

SLAC-PUB-7277  
hep-ph/9609215  
August 1996

CONF-960765--

## POWER CORRECTIONS, RENORMALONS AND RESUMMATION <sup>a</sup>


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I briefly review three topics of recent interest concerning power corrections, renormalons and Sudakov resummation: (a)  $1/Q$  corrections to event shape observables in  $e^+e^-$  annihilation, (b) power corrections in Drell-Yan production and (c) factorial divergences that arise in resummation of large infrared (Sudakov) logarithms in moment or 'real' space.

*To appear in the Proceedings of the  
28th International Conference  
on High Energy Physics,  
Warsaw, Poland, 25-31 July 1996*

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<sup>a</sup>Research supported by the Department of Energy under contract DE-AC03-76SF00515.

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# POWER CORRECTIONS, RENORMALONS AND RESUMMATION

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I briefly review three topics of recent interest concerning power corrections, renormalons and Sudakov resummation: (a)  $1/Q$  corrections to event shape observables in  $e^+e^-$  annihilation, (b) power corrections in Drell-Yan production and (c) factorial divergences that arise in resummation of large infrared (Sudakov) logarithms in moment or 'real' space.

## 1 Utilizing renormalons

Perturbative QCD relies on factorization. By this one implies that an observable that depends on at least one hard scale  $Q$  can be expanded in powers and logarithms of  $\Lambda/Q$ , where  $\Lambda$  is the intrinsic QCD scale. At least up to some order in  $1/Q$ , one must also be able to factor a 'short-distance' part from long-distance contributions, which are independent of the details of the hard process. At leading order in  $1/Q$ , the long-distance contribution can be absent, like in  $e^+e^-$  annihilation, or a product of parton distributions like for inclusive quantities in hadron-hadron collisions. Beyond leading order, little is known about power corrections, with exceptions like deep inelastic scattering. For event shapes or effects referred to as 'hadronization', it is not known how to express power corrections in terms of operators and asymptotic states.

The renormalon approach to power corrections uses the fact that the leading term in the power expansion already indicates the existence of power corrections, because the perturbative expansion of its short-distance coefficient diverges. This 'renormalon' divergence occurs, because certain higher-order diagrams (the simplest being 'bubble graphs') contain many powers of logarithms of a loop momentum, which make these diagrams sensitive to large distances.<sup>a</sup> Summing a divergent series requires a prescription. The prescription-dependence is best captured by the ambiguity of the Borel integral and takes the form  $(\Lambda/Q)^a \ln^b Q/\Lambda$ , where infrared (IR) renormalons yield positive integers for  $a$ . One can interpret the ambiguity as an ambiguity in separating long- and short-distances, much as the fac-

torization scale dependence in separating coefficient functions and parton distributions. Since the physical observable is unambiguous, the ambiguity in defining perturbation theory must be matched by power corrections and this determines their  $Q$ -dependence, but not their magnitude, just as the evolution of parton distributions is perturbatively calculable, but not their initial values.

The advantage of the method is that it is entirely perturbative, although to all orders. To some extent, the language inherited from studies of large-order perturbation theory is an unnecessary complication, since the set of diagrams that leads to a divergent series really only probes the IR sensitive regions of low-order skeleton-like graphs. It would be desirable to classify these regions systematically by extending standard methods of perturbative factorization<sup>1</sup> that identify logarithmic IR divergences to subleading, power-like IR sensitivity. Meanwhile, most calculations are done in the formal  $N_f \rightarrow \infty$ -limit, which selects diagrams with one chain of fermion loops at leading order. Provided the phase space of a cut fermion loop is integrated unweighted, the structure of power corrections inferred from renormalons in this approximation is equivalent to calculating the low-order diagrams with finite gluon mass  $\lambda$  and interpreting non-analytic terms in the small- $\lambda^2$  expansion as power corrections.<sup>2</sup>

The method has its limitations, precisely because it is purely perturbative. To go beyond classifying the expected power corrections for each observable separately, one needs additional assumptions, which do not follow from perturbative considerations alone, such as universality of power corrections, to relate different observables. And, of course, the IR sensitivity of Feynman diagrams might not exhaust all possibilities for power corrections.

<sup>a</sup>The logarithms also enhance the sensitivity to distances much shorter than  $1/Q$ . The corresponding ultraviolet renormalons will not be discussed here.

