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COMPUTER MODELING OF PIEZORESISTIVE GAUGES

MASTER

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COMPUTER MODELING OF PIEZORESISTIVE GAUGES*

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A computer model of a piezoresistive gauge subject to shock loading is developed. The time-dependent two-dimensional response of the gauge is calculated. The stress and strain components of the gauge are determined assuming elastic-plastic material properties.

The model is compared with experiment for four cases. An ytterbium foil gauge in a PPMA medium subjected to a 0.5 Gp plane shock wave, where the gauge is presented to the shock with its flat surface both parallel and perpendicular to the front. A similar comparison is made for a manganin foil subjected to a 2.7 Gp shock.

The signals are compared also with a calibration equation derived with the gauge and medium properties accounted for but with the assumption that the gauge is in stress equilibrium with the shocked medium.

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I. Introduction

This study of the dynamic and electrical response of piezoresistive gauges employs the two-dimensional, Lagrangian, finite difference code DYNA2D.[1] The code models the time dependent stress and strain state of the gauge foil. These results are then edited to give electrical response of the gauge as a function of time.

The basis for modeling the gauge signal are piezoresistance equations of the type discussed by Grady and Ginsburg.[2] These equations allow a separation of the gauge signal into a stress related component (reversible) and a strain related component (possibly hysteretic). The calculated signal is then compared with experiments.

The calculated gauge signals are compared with experiments. Two experiments involve ytterbium gauges embedded in a block of PMMA. One gauge foil is in a plane parallel to an incident 5.0 kbar plane shock. The second experiment is with the shock incident on the edge of the gauge foil. A similar calibration study is done with manganin gauge elements embedded in teflon subjected to a 27.5 kbar shock.

It is found that a von Mieses model of the material properties is sufficient to account for the gauge response. The results, however, are sensitive to the assumed values of yield strength.

This approach allows us to calculate the separate stress and strain components of the signals. Although the calibration equations, applied to the gauge as a whole, are invariant under rotation about the direction of current flow, experiments show different signals for different orientations. We are able to resolve this paradox by calculating the dynamic responses of both configurations.

II. Basic Equations

Under a uniform applied stress a piezoresistive material will undergo a resistance change due to change in the resistivity of the sample as well as to change in its dimensions. To first order in strain the resistance change in the direction of the third principal axis is:

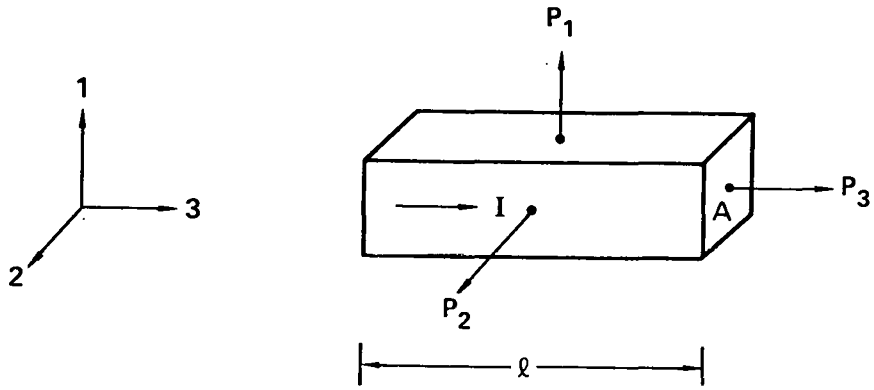
$$\frac{\delta R_3}{R_0} = \frac{\delta \rho}{\rho_0} + \frac{\delta \ell}{\ell_0} - \frac{\delta A}{A_0} \qquad (1)$$

The first term in Eq. (1) is expressed in terms of the piezoresistivity tensor. If the conductor has cubic crystal structure, the tensor has three independent elements. If, in addition, the sample

12s polycrystalline and isotropic only two of the elements are independent. We shall assume the gauge foils are isotopic and polycrystalline so that

$$\frac{\delta\rho}{\rho_o} = \pi_t (P_1 + P_2) + \pi_\ell P_3 \qquad (2)$$

where $P_{1, 2, 3}$ are the principal stresses in the gauge. We have chosen the length of the sample to lie along the third principal axis as shown in Fig. 1. The last two terms in Eq. (1) represent the effects of the change in length and the change in cross sectional area respectively of the sample.



$$\frac{\delta R}{R_0} = \pi_t(P_1 + P_2) + \pi_\ell P_3 + \frac{\delta\ell}{\ell_0} - \frac{\delta A}{A_0}$$

Fig. 1: Relationship between current, principal stresses, and sample dimentions in piezoresistance equation.

