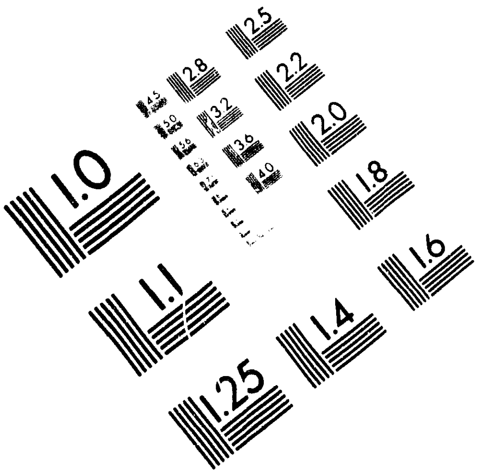

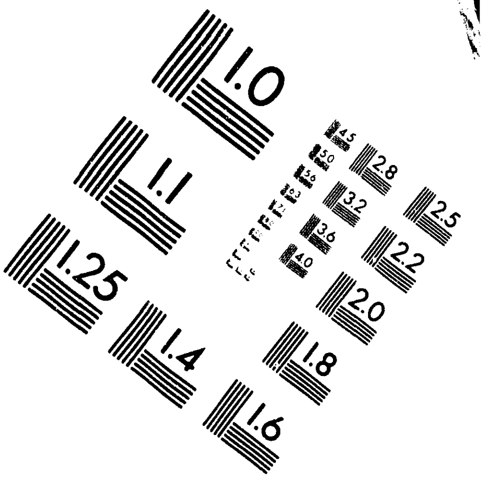

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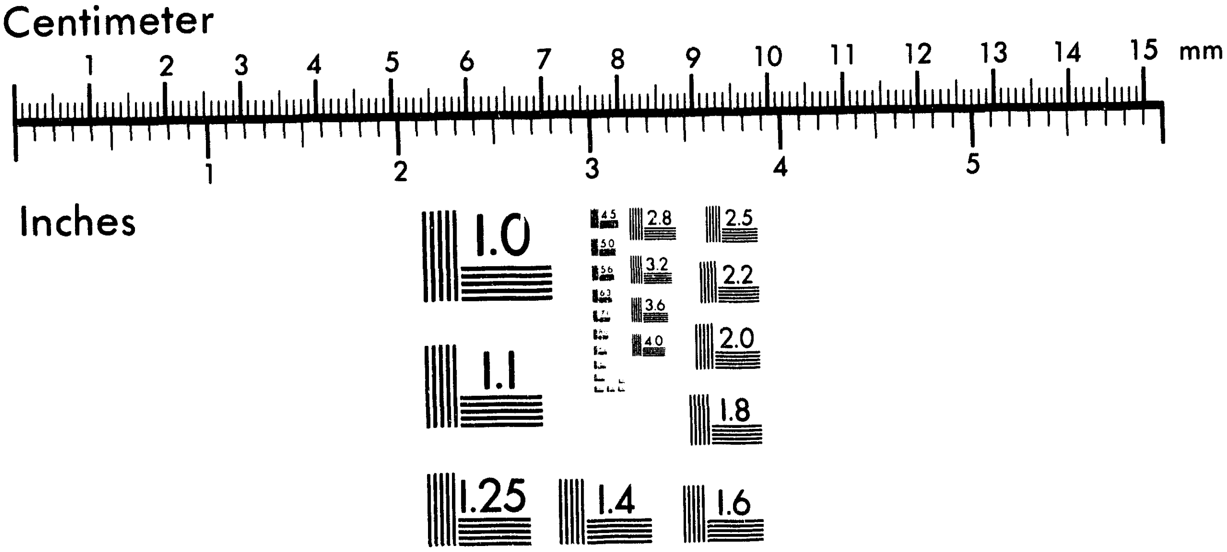

