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A DETERMINATION OF THE CHIRAL SU(4) X SU(4)

BREAKING PARAMETERS*

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ABSTRACT

We consider broken chiral SU(4) X SU(4) symmetry. From the observed mass spectrum of pseudoscalar charmed mesons, we are able to solve for the symmetry breaking parameters of the theory. We find that both vacuum and Hamiltonian breaking play an important role as far as charmed states are concerned. Purely from the masses of D and F mesons we deduce the current algebra mass ratio $\frac{m_c}{m_s} < 5$. This differs greatly from values obtained using linear or quadratic mass formulae. Considering η, η', and η_c mixing we further obtain a good solution with $\frac{m_c}{m_s} \simeq 3.2$ and $\frac{\langle\bar{c}c\rangle}{\langle\bar{u}u\rangle} \simeq 5.67$.

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I. INTRODUCTION

Recent observation[1,2,3] of charmed pseudoscalar mesons $D^{\pm}$, D^{0}, F^{+} as well as η_c prompts us to reexamine the question of how the chiral SU(4) x SU(4) symmetry is broken.[4] Current ideas of strong interactions based on Quantum Chromodynamics (QCD) and Unified Theories of Weak and Electromagnetic interactions suggest that chiral SU(4) x SU(4) symmetry is a Global symmetry of the Lagrangian associated with the flavor group. Further this symmetry is broken both by the vacuum and explicitly in the interaction Lagrangian by the quark mass terms which transform according to $(4,4^{*}) \oplus (4^{*},4)$ representation. Our knowledge of Quantum Chromodynamics is not yet at a stage which will allow us to calculate directly the true vacuum of the chiral group. Nevertheless, we can use current algebra techniques and the observed mass spectrum of the pseudoscalar mesons and their decay constants F_π, F_k, etc. to determine the properties of the vacuum as well as the mass ratios of the quarks.

The 'charm' quark to 'strange' quark mass ratio $\frac{m_c}{m_s}$ is in principle simply calculated from the knowledge of the ratio of SU(4) breaking along the 15 direction to that along the 8 direction. Thus if Hamiltonian for symmetry breaking is

$$H_{\text{symmetry breaking}} = -(\epsilon_0 u_0 + \epsilon_8 u_8 + \epsilon_{15} u_{15}) \tag{1.1}$$

then since $m_u, m_d, \ll m_s, m_c$ [5]

$$\frac{m_c}{m_s} \simeq \frac{4}{3\sqrt{2}} \frac{\epsilon_{15}}{\epsilon_8} + \frac{1}{3} \tag{1.2}$$

However, the estimates for $\frac{\epsilon_{15}}{\epsilon_8}$ vary widely depending on whether we use first order breaking formula to fit the masses or the masses squared. Thus if linear mass formula is employed we have $\frac{\epsilon_{15}}{\epsilon_8} \simeq 9.7$

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and if quadratic mass formula[6] is employed we find $\frac{\epsilon_{15}}{\epsilon_8} \simeq 21.6$. These yield for the ratio $\frac{m_c}{m_s}$ the values 9.5 and 20.7 respectively.

Further, neither fit is satisfactory because they yield the formulae for the pseudoscalar mesons,

$$\begin{array}{ccc} M_F - M_D & = & M_K - M_\pi \\ (.178\ GeV) & \neq & (.414\ GeV) \end{array}$$

or (1.3)

$$\begin{array}{ccc} M_F^2 - M_D^2 & = & M_K^2 - M_\pi^2 \\ (.695\ GeV^2) & \neq & (.283\ GeV^2) \end{array}$$

Thus we need a more accurate treatment of chiral breaking to yield more reliable estimate of $\frac{m_c}{m_s}$.

Work based on chiral SU(3) x SU(3) breaking[7] had revealed that

a) the lagrangian is approximately SU(2) x SU(2) invariant leading to a small number for the ratios $\frac{m_u}{m_s}$, $\frac{m_d}{m_s}$, and

b) the vacuum is to a good approximation a SU (3) singlet implying approximate equality of decay constants $F_\pi \simeq F_K$, as well as the $(\text{mass})^2$ octet broken formula for the pseudoscalar bosons. An extremely good solution[8] for all the parameters was obtained by solving the current algebra equations which included a general η-η' mixing with a single hypothesis on equality of renormalization constants $\sqrt{Z_i} \equiv \langle 0 | i \bar{q} \lambda_i \gamma_5 q | P_i \rangle$ where P_i denotes the i-th pseudoscalar meson. In this paper we wish to find a consistent set of solutions to the current algebra equations using similar technique in the case of SU(4) x SU(4) symmetry. We have found, however, that the requirement that $\sqrt{Z_i}$ are SU(4) symmetric is completely at variance with the mass spectrum of the pseudoscalar mesons. We shall show that such an assumption leads to an extremely small D-F mass splitting if the decay constants are assumed equal, and if the latter requirement is given up, a realistic value for $F_k \simeq 1.28$ leads to $M_F < M_D$, which is quite unacceptable. In the present work, we use the value of M_D and M_F as inputs and find that Z_i's have large SU(4) breaking. Purely from SU(3) symmetry of Z_i and the value of D and F meson masses, we establish that the ratio $\frac{m_c}{m_s} < 5$.

With further assumptions we are able to solve the coupled set of equations that characterize the model. We allow for a general mixing for η, η' and η_c . A surprise is that the vacuum is not a SU(4) singlet, although it is still to a good approximation SU(3) symmetric.

We have determined the ratios of quark masses and the decay constants. These are

$$\frac{m_s}{m_u} \simeq 33.5 \; ; \quad \frac{m_c}{m_s} \simeq 3.2 \tag{1.4}$$

and the decay constants are, in units of $F_\pi \simeq 92\text{MeV}$

$$F_K = 1.28, \quad F_D = 0.974, \quad F_F = 1.056 \tag{1.5}$$

We could have started with the group U(4) x U(4) instead of SU(4) x SU(4). This would lead to the U(1) problem discussed by Weinberg.[9] As noted by 't Hooft[10] this problem can be circumvented in QCD where presence of instantons leads to an anomaly in the divergence of $U_A(1)$ current. We have added an extra term to the divergence of the A_μ^0 current to take into account this effect. The net result is that it is possible to consider SU(4) x SU(4) algebra itself and solve for the unknown parameters. No constraint is imposed on this from the $U_A(1)$ sector. On the other hand, from the knowledge of the solution we can say something about the matrix elements of the U_A (1) breaking term.

In Section II, we set up the basic equations of the model. The section also serves to define our notations. In Section III, we show how simple assumptions on equality of Z_i give unacceptable values and in Section IV and V we present our technique for solving the set of equations. A discussion of our results is contained in Section VI.

II. THE FUNDAMENTAL EQUATIONS

The strong interaction Hamiltonian density is assumed to be of the form

$$H = H_0 - \epsilon_0 u_0 - \epsilon_8 u_8 - \epsilon_{15} u_{15} \tag{2.1}$$

where H_0 is invariant under chiral SU(4) x SU(4) symmetry, while the symmetry breaking terms u_i transform according to $(4^*, 4) \oplus (4, 4^*)$ representation. In terms of the quark model, these symmetry breaking terms are merely the mass terms of the quarks. We shall neglect isospin breaking effects due to lack of degeneracy of the mass of u and d quarks as well as conventional electromagnetic corrections, in this paper. The explicit relation between quark masses and ϵ_0 ; ϵ_8 and ϵ_{15} are easily found to be

$$m_u = m_d = -\left(\frac{\epsilon_0}{\sqrt{2}} + \frac{\epsilon_8}{\sqrt{3}} + \frac{\epsilon_{15}}{\sqrt{6}}\right) \tag{2.2a}$$

$$m_s = -\left(\frac{\epsilon_0}{\sqrt{2}} - \frac{2\epsilon_8}{\sqrt{3}} + \frac{\epsilon_{15}}{\sqrt{6}}\right) \tag{2.2b}$$

$$m_c = -\left(\frac{\epsilon_0}{\sqrt{2}} - \frac{3\epsilon_{15}}{\sqrt{6}}\right) \tag{2.2c}$$

Note that if $m_u \neq m_d$, then (2.2a) should have average mass of light quarks, $m_\ell = \frac{1}{2}(m_u + m_d)$ on the left hand side.

