS}}(e^{-5/6}Q)\left(1 - \frac{2C_A}{3}\frac{\alpha_{\overline{MS}}}{\pi} + \cdots\right), \tag{4}$$

above or below any quark mass threshold. The factor $e^{-5/6} \simeq 0.4346$ is the ratio of commensurate scales between the two schemes to this order. It arises because of the

conventions used in defining the modified minimal subtraction scheme. The scale in the $\overline{MS}$ scheme is thus a factor ~ 0.4 smaller than the physical scale. The coefficient $2C_A/3$ in the NLO term is a feature of the non-Abelian couplings of QCD; the same coefficient would occur even if the theory were conformally invariant with $\beta_0 = 0$. Recent lattice calculations have provided strong constraints on the normalization and shape of $\alpha_V(Q^2)$. [19] The J/ψ and Υ spectra have been used to determine the normalization:

$$\alpha_V^{(3)}(8.2 \text{ GeV}) = 0.196(3), \tag{5}$$

where the effective number of light flavors is $n_f = 3$. The corresponding modified minimal subtraction coupling evolved to the Z mass using Eq. (4) is given by

$$\alpha_{\overline{MS}}^{(5)}(M_Z) = 0.115(2). \tag{6}$$

This value is consistent with the world average of 0.117(5), but is significantly more precise. These results are valid up to NLO.

Ji, Pang, Robertson, and I [20] have recently analyzed the pion transition form factor $F^{\gamma^*\gamma} \to \pi^0$ obtained from $e\gamma \to e'\pi^0$, the timelike pion form obtained from $e^+e^- \to \pi^+\pi$, and the $\gamma\gamma \to \pi^+\pi^-$ processes, all at NLO in α_V. The assumption of a nearly constant coupling in the hard scattering amplitude at low scales provides an explanation for the phenomenological success of dimensional counting rules for exclusive processes; *i.e.*, the power-law fall-off follows the nominal scaling of the hard scattering amplitude $\mathcal{M}_{\rm had} \sim T_H \sim [p_T]^{4-n}$ where n is in the total number of incident and final fields entering T_H. The transition form factor has now been measured up to $Q^2 < 8$ GeV^2 in the tagged two-photon collisions $e\gamma \to e'\pi^0$ by the CLEO and CELLO collaborations. In this case the amplitude has the factorized form

$$F_{\gamma M}(Q^2) = \frac{4}{\sqrt{3}} \int_0^1 dx \phi_M(x, Q^2) T_{\gamma \to M}^H(x, Q^2), \tag{7}$$

where the hard scattering amplitude for $\gamma\gamma^* \to q\overline{q}$ is

$$T_{\gamma M}^H(x, Q^2) = \frac{1}{(1-x)Q^2} \left(1 + \mathcal{O}(\alpha_s)\right). \tag{8}$$

The leading QCD corrections have been computed by Braaten [21]; however, the NLO corrections are necessary to fix the BLM scale at LO. Thus it is not yet possible to rigorously determine the BLM scale for this quantity. We shall here assume that

this scale is the same as that occurring in the prediction for F_π. For the asymptotic distribution amplitude we thus predict

$$Q^2 F_{\gamma\pi}(Q^2) = 2 f_\pi \left(1 - \frac{5}{3}\frac{\alpha_V(Q^*)}{\pi}\right). \tag{9}$$

As we shall see, given the phenomenological form of α_V we employ (discussed below), this result is not terribly sensitive to the precise value of the scale.

An important prediction resulting from the factorized form of these results is that the normalization of the ratio

$$\begin{aligned} R_\pi(Q^2) &\equiv \frac{F_\pi(Q^2)}{4\pi Q^2 |F_{\pi\gamma}(Q^2)|^2} && (10)\\ &= \alpha_{\overline{MS}}(e^{-14/6}Q)\left(1 - 0.56\frac{\alpha_{\overline{MS}}}{\pi}\right) && (11)\\ &= \alpha_V(e^{-3/2}Q)\left(1 + 1.43\frac{\alpha_V}{\pi}\right) && (12)\\ &= \alpha_R(e^{5/12-2\zeta_3}Q)\left(1 - 0.65\frac{\alpha_R}{\pi}\right) && (13) \end{aligned}$$

is formally independent of the form of the pion distribution amplitude. The $\alpha_{\overline{MS}}$ correction follows from combined references [21, 22, 23]. The next-to-leading correction given here assumes the asymptotic distribution amplitude.