In the past two years, these ideas have been applied to observables that do not admit an operator product expansion, such as event shapes in  $e^+e^-$  annihilation, jet observables and the Drell-Yan cross section. In these cases, IR renormalons provide genuinely new information about power corrections. This talk gives a somewhat qualitative overview and emphasizes the outstanding issues.

## 2 Power corrections to event shape variables

Event shapes are constructed from IR safe weights on hadronic final states in  $e^+e^-$  collisions. Thrust, for example, is defined as  $T = \max_{\vec{n}} (\sum_i |\vec{p}_i \cdot \vec{n}|) / \sum_i |\vec{p}_i|$ . The theoretical prediction is computed in terms of parton momenta, while hadron momenta are measured. Matching partons and hadrons is dealt with as a hadronization correction, which is obtained from Monte Carlo programs and accounted for in determinations of  $\alpha_s$  from event shapes. The fragmentation models built into Monte Carlo programs lead to hadronization corrections that vary like  $1/Q$  with the cms energy  $Q$ .

Theoretically one considers hadronization as a soft parton phenomenon that takes over from the parton shower at a certain typical hadronic scale  $\mu_h$ . This separation scale is not uniquely fixed and the boundary between perturbation theory and hadronization is vague. Thus, probing the boundary of perturbation theory with renormalons may tell us more about hadronization. Event shape variables have been computed to this end both with a finite mass gluon in the lowest order gluon emission diagram<sup>3,4,5</sup> and in the approximation of a single chain of fermion loops<sup>6</sup>, in which case the region of small invariant mass of the  $q\bar{q}$  pair in a cut fermion loop is the important one. The two methods are not equivalent in this case, because the invariant mass distribution depends on how each particular event shape weights the  $q\bar{q}$  phase space.

The calculation of the average  $\langle 1 - T \rangle$  with finite gluon mass leads to a  $1/Q$ -correction

$$\langle 1 - T \rangle_{1/Q} = K \cdot \frac{\lambda}{Q}. \quad (1)$$

The  $1/Q$ -correction comes only from the two-jet region  $T \rightarrow 1$ , when the gluon momentum be-

comes very small. Multiple gluon emission diagrams modify the constant  $K$ , but only if all gluons are soft, so that again  $T \rightarrow 1$ . Consequently, if the two-jet region is excluded from the average over  $T$ , we expect a smaller hadronization correction,

$$\frac{\langle 1 - T \rangle_{1/Q, 0.5 < T < 0.8}}{\langle 1 - T \rangle_{1/Q, \text{all } T}} \propto \alpha_s(Q). \quad (2)$$

The same conclusion applies to the heavy jet mass average  $\langle M_h^2 \rangle$ . The DELPHI collaboration has re-analyzed<sup>7</sup> the energy dependence of event shapes by adding  $1/Q$  and  $1/Q^2$  terms to the next-to-leading order perturbative expression (evaluated at scale  $\mu = Q$ ) and by fitting the coefficients of the power corrections to the data. No hadronization correction from Monte Carlo programs is applied. Some of the results are reproduced in Tab. 1 and agree qualitatively with the above predictions. The energy-energy correlation (EEC) is predicted<sup>6</sup> to have  $1/Q$ -corrections at all angles, because the soft gluon region contributes at all angles. It is also easy to see that the three-jet rates  $R_3$  computed from the JADE clustering algorithm have  $1/Q$ -corrections, while the Durham algorithm should have only  $1/Q^2$ -corrections, because it weights the region of soft gluons quadratically with their energy rather than linearly. The DELPHI analysis does not have enough data points to test this prediction for the Durham algorithm.

The examination of expected power corrections provides some guidance to selecting 'good' event shapes, the good ones being those less sensitive to hadronization. To go further, one has to make the stronger assumption that hadronization corrections in the two-jet limit are universal<sup>4,5</sup>. This implies that although the constants  $K$  above are not calculable for any observable, their ratio for different observables is calculable, because multiple soft parton emission modifies  $K$  in a universal way. Thus, fitting a  $1/Q$ -correction to one observable would determine the hadronization parameter once and for ever. The assumption of universality could be justified diagrammatically if an event shape variable did not resolve the soft parton kinematics, which in fact it does.<sup>6</sup> For example, in the two-jet region  $1 - T \approx (M_h^2/Q^2) + (M_l^2/Q^2)$ , where  $M_l$  is the light jet mass. If we now consider the diagram where a single emitted gluon splits into a  $q\bar{q}$  pair, we find  $1 - T = M_h^2/Q^2$  if both quarks are emitted into the same hemisphere, and

Table 1: Fits to the  $Q$ -dependence of event shape variables.  $\alpha_s(M_Z)$ , the coefficient of a  $1/Q$ -term,  $C_1$ , and  $1/Q^2$ -term,  $C_2$  (not quoted) are fitted to obtain the second entry for each observable. For the first entry  $C_2$  is fixed to zero.

