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MASTER

THE USE OF RUN STATISTICS TO VALIDATE TENSILE TESTS*

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ABSTRACT

In tensile testing of irradiated graphites, it is difficult to assure alignment of sample and train for tensile measurements. By recording location of fractures, run (sequential) statistics can readily detect lack of randomness. The technique is based on partitioning binomial distributions.

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THE USE OF RUN STATISTICS TO VALIDATE TENSILE TESTS*

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Tensile testing of brittle materials is known to be extremely sensitive to sample and apparatus alignment, and this sensitivity is most easily detected by fracture location. Normally, strain-instrumented samples are periodically run to check the apparatus, but in the case of small samples fully passive instrumentation is difficult if not impossible to impose on the sample. In the testing of irradiated graphite these problems become extremely severe, and hence a test was sought to detect misalignments through other means.

The number of fractures occurring in a given region of the gauge section is governed by binomial statistics, and the fluctuations from the most probable value are broad and continue to broaden as the number of specimens increases. Hence, direct statistics on fracture location is not a sensitive test. On the other hand, run statistics is extremely sensitive, and the power of this test is not only large but relatively stable as the number of tests decreases. Hence, it forms an ideal real-time technique for following the randomness of fracture location.

A run is defined as a serial occurrence of the "same" event to the exclusion of all other events, and the run terminates as soon as any other event occurs. Here, the calculation of run probabilities involves rather elaborate partitioning of binomial distributions, a problem first solved by Olmstead.¹ We are interested in two cases: the probability Pr_1 that a run of length r will occur from N measurements on one preselected side of a median, and the probability $Pr_{1,1}$ that

such a run will occur on either side of the median. To calculate the latter, we will need also to calculate the probability Pr_2 that the same length run will occur on both sides of the median.

The first result (one preselected side) is

$$Pr_1(r; N, m) = \frac{1}{\binom{N}{n}} \sum_{j=1}^{[k]} (-1)^{j+1} \binom{n+1}{j} \binom{N-jr}{n}$$

where

$$[k] = \text{largest integer } \leq n/r,$$

$$n = N/2, N \text{ even.}$$

The results are given in Table 1 and plotted in Fig. 1 for various sample sizes. We note the probable run length is small and increases slowly with N , both desirable for a real-time data analysis.

To calculate $Pr_{1,1}$ we must first obtain Pr_2 . This is given by

$$Pr_2(r; N, m) = \frac{1}{\binom{N}{n}} \sum_{j=1}^{n-r+1} [B_r(n, s) + B_r(n, s+1)]^2,$$

where

$$B_r(n, s) = \sum_{j=1}^{[k]} (-1)^{j+1} \binom{s}{j} \binom{(n-1) - j(r-1)}{s-1},$$

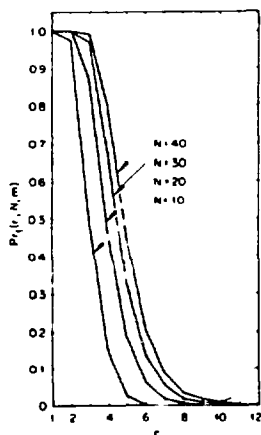


Fig. 1. Probability of a Run of Length r or Longer on a Preselected Side of the Median.

Table 1. Probability of a Run of Length r or Longer for a Sample of Size N

| r | N | One Preselected Side of Median | | | | Either Side of Median | | | |
|-----|-----|--------------------------------|--------|--------|--------|-----------------------|--------|--------|--------|
| | | 10 | 20 | 30 | 40 | 10 | 20 | 30 | 40 |
| 1 | | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 2 | | 0.9762 | 0.9999 | 1.0000 | 1.0000 | 0.9921 | 1.0000 | 1.0000 | 1.0000 |
| 3 | | 0.5000 | 0.8697 | 0.9680 | 0.9922 | 0.6667 | 0.9556 | 0.9947 | 0.9993 |
| 4 | | 0.1429 | 0.4571 | 0.6679 | 0.7988 | 0.2302 | 0.6401 | 0.8387 | 0.9298 |
| 5 | | 0.0238 | 0.1785 | 0.3252 | 0.4495 | 0.0397 | 0.2934 | 0.4997 | 0.6498 |
| 6 | | | 0.0596 | 0.1362 | 0.2073 | | 0.1063 | 0.2319 | 0.3465 |
| 7 | | | 0.0170 | 0.0356 | 0.0870 | | 0.0316 | 0.0930 | 0.1575 |
| 8 | | | 0.0040 | 0.0176 | 0.0344 | | 0.0074 | 0.0334 | 0.0650 |
| 9 | | | 0.0006 | 0.0056 | 0.0129 | | 0.0012 | 0.0108 | 0.0250 |
| 10 | | | 0.0001 | 0.0016 | 0.0046 | | 0.0001 | 0.0031 | 0.0090 |
| 11 | | | | 0.0004 | 0.0015 | | | 0.0008 | 0.0030 |
| 12 | | | | 0.0001 | 0.0005 | | | 0.0002 | 0.0009 |
| 13 | | | | | 0.0001 | | | | 0.0003 |
| 14 | | | | | | | | | 0.0001 |

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and

$[k]$ = largest interger equal to or less than k_1 or k_2 :

$$k_1 = \frac{n-s}{r-1}, k_2 = s;$$

with the proviso

$$B_r(n,0) = 0, B_r(n,n-r+2) = 0.$$

We can now obtain $Pr_{1,1}$ by taking out the redundant counts in Pr_1 :

$$Pr_{1,1} = 2Pr_1 - Pr_2.$$

We also tabulate $Pr_{1,1}$ in Table 1 and again note that the probabilities die away rapidly, although not as quickly as for Pr_1 .