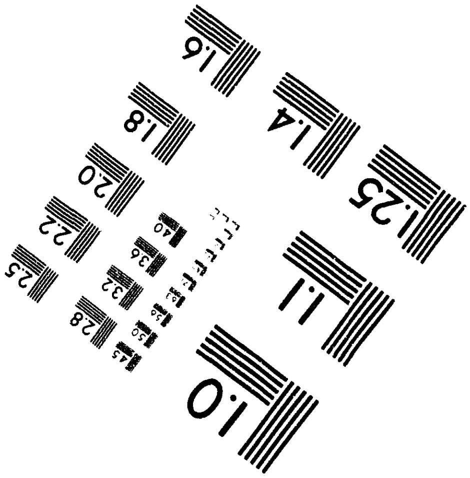

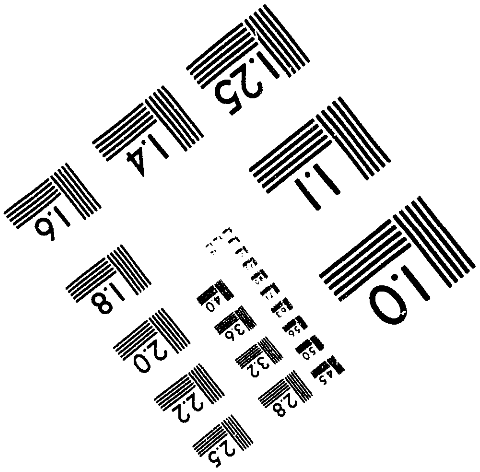

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Fin-Efficiency Calculation for Condensation in the Presence of Noncondensable Gases

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ABSTRACT

Calculation of the fin efficiency is difficult for condensation in the presence of noncondensable gases due to the spatial variation of the interfacial temperature. An analysis was carried out to develop a simplified method to calculate the fin efficiency for condensation of a vapor in the presence of noncondensable gases. The analysis includes the variation in the interfacial temperature along the fin surface. Appropriate assumptions are made to simplify the coupled heat-conduction equation in the fin and the heat/mass fluxes at the interface. The resulting expression for the fin efficiency includes mass-flux parameters, and it is similar to the common expression used for single-phase flow.

INTRODUCTION

Plate-fin heat exchangers are being considered for many condenser applications. They are commonly used for the gas-separation process (Lenfestey, 1971), because they can provide a high thermal performance to obtain a low mean-temperature difference, essential for the gas-separation process. Haseler (1980) discussed condensation of nitrogen in a plate-fin heat exchanger. Bell (1991) provided an overview of possible use of plate-fin heat exchangers in the process industries. Plate-fin heat exchangers are also considered for the heat-pump system using nonazeotropic refrigerant mixtures (Mitsuda et al., 1980; Shotani, 1980). Properly designed plate-fin heat exchangers would reduce the adverse effects of the mass-transfer resistance by keeping the liquid and vapor phases in equilibrium. The brazed plate-fin condenser was considered to be a leading candidate for the Ocean Thermal Energy Conversion (OTEC) system, where high-performance heat exchangers are essential for maintaining a low mean-temperature difference. Yung et al. (1983) and Panchal (1984) analyzed the experimental data for condensation of ammonia in serrated- and straight-fin heat exchangers, respectively. In a design study by Panchal and Rabas (1991), a plate-fin condenser with a multi-water-pass configuration was designed for condensation of steam at about 1.4 kPa pressure with high concentration of air for an

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Open-Cycle OTEC plant. Maximizing the air to steam ratio at the outlet with minimum pressure drop was the key design feature in this study. Plate-fin heat exchangers are considered for absorption of ammonia in the aqueous solution for the absorption heat-pump system. Machielsen (1990) found that the thermal performance, measured in terms of the rate of absorption at a given mean temperature difference, for the plate-fin absorber was two to three time that for a conventional shell-and-tube unit. Gopin (1976) tested a plate-fin heat exchanger for condensation of refrigerants.

The plate-fin configuration is particularly suited for condensation of multicomponent vapor mixtures and absorption with a large heat of mixing. In such cases, maintaining the two phases together is quite important to minimize the adverse effects of nonequilibrium conditions, and possible temperature pinch in the unit. The plate-fin configuration can provide flow passages that keep two phases together. In addition, an appropriately designed finned surface can enhance the process of simultaneous heat and mass transfer. Panchal and Rabas (1991) used increasing fin density along the condensation path (see Figure 1) for condensation of low pressure (1.4 kPa) steam in the presence of air. The fin density and geometry can be designed to gain the maximum possible benefit of the finned surface without a significant increase in the pressure drop, which is an important design issue for vacuum condensers.