If the strain on the piezoresistive sample is hydrostatic, Eq. (1) with Eq. (2), and $-P = P_1 = P_2 = P_3$ gives

$$\frac{\delta R_3}{R_o} = -(\pi_\ell + 2\pi_t) P + \frac{1}{3B} P \qquad (3)$$

where B is the bulk modulus of the sample. Eq. (3) is useful for obtaining a condition on the tensor elements and . A second condition is necessary to completely determine the piezoresistive properties of the sample.

If we assume the gauge sample is hit by a planar shock wave propagating in a direction orthogonal to the direction of current flow, we can assume the gauge undergoes a condition of uniaxial strain, which is expressed by

$$P_3 = P_2 = \frac{\nu}{1-\nu} P_1 \qquad (4)$$

where is Poissons ratio. For a foil in equilibrium with these stresses the gauge signal will be

$$\frac{\delta R_3}{R_o} = \left[(\pi_\ell - \pi_t) \frac{\nu}{1-\nu} + \pi_t \frac{1+\nu}{1-\nu}\right] P_1 - \frac{1}{3B} \frac{1+\nu}{1-\nu} P_1 \qquad (5)$$

According to the von Mieses yield criterion, for stresses above the onset of plasticity

$$\nu = \frac{1- |P_1|/Y}{1-2|P_1|/Y} \qquad (6)$$

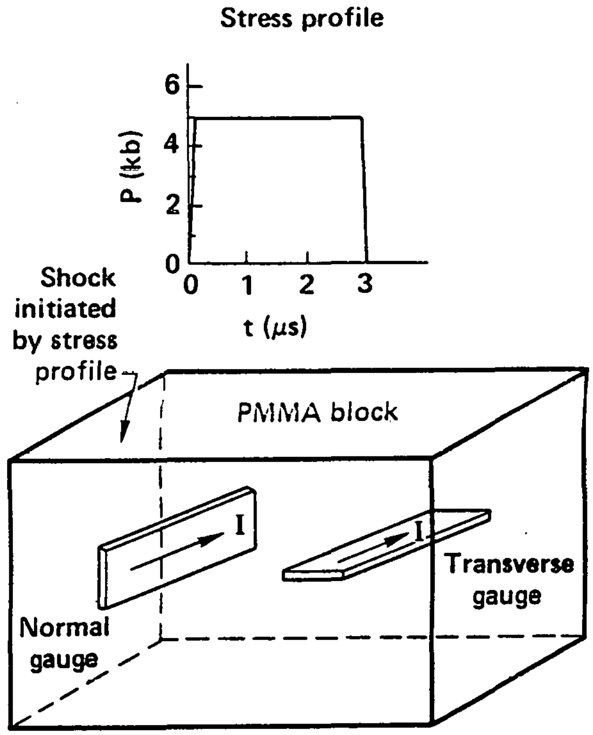
and below the yield stress, takes on its low stress value. Eqs. (5) and (6) together can be used to estimate the response of a gauge under shock conditions above and below the Hugoniot elastic limit.

No assumption about the orientation of the gauge with respect to the shock front has been made in Eq. (5) except the direction of current flow is fixed orthogonal to the direction of shock propagation. It has been observed for foil gauges that rotating the plane of the foil with respect to the plane of the shock has a substantial effect on the signal.[3,4] Our calculations show the assumption of stress equilibrium underlying Eq. (5) does not obtain in the transverse orientation.

III. Code Calculations

The transverse dimensions of the foils in this study are much smaller than their length. The ytterbium foils are 2.5 x 0.2 x 0.005 cm and the manganin foils are 0.2 x 0.07 x 0.0025 cm. In order to model these gauges on a two dimensional computer code we shall assume the gauges are of infinite length. Thus, we can model effects associated with the edge of the foil but not effects associated with the ends.