The generators of SU(4) x SU(4) can be expressed in terms of F^i and F_5^i, the vector and axial generators, which are defined as usual by

$$F^i(t) = \int_{x_0=t} d^3x\, V_0^i(x) \qquad i = (0, \cdots, 15) \tag{2.3}$$

$$F_5^i(t) = \int_{x_0=t} d^3x\, A_0^i(x)$$

The scalar densities $u_i\ (\equiv \bar{q}\lambda_i q)$ and pseudoscalar densities $v_i\ (\equiv i\bar{q}\lambda_i\gamma^5 q)$ satisfy the equal time commutation rules.

$$[F^i(t), u^j(x)]_{x_0=t} = i f_{ijk} u^k(x)$$
$$[F^i(t), v^j(x)]_{x_0=t} = i f_{ijk} v^k(x)$$
$$[F_5^i(t), u^j(x)]_{x_0=t} = -i d_{ijk} v^k(x) \qquad (2.4)$$
$$[F_5^i(t), v^j(x)]_{x_0=t} = i d_{ijk} u^k(x)$$

The current divergences are given by

$$\partial^\mu V_\mu^i = -i [F^i(t), H(x)]_{x_0=t} \qquad (2.5\,a)$$

$$\partial^\mu A_\mu^i = -i [F_5^i(t), H(x)]_{x_0=t} \qquad (2.5\,b)$$

From Equations (2.1) and (2.5) these are found to be

$$\partial^\mu V_\mu^i = -\epsilon_8 f_{i8k} u^k - \epsilon_{15} f_{i15k} u^k \qquad (2.6\,a)$$

$$\partial^\mu A_\mu^i = \epsilon_0 d_{i0k} v^k + \epsilon_8 d_{i8k} v^k + \epsilon_{15} d_{i15k} v^k + \delta_{i0} V \qquad (2.6\,b)$$

where $V = \frac{g^2}{16\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu a}$ is the anomaly in A_μ^0 current for QCD and is a SU(4) singlet operator.

We take matrix elements of Eq. (2.7) between vacuum and single pseudoscalar or scalar meson states. Following conventional definitions are employed

$$\langle 0 | A_\mu^{1,2,3} | \pi \rangle = i p_\mu F_\pi$$
$$\langle 0 | A_\mu^{4,5,6,7} | K \rangle = i p_\mu F_K$$
$$\langle 0 | A_\mu^{9,10,11,12} | D \rangle = i p_\mu F_D$$
$$\langle 0 | A_\mu^{13,14} | F \rangle = i p_\mu F_F$$
$$\langle 0 | A_\mu^i | \eta, \eta', \eta_c \rangle = i p_\mu F^i_{\eta,\eta',\eta_c}$$
$$(i = 0, 8 \text{ or } 15)$$

$$\langle 0 | V_\mu^4 | K \rangle = i p_\mu F_K$$

$$\langle 0 | V_\mu^{9,11} | S_{10,12} \rangle = - \langle 0 | V_\mu^{10,12} | S_{9,11} \rangle = i p_\mu F_{S_D}$$

$$\langle 0 | V_\mu^{13} | S_{14} \rangle = - \langle 0 | V_\mu^{14} | S_{13} \rangle = i p_\mu F_{S_F} \qquad (2.7)$$

Our currents are so renormalized that $F_\pi \simeq 92$ MeV. States S_D and S_F are members of the scalar 15-plet.

$$\langle 0 | v^{1,2,3} | \pi \rangle = \sqrt{Z_\pi}$$

$$\langle 0 | v^{4,5,6,7} | K \rangle = \sqrt{Z_K}$$

$$\langle 0 | v^{9,10,11,12} | D \rangle = \sqrt{Z_D}$$

$$\langle 0 | v^{13,14} | F \rangle = \sqrt{Z_F}$$

$$\langle 0 | v^i | \eta, \eta', \eta_c \rangle = \sqrt{Z^i_{\eta,\eta',\eta_c}} \qquad (i = 0, 8 \text{ or } 15)$$

$$\langle 0 | u^{4,5,6,7} | \kappa \rangle = \sqrt{Z_\kappa}$$

$$\langle 0 | u^{9,10,11,12} | S_D \rangle = \sqrt{Z_{S_D}}$$

$$\langle 0 | u^{13,14} | S_F \rangle = \sqrt{Z_{S_F}}$$

$$\langle 0 | V | \eta, \eta', \eta_c \rangle = g_{\eta,\eta',\eta_c} \qquad (2.8)$$

We then find for pseudoscalar mesons

$$\frac{M_n^2 F_n}{\sqrt{Z_n}} = \frac{\epsilon_0}{\sqrt{2}} + \frac{\epsilon_8}{\sqrt{3}} + \frac{\epsilon_{15}}{\sqrt{6}}$$

$$\frac{M_K^2 F_K}{\sqrt{Z_K}} = \frac{\epsilon_0}{\sqrt{2}} - \frac{\epsilon_8}{2\sqrt{3}} + \frac{\epsilon_{15}}{\sqrt{6}}$$

$$\frac{M_D^2 F_D}{\sqrt{Z_D}} = \frac{\epsilon_0}{\sqrt{2}} + \frac{\epsilon_8}{2\sqrt{3}} - \frac{\epsilon_{15}}{\sqrt{6}}$$

$$\frac{M_F^2 F_F}{\sqrt{Z_F}} = \frac{\epsilon_0}{\sqrt{2}} - \frac{\epsilon_8}{\sqrt{3}} - \frac{\epsilon_{15}}{\sqrt{6}}$$

$$M_{\eta_i}^2 F_{\eta_i}^8 = \left(\frac{\epsilon_0}{\sqrt{2}} - \frac{\epsilon_8}{\sqrt{3}} + \frac{\epsilon_{15}}{\sqrt{6}}\right)\sqrt{Z_{\eta_i}^8} + \frac{\epsilon_8}{\sqrt{6}}\sqrt{Z_{\eta_i}^{15}} + \frac{\epsilon_8}{\sqrt{2}}\sqrt{Z_{\eta_i}^0}$$

$$M_{\eta_i}^2 F_{\eta_i}^{15} = \left(\frac{\epsilon_0}{\sqrt{2}} - 2\frac{\epsilon_{15}}{\sqrt{6}}\right)\sqrt{Z_{\eta_i}^{15}} + \frac{\epsilon_{15}}{\sqrt{2}}\sqrt{Z_{\eta_i}^0} + \frac{\epsilon_8}{\sqrt{6}}\sqrt{Z_{\eta_i}^8}$$

$$M_{\eta_i}^2 F_{\eta_i}^0 = \frac{\epsilon_0}{\sqrt{2}}\sqrt{Z_{\eta_i}^0} + \frac{\epsilon_8}{\sqrt{2}}\sqrt{Z_{\eta_i}^8} + \frac{\epsilon_{15}}{\sqrt{2}}\sqrt{Z_{\eta_i}^{15}} + g_{\eta_i}$$

$$(\eta_i = \eta, \eta' \text{ or } \eta_c) \tag{2.9}$$

Similarly, we find for scalar mesons

$$\frac{M_\kappa^2 F_\kappa}{\sqrt{Z_\kappa}} = \frac{\sqrt{3}}{2}\epsilon_8$$

$$\frac{M_{S_D}^2 F_{S_D}}{\sqrt{Z_{S_D}}} = \frac{\epsilon_8}{2\sqrt{3}} + 2\frac{\epsilon_{15}}{\sqrt{6}}$$

$$\frac{M_{S_F}^2 F_{S_F}}{\sqrt{Z_{S_F}}} = -\frac{\epsilon_8}{\sqrt{3}} + 2\frac{\epsilon_{15}}{\sqrt{6}} \tag{2.10}$$