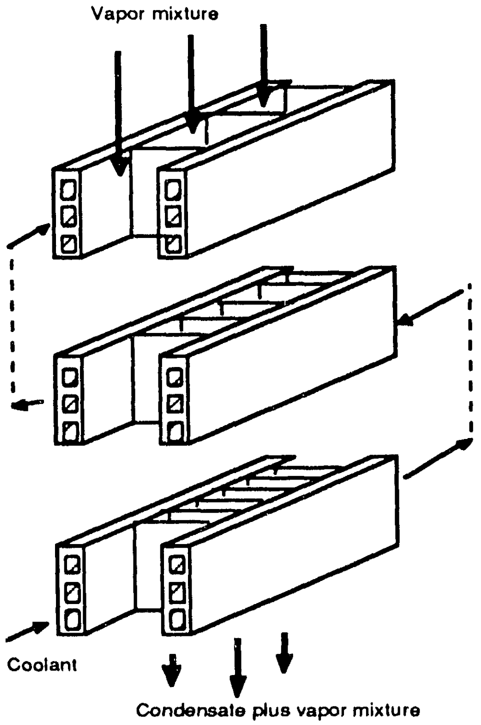

Figure 1. Elemental section of condenser showing increasing fin density along condensation path.

For applications described above, it is necessary to determine the thermal performance of the fin with simultaneous heat and mass transfer. Expressions have been derived for single-phase heat transfer for various fin geometries and prevailing conditions. Recently, Shah (1991) reviewed fin-efficiency calculations for various sets of conditions including varying heat-transfer coefficient, thermal conductivity, and local fluid temperature. The fin efficiency was calculated by fitting a quadratic equation to the temperature or heat-transfer coefficient profiles along the fin height. Such a method cannot be applied to calculate the fin efficiency for condensation in the presence of noncondensable gases, because the thermal conduction in the finned surface and the local heat and mass transfer are coupled.

In the present study, an analysis was carried out to develop a simplified method for calculating the fin efficiency for condensation of vapor in the presence of noncondensable gases. The calculation of the fin efficiency is difficult for condensation in the presence of noncondensable gases due to the spatial variation of the interfacial temperature. A detailed numerical technique can be developed; however, such a method will require significant computing time, because the fin efficiency is a part of the integration method used for calculating the local heat and mass fluxes for an elemental section of the condenser. Appropriate assumptions are made to simply the coupled heat conduction equation in the fin and heat/mass fluxes at the interface. The derived expression should be helpful for calculating the fin efficiency, thereby determining an optimum fin geometry for a given set of local conditions.

DERIVATION OF FIN EFFICIENCY

A schematic diagram of a rectangular fin with condensation film is shown in Figure 2. Assuming one dimensional heat conduction in the fin, a heat balance across an incremental section, dx, yields the following:

$$-(k_f t_f)\frac{d^2T}{dx^2} = 2h_c(T_i - T) = 2[H_g(T_g - T_i) + N(h^v_b - h^l_i)] \tag{1}$$

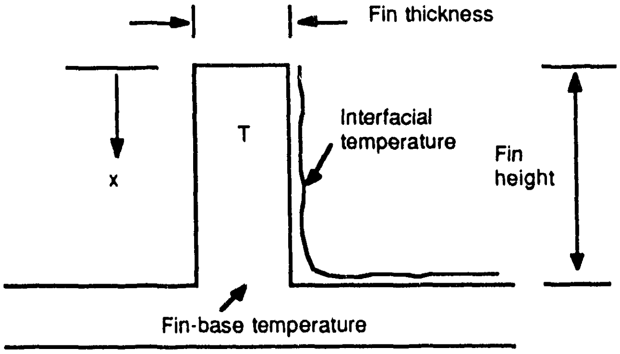

Figure 2. Schematic diagram showing fin geometry.

Equation 1 can be solved numerically along the fin surface with the appropriate boundary conditions. In the present study, Eqn. 1 is simplified by incorporating the following assumptions:

1. Heat- and mass-transfer coefficients, including high mass-flux corrections, are constant along the fin surface,
2. Condensate film coefficient is relatively high, i.e. T_i is equal to T along the fin surface.