Figure 2 represents schematically the two experiments with ytterbium gauges performed by Gupta et al.[3] We model these experiments separately with 2000 elements in each gauge in the form of a filament parallel with the current flow. The zoning is uniform throughout the gauge foil. The resistance of the foil is the resistance of its elements in parallel.



Gauge dimensions: 2.5 × 0.2 × 0.005 cm

Fig. 2: Diagram of computer model of ytterbium gauge experiments. Shock is initiated by incident pressure on the left hand surface of PMMA block.

If the foil is composed of N elements all of the same size, the relative change in resistance of the foil is

$$\frac{\delta R}{R_o} = N\left(\sum_{i=1}^{N} \frac{1 + \delta A^i/A_o}{1 + \delta\rho^i/\rho_o}\right)^{-1} - 1 \qquad (7)$$

The sum is taken over all gauge elements and the change in resistivity of each element $\delta\rho^i/\rho_o$ is calculated using Eq. (2).

Material Properties

The results of these calculations depend strongly on the constitutive properties of the foils and the matrix in which they are embedded. Of course, the calculated gauge signals are sensitive to the choice of piezoresistive coefficients. Grady and Ginsberg[2] found that their measurements with ytterbium foils were consistent with and being equal. Combining this result with the hydrostatic measurements Lilley and Stephens[5] one obtains

$$\pi_t = \pi_\ell = -2.05 \times 10^{-2} - 7.8 \times 10^{-4} P \text{ (pressure in k bar)} \quad (8)$$

for ytterbium.

For PMMA, we use a pressure volume relationship taken from high strain rate experiments of Gupta et. al.[6] and assign an elastic limit of 4.2 kbar[7].

The pressure volume relationship for ytterbium used in our calculation comes from the low pressure shock velocity-particle velocity data of Gust and Royce.[8] The yield strength of ytterbium is taken to be 0.5 kbar.[9]

The Hugoniot pressure-volume relationship for manganin is taken from the shock velocity-particle velocity data of Keough and Wong.[10] New measurements[11] of the Hugoniot elastic limit give a yield strength for manganim of 5.58 kbar.

The manganin gauge foils were embedded in a block of PTFE (teflon). The constitutive properties were taken from a standard form for the equation of state of teflon in use at Lawrence Livermore Laboratory.[7] The pertinent constitutive properties used in our calculations are summarized in Table I.

MATERIAL	SHEAR MODULUS	YIELD STRENGTH	HUGONIOT DATA
PMMA	23	4.2	$p = 68 + .68^2 + 2430^3$
YTTERBIUM	73	0.5	$U_s = 1.49 + .78U_p$ (cm/μs)
PTFE	35.5	2.0	$p = 60.7 + 57.7^2 + 13.3^3$
MANGANIN	419	5.58	$U_s = 3.79 + 1.73\ U_p$ (cm/μs)

Table 1: Material properties used in calculation of gauge response. Units are kbar and cm/μs - $\mu = \rho/\rho_0 - 1$.

IV. Results

A. Ytterbium Gauges

The results of our calculation for ytterbium gauges is summarized in Fig. 3. The strain component of the gauge signal is 0.028 for the normal gauge end 0.020 for the transverse gauge. The strain is purely volumetric in a two-dimensional calculation so gauge stretching does not occur. Clearly such deformation is not necessary to account for the signal.