The above equations are exact consequences of the model. Further equations are obtained by single particle saturation which becomes exact in the Nambu-Goldstone limit, i.e., $\varepsilon_i \to 0$. Then the only breaking of symmetry is in the vacuum, resulting in massless bosons that saturate the commutation rules. The corrections to these results are expected to be of order ε and except for charmed states, we might expect these to be quite small. Here we assume the validity of all the relations and appeal to future experiments as a way of establishing them. Defining $\langle 0|u_i|0\rangle \equiv \delta_i$, we have for pseudoscalar mesons

$$F_\pi \sqrt{Z_\pi} = \frac{\delta_0}{\sqrt{2}} + \frac{\delta_8}{\sqrt{3}} + \frac{\delta_{15}}{\sqrt{6}}$$

$$F_K \sqrt{Z_K} = \frac{\delta_0}{\sqrt{2}} - \frac{\delta_8}{2\sqrt{3}} + \frac{\delta_{15}}{\sqrt{6}}$$

$$F_D \sqrt{Z_D} = \frac{\delta_0}{\sqrt{2}} + \frac{\delta_8}{2\sqrt{3}} - \frac{\delta_{15}}{\sqrt{6}}$$

$$F_F \sqrt{Z_F} = \frac{\delta_0}{\sqrt{2}} - \frac{\delta_8}{\sqrt{3}} - \frac{\delta_{15}}{\sqrt{6}}$$

$$F^8_\eta \sqrt{Z^8_\eta} + F^8_{\eta'} \sqrt{Z^8_{\eta'}} + F^8_{\eta_c} \sqrt{Z^8_{\eta_c}} = \frac{\delta_0}{\sqrt{2}} - \frac{\delta_8}{\sqrt{3}} + \frac{\delta_{15}}{\sqrt{6}}$$

$$F^8_\eta \sqrt{Z^0_\eta} + F^8_{\eta'} \sqrt{Z^0_{\eta'}} + F^8_{\eta_c} \sqrt{Z^0_{\eta_c}} = \frac{\delta_8}{\sqrt{2}}$$

$$F^8_\eta \sqrt{Z^{15}_\eta} + F^8_{\eta'} \sqrt{Z^{15}_{\eta'}} + F^8_{\eta_c} \sqrt{Z^{15}_{\eta_c}} = \frac{\delta_8}{\sqrt{6}}$$

$$F^{15}_\eta \sqrt{Z^0_\eta} + F^{15}_{\eta'} \sqrt{Z^0_{\eta'}} + F^{15}_{\eta_c} \sqrt{Z^0_{\eta_c}} = \frac{\delta_{15}}{\sqrt{2}}$$

$$F^{15}_{\eta'} \sqrt{Z^8_{\eta'}} + F^{15}_\eta \sqrt{Z^8_\eta} + F^{15}_{\eta_c} \sqrt{Z^8_{\eta_c}} = \frac{\delta_8}{\sqrt{6}}$$

$$F^{15}_\eta \sqrt{Z^{15}_\eta} + F^{15}_{\eta'} \sqrt{Z^{15}_{\eta'}} + F^{15}_{\eta_c} \sqrt{Z^{15}_{\eta_c}} = \frac{\delta_0}{\sqrt{2}} - 2\,\frac{\delta_{15}}{\sqrt{6}} \qquad (2.11)$$

Similar equations for scalar bosons can also be written. However, pole saturation can not be justified for these because in the chiral limit the masses of scalar bosons diverge. For the sake of completeness, we only list them but shall not make use of them in this paper.

$$F_\kappa \sqrt{Z_\kappa} = \frac{\sqrt{3}}{2}\,\delta_8$$

$$F_{S_D} \sqrt{Z_{S_D}} = \frac{\delta_8}{2\sqrt{3}} + 2\,\frac{\delta_{15}}{\sqrt{6}}$$

$$F_{S_F} \sqrt{Z_{S_F}} = -\frac{\delta_8}{\sqrt{3}} + 2\,\frac{\delta_{15}}{\sqrt{6}} \qquad (2.12)$$

It is also possible to derive relations by considering commutators of generators with the divergences of currents, and then taking their vacuum expectation values. These relations are also obtained from Eqns. (2.9) and (2.11) by eliminating $\sqrt{Z_1}$. We list these for completeness

$$M_\pi^2 F_\pi^2 = \left(\frac{\epsilon_0}{\sqrt2} + \frac{\epsilon_8}{\sqrt3} + \frac{\epsilon_{15}}{\sqrt6}\right)\left(\frac{\delta_0}{\sqrt2} + \frac{\delta_8}{\sqrt3} + \frac{\delta_{15}}{\sqrt6}\right)$$

$$M_K^2 F_K^2 = \left(\frac{\epsilon_0}{\sqrt2} - \frac{\epsilon_8}{2\sqrt3} + \frac{\epsilon_{15}}{\sqrt6}\right)\left(\frac{\delta_0}{\sqrt2} - \frac{\delta_8}{2\sqrt3} + \frac{\delta_{15}}{\sqrt6}\right)$$

$$M_D^2 F_D^2 = \left(\frac{\epsilon_0}{\sqrt2} + \frac{\epsilon_8}{2\sqrt3} - \frac{\epsilon_{15}}{\sqrt6}\right)\left(\frac{\delta_0}{\sqrt2} + \frac{\delta_8}{2\sqrt3} - \frac{\delta_{15}}{\sqrt6}\right)$$

$$M_F^2 F_F^2 = \left(\frac{\epsilon_0}{\sqrt2} - \frac{\epsilon_8}{\sqrt3} - \frac{\epsilon_{15}}{\sqrt6}\right)\left(\frac{\delta_0}{\sqrt2} - \frac{\delta_8}{\sqrt3} - \frac{\delta_{15}}{\sqrt6}\right)$$

$$\left(M_\eta F_\eta^8\right)^2 + \left(M_{\eta'} F_{\eta'}^8\right)^2 + \left(M_{\eta_c} F_{\eta_c}^8\right)^2 = \left(\frac{\epsilon_0}{\sqrt2} - \frac{\epsilon_8}{\sqrt3} + \frac{\epsilon_{15}}{\sqrt6}\right)\left(\frac{\delta_0}{\sqrt2} - \frac{\delta_8}{\sqrt3} + \frac{\delta_{15}}{\sqrt6}\right) + \frac{2}{3}\epsilon_8\delta_8$$

$$\left(M_\eta F_\eta^{15}\right)^2 + \left(M_{\eta'} F_{\eta'}^{15}\right)^2 + \left(M_{\eta_c} F_{\eta_c}^{15}\right)^2 = \left(\frac{\epsilon_0}{\sqrt2} - 2\frac{\epsilon_{15}}{\sqrt6}\right)\left(\frac{\delta_0}{\sqrt2} - 2\frac{\delta_{15}}{\sqrt6}\right) + \frac{\epsilon_8\delta_8}{6} + \frac{\epsilon_{15}\delta_{15}}{2}$$