We emphasize that when we relate R_π to α_V we relate observable to observable and thus there is no scheme ambiguity. Furthermore, effective charges such as α_V are defined from physical observables and thus must be finite even at low momenta. A number of proposals have been suggested for the form of the QCD coupling in the low-momentum regime. For example, Petronzio and Parisi [24] have argued that the coupling must freeze at low momentum transfer in order that perturbative QCD loop integrations be well defined. Mattingly and Stevenson [25] have incorporated such behavior into their parameterizations of α_R at low scales. Gribov [26] has presented novel dynamical arguments related to the nature of confinement for a fixed coupling at low scales. Zerwas [27] has noted the heavy quark potential must saturate to a Yukawa form since the light-quark production processes will screen the linear confining potential at large distances. Cornwall [28] and others [29, 30] have argued that the gluon propagator will acquire an effective gluon mass m_g from nonperturbative dynamics, which again will regulate the form of the effective couplings

at low momentum. We shall adopt the simple parameterization

$$\alpha_V(Q) = \frac{4\pi}{\beta_0 \ln\left(\frac{Q^2+4m_g^2}{\Lambda_V^2}\right)}, \tag{14}$$

which effectively freezes the α_V effective charge to a finite value for $Q^2 \leq 4m_g^2$.

We can use the non-relativistic heavy quark lattice results [19, 31] to fix the parameters. A fit to the lattice data of the above parameterization gives $\Lambda_V = 0.16$ GeV if we use the well-known momentum-dependent n_f [32]. Furthermore, the value $m_g^2 = 0.19$ GeV^2 gives consistency with the frozen value of α_R advocated by Mattingly and Stevenson [25]. Their parameterization implies the approximate constraint $\alpha_R(Q)/\pi \simeq 0.27$ for $Q = \sqrt{s} < 0.3$ GeV, which leads to $\alpha_V(0.5 \text{ GeV}) \simeq 0.37$ using the NLO commensurate scale relation between α_V and α_R. The resulting form for α_V is shown in Fig. 1. The corresponding predictions for α_R and $\alpha_{\overline{MS}}$ using the CSRs at NLO are also shown. Note that for low Q^2 the couplings, although frozen, are large. Thus the NLO and higher-order terms in the CSRs are large, and inverting them perturbatively to NLO does not give accurate results at low scales. In addition, higher-twist contributions to α_V and α_R, which are not reflected in the CSR relating them, may be expected to be important for low Q^2 [33].

It is clear that exclusive processes such as the photon to pion transition form factors can provide a valuable window for determining the magnitude and the shape of the effective charges at quite low momentum transfers. In particular, we can check consistency with the α_V prediction from lattice gauge theory. A complimentary method for determining α_V at low momentum is to use the angular anisotropy of $e^+e^- \to Q\overline{Q}$ at the heavy quark thresholds [34]. It should be emphasized that the parameterization (14) is just an approximate form. The actual behavior of $\alpha_V(Q^2)$ at low Q^2 is one of the key uncertainties in QCD phenomenology.

As we have emphasized, exclusive processes are sensitive to the magnitude and shape of the QCD couplings at quite low momentum transfer: $Q_V^{*2} \simeq e^{-3}Q^2 \simeq Q^2/20$ and $Q_R^{*2} \simeq Q^2/50$ [35]. The fact that the data for exclusive processes such as form factors, two photon processes such as $\gamma\gamma \to \pi^+\pi^-$, and photoproduction at fixed $\theta_{c.m.}$ are consistent with the nominal scaling of the leading-twist QCD predictions (dimensional counting) at momentum transfers Q up to the order of a few GeV can be immediately understood if the effective charges α_V and α_R are slowly varying