Observable	$C_1/\text{GeV}$	$\alpha_s(M_Z)$
$\langle 1 - T \rangle$	$0.82 \pm 0.07$ $0.83 \pm 0.20$	$0.123 \pm 0.002$ $0.122 \pm 0.004$
$\int_{0.2}^{0.5} dT (1 - T)$	$0.37 \pm 0.05$ $0.20 \pm 0.05$	$0.121 \pm 0.008$ $0.134 \pm 0.003$
$\langle M_h^2/E_{vis}^2 \rangle$	$0.54 \pm 0.08$ $0.75 \pm 0.26$	$0.121 \pm 0.002$ $0.116 \pm 0.006$
$\int_{0.1}^{0.4} dM_h (M_h^2/E_{vis}^2)$	— $-0.01 \pm 0.03$	— $0.123 \pm 0.000$
$\langle M_d^2/E_{vis}^2 \rangle$	$0.19 \pm 0.04$ $0.10 \pm 0.05$	$0.094 \pm 0.003$ $0.097 \pm 0.003$
$\int_{-0.5}^{0.5} d \cos \Theta EEC$	$1.68 \pm 0.05$ $0.27 \pm 0.23$	$0.115 \pm 0.002$ $0.137 \pm 0.004$
$R_3^J(y_{cut} = 0.08)$	$0.44 \pm 0.15$ $-3.59 \pm 0.55$	$0.107 \pm 0.001$ $0.123 \pm 0.002$
$R_3^D(y_{cut} = 0.04)$	$-0.67 \pm 0.49$ $-2.53 \pm 3.15$	$0.126 \pm 0.004$ $0.137 \pm 0.019$

$1 - T = 2M_h^2/Q^2$  if they are emitted into opposite hemispheres. Thus, there is no unique relation between  $1 - T$  and  $M_h^2$ , even if both quarks are soft. Universality could still hold in an approximate sense, if, as advocated in Ref. <sup>4</sup>, the strong coupling approaches a finite and not too large value in the infrared. In this case, the diagram just discussed is higher order in the IR coupling. In this scenario, the  $1/Q$ -correction to  $M_l^2$  should be smaller than for  $M_h^2$ , because it arises only at second order. The small power correction for the average  $M_d^2 = M_h^2 - M_l^2$  does not support this picture, although the small fit value for  $\alpha_s(M_Z)$  indicates that the corresponding  $C_1$  in Tab. 1 might not be too reliable.

Eventually, universality should be subjected to experimental tests. In this respect, it would be interesting to obtain the coefficients  $C_1$  in Tab. 1 with  $\alpha_s$  fixed to a unique value. As a matter of principle, the power corrections obtained by renormalon methods are synonymous with large perturbative corrections in higher orders. If large coefficients are a practical concern, the divergent piece of the series should be separated and discarded,

so that it is absorbed into the power correction, leaving an unambiguous perturbative series. A procedure of this sort has been proposed in <sup>4</sup> and applied with some success to  $\langle 1 - T \rangle$ , the average  $C$ -parameter and  $\sigma_L$ . Another question of importance for testing universality is to what extent the power corrections in Tab. 1 effectively parameterize higher order corrections in perturbation theory, which would in principle be calculable, leaving a rather small ‘true’ hadronization correction. In Ref. <sup>8</sup> it was argued that higher order corrections, summed up to the point where the series diverges, can well mimic the shape of a  $1/Q$ -correction. An equivalent effect is obtained, if one expresses the second order perturbative prediction in terms of  $\alpha_s(Q^*)$  with  $Q^* \sim 0.1Q$ . Such a low scale is not unnatural for event shape observables, since they are dominated by the soft-collinear region, where the scale is set by the transverse momentum of the emitted gluon rather than  $Q$ . From this point of view, the question of whether universality holds is less important than determining the higher-order perturbative corrections or correct scale for each event shape.

In principle, the universality assumption could also be invoked to relate two non-IR safe event shapes to each other. This would circumvent the difficulty of having to extract subdominant power corrections to test universality.

