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THE STUDY OF EFFECTS OF SMALL PERTURBATIONS ON CHAOTIC SYSTEMS

Progress Report

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INTRODUCTION

The following report summarizes our research activities under the U.S. Department of energy, Scientific Computing Staff Office of Energy Research, Contract No. DE-FG05-87ER25036. We have organized the report under the following categories:

I. Work done in the period January 1990 through December 1990.

A. Summary of the research done.

B. Papers published in refereed journals or refereed proceedings.

II. Planned work for the period January 1991 through December 1991.

III. Appended reprints.

I. WORK DONE IN THE PERIOD JANUARY 1990 THROUGH DECEMBER 1990

A. Summary of the Research Done.

1. "Controlling Chaos"

The control of chaos is a major component in our DOE funded research program. In this work we consider both that chaos is common in nonlinear dynamical systems and that, in practical applications, one is often interested in periodic and predictable behavior. Thus, we address the following question: Given a chaotic attractor, how can one obtain improved performance and a desired attracting-periodic motion without eventually changing the system? The key observation to answer this question is to realize that a chaotic attractor typically has embedded within it an infinite number of periodic orbits. Our method then converts a motion on a chaotic attractor to a desired, accordingly to some performance criteria, attracting time-periodic motion by making only small time-dependent perturbations in an accessible system parameter. By using delay coordinate embedding, the method is applicable to experimental situations in which the dynamical equations are not known. Other issues also investigated include the time required to achieve control, the effect of imperfect system identification, and the effect of noise.

2. "Shadowing and Noise Reduction"

For chaotic processes, neighboring trajectories diverge exponentially from each other. Thus, numerical trajectories, with their inherent round-off error, diverge from the true trajectories. We are faced then with the following central question when interpreting numerical results: For a physical system which exhibits chaos, in what sense does a numerical study reflect the true dynamics of the actual system? We have devised a rigorous procedure to prove whether there exists a true trajectory which stays near or shadows the noisy trajectory for a long time. Our technique to prove shadowing involves a combination of containment and refinement. The refinement part of the proof is being used by other groups (Los Alamos national laboratory and Naval Surface Warfare Center) as a technique for noise reduction. However, to apply this noise reduction technique one needs to know the equations that generate the noisy data explicitly. To bypass this limitation, we developed a novel method for noise reduction in chaotic experimental data where no knowledge of the dynamical equations is needed.

3. "Chaotic Scattering"

Open Hamiltonian systems (i.e., one for which the energy surface is unbounded) can have chaotic orbits for initial conditions on a set of Lebesgue measure zero. The chaotic behavior is caused by the presence of a fractal invariant set of unstable bounded orbits. In our work we considered a novel type of bifurcation to chaos in which the chaotic invariant set arises abruptly as the particle energy decreases from above a critical value to below. We also predicted analytically the fractal dimension of the invariant set near this abrupt bifurcation. We have also studied chaotic scattering in higher dimensional systems. In particular, we considered the scatterer to consist of four reflecting spheres at the vertices of a regular tetrahedron.

4. "Random Maps"

We introduced the "snapshot" technique to study the qualitative and quantitative properties of attractors of random maps. By a random map we mean that the parameters that occur in the map vary randomly according to some probability distribution. By a snapshot attractor we mean the measure resulting from many iterations of a cloud of initial conditions viewed at a single instant (i.e., iteration). In our work we investigated the multifractal properties of these snapshot attractors. This work was applied to problems in the convection of particles by chaotic fluid flows. In particular, we studied the conditions under which fractal concentration of particles (aerosols and bubbles) in configuration (physical) space is possible.

5. "Magnetic Dynamo"

This problem was motivated by the desire to explain the observed prevalence of magnetic fields in nature (e.g., planets, stars and galaxies). We argue that the dynamo problem can be reduced to one in the chaotic dynamics of fluid element trajectories.

6. "AIDS Transmission"

We presented a set of models that show that essential knowledge is missing for predicting how quickly the HIV infection will spread through the heterosexual population. A variety of models are consistent with the data available from monogamous relationships. However, these models make strikingly different predictions for non-monogamous relationships. The highest plausible rate of ransmission (in non-monogamous situations) is obtained with the bipolar risk model, where each sexually active seropositive male infects 18% of his partners and each seropositve female infects 8% of her partners.

B. Papers Published in Refereed Journals or Refereed Proceedings.

1. "Controlling Chaos", E. Ott, C. Grebogi, and J. A. Yorke, Phys. Rev. Lett. **64**, 1196-1199(1990).
2. "Controlling Chaotic Dynamical Systems", E. Ott, C. Grebogi, and J. A. Yorke, in *CHAOS : Soviet – American Perspectives in Nonlinear Science*, ed. D. K. Campbell (American Institute of Physics, New York, 1990), pp. 153-172.
3. "Shadowing of Physical Trajectories in Chaotic Dynamics: Containment and Refinement", C. Grebogi, S. M. Hammel, J. A. Yorke, and T. Sauer, Phys. Rev. Lett. **65**, 1527-1530(1990).
4. "Noise Reduction: Finding the Simplest Dynamical System Consistent with the Data", E. J. Kostelich and J. A. Yorke, Physica **41D**, 183-196(1990).
5. "Bifurcation to Chaotic Scattering", S. Bleher, C. Grebogi, and E. Ott, Physica **46D**, 87-121(1990).
6. "Chaotic Scattering in Several Dimensions", Q. Chen, M. Ding, and E. Ott, Phys. Lett. **145A**, 93-100(1990).
7. "Multifractal Properties of Snapshot Attractors of Random Maps", F. J. Romeiras, C. Grebogi, and E. Ott, Phys. Rev. A **41**, 784-799(1990).
8. "Fractal Structure in Physical Space in the Dispersal of Particles in Fluids", L. Yu, C. Grebogi, and E. Ott, in *Nonlinear Structure in Physical Systems*, Ed. L. Lam and H. C. Morris (Springer-Verlag, New York, 1990), pp. 223-231.
9. "Cross Sections of Chaotic Attractors", Q. Chen and E. Ott, Phys. Lett. **147A**, 450-454(1990).

10. "The Fast Kinematic Magnetic Dynamo and the Dissipationless Limit", J. M. Finn and E. Ott, Phys. Fluids B **2**, 916-926(1990).

11. "Chaotic Flows and Fast Magnetic Dynamos", E. Ott and J. M. Finn, Comm. Plasma Phys. **3**, 113-117(1990).

12. "Modeling Non-monogamous Heterosexual Transmission of AIDS", I. Kramer, J. A. Yorke, and E. D. Yorke, Mathl. Comput. Modelling **13**, 99-107(1990).

III. APPENDED REPRINTS

(removed and cycled separately) –

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