$$M_\eta^2 F_\eta^8 F_\eta^{15} + M_{\eta'}^2 F_{\eta'}^8 F_{\eta'}^{15} + M_{\eta_c}^2 F_{\eta_c}^8 F_{\eta_c}^{15} = \left(\frac{\epsilon_0}{\sqrt2} - \frac{\epsilon_8}{\sqrt3} + \frac{\epsilon_{15}}{\sqrt6}\right)\frac{\delta_8}{\sqrt6} + \frac{\epsilon_8}{\sqrt6}\left(\frac{\delta_0}{\sqrt2} - 2\frac{\delta_{15}}{\sqrt6}\right) + \frac{\epsilon_8\delta_{15}}{2}$$

$$M_\kappa^2 F_\kappa^2 = \frac{3}{4}\epsilon_8\delta_8$$

$$M_{S_D}^2 F_{S_D}^2 = \frac{\epsilon_8\delta_8}{12} + 2\frac{\epsilon_{15}\delta_{15}}{3} + \frac{1}{3\sqrt2}\left(\epsilon_8\delta_{15} + \epsilon_{15}\delta_8\right)$$

$$M_{S_F}^2 F_{S_F}^2 = \frac{\epsilon_8\delta_8}{3} - \frac{\sqrt2}{3}\left(\epsilon_8\delta_{15} + \epsilon_{15}\delta_8\right) + \frac{2}{3}\epsilon_{15}\delta_{15} \qquad (2.13)$$

The basic problem we address to ourselves is to solve these equations with reasonable assumptions. The result will be to determine symmetry breaking parameters, i.e., $\varepsilon_8/\varepsilon_0$, $\varepsilon_{15}/\varepsilon_0$, and δ_8/δ_0, δ_{15}/δ_0 as well as decay constants F_i's. In the next section we examine some simple but experimentally inadmissible solutions and then in Section IV and V we make only the most plausible assumptions to solve these equations.

III. SIMPLE SOLUTIONS AND PROBLEMS

The set of equations obtained in Section II clearly involve too many unknown parameters to obtain a complete solution. In this section we shall make some simplifying assumptions to illustrate the difficulty in obtaining physically meaningful solutions. The simplest assumption is the generalization of Gell-Mann, Oakes and Renner[7] solution to this enlarged group. The assumption is that the vacuum is a SU(4) singlet, i.e.

$$\langle u_0 \rangle \equiv \delta_0 \neq 0$$

$$\langle u_8 \rangle \equiv \delta_8 = 0 \quad ; \quad \langle u_{15} \rangle \equiv \delta_{15} = 0 \tag{3.1}$$

and further that SU(4) symmetry is good for $\sqrt{Z}$'s i.e., $Z_\pi = Z_K = Z_D = Z_F = Z_8 = Z_{15}$. We also allow the possibility of η-η_c mixing because u_8 in the Hamiltonian mixes the 8 and the 15 components of the 15 representation. We shall, however, following G - O - R, neglect any singlet (η') mixing. Thus

$$\sqrt{Z^8_\eta} = \sqrt{Z^{15}_{\eta_c}} = \sqrt{Z_\pi}\,\cos\theta$$

$$\sqrt{Z^8_{\eta_c}} = -\sqrt{Z^{15}_\eta} = \sqrt{Z_\pi}\,\sin\theta \tag{3.2}$$

A consistent set of solutions is then obtained to all the equations in Section II. The mass sum-rules are

$$M_F^2 - M_D^2 = M_K^2 - M_\pi^2$$

$$4M_K^2 - M_\pi^2 = 3\left(M_\eta^2\cos^2\theta + M_{\eta_c}^2\sin^2\theta\right)$$

$$-\left(M_K^2 - M_\pi^2\right)\frac{2\sqrt{2}}{3} = \left(M_{\eta_c}^2 - M_\eta^2\right)\sin 2\theta$$

$$9M_D^2 + M_K^2 - 4M_\pi^2 = 6\left(M_\eta^2\sin^2\theta + M_{\eta_c}^2\cos^2\theta\right) \tag{3.3}$$

These are 4 equations involving seven variables, 6 masses and 1 mixing angle. A general feature of these equations is that because of large η_c mass (~2.6 GeV) we have

$$M_D \simeq \sqrt{\tfrac{2}{3}}\, M_{\eta_c} = 2.13 \text{ GeV}$$

$$M_F - M_D \simeq \frac{M_K^2}{M_F + M_D} \simeq 60 \text{ MeV}$$

(3.4)

The experimental value of M_D is, however, much lower, 1862 MeV,[2] while $M_F - M_D$ is closer to 180 MeV.[3,2] The source of the problem can be traced to (mass)^2 sum rules that emerge with our simplifying assumptions, while the heavier masses are fit better with a linear mass formula. Thus,

$$M_D \simeq \frac{2}{3} M_{\eta_c}$$

(3.5)

Inclusion of η' in our mixing scheme does not change the basic situation. The matrix $\sqrt{Z^i_{\eta,\eta',\eta_c}}$ ($i = 0, 8$ or 15) is then a 3X3 orthogonal matrix, and Eq. (2.11) yields the solution (remembering $\delta_8 = \delta_{15} = 0$)

$$\frac{F^i_{\eta,\eta',\eta_c}}{F_\pi} = \frac{\sqrt{Z^i_{\eta,\eta',\eta_c}}}{\sqrt{Z_\pi}} \qquad (i = 0, 8 \text{ or } 15)$$

(3.6)

Now solving the Eq. (2.9) is equivalent to the diagonalization of the 3X3 η, η', η_c mass matrix. We identify the physical states as

$$|\eta\rangle = Z_\pi^{-1/2}\left[\sqrt{Z^0_\eta}\,|P_0\rangle + \sqrt{Z^8_\eta}\,|P_8\rangle + \sqrt{Z^{15}_\eta}\,|P_{15}\rangle\right]$$

$$|\eta'\rangle = Z_\pi^{-1/2}\left[\sqrt{Z^0_{\eta'}}\,|P_0\rangle + \sqrt{Z^8_{\eta'}}\,|P_8\rangle + \sqrt{Z^{15}_{\eta'}}\,|P_{15}\rangle\right]$$

$$|\eta_c\rangle = Z_\pi^{-1/2}\left[\sqrt{Z^0_{\eta_c}}\,|P_0\rangle + \sqrt{Z^8_{\eta_c}}\,|P_8\rangle + \sqrt{Z^{15}_{\eta_c}}\,|P_{15}\rangle\right]$$

(3.7)

where $|P_i\rangle$ are SU(4) symmetric un-mixed states. Since operator V is a SU(4) singlet, it is reasonable to assume that matrix elements of V in lowest order perturbation theory, are

$$\langle 0|V|\eta\ \eta',\eta_c\rangle \equiv g_{\eta,\eta',\eta_c} = \mu^2\sqrt{Z^0_{\eta,\eta',\eta_c}}$$

(3.8)

where μ^2 is an arbitrary constant.