### 3 Drell-Yan production and Sudakov resummation

Drell-Yan (DY) production, apart from its phenomenological significance, is theoretically interesting, because one can kinematically realize the situation of a process with two hard scales. In the following, we consider the partonic DY cross section  $\hat{\sigma}_{DY}$  (after collinear subtractions) in the region  $Q \gg Q(1 - z) \gg \Lambda$ , where  $z = Q^2/\hat{s}$ ,  $Q^2$  being the mass of the DY pair and  $\hat{s}$  the partonic cms energy. The second scale  $Q(1 - z)$  can be identified with the energy available to parton emission into the final state. Since  $Q(1 - z) \ll Q$ , these partons are referred to as soft, although they are not soft in terms of the QCD scale  $\Lambda$ . Taking moments in  $z$  (roughly, this replaces  $1/(1 - z)$  by  $N$ ), one obtains two powers of  $\ln N$  for each power of  $\alpha_s \equiv \alpha_s(Q)$ , so that the actual expansion parameter of the hard cross section is  $\alpha_s \ln^2 N$ . Thus, in higher orders, one has two sources of large cor-

rections, Sudakov logarithms, related to the scale  $Q(1-z)$  and renormalon factorials, related to the scale  $\Lambda$ . One may ask how this complication affects the arguments that lead to the identification of power corrections through renormalons.

This question has been addressed in Refs. <sup>9,10,11</sup>. Starting from

$$\ln \hat{\sigma}_{DY}(N) = \frac{2C_F}{\pi} \int_0^1 dz \frac{z^N - 1}{1-z} \int_{Q^2(1-z)}^{Q^2(1-z)^2} \frac{dk_\perp^2}{k_\perp^2} \alpha_s(k_\perp), \quad (3)$$

which resums all leading logs  $\ln N (\alpha_s \ln N)^k$  in the DIS scheme, one finds <sup>9,10</sup> that the integral has a renormalon ambiguity of order  $N\Lambda/Q$  from the region of large  $z$ . However, the corresponding  $n!$  occurs in far subleading logarithms, beyond the accuracy to which (3) was derived. Keeping all subleading logarithms essentially implies that the hard cross section is evaluated exactly. In Ref. <sup>11</sup> this has been done in the approximation of a single chain, interpreted as an approximation to the logarithm of the cross section. Using the equivalence of this approximation with taking an explicit IR cut-off, we choose, for illustration, a lower cut-off  $\mu$  on the emitted gluon energy. Omitting all terms that can not give rise to a  $\mu/Q$ -correction (which allows us to ignore virtual corrections and collinear subtractions), one obtains instead of the right hand side of (3)

$$\frac{2C_F\alpha_s}{\pi} \int_0^1 dz z^{N-1} \int_{\mu^2}^{Q^2(1-z)^2/4} \frac{dk_\perp^2}{k_\perp^2} \frac{1}{\sqrt{(1-z)^2 - 4k_\perp^2/Q^2}} \quad (4)$$

which reduces to the structure of (3) in the double-logarithmic, soft-collinear limit, if  $k_\perp$  is set to zero in the square root. In this limit, consistent with the previous result, the integral contains a  $\mu/Q$ -term in the expansion in small  $\mu$ . However, the  $k_\perp$ -term is crucial and can not be neglected precisely in the region  $z \rightarrow 1$ , where the  $\mu/Q$ -term originates from. Keeping  $k_\perp$  in the square root, one finds that  $1/Q$ -corrections are absent and that the leading power correction is of order  $(N\mu/Q)^2$ . The cancelation of the  $1/Q$ -term emphasizes that leading power corrections stem from soft gluons, but small angle (collinear) and large angle ( $k_\perp \sim k_0 \sim Q(1-z)$ ) emission are both important. <sup>11</sup> This might appear surprising, because large angle soft emission is usually con-

sidered suppressed, leading to angular ordering in parton cascades. But only logarithmic enhancements of matrix elements cancel at large angles and no conclusion follows for power corrections.

Returning to resummation of Sudakov logarithms, we conclude that there is no direct connection between resummation of  $\ln N$  terms and power corrections, which are 'buried' among an infinite number of subleading logs. Roughly speaking, this is so, because  $Q \gg Q(1-z) \gg \Lambda$  and Sudakov resummation is concerned only with the first inequality, while power corrections are associated with the smallest scale  $\Lambda$ . Nevertheless, one would like to formulate the resummation procedure in such a way that it does not introduce stronger power corrections than required. Given the hierarchy of scales, it is useful to think about the problem from the effective field theory point of view, which deals with scales sequentially. Thus, one would first 'integrate out' momenta larger than  $Q(1-z)$  and deal with Sudakov resummation in this step. At that lower scale, we can consider power corrections, which then appear as the ratio  $\Lambda/(Q(1-z))$  ( $N\Lambda/Q$  in moment space).