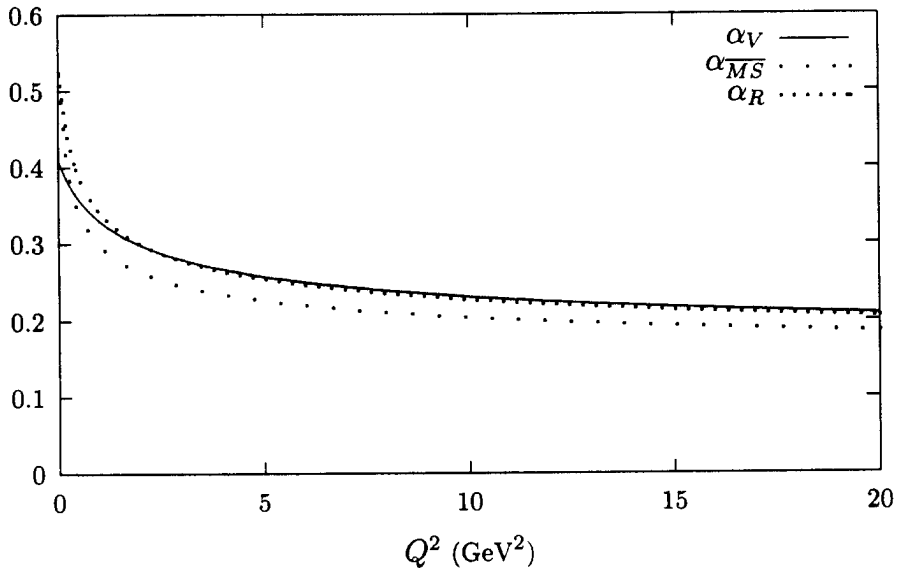

Figure 1: The coupling function $\alpha_V(Q^2)$ as given in Eq. (14). Also shown are the corresponding predictions for $\alpha_{\overline{MS}}$ and α_R following from the NLO commensurate scale relations.

at low momentum. The scaling of the exclusive amplitude then follows that of the subprocess amplitude T_H with effectively fixed coupling. Note also that the Sudakov effect of the end point region is the exponential of a double log series if the coupling is frozen, and thus is strong.

In Fig. 2, we compare the recent CLEO data [7] for the photon to pion transition form factor with the prediction

$$Q^2 F_{\gamma\pi}(Q^2) = 2 f_\pi \left(1 - \frac{5}{3}\frac{\alpha_V(e^{-3/2}Q)}{\pi}\right). \tag{15}$$

The flat scaling of the $Q^2 F_{\gamma\pi}(Q^2)$ data from $Q^2 = 2$ to $Q^2 = 8$ GeV^2 provides an important confirmation of the applicability of leading twist QCD to this process. The magnitude of $Q^2 F_{\gamma\pi}(Q^2)$ is remarkably consistent with the predicted form, assuming the asymptotic distribution amplitude and including the LO QCD radiative correction with $\alpha_V(e^{-3/2}Q)/\pi \simeq 0.12$. Radyushkin [36], Ong [37] and Kroll [38] have also noted that the scaling and normalization of the photon-to-pion transition form factor tends

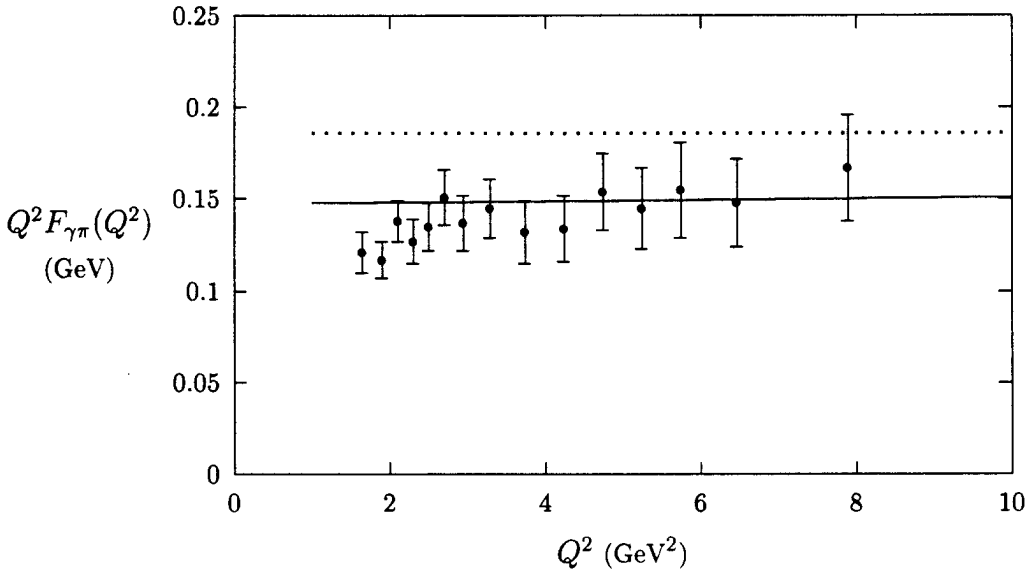

Figure 2: The $\gamma \to \pi^0$ transition form factor. The solid line is the full prediction including the QCD correction [Eq. (15)]; the dotted line is the LO prediction $Q^2F_{\gamma\pi}(Q^2) = 2f_\pi$.