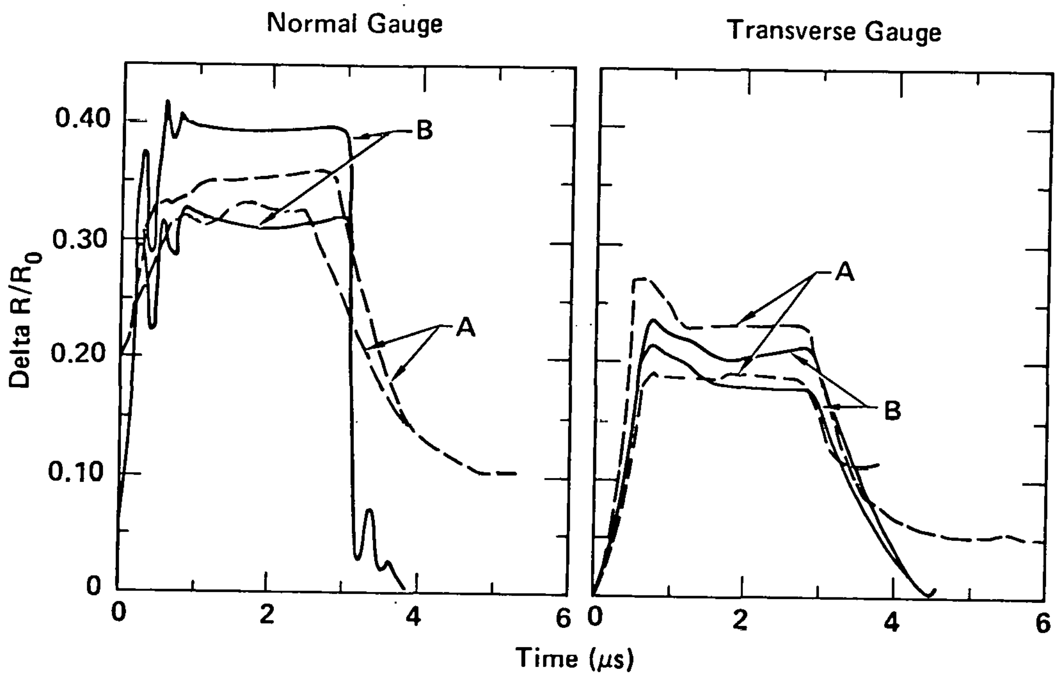


Fig. 3: Comparison of measured response of ytterbium gauges (A) with calculated response (B). The two calculated curves show the effect of varying the yield strength of ytterbium. The higher curves correspond to 0.5 Kbar and the lower curves to 1.0 kbar.

The "free field" stress state in the PMMA block is shown in Fig. 4. The longitudinal principal stress is -5 kb, the transverse principal stress is -2.8 kbar, and the average stress is 3.6 kbar. In the gauge itself, a stress state temporarily higher than 5 karb occurs on the front surface as the flow stagnates. Unlike the "free field" stress state, the stress field inside the gauge is nearly isotropic since the ytterbium is well into the plastic condition. Equation (6) assigns the ytterbium a Poisson ratio of 0.47 at a stress level of 5 kbar.

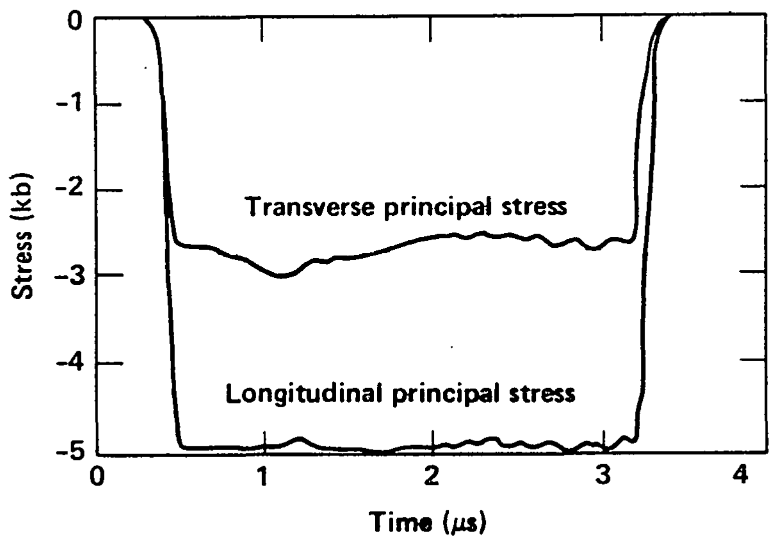


Fig. 4: Stress field in PMMA before interacting with gauge.

The stress field in the transverse gauge is more complicated. It is nearly isotropic, but the average stress (which is responsible for the piezoresistance under the assumptions of Eq. (8)) varies from leading edge to trailing edge as shown in Fig. 5. The shock wave propagates more rapidly in the PMMA than in the foil. As a result the transverse gauge is not uniformly stressed in contrast with the normal gauge for which the stress field is uniform.