The effective fields for fast quarks interacting with soft gluons are expressed as eikonal or Wilson lines. When  $N \gg 1$ , the hard Drell-Yan cross section (omitting for simplicity collinear subtractions) factorizes <sup>12,13</sup> as  $\hat{\sigma}_{DY}(N, Q) = H_{DY}(Q, \mu) S_{DY}(Q/N, \mu)$ , where the scales  $Q$  and  $Q/N$  are separated. The 'soft part'  $S$  satisfies a renormalization group equation in  $\mu$  that can be used to sum logarithms in  $N$ , because  $S$  depends only on the single dimensionless ratio  $Q/(N\mu)$ . The solution to the RGE is expressed as

$$\begin{aligned} \hat{\sigma}_{DY} &= H_{DY}(Q) \cdot S_{DY}(1, \alpha_s(Q/N)) \cdot \\ &\exp \left( \int_{Q^2/N^2}^{Q^2} \frac{dk_t^2}{k_t^2} \left[ \Gamma_{eik}(\alpha_s(k_t)) \ln \frac{k_t^2 N^2}{Q^2} \right. \right. \\ &\quad \left. \left. + \Gamma_{DY}(\alpha_s(k_t)) \right] \right). \end{aligned} \quad (5)$$

The three factors on the r.h.s. correspond to coefficient function, matrix element and anomalous dimension factor in the effective field theory language. The analysis of Ref. <sup>11</sup> shows that in the  $\overline{\text{MS}}$  scheme the universal eikonal anomalous dimension and DY specific anomalous dimension  $\Gamma_{DY}$  are analytic functions for small  $\alpha_s$ , so that

the exponential factor does not contain any renormalons (power corrections) at all. These enter only through the boundary conditions at the lower and higher scale and turn out to be  $(N\Lambda/Q)^2$  as stated before. Notice also that when  $N > Q/\Lambda$ , the exponent becomes ill-defined. For such large moments, the language of power corrections loses its meaning ( $N\Lambda/Q \sim 1$ ), the energy of soft gluons becomes of order  $\Lambda$  and any short-distance expansion fails.

Whether the absence of  $1/Q$ -corrections for Drell-Yan production persists beyond the approximation of a single chain (one-gluon emission), is an unsolved problem. In Ref. <sup>14</sup>, the cancelation at leading order has been reproduced as a consequence of the KLN and Low theorems and it has been argued to hold at the level of two-gluon emission as well. On the other hand, in the language of Wilson lines, emphasized in Refs. <sup>10,15</sup>, a  $1/Q$ -correction could be naturally accommodated by a certain operator constructed from Wilson lines, although it vanishes at leading order. If, as suspected in Ref. <sup>15</sup>, Glauber gluons constitute a new potential source of  $1/Q$ -corrections at higher orders, the validity of the eikonal approximation and Wilson line treatment to power-like accuracy would have to be re-examined. In principle, the possibility exists <sup>11</sup> that the cancelation of  $1/Q$  terms occurs between (5) and terms dropped in (5), although there is no indication of it at leading order. Since, as explained above, the problem of resummation is disconnected from the problem of power corrections, one should be able to establish equivalence of the renormalon analysis for the DY cross section with the analysis of power corrections in terms of multi-parton correlation functions.<sup>16</sup> The present discussion indicates that the optimal language for problems with two scales might yet have to be found.

#### 4 The Sudakon: $x$ -space vs. $N$ -space resummation

As pointed out in Ref. <sup>17</sup>, factorial divergence, not related to renormalons, can appear when one converts resummed distributions in moment space ( $N$ -space) back to 'real' space ( $x$ -space; note that  $x$  replaces  $z$  in the previous section). Typically, a

physical quantity is given as an integral

$$\sigma(\tau) = \int_0^1 dx W(\tau/x) \hat{\sigma}(x), \quad (6)$$

where  $\hat{\sigma}$  could be the partonic Drell-Yan cross section – in which case  $W$  is the parton luminosity and  $\tau = Q^2/s$  – or the thrust distribution, or the lepton energy distribution in semileptonic  $B \rightarrow X_u l \nu$  decay, for example. Let  $x$  be a generic variable, such that  $x \rightarrow 1$  corresponds to the soft gluon region. If the weight function  $W$  constrains  $x$  to the region of large  $x$ , but such that  $1-x$  is still large compared to  $\Lambda/Q$ , where  $Q$  is the generic hard scale, the perturbative expansion of  $\sigma(\tau)$  contains the usual renormalon factorials and the corresponding poles in the Borel transform.