These two assumptions are valid for cases where the mass transfer is the controlling resistance. The first assumption is reasonable for most conditions. The second assumption is justified, because h_c is generally high as compared with the vapor-side mass-transfer resistance from the vapor bulk to the interface. Applying these two assumptions, Eqn. 1 can be written as follows:

$$-(k_f t_f)\frac{d^2T}{dx^2} = 2[H_g(T_g - T_i) + N(h^v_b - h^l_i)] \tag{2}$$

The molar flux, N, can be expressed as follows:

$$N = k_g\, ln[(1 - y_i)/(1 - y_b)] = K_g(y_b - y_i) \tag{3}$$

An expression will now be developed to relate the interfacial composition, y_i, to the fin-metal temperature. The interfacial composition is related to the vapor partial pressure, or $y_i = P_i/P_{total}$, where P_i is the vapor pressure at temperature T_i, and P_{total} is the total pressure. It is further assumed that the vapor pressure can be linearized over the small temperature range prevailing over the fin surface. As a result, $P_i = C\ T_i$, with the second assumption above, or $T_i = T$. These assumptions yield the following expression:

$$y_i = \ P_i/P_{total} = C\ T/P_{total} = C'\ T \tag{4}$$

Inserting Eqns.. 3 and 4 into 2 yields the following:

$$-(k_f\ t_f)\ \frac{d^2T}{dx^2} = 2\ [H_g\,(T_g - T_i)\ +\ K_g\,(h^v{}_b\ -\ h^l{}_i)\,(y_b - CT)] \tag{5}$$

This differential equation will now be solved with the boundary conditions cf zero heat flux at $x = 0$ and $T = T_0$ at the fin base. The first boundary condition is the symmetry condition at the tip of the fin. Next, dimensionless variables are defined as follows:

$\Theta = (T\ -\ T_c)/(T_0\ -\ T_c)$, and $\eta = x/(b/2)$

where T_c and T_0 are the coolant and fin-base temperatures, respectively. Substitution of these dimensionless variables into Eqn. 5 and rearrangement of terms yields

$$\frac{d^2\Theta}{dh^2} - \phi^2\,\Theta = -\phi^2\,(y_b/C'\ -\ T_c)/(T_0\ -\ T_c) \tag{6}$$

which is subject to the following boundary conditions:

$$\Theta = 1 \text{ at } \eta = 1, \text{and } d\Theta/dh\ = 0 \text{ at } \eta = 0 \tag{7}$$

where:

$$\phi^2\ = \{2\,b^2\,[H_g + K_g\ C'\,(h^v{}_b\ -\ h^l{}_i)]\}/(k_f\ t_f) \tag{8}$$

An assumption is made that ϕ is constant along the fin height and T_g can be approximated to y_b/C', which is true for saturated bulk conditions. It is suggested that for relatively high concentration of gases ($y_b > 0.8$), replace y_b/C' with T_g. Note that ϕ and C' are calculated at each point by using local conditions. The solution to Eqn. 6, with the above boundary conditions and ϕ assumed to be constant, yields temperature distribution along the fin height:

$$\Theta = (T_0 - y_b/C')/(T_0 - T_c)\ \cosh(\phi\,\eta)/\cosh(\phi) + (y_b/C' - T_c)/(T_0\ -\ T_c) \tag{9}$$

At $x = b$, a heat-flux balance gives the following:

$$-\ k_f\,dT/dx\ = U\,(T_0\ -\ T_c) \tag{10}$$

where U is the overall heat-transfer coefficient between the fin base ($x = b/2$) and the coolant. In dimensionless form, Eqn. 10 becomes

$$d\Theta/d\eta\ = -\,U\,b/k_f \tag{11}$$

Differentiating Eqn. 9 and substituting this result into Eqn. 11 yields an expression that can be used to solve for temperature at the fin base, T_0.

$$(T_0 - y_b/C')/(T_0 - T_c)\ \phi \tanh(\phi) = - U\ b/k_f \tag{12}$$

Because ϕ contains interfacial conditions to be calculated at the fin-base temperature, an iterative approach is needed to calculate T_0 with Eqn. 12. Once the fin-base temperature is determined for specific bulk conditions, all the necessary parameters can be evaluated, and the heat and mass fluxes can be determined. The fin efficiency is calculated by taking the ratio of heat flux on the finned surface to that for the primary-wall surface.