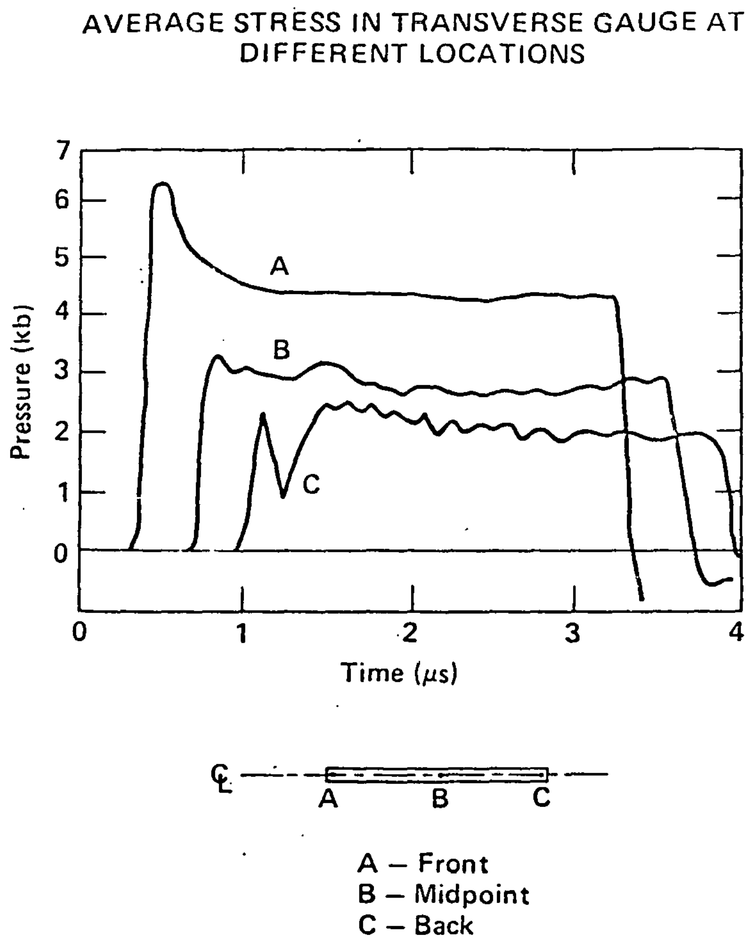


Fig. 5: Average Stress in Transverse Gauge at Different Locations

B. Manganin Gauges

The principal differences between these experiments and the ones just discussed for ytterbium are material properties, foil dimensions, and shock strength. The manganin foils are 0.07 x 0.2 x 0.0025 cm, embedded in teflon and subjected to 27.5 kbar planar shock. A comparison of these calculations with experiment are shown in Fig. 6.