Consider now the double logarithmic approximation with fixed coupling  $\alpha_s$ , in which all  $(\alpha_s \ln^2 N)^k$  have been resummed in moment space. It is perfectly consistent to perform the inverse Mellin transformation to  $x$ -space in the same approximation. Then  $\hat{\sigma}(x) \approx \exp(c\alpha_s \ln^2(1-x))$  with some constant  $c$ . Expanding  $\sigma(\tau)$  in  $\alpha_s$ , one finds the divergent series

$$\begin{aligned} & \int_0^1 dx W(\tau/x) \exp(c\alpha_s \ln^2(1-x)) \\ & \sim \sum_n F_n(\tau) (4c)^n n! \alpha_s^{n+1} \end{aligned} \quad (7)$$

independent of  $W$ , as long as it does not depend exponentially on  $x$ . In practical cases  $c$  is such that this series diverges much faster than expected from renormalons. Continuing the tradition of misnomers, I call the corresponding singularity in the Borel plane a Sudakon pole. It is unphysical and appears, because in the process of resummation, one has dropped terms, which after integration over  $x$  give equally large contributions and cancel the singularity.

On physical grounds, one expects Sudakov suppression, so  $c$  is negative. Then the integral in (7) can just be done and there is no reason to re-expand it in  $\alpha_s$ . Even in this case, however, resummation has modified the analytic structure of the Borel transform, although the spurious pole is Borel summable. While  $c$  is indeed negative for the thrust distribution or lepton energy distribution in  $B$  decay, mentioned above, it is in fact positive

for Drell-Yan production (and other hadronic collisions) in the conventional subtraction schemes, because the subtracted product of partonic structure functions shows stronger Sudakov suppression than the partonic Drell-Yan cross section. In this case, the integral in (7) does not exist and must be defined by truncating the divergent expansion at its minimal term. If, as usual, one interprets the size of the minimal term as an uncertainty in defining the integral, one finds that this uncertainty is of order  $(\Lambda/Q)^{\beta_0/4c}$ . As  $\beta_0/(4c)$  can be much smaller than one<sup>17</sup>, this uncertainty is large. Alternatively, the integral can be defined by excluding the region of large  $x$ . Truncation of the series is equivalent to a cut-off in  $x$  that corresponds to excluding gluons with energy larger than  $\Lambda \cdot (Q/\Lambda)^{1-\beta_0/(2c)}$  from a perturbative treatment. This cut grows with  $Q$  and is much larger than the expected limit of order  $\Lambda$  for perturbative gluons. In fact, the energy cut corresponds to the position of the Sudakov peak of the  $x$ -distribution, if  $c$  were negative.

Although the sketched procedure is consistent from the point of view of summing logarithms in  $1-x$ , it is desirable to formulate resummation for integrated quantities as in (6) such that factorial behaviour in their expansions is consistent with the expected renormalon structure. In Ref.<sup>17</sup> it is proposed to perform the inverse Mellin transform of the double-log resummed  $\hat{\sigma}(N)$  back to  $x$ -space exactly, keeping all subleading logarithms. In this procedure, resummation by itself does not introduce any factorial behaviour, as suggested also by the discussion of Sect. 3. Exact evaluation of the inverse Mellin transformation keeps exactly those subleading logarithms, which are necessary to cancel the unphysical Sudakov pole. This, together with the discussion of  $1/Q$ -corrections before, emphasizes that often it is not sufficient to perform resummations that are consistent to a certain logarithmic accuracy. Different treatments of subleading logarithms can result in numerically important different constant terms. Renormalon considerations can help to decide whether such terms are spurious or not.

### Acknowledgments

I am grateful to Volodya Braun for many discussions related to the subject of this article and Arthur Hebecker for comments on the text. This

work is supported by the Department of Energy under contract DE-AC03-76SF00515.

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M97009095



Report Number (14) SLAC-PUB--7277  
CONF-960765--  
HEP-PH--9609215

Publ. Date (11) 199608  
Sponsor Code (18) DOE/ER, XF  
UC Category (19) UC-414, DOE/ER

DOE