Equation (2.9) can be written as a matrix equation

$$\underline{M}^2\sqrt{Z} = \sqrt{Z}\,M^2$$

where

$$\underline{M}^2 = \begin{pmatrix} M^2_\eta & 0 & 0 \\ 0 & M^2_{\eta'} & 0 \\ 0 & 0 & M^2_{\eta_c} \end{pmatrix}$$

(3.9a)

is a diagonal matrix.

$$M^2 = \langle P_i | H | P_j \rangle$$

$$= \begin{pmatrix} \mu^2 + \frac{\epsilon_0}{\sqrt{2}} & \frac{\epsilon_8}{\sqrt{2}} & \frac{\epsilon_{15}}{\sqrt{2}} \\ \frac{\epsilon_8}{\sqrt{2}} & \frac{\epsilon_0}{\sqrt{2}} - \frac{\epsilon_8}{\sqrt{3}} + \frac{\epsilon_{15}}{\sqrt{6}} & \frac{\epsilon_8}{\sqrt{6}} \\ \frac{\epsilon_{15}}{\sqrt{2}} & \frac{\epsilon_8}{\sqrt{6}} & \frac{\epsilon_0}{\sqrt{2}} - 2\frac{\epsilon_{15}}{\sqrt{6}} \end{pmatrix} \tag{3.9b}$$

and

$$\sqrt{Z} = \begin{pmatrix} \sqrt{Z^0_\eta} & \sqrt{Z^8_\eta} & \sqrt{Z^{15}_\eta} \\ \sqrt{Z^0_{\eta'}} & \sqrt{Z^8_{\eta'}} & \sqrt{Z^{15}_{\eta'}} \\ \sqrt{Z^0_{\eta_c}} & \sqrt{Z^8_{\eta_c}} & \sqrt{Z^{15}_{\eta_c}} \end{pmatrix} \tag{3.9c}$$

Eq. (3.9b) is then written as

$$M^2 = \begin{pmatrix} \mu^2 + \frac{1}{2}(M_D^2 + M_K^2) & -\frac{\sqrt{2}}{\sqrt{3}}(M_K^2 - M_\pi^2) & \frac{1}{2\sqrt{3}}(M_K^2 + 2M_\pi^2 - 3M_D^2) \\ -\frac{\sqrt{2}}{\sqrt{3}}(M_K^2 - M_\pi^2) & \frac{1}{3}(4M_K^2 - M_\pi^2) & -\frac{\sqrt{2}}{3}(M_K^2 - M_\pi^2) \\ \frac{1}{2\sqrt{3}}(M_K^2 + 2M_\pi^2 - 3M_D^2) & -\frac{\sqrt{2}}{3}(M_K^2 - M_\pi^2) & \frac{1}{6}(9M_D^2 + M_K^2 - 4M_\pi^2) \end{pmatrix} \tag{3.9d}$$

This matrix is easily diagonalized as a function of μ^2.

Using M_π = 135 MeV, M_K = 4.96 MeV and M_D = 1862 MeV we, however, find that <u>no value</u> of μ^2 give masses of η, η' and η_c that are close to the experimental values. All results being expressed in MeV, our results are:

μ	M_η	M_{η_c}	M_η
1909.2	549	2879	1569
1653.4	543	2802	1413
954.6	493	2677	953

Further, we still have $M_F - M_D$ = 60 MeV, which is far from the experimental value 180 MeV. Thus, we are forced to give up our assumption of vacuum being a SU(4) singlet. We next attempt a solution that admits nonvanishing δ_8 and δ_{15} though still preserve the SU(4) symmetry of Z's. The $\sqrt{Z}$ mixing matrix in Eqn.(3.9c) is taken as

$$\sqrt{Z_\pi}\begin{pmatrix} -\sin\theta\cos\phi & \cos\theta & \sin\theta\sin\phi \\ -\sin\psi\sin\phi + \cos\theta\cos\phi\cos\psi & \sin\theta\cos\psi & -\sin\psi\cos\phi - \cos\theta\sin\phi\cos\psi \\ \cos\psi\sin\phi + \cos\theta\cos\phi\sin\psi & \sin\theta\sin\psi & \cos\psi\cos\phi - \cos\theta\sin\phi\sin\psi \end{pmatrix} \tag{3.10}$$

The set of Eqns.(2.9, 2.11) can now be solved numerically on a computer. This approach, however, leads to a problem of mass reversal, i.e. M_D comes out greater than M_F. The source of the problem can be seen easily. From Eqns. (2.9) and (2.11) we have since Z's are equal

$$F_F - F_D = F_K - F_\pi$$

$$M_F^2 F_F - M_D^2 F_D = M_K^2 F_K - M_\pi^2 F_\pi$$

(3.11)

Simplifying we obtain

$$M_F^2 - M_D^2 \simeq \frac{F_K}{F_D}\left[M_K^2 - M_F^2\left(\frac{F_K - 1}{F_K}\right)\right]$$

(3.12)

We then see that for $F_K \simeq 1.28$. the right side is negative provided F_D has the same sign as F_K. This is expected from SU(4) symmetry, and also found to be true in the numerical solution.

We are forced, thus, to abandon the assumption of equality of $\sqrt{Z}$'s Nevertheless, from SU(3) X SU(3) solution we know that the equality for $\sqrt{Z}$ among SU(3) members is a good assumption. Thus we can retain the restrictive assumption $\sqrt{Z_\pi} = \sqrt{Z_K} = \sqrt{Z_8}$ and $\sqrt{Z_D} = \sqrt{Z_F}$. The simplest assumption to make now is that the vacuum is a SU(4) singlet, i.e. $\delta_8 = \delta_{15} = 0$. We can assume that $\sqrt{Z}$ are SU(4) broken along the 15 direction, i.e.

$$\langle 0 | V_\alpha | P_\beta \rangle = c_1 \delta_{\alpha\beta} + c_2 \delta_{\alpha c} \delta_{\beta c} + c_3 d_{15\alpha\beta}$$

(3.13)

where $|P_\beta\rangle$ are pure SU(4) states. The states, η, η', η_c are taken as linear combinations of $|P_0\rangle$, $|P_8\rangle$ and $|P_{15}\rangle$. The Eqns. (2.9) and (2.11) can now be solved on a computer, and we again find no acceptable solution. We thus are forced to consider the very general case of symmetry breaking in $\sqrt{Z}$'s as well as in the vacuum. In the next section we shall see the restriction that emerges from purely D and F masses on the nature of symmetry breaking.

IV. CONTSTRAINTS FROM D AND F MESON MASSES

In this section we obtain powerful constraints on the solution to Eqns.(2.9) and (2.11) that arise purely from our knowledge of D and F meson masses and the weak assumptions that the wave function renormalization constants, $\sqrt{Z}$ are SU(3) symmetric. The latter is verified to a good extent from previous work on SU(3) X SU(3) breaking. Consider the subset of Eqns.(2.9) and (2.11) which arise from π, K, D, and F meson pole saturation. We set $\sqrt{Z_\pi} = \sqrt{Z_K}$ and $\sqrt{Z_D} = \sqrt{Z_F}$.

We prefer to write these equations in terms of the masses of quarks and quark expection values. Relations between ε_i and masses are given in Eqns.(2.2) and the vacuum expectation values δ_i are related to $\langle \bar{q}q \rangle$ by

$$\delta_i \equiv \langle \bar{q} \lambda_i q \rangle$$

$$-\frac{2 M_\pi^2 F_\pi}{\sqrt{Z_\pi}} = m_u + m_d = 2 m_u$$

$$-\frac{2 M_K^2 F_K}{\sqrt{Z_\pi}} = m_u + m_s$$

$$-\frac{2 M_D^2 F_D}{\sqrt{Z_D}} = m_u + m_c$$

$$-\frac{2 M_F^2 F_F}{\sqrt{Z_D}} = m_s + m_c$$

$$F_\pi \sqrt{Z_\pi} = \langle \bar{u}u \rangle + \langle \bar{d}d \rangle = 2 \langle \bar{u}u \rangle$$

$$F_K \sqrt{Z_\pi} = \langle \bar{u}u \rangle + \langle \bar{s}s \rangle$$

$$F_D \sqrt{Z_D} = \langle \bar{c}c \rangle + \langle \bar{u}u \rangle$$

$$F_F \sqrt{Z_D} = \langle \bar{c}c \rangle + \langle \bar{s}s \rangle \tag{4.1}$$