An additional, but simplified, analysis for the fin efficiency can be derived. The basis of this simplified analysis is that $T_0 = T_c$, which follows when U in Eqn. 10 is very large. The fin efficiency derived below can still be used with respect to the base temperature. The base temperature is then determined by an iterative process commonly used for condensation in the presence of noncondensable gases. The simplified analysis proceeds as follows:

Let $\Theta = (T - T_m)/(T_0 - T_m)$

where T_m is the reference fin temperature at the tip. The resulting temperature distribution obtained by solving Eqn. 6 is now as follows:

$$\Theta = (T_0 - y_b/C')/(T_0 - T_m) \cosh(\phi\ \eta)/\cosh(\phi) + (y_b/C' - T_m)/(T_0 - T_m) \tag{13}$$

Because $\Theta = 0$ at $\eta = 0$, the fin-tip temperature is as follows:

$$T_m = y_b/C' + (T_0 - y_b/C')/\cosh(\phi) \tag{14}$$

This equation will now be used to obtain an expression of the fin efficiency. By definition, the fin efficiency is expressed as

$$\eta_f = [\ - (k_f\ t_f)\ dT/dx\ @\ x = b]/Q_{total} \tag{15}$$

where:

$$Q_{total} = 2(b/2)\ [H_g\ (T_g - T_0) + N\ (h^v{}_b - h^l{}_i)]$$

$$= 2(b/2)\ [H_g\ (T_g - T_0) + (h^v{}_b - h^l{}_i)\ K_g\ (y_b - y_0)] \tag{16}$$

The fin-base heat flux can be written as follows:

$$- (k_f\ t_f)\ dT/dx = - (k_f\ t_f)\ d\Theta/d\eta\ (T_0 - T_m)/(b/2)\ @\ x = b \tag{17}$$

where:

$$d\Theta/d\eta = (T_0 - y_b/C')/(T_0 - T_m)\ \phi \tanh(\phi)\ @\ x = b \tag{18}$$

The combination of these two expressions yields the following fin-efficiency expression:

$$\eta_f = \tanh(\phi) / \phi \tag{19}$$

where ϕ is given by Eqn. 8. This expression is identical to that for the single-phase heat transfer with a constant heat-transfer coefficient.

CALCULATION ALGORITHM

The calculation algorithm for the fin efficiency is an integral part of the iterative method used to determine the interfacial conditions for the primary-wall surface. The condensate-film distribution between the primary wall and the finned surface is difficult to determine; therefore, an uniform film thickness is generally assumed. However, the rate of condensation for the primary wall and the finned surface are separately calculated. Note that the bulk conditions at a given axial location are known from the integration procedure. The calculation algorithm for the fin efficiency is summarized as follows:

1. Calculate physical and transport properties at the bulk conditions,
2. Calculate the heat- and mass-transfer coefficients,
3. Determine the interfacial conditions for the primary wall,
4. Use interfacial concentration, y_i, for calculating the high mass-flux mass-transfer coefficient, K_g.

To use Eqn. 12 for calculation of the fin-base temperature, continue as follows:

5. Assume the fin-base temperature, T_0,
6. Calculate C' and h^l_i at T_0,
7. Calculate ϕ,
8. Solve Eqn. 12 with the Newton-Raphson method,
9. Calculate the heat flux at the fin-base.

To use the fin-efficiency expression, Eqn. 19,

5. Calculate parameters in Eqn. 19,
6. Correct the finned area by applying the fin efficiency,
7. Calculate the effective heat flux by using the corrected finned area.

RESULTS AND DISCUSSION

The two methods of calculating the fin efficiency are illustrated with an example dealing with condensation of steam in the presence of air. The fin geometry and operating conditions are shown in Table 1. The effects of wall resistance was assumed to be insignificant because the material of construction for the condenser was an aluminum alloy.

