THE PLASMA PHSYICS OF TRAPPING OF COSMIC RAYS AROUND SUPERNOVA

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PPPL-1558 June 1979

Presented at the 16th International Cosmic Ray Conference, Kyoto, Japan, 6-18 August 1979.

The Plasma Physics of Trapping of Cosmic Rays Around Supernova

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We discuss two problems associated with the question of whether a large number of cosmic rays produced in the early stages of expansion of a supernova bubble would be trapped in it.

The problems we consider are: (1) the determination of the manner in which the cosmic ray anisotropy and the Alfvén wave energy density, driven unstable by this anisotropy evolve towards equilibrium balance. (2) How can the singularity associated with standard quasilinear theory for resonant interaction, which occurs for pitch angles near 90°, be treated by inclusion of other nonlinear effects? Are wave amplitudes of turbulence estimated from the results of problem (1) large enough that we may conclude that trapping still occurs when this singularity is properly treated? As pointed out recently[1] this problem is exacerbated by cyclotron damping of waves resonant with near 90° pitch angle particles. As we show the problem, although serious, does not prevent trapping under most conditions that one can imagine for the expanding supernova bubble and for the number of cosmic rays injected into it.

Assume that cosmic rays are produced in the earlier stages of a supernova explosion. Assume, further, that they have managed by some mechanism to move across the magnetic field from the debris region onto the magnetic field lines in the shock-heated interstellar medium. In the absence of instabilities, they could propagate freely along the field lines and escape into the cool unshocked interstellar medium. Those cosmic rays with small pitch angle will move faster and reach regions beyond those of the rest of the cosmic rays. The resulting cosmic ray distribution will thus develop an anisotropy distribution with anisotropy δ, which builds up at a rate ν_1 that is of order c/L where c is the speed of light and L a characteristic scale length.

Because of the presence of this anisotropy parallel propagating Alfvén waves of wave number k will be unstable and will amplify with growth rate $\nu_2\delta$ of order[2]

$$\nu_2\delta \approx \Omega_0 \frac{n_{cr}(>p)}{n} \frac{c}{v_A}\delta \equiv \Sigma\delta \quad , \tag{1}$$

where Ω_0 is the nonrelativistic ion gyrofrequency, n is the background density, $n_{cr}(>p_k)$ the number of cosmic rays with total momentum greater than $p_k \equiv eB/kc$ and v_A is the Alfvén speed.

As the intensity $\mathcal{E} = (\delta B)^2_k/B^2$ of fluctuating magnetic fields build ups, they in turn pitch-angle scatter those cosmic rays whose momentum satisfy

$$\mu p = eB/kc \tag{2}$$

where $\mu = \cos\theta$ and θ is the pitch angle. This rate is of order[2] $(\Delta\theta)/t \approx \Omega\mathcal{E}_k\delta$ where Ω is the relativistic ion gyration frequency. $\mathcal{E}_k$ has been normalized so that the total relative fluctuation energy $(\delta B)^2/B^2 = \int_0^\infty \varepsilon_k d\log k$. Thus, anisotropy will be reduced at a rate

$$-\nu_3\delta = -\Omega\varepsilon\delta \quad . \tag{3}$$

The waves will build up until their amplification is balanced by damping processes. In the supernova bubble there is no linear damping of parallel propagating Alfvén waves, except for cyclotron damping at large k . The principle damping is nonlinear Landau damping which reduces ε at a rate of order[3]

$$-\nu_4\varepsilon^2 = \Omega_o \frac{v_i}{c}\varepsilon^2 \tag{4}$$

where v_i is the thermal velocity.

Let us apply these results to describe the time evolution of δ and ε after the cosmic rays have been released in the supernova bubble. We have

$$\frac{d\delta}{dt} = \nu_1 - \nu_3\varepsilon\delta \quad , \tag{5a}$$

$$\frac{d\varepsilon}{dt} = \nu_2\varepsilon\delta - \nu_4\varepsilon^2 \quad . \tag{5b}$$

These equations refer to δ at a particular momentum and ε at a particular k . The right-hand sides vanish when

$$\varepsilon = \varepsilon_o \equiv \left(\frac{\nu_2}{\nu_4}\frac{\nu_1}{\nu_3}\right)^{1/2} \quad , \quad \delta = \delta_o \equiv \left(\frac{\nu_4}{\nu_2}\frac{\nu_1}{\nu_3}\right)^{1/2} \quad . \tag{6}$$

Choosing these as units, $\varepsilon = \varepsilon_o\varepsilon'$, $\delta = \delta_o\delta'$ and setting $t = t_o\tau \equiv (\nu_3/\nu_1\nu_2\nu_4)^{1/2}\tau$ we have

$$\frac{d\delta'}{d\tau} = \frac{\nu_2}{\nu_4}(1-\delta'\varepsilon') = \mu(1-\delta'\varepsilon') \quad , \tag{7a}$$

$$\frac{d\varepsilon'}{d\tau} = \delta'\varepsilon' - \varepsilon'^2 \quad . \tag{7b}$$