to favor the asymptotic form for the pion distribution amplitude and rules out broader distributions such as the two-humped form suggested by QCD sum rules [39]. One cannot obtain a unique solution for the non-perturbative wavefunction from the $F_{\pi\gamma}$ data alone. However, we have the constraint that

$$\frac{1}{3}\langle\frac{1}{1-x}\rangle\left[1-\frac{5}{3}\frac{\alpha_V(Q^*)}{\pi}\right] \simeq 0.8 \tag{16}$$

(assuming the renormalization scale we have chosen in Eq. (9) is approximately correct). Thus one could allow for some broadening of the distribution amplitude with a corresponding increase in the value of α_V at low scales.

We have also analyzed the $\gamma\gamma \to \pi^+\pi^-, K^+K^-$ data. These data exhibit true leading-twist scaling (Fig. 3), so that one would expect this process to be a good test of theory. One can show that to LO

$$\frac{\frac{d\sigma}{dt}(\gamma\gamma \to \pi^+\pi^-)}{\frac{d\sigma}{dt}(\gamma\gamma \to \mu^+\mu^-)} = \frac{4|F_\pi(s)|^2}{1-\cos^4\theta_{c.m.}} \tag{17}$$

in the CMS, where $dt = (s/2)d(\cos\theta_{c.m.})$ and here $F_\pi(s)$ is the *time-like* pion form factor. The ratio of the time-like to space-like pion form factor for the asymptotic distribution amplitude is given by

$$\frac{|F_\pi^{(\text{timelike})}(-Q^2)|}{F_\pi^{(\text{spacelike})}(Q^2)} = \frac{|\alpha_V(-Q^{*2})|}{\alpha_V(Q^{*2})}. \tag{18}$$

If we simply continue Eq. (14) to negative values of Q^2 then for $1 < Q^2 < 10$ GeV^2, and hence $0.05 < Q^{*2} < 0.5$ GeV^2, the ratio of couplings in Eq. (18) is of order 1.5. Of course this assumes the analytic application of Eq. (14). Thus if we assume the asymptotic form for the distribution amplitude, then we predict $F_\pi^{(\text{timelike})}(-Q^2) \simeq (0.3\ \text{GeV}^2)/Q^2$ and hence

$$\frac{\frac{d\sigma}{dt}(\gamma\gamma \to \pi^+\pi^-)}{\frac{d\sigma}{dt}(\gamma\gamma \to \mu^+\mu^-)} \simeq \frac{.36}{s^2}\frac{1}{1-\cos^4\theta_{c.m.}}. \tag{19}$$

The resulting prediction for the combined cross section $\sigma(\gamma\gamma \to \pi^+\pi^-, K^+K^-)$[†] is shown in Fig. 3, along with CLEO data [7]. Considering the possible contribution of the resonance $f_2(1270)$, the agreement is reasonable.

We also note that the normalization of α_V could be larger at low momentum than our estimate. This would also imply a broadening of the pion distribution amplitude compared to its asymptotic form since one needs to raise the expectation value of $1/(1-x)$ in order to maintain consistency with the magnitude of the $F_{\gamma\pi}(Q^2)$ data. A full analysis will then also require consideration of the breaking of scaling from the evolution of the distribution amplitude. In any case, we find no compelling argument for significant higher-twist contributions in the few GeV regime from the hard scattering amplitude or the endpoint regions, since such corrections violate the observed scaling behavior of the data.

The analysis we have presented here suggests a systematic program for estimating exclusive amplitudes in QCD (including exclusive B-decays) which involve hard scattering. The central input is $\alpha_V(0)$, or

$$\overline{\alpha_V} = \frac{1}{Q_0^2}\int_0^{Q_0^2} dQ'^2 \alpha_V(Q'^2), \quad Q_0^2 \leq 1\ \text{GeV}^2, \tag{20}$$

[†]The contribution from kaons is obtained at this order simply by rescaling the prediction for pions by a factor $(f_K/f_\pi)^4 \simeq 2.2$.