Table 1. Fin geometry and operating conditions

Parameters	Value	Units
Fin Geometry		
Fin height	7.62	mm
Fin thickness	0.82	mm
Fin thermal conductivity	o.19	kW/m K
Operating Conditions		
Steam velocity	36.6	m/s
Pressure	33.8	kPa
Condensate coefficient for primary wall	8.5	kW/m^2 K
Coolant temperature	10.0	°C
Noncondensable gas concentration	0.3	weight fraction
Coolant velocity	3.0	m/s

The effects of noncondensable gas concentration, coolant velocity, fin height, and fin thermal conductivity on the fin efficiency are determined. The fin-efficiency values calculated by the two methods represented by Eqns.

12 and 19 are compared and the predicted effects of controlling parameters are determined. The effects of noncondensable gas concentration on the fin efficiency are shown in Figure 3. In addition to the two methods described above, the two limiting cases for calculating the fin efficiency are also shown in Figure 3. In one of the limiting cases, pure gas is assumed and the fin efficiency is calculated by using the commonly used single-phase equation equivalent of Eqn. 19, but the dimensionless parameter, ϕ is calculated differently. In the other limiting case, pure steam is assumed to condense and a value for the condensate coefficient for the finned surface is adjusted to obtain the limiting fin-efficiency value at zero gas concentration by the two methods developed in the present analysis. The resulting condensate coefficient is found to be 51 kW/m^2 K.

The computed results presented in Figure 3 show that the fin efficiency calculated by the two methods start with a limiting value at zero gas concentration. As the gas-concentration value is increased, the fin efficiency calculated by the simplified Eqn. 19 increases rapidly as compared with that calculated by the heat flux method of Eqn. 12. They both are expected to reach asymptotically to the value for pure gas as the gas concentration is increased to a higher value. The fin efficiency calculated by Eqn. 12 is lower than that calculated by Eqn. 19 for the set of condition used in the present analysis.

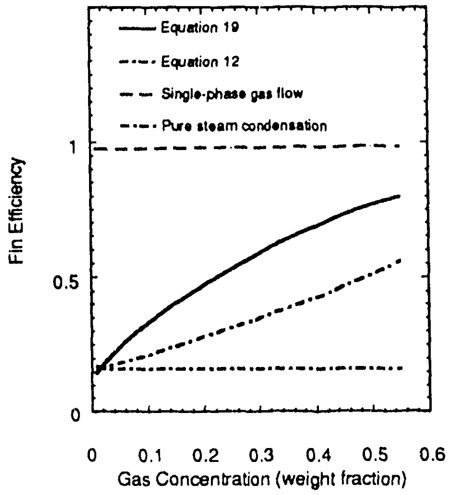

Figure 3. Effects of noncondensable gas concentration on fin efficiency.

In order to derive the simplified form of Eqn. 19, an assumption is made that the coolant heat-transfer resistance is negligible. Figure 4 shows that the relative effects of the above assumption for three values of the gas concentrations. The results show that the error produced by this assumption depends on the gas concentration for a given set of fin geometry. Low coolant velocity gives a high fin-base temperature that adversely affects the temperature distribution in the finned surface and leads to low fin efficiency. The nonlinear dependence of the heat and mass fluxes on the interfacial temperature causes such interactive effects, which are not observed for the single-phase heat transfer.

The effects of fin geometry expressed in terms of parameter, ϕ are shown in Figure 5, where the fin height and fin thermal conductivity are independently varied. It is interesting to note that the effects of thermal conductivity and fin height cannot be directly correlated with parameter, ϕ, with the method represented by Eqn. 12. As expected, the fin efficiency calculated by Eqn. 19 can be correlated with parameter, ϕ. As shown for above cases, the fin efficiency calculated by Eqn. 12 is lower than that calculated by Eqn. 19 for the selected set of conditions. The effects of fin height on the fin efficiency, as predicted by Eqn. 12, are more dominant than those of the thermal conductivity. Difference in the fin efficiency calculated by the two methods diminishes as the thermal conductivity is reduced to below a value of about 30 kW/ m K.