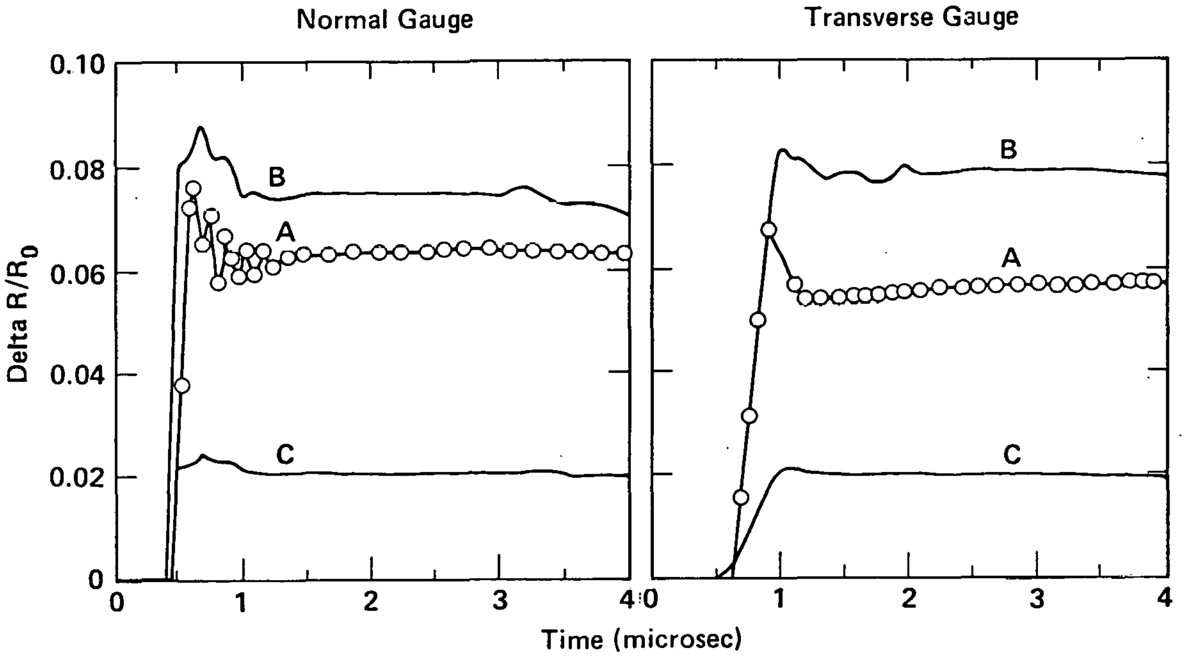


Fig. 6: Comparison of measured signals in manganin, A, with calculated signals, B, for normal and transverse orientations. Curves marked C, are the strain component of the signal.

The calculated gauge signals are obtained using

$$\pi_t = \pi_\ell = -7.042 \times 10^{-4} - 1.101 \times 10^{-7}\, p \quad \text{(pressure in k bar)} \qquad (9)$$

The calculated results are about 14% too high for the normal gauge.

Varying the pressure-volume relation and the yield strength of manganin though reasonable values produces only 2 to 3% change in the calculated gauge signal. To account for the discrepancy between calculation and experiment by changes in the piezoresistance tensor elements would require the improbable values of $_t$ +7 and -16 (k bar $)^{-1}$. Thus, by elimination we conclude the discrepancy between measurement and calculation of the manganin gauge signals are most likely due to three-dimensional effects. Referring to Eq. (1) we see that on DYNA2D the lengthwise strains in each element are zero, and the areal strains are equal to the volumetric strains. However, in an isotropic stress field, which is approximately the case in the gauge foil, we expect the principal strains will be approximately equal for a correct three-dimensional calculation. The strain signal, in such a calculation would be one third the volumetric strain:

$$\frac{\delta \ell}{\ell_0} - \frac{\delta A}{A_0} = \epsilon_3 - \epsilon_2 - \epsilon_1 \approx -\epsilon_3 \approx -\frac{1}{3}\,\frac{\delta V}{V_0} \qquad (10)$$

These arguments lead us to expect that a three-dimensional calculation of the gauge signal would yield about one third the value obtained on DYNA2D bringing the calculated normal gauge signal into excellent agreement with measurement.

The calculation of the transverse manganin gauge has the added difficulty that the small foil area presented to the incident shock resulted in flow of the PTFE around the foil before the gauge could be accelerated to the local flow velocity. This caused additional stresses in the foil due to surface traction at the manganin - PTFE interface. As a result, the calculated gauge signal for the transverse foil is higher than for the normal foil. This is in direct contradiction to the measurements which show a lower signal in the transverse configuration.

Our DYNA2D calculations keep the nodes at the manganin -PTFE interface tied when in fact they should slip. We were unsuccessful in finding a slip-surface algorithm which would remain stable under the conditions of the problem. Thus, we are unable as yet to model a transverse gauge response when there is significant flow around the gauge foil. This problem does not seem to affect the calculations of gauges in normal orientation.

In conclusion, we remark that modeling piezoresistive gauges on a computer has shown itself to be a useful tool for interpreting some of the outstanding ambiguities in the gauge signals. The effect of orientation, and the assumptions of stress equilibrium between foil and medium can be successfully studied this way. This technique is primarily limited by uncertainties in the constitutive properties of the foils and medium. We believe present three-dimensional codes such as DYNA3D can account for the strain effects more accurately than DYNA2D. We also believe that the problem presented by flow around the gauge can be solved.

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