In principle the test now consists of randomizing the samples in both the locations from the source and the position in the test apparatus, tracking the position of fracture, and questioning the results if a run of unlikely length (say $Pr < 0.2$) occurs as the tests proceed. We divide the gage length into four equal cells (a, b, c, d), testing (a+b) against (c+d) for skewness of fracture and (a+d) against (b+c) for kurtosis. In the skewness test, the median becomes the center of the specimen; in the kurtosis test, the median becomes the demarcation planes between the two ends and the (two-celled) center. No ties are permitted; each fracture must be assigned to a cell since a tie leaves the question of a run continuing or terminating unanswerable.

Of the various sets of data examined by the author, three have particular pertinence here. The first set of data² was taken at Southern Research Institute (SoRI) on a high-quality aerospace graphite utilizing their high-temperature air-bearing apparatus and longitudinally machined specimens. Hence the experimental techniques are unimpeachable. Conversely, the specimens were not randomized - serial sets of four from each billet were employed - and hence do not fully meet the requirements of our test. The totality of 284 measurements was partitioned sequentially into seven sets of 10, 20, 30, and 40 specimens, and the results for kurtosis (runs with preferential fracture at ends or center) are given in the first part of Table 2. Runs with a probability of occurrence of less than 20% under Pr_1 have been circled. The persistence of end-kurtosis through data set 6 and, to a lesser extent, set 5 is typical: every set of data examined has shown this tendency. This implies either that our rather arbitrarily chosen 20% probability is overly severe or that the slight stress concentration (~2%) due to tapering of the specimen from grip region to gage region is detectable. We retain our 20% criterion, preferring to err on the side of conservatism.

The tests illustrated in the center and bottom portions of Table 2 were designed to compare small irradiated and/or oxidized graphite specimens with no taper and glued-on ends against standard ASTM cylindrical dogbone-type specimens and represent the type of experiment³

for which this statistical procedure was devised. The run statistics are comparable to the SoRI data analyzed above. The specimens were fully randomized in the General Atomic (GA) tests. The first 12 specimens of H451 graphite measured gave a kurtosis run (at ends) of 9. This was traced to misalignment of the glued-on ends. The data given in Table 2 represent results after this fault was corrected. As with the SoRI data, the GA data continues to show a tendency toward end-kurtosis.

Lying behind these data are the actual occurrence of runs less than or equal to the longest. Occurrence of such secondary long runs was most pronounced for the nuclear graphite cylindrical samples and least pronounced for the SoRI data. Repartitioning the binomial distributions to get the probabilities of less than maximum runs would be an almost impossibly tedious task.

We emphasize this statistical run test is a validation test only in a positive case. Runs longer than "probably" can always occur as a fluctuation, and our arbitrary probability limit of 20% appears overly severe. Hence in the negative case (the occurrence of improbably long runs) the test serves as an alert to possible error, not a definitive finding of error.

References

1. Olmstead, P. S., *Bell System Technical Journal*, 37 p. 55 (1958). The equations given by Olmstead contain typographical errors, but his numerical results are correct.
2. Starrett, H. S., Southern Research Institute. Private communication.
3. Beavan, L. A., General Atomic Company. Private communication.

Table 2. Observed Lengths for One-Sided Runs at Several Sampling Levels

| Sampling Level N | Central (Kurtosis) | | | | Ends (Kurtosis) | | | |
|--|--------------------|----|----|----|-----------------|----|----|----|
| | 10 | 20 | 30 | 40 | 10 | 20 | 30 | 40 |
| Set 1 | 4 | 4 | ④ | 4 | 1 | 1 | 5 | 5 |
| 2 | 1 | 3 | ⑥ | 6 | ④ | 4 | 4 | 4 |
| 3 | 2 | 2 | 2 | 4 | ④ | 4 | 4 | 4 |
| 4 | 2 | 2 | 2 | 4 | 3 | 3 | 4 | 6 |
| 5 | 2 | ⑧ | ⑧ | ⑧ | ⑥ | ⑥ | ⑥ | 6 |
| 6 | 1 | 3 | 3 | 4 | ⑧ | ⑧ | ⑧ | ⑧ |
| 7 | 3 | 3 | 3 | 3 | 3 | 3 | 4 | 4 |
| Tested by SoRI. Aerospace graphite. | | | | | | | | |
| Cylinder ^a | 2 | 2 | 2 | 2 | ⑤ | ⑤ | ⑦ | ⑦ |
| ASTM ^b | 2 | 2 | 2 | 6 | 3 | 3 | 3 | 5 |
| Cylinder ^c | ④ | ⑥ | ⑥ | 6 | ④ | ⑥ | ⑥ | 6 |
| ASTM ^c | 1 | 4 | 4 | 4 | 3 | 3 | 3 | ⑦ |
| Tested by General Atomic. Nuclear grade PGX. | | | | | | | | |
| Cylinder ^a | 2 | 2 | 3 | 3 | 3 | 3 | 5 | 5 |
| ASTM ^b | 2 | 2 | 4 | 4 | ④ | 4 | 4 | 4 |
| Cylinder ^c | ④ | 4 | 4 | 4 | ④ | ⑤ | 5 | 5 |
| ASTM ^c | ④ | 4 | 4 | 4 | 3 | 3 | ⑥ | 6 |

Tested by General Atomic. Nuclear grade H451.

^a1/2-in. diam ^b3/8-in. diam ^c1/4-in. diam