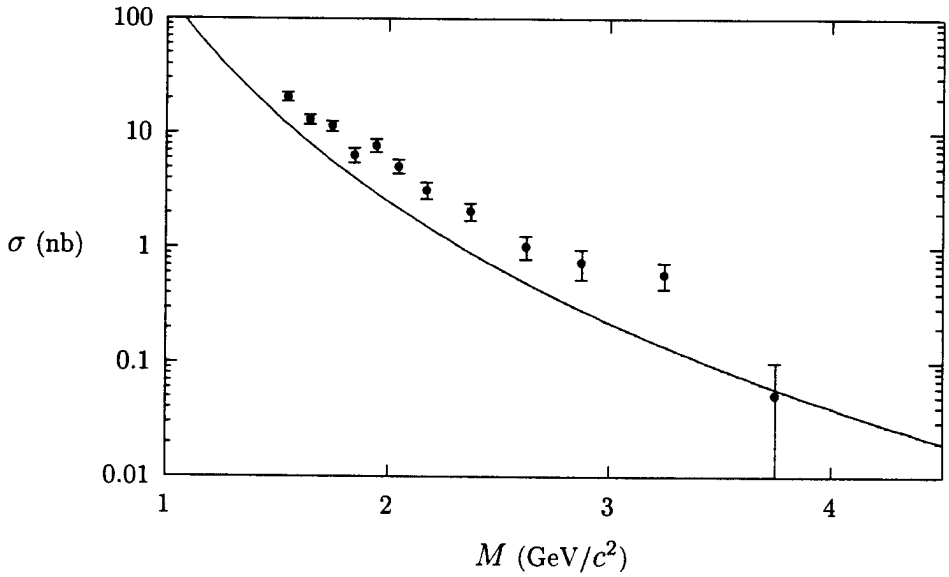

Figure 3: Two-photon annihilation cross section $\sigma(\gamma\gamma \to \pi^+\pi^-, K^+K^-)$ as a function of CMS energy, for $|\cos\theta^*| < 0.6$.

which largely controls the magnitude of the underlying quark-gluon subprocesses for hard processes in the few-GeV region. In this work, the mean coupling value for $Q_0^2 \simeq 0.5$ GeV^2 is $\overline{\alpha_V} \simeq 0.38$. The main focus will then be to determine the shapes and normalization of the process-independent meson and baryon distribution amplitudes.

3 Conclusions

The leading-twist scaling of the observed cross sections for exclusive two-photon processes and other fixed θ_{cm} reactions can be understood if the effective coupling $\alpha_V(Q^*)$ is approximately constant in the domain of Q^* relevant to the underlying hard scattering amplitudes. In addition, the Sudakov suppression of the long-distance contributions is strengthened if the coupling is frozen because of the exponentiation of a double log series. We have also found that the commensurate scale relation connecting the heavy quark potential, as determined from lattice gauge theory, to the photon-to-pion transition form factor is in excellent agreement with $\gamma e \to \pi^0 e$ data assuming that

the pion distribution amplitude is close to its asymptotic form $\sqrt{3} f_\pi x(1-x)$. We also reproduce the scaling and approximate normalization of the $\gamma\gamma \to \pi^+\pi^-, K^+K^-$ data at large momentum transfer. However, the normalization of the space-like pion form factor $F_\pi(Q^2)$ obtained from electroproduction experiments is somewhat higher than that predicted by the corresponding commensurate scale relation. This discrepancy may be due to systematic errors introduced by the extrapolation of the $\gamma^* p \to \pi^+ n$ electroproduction data to the pion pole.

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References

[1] S. J. Brodsky and G. P. Lepage, *Phys. Rev. Lett.* **53**, 545 (1979); *Phys. Lett.* **87B**, 359 (1979); G. P. Lepage and S. J. Brodsky, *Phys. Rev.* **D22**, 2157 (1980).

[2] I.F. Ginzburg, D.Yu. Ivanov, and V.G. Serbo these proceedings, and e-Print hep-ph/9508309.

[3] H. Paar, these proceedings.

[4] M. Chanowitz, *Nucl. Instrum. Meth.* **A355** 42, (1995), e-Print hep-ph/9407231, and references therein.

[5] S. Klein, e-Print, nucl-th/9707008 and these proceedings.

[6] S. J. Brodsky and P. M. Zerwas, *Nucl. Instrum. Meth.* **A355** 19, (1995), e-Print hep-ph/9407362, and references therein. Se also G. Jikia, these proceedings.