We assume the masses of mesons M_π=135 MeV, M_K = 496 MeV, M_D = 1862 MeV, M_F = 2039.5 MeV and the ratio F_K/F_π = 1.28. The following relations can now be easily derived.

$$\frac{M_F^2}{M_D^2} = \frac{\left[\frac{m_c}{m_s} + 1\right]\left[\frac{\langle \bar{c}c \rangle}{\langle \bar{u}u \rangle} + 1\right]}{\left[\frac{m_c}{m_s} + \frac{m_u}{m_s}\right]\left[\frac{\langle \bar{c}c \rangle}{\langle \bar{u}u \rangle} + \frac{\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle}\right]}$$

$$\frac{\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle} = 2\,\frac{F_K}{F_\pi} - 1 = 1.56$$

$$\frac{m_s}{m_u} = 2\,\frac{M_K^2 F_K}{M_\pi^2 F_\pi} - 1 \simeq 33.4$$

(4.2)

Thus to a good approximation

$$\frac{M_F^2}{M_D^2} = \frac{\left[\frac{m_c}{m_s} + 1\right]\left[\frac{\langle\bar{c}c\rangle}{\langle\bar{u}u\rangle} + 1\right]}{\frac{m_c}{m_s}\left[\frac{\langle\bar{c}c\rangle}{\langle\bar{u}u\rangle} + 1.56\right]} \simeq 1.2 \qquad (4.3)$$

Note this relation is insensitive to assumed equality of m_u and m_d.

We plot the ratio m_c/m_s as a function $\langle\bar{c}c\rangle / \langle\bar{u}u\rangle$ in Fig. 1. Since $\langle\bar{s}s\rangle / \langle\bar{u}u\rangle$ is positive we expect $\langle\bar{c}c\rangle / \langle\bar{u}u\rangle$ to be positive and large, since the symmetry breaking arises from large m_c. As $\langle\bar{c}c\rangle/\langle\bar{u}u\rangle \to \infty$ we observe $m_c/m_s \to 5$. Thus for all physically meaningful values of $\langle\bar{c}c\rangle/\langle\bar{u}u\rangle$ we deduce the condition

$$\frac{m_c}{m_s} < 5 \qquad (4.4)$$

This conclusion is very different from the result that follows from the quadratic mass formula[6] that yields $m_c/m_s = 20.7$, or linear mass formula which gives $m_c/m_s = 9.5$. Some support for a small value for m_c/m_s comes from consideration of renormalization group in QCD where Georgi and Politzer[11] have deduced the value $m_c/m_s \simeq 4$. We can make further progress only after an estimate of $\langle\bar{c}c\rangle/\langle\bar{u}u\rangle$ or equivalently the ratio δ_{15}/δ_0.

A model for vacuum breaking which assumes a linear relation

$$\langle\bar{q}q\rangle = \langle\bar{q}q\rangle_0 + m_q A \qquad (4.5)$$

would yield

$$\frac{\langle \bar{c}c \rangle}{\langle \bar{u}u \rangle} = 1 + \frac{m_c}{m_s}\left(\frac{\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle} - 1\right)$$

$$= 1 + (0.56)\frac{m_c}{m_s} \qquad (4.6)$$

The solution to Eqns. (4.3) and (4.6) lead to $m_c/m_s \simeq 2.4$ and $\langle \bar{c}c \rangle / \langle \bar{u}u \rangle = 2.3$. However, this value for m_c/m_s seems rather low and linear breaking can not be justified. In the next section we shall consider the remaining equations involving η, η', η_c mixing and solve for $\langle \bar{c}c \rangle / \langle \bar{u}u \rangle$. Our conclusion is that $\langle \bar{c}c \rangle / \langle \bar{u}u \rangle \gtrsim 5.6$.

This leads to

$$\frac{m_c}{m_s} \gtrsim 3.2 \qquad (4.7)$$

V. GENERAL SOLUTION

In this section we obtain phenomenological solution of the Eqns. (2.9) and (2.11) by considering the equations involving η, η' and η_c mixing in addition to constraints obtained in the last section.

Reviewing, we find that the equations resulting from considerations of π, K, D, and F mesons have yielded a considerable amount of information. They involve 6 equations with 9 unknowns and we obtain values of the 6 symmetry breaking parameters, ε_0, ε_8, ε_{15}, δ_0, δ_8, and δ_{15} and the decay constants F_D and F_F if we know one unknown which can be chosen to be $K \equiv \frac{\delta_{15}}{\sqrt{2}\,\delta_3}$. The value of K in terms of $\frac{\langle \bar{c}c \rangle}{\langle \bar{u}u \rangle}$ can be written as,

$$K \equiv \frac{\delta_{15}}{\sqrt{2}\,\delta_8} = \frac{3}{4}\,\frac{\left[\frac{\langle\bar{c}c\rangle}{2\langle\bar{u}u\rangle} - \frac{1}{2} - \frac{1}{3}(F_K - 1)\right]}{\left[\frac{\langle\bar{s}s\rangle}{2\langle\bar{u}u\rangle} - \frac{1}{2}\right]}$$

$$= 2.6786\left[\frac{\langle\bar{c}c\rangle}{2\langle\bar{u}u\rangle} - .593\right] \tag{5.1}$$

where we expect K to be large and positive number.

We now turn to the η-mixing Eqns. (2.9) and (2.11) and obtain solutions as a function of K. We shall see that not all values of K are allowed.

Consider Eqn (2.9). We can eliminate F's which are involved linearly in favour of $\sqrt{Z}$'s. It is useful here to define new variables x_1, x_2, x_3, ... x_6.

$$x_1 = \frac{Z^0_\eta}{M_\eta^2} + \frac{Z^0_{\eta'}}{M_{\eta'}^2} + \frac{Z^0_{\eta_c}}{M_{\eta_c}^2}$$

$$x_2 = \frac{Z^8_\eta}{M_\eta^2} + \frac{Z^8_{\eta'}}{M_{\eta'}^2} + \frac{Z^8_{\eta_c}}{M_{\eta_c}^2}$$

$$x_3 = \frac{Z^{15}_\eta}{M_\eta^2} + \frac{Z^{15}_{\eta'}}{M_{\eta'}^2} + \frac{Z^{15}_{\eta_c}}{M_{\eta_c}^2}$$

$$x_4 = \frac{\sqrt{Z^0_\eta Z^8_\eta}}{M_\eta^2} + \frac{\sqrt{Z^0_{\eta'} Z^8_{\eta'}}}{M_{\eta'}^2} + \frac{\sqrt{Z^0_{\eta_c} Z^8_{\eta_c}}}{M_{\eta_c}^2}$$

$$X_5 = \frac{\sqrt{Z^{0}_{\eta} Z^{15}_{\eta}}}{M_{\eta}^{2}} + \frac{\sqrt{Z^{0}_{\eta'} Z^{15}_{\eta'}}}{M_{\eta'}^{2}} + \frac{\sqrt{Z^{0}_{\eta_c} Z^{15}_{\eta_c}}}{M_{\eta_c}^{2}}$$

$$X_6 = \frac{\sqrt{Z^{8}_{\eta} Z^{15}_{\eta}}}{M_{\eta}^{2}} + \frac{\sqrt{Z^{8}_{\eta'} Z^{15}_{\eta'}}}{M_{\eta'}^{2}} + \frac{\sqrt{Z^{8}_{\eta_c} Z^{15}_{\eta_c}}}{M_{\eta_c}^{2}} \qquad (5.2)$$