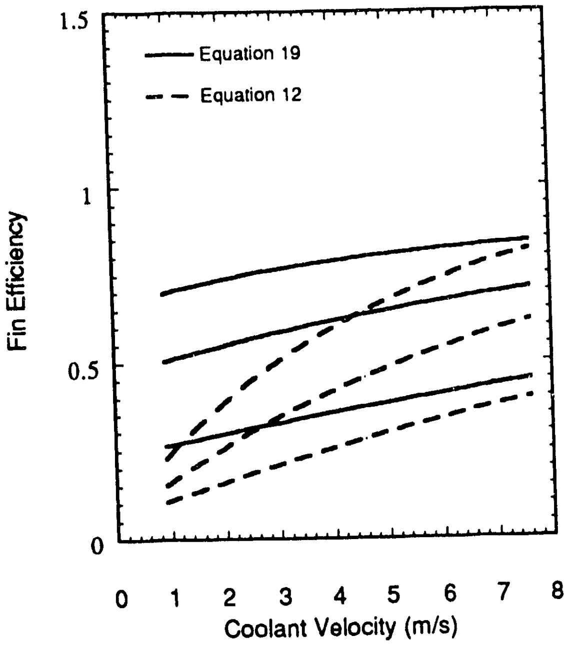

Figure 4. Effects of coolant velocity on fin efficiency.

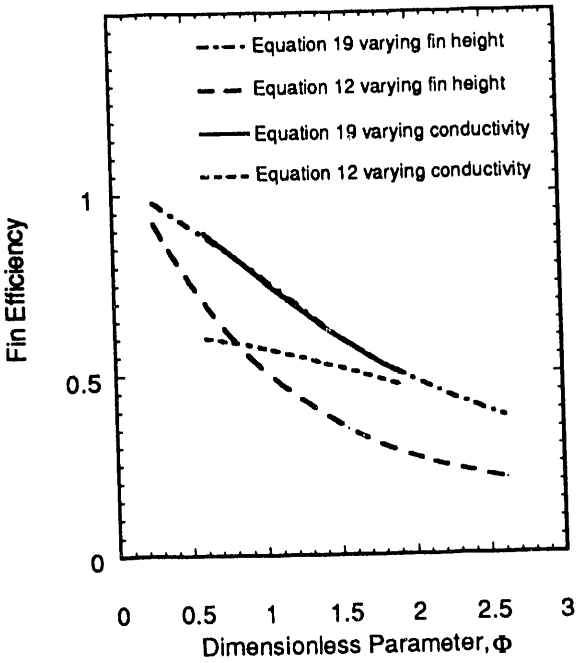

Figure 5. Fin efficiency as a function of dimensionless parameter Φ.

The results discussed above illustrate the relative effects of key parameters on the fin efficiency for condensation in the presence of noncondensable gases. The fin-efficiency calculation method represented by Eqn. 12 is recommended; however, the simplified method can be used, provided that the effects of the coolant heat-transfer resistance are determined for a reference set of conditions.

SUMMARY

A simplified expression is developed to calculate the fin efficiency for condensation in the presence of noncondensable gases. It can be useful for determining an optimum fin geometry and density for a given application. Further analysis is required to determine the fin-efficiency for condensation of binary and multicomponent vapor mixtures.

NOMENCLATURE

b = fin height (mm)
C' = equilibrium constant (°C^{-1})
h_c = condensate heat-transfer coefficient (kW/m^2 K)
H_g = gas-side heat transfer coefficient (kW/^{2} K)
h^v = vapor-phase partial molar enthalpy (kJ/mol)
h^l = liquid-phase partial molar enthalpy (kJ/mol)
k = thermal conductivity of fin material (kW/m K)
k_g = mass-transfer coefficient (mol/m^2 s)
K_g = high mass-flux mass transfer coefficient (mol/m^2 s)
= $k_g \, \mathit{ln}[(1 - y_i)/(1 - y_b)/(y_b - y_i)$
t_f = fin thickness (mm)
T = fin-metal temperature (°C)
T = temperature (°C)
Q_{total} = heat flux assuming fin efficiency equal to 1 (kW/m^2)
N = molar flux (mol/m^2 s)
x = coordinate along the fin height (mm)
y = vapor mole fraction

Subscript

b = bulk
c = coolant
g = gas phase
i = interface
f = fin
m = reference point
0 = fin base

Greek symbols

ϕ = dimensionless parameter defined in Eqn. 8
Θ = dimensionless temperature
η = dimensionless x coordinate
η_f = fin efficiency

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