[7] J. Dominick, *et al.*, Phys. Rev. **D50**, 3027 (1994). See also, J. Gronberg *et al.* CLNS-97-1477, (1997), e-Print hep-ex/9707031 and V. Savinov, these proceedings.

[8] A. V. Efremov and A. V. Radyushkin, *Theor. Math. Phys.* **42**, 97 (1980).

[9] S. J. Brodsky and G. P. Lepage, in *Perturbative Quantum Chromodynamics*, A. H. Mueller, Ed. (World Scientific, 1989).

[10] H. J. Lu and S. J. Brodsky, *Phys. Rev.* **D48**, 3310 (1993).

[11] C.-R. Ji, A. Pang, and A. Szczepaniak, *Phys. Rev.* **D52**, 4038 (1995).

[12] A. H. Mueller, *Nucl. Phys.* **B250**, 327 (1985).

[13] P. Ball, M. Beneke and V. M. Braun, *Phys. Rev.* **D52**, 3929 (1995).

[14] S. J. Brodsky, G. P. Lepage, and P. B. Mackenzie, *Phys. Rev.* **D28**, 228 (1983).

[15] G. P. Lepage and P. B. Mackenzie, *Phys. Rev.* **D48**, 2250 (1993).

[16] M. Neubert, *Phys. Rev.* **D51**, 5924 (1995); *Phys. Rev. Lett.* **76**, 3061 (1996).

[17] C.-R. Ji, A. Sill and R. Lombard-Nelsen, *Phys. Rev.* **D36**, 165 (1987).

[18] C.-R. Ji and F. Amiri, *Phys. Rev.* **D42**, 3764 (1990).

[19] C. T. H. Davies *et. al.*, *Phys. Rev.* **D52**, 6519 (1995).

[20] S. J. Brodsky, C.-R. Ji, A. Pang, and D. Robertson, SLAC-PUB-7473, (1997).

[21] E. Braaten and S.-M. Tse, *Phys. Rev.* **D35**, 2255 (1987).

[22] F. M. Dittes and A. V. Radyushkin, *Sov. J. Nucl. Phys.* **34**, 293 (1981); *Phys. Lett.* **134B**, 359 (1984).

[23] R. D. Field, R. Gupta, S. Otto and L. Chang, *Nucl. Phys.* **B186**, 429 (1981).

[24] G. Parisi and R. Petronzio, *Phys. Lett.* **95B**, 51 (1980).

[25] A. C. Mattingly and P. M. Stevenson, *Phys. Rev.* **D49**, 437 (1994).

[26] V. N. Gribov, Lund Report No. LU-TP 91-7, 1991 (unpublished).

[27] K. D. Born, E. Laermann, R. Sommer, P. M. Zerwas, and T. F. Walsh, *Phys. Lett.* **329B**, 325 (1994).

[28] J. M. Cornwall, *Phys. Rev.* **D26**, 1453 (1982).

[29] A. Donnachie and P. V. Landshoff, *Nucl. Phys.* **B311**, 509 (1989).

[30] M. Gay Ducati, F. Halzen and A. A. Natale, *Phys. Rev.* **D48**, 2324 (1993).

[31] A. X. El-Khadra, G. Hockney, A. Kronfeld and P. B. Mackenzie, *Phys. Rev. Lett.* **69**, 729 (1992).

[32] D. V. Shirkov and S. V. Mikhailov, *Z. Phys.* **C63**, 463 (1994).

[33] V. M. Braun, "QCD renormalons and higher twist effects," **hep-ph/9505317**.

[34] S. J. Brodsky, A. H. Hoang, J. H. Kuhn and T. Teubner, *Phys. Lett.* **359B**, 355 (1995).

[35] N. Isgur and C. H. Lewellyn-Smith, *Phys. Rev. Lett.* **52**, 1080 (1984); *Phys. Lett.* **217B**, 535 (1989); *Nucl. Phys.* **B317**, 526 (1989).

[36] A. V. Radyushkin, *Acta Phys. Polon.* **B26**, 2067 (1995).

[37] S. Ong, *Phys. Rev.* **D52**, 3111 (1995).

[38] P. Kroll and M. Raulfs, *Phys. Lett.* **387B**, 848 (1996).

[39] V. L. Chernyak and A. R. Zhitnitsky, *Phys. Rep.* **112**, 173 (1984).

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