The new equations which take the place of η-mixing equations in Eq. (2.11) are then written in a matrix form, as:

$$\begin{bmatrix} 0 & \frac{\epsilon_0}{\sqrt{2}} - \frac{\epsilon_8}{\sqrt{3}} + \frac{\epsilon_{15}}{\sqrt{6}} & 0 & \frac{\epsilon_8}{\sqrt{2}} & 0 & \frac{\epsilon_8}{\sqrt{6}} \\ \frac{\epsilon_8}{\sqrt{2}} & 0 & 0 & \frac{\epsilon_0}{\sqrt{2}} - \frac{\epsilon_8}{\sqrt{3}} + \frac{\epsilon_{15}}{\sqrt{6}} & \frac{\epsilon_8}{\sqrt{6}} & 0 \\ 0 & 0 & \frac{\epsilon_8}{\sqrt{6}} & 0 & \frac{\epsilon_8}{\sqrt{2}} & \frac{\epsilon_0}{\sqrt{2}} - \frac{\epsilon_8}{\sqrt{3}} + \frac{\epsilon_{15}}{\sqrt{6}} \\ \frac{\epsilon_{15}}{\sqrt{2}} & 0 & 0 & \frac{\epsilon_8}{\sqrt{6}} & \frac{\epsilon_0}{\sqrt{2}} - 2\frac{\epsilon_{15}}{\sqrt{6}} & 0 \\ 0 & \frac{\epsilon_8}{\sqrt{6}} & 0 & \frac{\epsilon_{15}}{\sqrt{2}} & 0 & \frac{\epsilon_0}{\sqrt{2}} - 2\frac{\epsilon_{15}}{\sqrt{6}} \\ 0 & 0 & \frac{\epsilon_0}{\sqrt{2}} - 2\frac{\epsilon_{15}}{\sqrt{6}} & 0 & \frac{\epsilon_{15}}{\sqrt{2}} & \frac{\epsilon_8}{\sqrt{6}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} \frac{\delta_0}{\sqrt{2}} - \frac{\delta_8}{\sqrt{3}} + \frac{\delta_{15}}{\sqrt{6}} \\ \frac{\delta_8}{\sqrt{2}} \\ \frac{\delta_8}{\sqrt{6}} \\ \frac{\delta_{15}}{\sqrt{2}} \\ \frac{\delta_8}{\sqrt{6}} \\ \frac{\delta_0}{\sqrt{2}} - 2\frac{\delta_{15}}{\sqrt{6}} \end{bmatrix}$$

(5.3)

An examination of ε-matrix reveals that the determinant of the matrix is zero, and actually only 5 of the 6 equations are linearly independent. So, it is possible to find the value of the 5 of the 6 x's as a function of one of them (chosen as x_2) and δ's, which are known for any given value of K.

Now, since,

$$0 \leq Z^8_{\eta_i} \leq 1 \quad \text{and} \quad \sum_i Z^8_{\eta_i} = Z^8 = 1$$

$$(\eta_i = \eta, \eta' \text{ or } \eta_c) \tag{5.4}$$

the bounds on x_2 are known to be:

$$\frac{1}{M^2_{\eta_c}} \leq x_2 \leq \frac{1}{M^2_\eta} \tag{5.5}$$

Further, x_2 must be close to $\frac{1}{M_\eta^{\,2}}$ because η is known to be nearly an octet.

Once the x's are known we have 6 equations [Eqn. (5.2)] for the 9 Z's. There is one constraint that $\sum_{\eta_i} Z^8_{\eta_i} = 1$. So we need to postulate 2 more reasonable constraints to solve for the individual Z's. Although there exists a lot of choices to select two such constraints, we, here, investigate the one that seems the most reasonable. We demand that η and η′ do not contain any charm quarks. Since $\bar{c}c \propto \bar{q}(\lambda_0 - \sqrt{3}\lambda_{15})q$, this requirement leads to

$$\langle 0 | V_0 - \sqrt{3}\, V_{15} | \eta \rangle = 0 \quad \text{or} \quad \sqrt{Z^0_\eta} = \sqrt{3}\sqrt{Z^{15}_\eta}$$

$$\text{and} \quad \langle 0 | V_0 - \sqrt{3}\, V_{15} | \eta' \rangle = 0 \quad \text{or} \quad \sqrt{Z^0_{\eta'}} = \sqrt{3}\sqrt{Z^{15}_{\eta'}} \tag{5.6}$$

Then the Z's and from them the F's are obtained as functions of $K = \frac{\delta_{15}}{\sqrt{2}\delta_8}$ and x_2. The solutions are found numerically by choosing particular values of K and letting x_2 vary near $\frac{1}{M_\eta^2}$. It was found that the set of equations yield consistent physical solutions only for a very narrow range of x_2. Besides, the solutions do not vary much in this range. This practically makes the whole set of solutions depend only on the value of K. The least value of K for which solutions were found is around 6. We present a Table (Table I) to show the variation of the solutions as a function of K. K = 6 implies a large SU(4) vacuum breaking ($\delta_{15} < 0$; $\delta_{15} \simeq 8.5\ \delta_8$) compared to the almost symmetric vacuum found in broken chiral SU(3) X SU(3) models.

Although we can not determine the value of K from these equations, we feel that K = 6~7 represents a reasonable solution. Larger values of K would mean extremely large SU(4) breaking in the vacuum which would not be reasonable in view of the validity of approximate SU(4) classification of states. We list below the various symmetry breaking parameters, as well as the decay constants that emerge from our solution. (Complete solution can be found in Appendix A)

$$K = 6 \qquad \frac{\delta_{15}}{\delta_8} \simeq 8.5 \qquad \frac{m_c}{m_s} \simeq 3.2$$

$$\frac{\langle \bar{c}c \rangle}{\langle \bar{u}u \rangle} \simeq 5.67 \qquad \frac{\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle} \simeq 1.56 \qquad \frac{m_s}{m_u} \simeq 33.5$$

$$\frac{F_D}{F_\pi} \simeq 0.974 \qquad \frac{F_F}{F_\pi} \simeq 1.056 \qquad \frac{F_K}{F_\pi} \simeq 1.28$$

We notice that although there is large SU(4) breaking both in the Hamiltonian and in the vacuum, F's retain their approximate SU(4) symmetry. This prediction can be tested experimentally by direct measurement of F_D and F_F.

From our solutions we can also obtain the expectation value of the operator V between vacuum and η, η' or η_c states. From Eqn. (2.9) we find, in $(GeV)^3$

$$g_\eta = 0.0231$$

$$g_{\eta'} = -0.6191$$

$$g_{\eta_c} = 0.0014$$

Since these are expectation values of SU(4) singlet operator which arises from QCD effects, it may be possible to verify them from direct calculation in the future. Here we observe that the contribution from η and η_c are small because these states are not predominately singlets, while η' is large as expected.

VI. RESULTS

We have found a good phenomenological solution for the broken chiral SU(4) X SU(4) model that incorporates the masses of the charmed pseudoscalar mesons D and F and η_c exactly. The values for the symmetry breaking parameters reveal that the vacuum is not a SU(4) singlet and a large value for the ratio of the vacuum expectations of the scalar densities, u_{15} to u_8 was observed. Further, the renormalization constants $\sqrt{Z_i}$'s for the pseudoscalar meson wave functions are found not to be SU(4) symmetric although the SU(3) symmetry is preserved.

From the observed D and F meson masses we reached a strong constraint on the mass ratio $\frac{m_c}{m_s}$ of the 'current' quarks $\frac{m_c}{m_s} < 5$. With two more plausible assumptions, namely that η and η' do not contain any charm quarks, we obtain $\frac{m_c}{m_s} \gtrsim 3.2$, $F_D \gtrsim 0.974\, F_\pi$ and $F_F \gtrsim 1.056\, F_\pi$. This value of $\frac{m_c}{m_s}$ comes very close to the value Georgi and Politzer[11] found from renormalization group consideration in QCD. This value differs sharply from the linear or quadratic mass fitting for SU(4) multiplets, both of which give much larger values for this ratio.

Table I

$K = \frac{\delta_{15}}{\sqrt{2}\delta_8}$	F_D/F_π	F_F/F_π	$\frac{\sqrt{Z}_D}{\sqrt{Z}_\pi}$	$\frac{m_s}{m_u}$	$\frac{m_c}{m_s}$	$\frac{\langle\bar{s}s\rangle}{\langle\bar{u}u\rangle}$	$\frac{\langle\bar{c}c\rangle}{\langle\bar{u}u\rangle}$
0	0.430	0.540	2.545		1.884		1.186
1	0.541	0.644	2.711		2.233		1.933
2	0.642	0.793	2.867		2.508		2.679
3	0.734	0.827	3.015		2.730		3.428
4	0.819	0.908	3.157		2.914		4.173
5	0.899	0.984	3.292		3.058		4.919
6	0.974	1.056	3.421	33.5	3.199	1.56	5.656
7	1.045	1.124	3.547		3.311		6.413
8	1.113	1.189	3.667		3.410		7.159
9	1.177	1.251	3.784		3.496		7.906
10	1.238	1.310	3.898		3.572		8.653
11	1.297	1.367	4.008		3.640		9.399
12	1.354	1.422	4.115		3.701		10.146

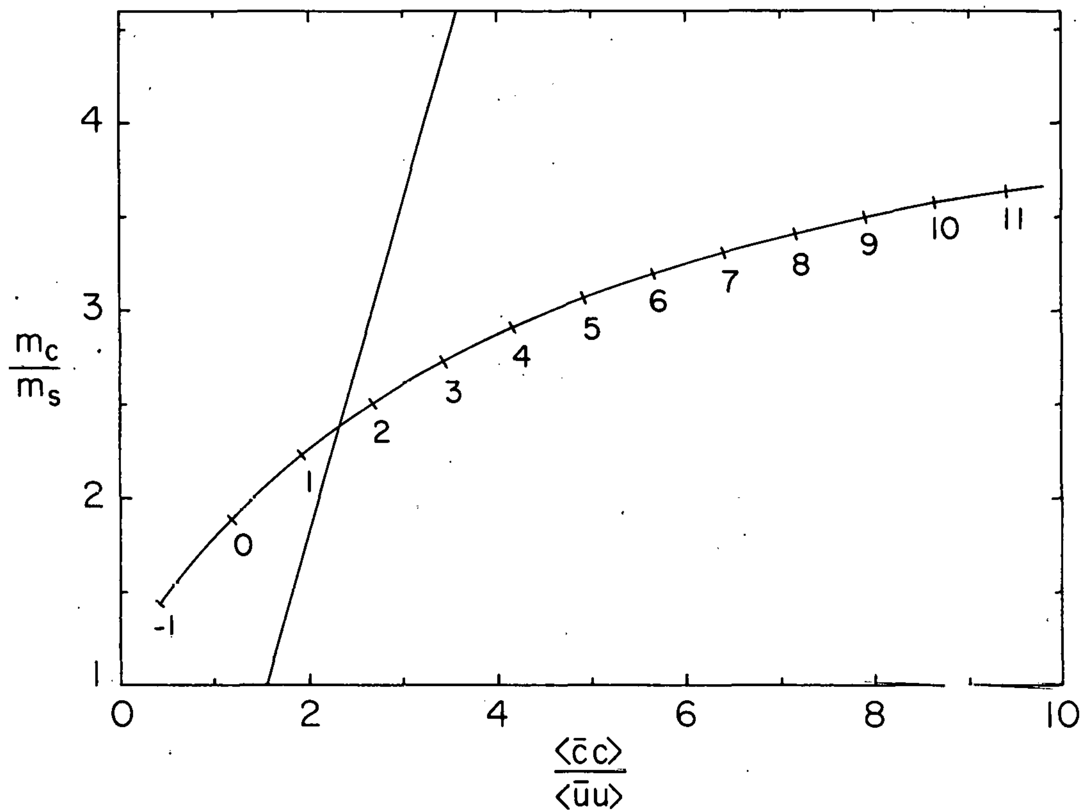
APPENDIX A

In Section IV and V we have discussed how the solution to Eqns. (2.9) and (2.11) is obtained. Our inputs were the masses M_π =135 MeV, M_K =496 MeV, M_D =1.862 GeV, M_F = 2.0395 GeV, M_η = 549 MeV, $M_{\eta'}$ = 958 MeV, $M_{\eta c}$ = 2.83 GeV; the decay constants of π and K mesons in units of F_π, F_π = 1, F_K = 1.28. Here we list the complete set of parameters for our solution with $K \equiv \frac{\delta_{15}}{\sqrt{2}\,\delta_8} = 6$ in units of $\sqrt{Z_\pi}$, F_π and M_π.

	$\varepsilon_0 = 50.52$	$\varepsilon_8 = -18.80$	$\varepsilon_{15} = -58.47$
	$\delta_0 = 3.262$	$\delta_8 = -0.3233$	$\delta_{15} = -2.4743$
	$\frac{\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle} = 1.56$	$\frac{\langle \bar{c}c \rangle}{\langle \bar{u}u \rangle} = 5.67$	
	$\frac{m_s}{m_u} = 33.5$	$\frac{m_c}{m_s} = 3.2$	
$F_\pi = 1$	$F_K = 1.28$	$F_D = 0.974$	$F_F = 1.056$
	$F^8_\eta = 1.456$	$F^8_{\eta'} = 0.0611$	$F^8_{\eta_c} = 0$
	$F^{15}_\eta = -0.4998$	$F^{15}_{\eta'} = -.0233$	$F^{15}_{\eta_c} = -1.019$
	$F^0_\eta = -.0346$	$F^0_{\eta'} = -7.369$	$F^0_{\eta_c} = 0.5884$
$\sqrt{Z_\pi} = 1$	$\sqrt{Z_K} = 1$	$\sqrt{Z_D} = 3.42$	$\sqrt{Z_F} = 3.42$
	$\sqrt{Z^0_\eta} = -0.1507$	$\sqrt{Z^0_{\eta'}} = -0.1542$	$\sqrt{Z^0_{\eta_c}} = 1.981$
	$\sqrt{Z^8_\eta} = 0.943$	$\sqrt{Z^8_{\eta'}} = 0.016$	$\sqrt{Z^8_{\eta_c}} = -0.333$
	$\sqrt{Z^{15}_\eta} = -0.087$	$\sqrt{Z^{15}_{\eta'}} = -0.039$	$\sqrt{Z^{15}_{\eta_c}} = -4.416$
	$g_\eta = 13.748$	$g_{\eta'} = -369.26$	$g_{\eta c} = 0.8106$

FIGURE CAPTION

Fig. 1. The curve represents the variation of $\frac{m_c}{m_s}$ as a function of $\frac{\langle\bar{c}c\rangle}{\langle\bar{u}u\rangle}$ which is obtained by using the masses of D and F mesons as input. The numbers on the curve are the values of the parameter K defined to be $\frac{\delta_{15}}{\sqrt{2}\,\delta_3}$. The straight line represents linear breaking for the vacuum expectation values.



4
3
2
1
$\frac{m_c}{m_s}$
0
2
4
6
8
10
$\frac{\langle \bar{c}c \rangle}{\langle \bar{u}u \rangle}$
-1
0
1
2
3
4
5
6
7
8
9
10
11

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