We trace the time behavior of the turbulent state in a $\varepsilon' - \delta'$ diagram in Fig. 1. Since $\mu = \nu_2/\nu_4 \gg 1$, the motion in δ' is large compared to that in ε' and the state tends first to the hyperbola $\delta'\varepsilon' = 1$, and subsequently moves at a slower rate along this hyperbola to the equilibrium point $\varepsilon' = \delta' = 1$. (Physically, δ evolves until a balance is reached between quasilinear scattering and growth due to the tendency for escape.) We determine the behavior assuming $\delta' = 0$, $\varepsilon' = \varepsilon_1' \ll \mu^{-1/2}$ initially. Under these conditions $\varepsilon' \ll \delta'$, and, if we neglect ε'^2 in Eq. (7a), the solution for ε' and δ' is

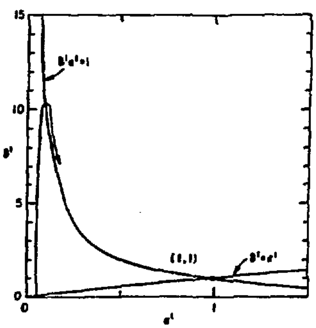

Figure 1

$$\frac{1}{\mathcal{E}'\sqrt{\mu}} = \exp -\frac{1}{2}\left(\frac{\delta'+\mathcal{E}'\mu}{\mu}\right)^2 \left\{\text{const} + \int_0^{(\delta'+\mathcal{E}'\mu)/\sqrt{\mu}} \exp(v'^2/2)dv'\right\} \tag{8a}$$

$$\mu\mathcal{E}' + \delta' = \mu\tau \quad . \tag{8b}$$

We see that δ', $\mathcal{E}'$ approach the $\delta'\mathcal{E}' = 1$ curve in a time $t_o/\sqrt{\mu}$ at $\mathcal{E}' = |\log(\mathcal{E}_i\sqrt{\mu})| << 1$. $\mathcal{E}'$ then varies as $\tanh(\tau+\tau_o)$, approaching equilibrium in a time t_o, that is short compared to the time of evolution of the bubble and the time for cosmic rays to escape.

From this point on, we assume that the nonlinear equilibrium state has been reached and we consider the problem of pitch-angle scattering through 90°. As μ decreases, we see from Eq. (2) that k approaches infinity. Strong linearized damping by cyclotron resonance suppresses all wave numbers greater than $k_o = \Omega_o/v_i$, or $\mu < \mu_c \equiv v_i/c$. For $\mu < \mu_c$, there can be no accumulative small-angle scattering as described by quasilinear theory. It is necessary for a single Alfvén wave packet to scatter a particle through an angle $\Delta\theta \approx v_i/c$ in order to actually turn the cosmic rays around and achieve trapping. As shown below, this occurs when

$$\varepsilon\left(\frac{k}{\mu_c}\right) \geq \left(A\frac{v_i}{c}\right)^2 \tag{9}$$

where A is a constant of order somewhat smaller than unity. Substituting $\mathcal{E} = \mathcal{E}_o$ from the above discussion, we obtain from Eqs. (1), (3), (4), and (6) that trapping will occur if

$$\mathcal{E}_o = \left(\frac{\nu_1\nu_2}{\nu_3\nu_4}\right)^{1/2} > \left(\frac{A\,v_i}{c}\right)^2 \tag{10}$$

or roughly

$$\Sigma\frac{c}{\Omega L} > \left(\frac{v_i}{c}\right)^5 A^4 \quad . \tag{11}$$

To get an idea of the magnitude, of this criterion for trapping, assume that 10^{52} cosmic rays are uniformly distributed in a sphere of radius R parsecs and that v_i is determined from $(4\pi R^3/3)n_o' m' v_i^2/2 = \frac{1}{2}E_{51}10^{51}$ ergs, where $E_{51}10^{51}$ is the energy of the supernova. Criterion (11) then becomes

$$R^{3.5}n^2 > 1.8\,E_{51}^{2.5}\varepsilon^{.5}A^4 \qquad \text{(trapping)} \tag{12}$$

where ε is the energy of the cosmic rays in GeV and we have assumed an integral spectrum of $\varepsilon^{-1.5}$.

Thus, if R is of order unity and either n is smaller than 1, or ε is larger than 1, we expect the cosmic rays to escape from the bubble. However, even under these circumstances, they would only escape into the surrounding cool unshocked interstellar medium. Again, $\mathcal{E}$ and δ would quickly approach the equilibrium values given by Eq. (6) and scattering through 90° would occur if

$$\frac{1}{R^2 n} > 4\times 10^{-10}\varepsilon^{.5}T_6^{2.5} \tag{13}$$

where T_6 is the temperature of the surrounding medium in millions of degrees. The condition for reversal of direction of the cosmic rays is satisfied by a large margin (because v_i/c is so small) and the cosmic rays reenter the hot bubble. This situation persists until the radius of the bubble R is large enough to satisfy Eq. (12) after which the cosmic rays will be trapped in the bubble itself.

We now discuss the scattering through 90° and attempt to estimate A. In order to do this, we choose a wave packet for $\delta B/B$ on the shortest-wavelength scale $k_o \equiv \Omega/\mu_c c$,

$$\frac{\delta B_x}{B} = \delta_o \exp(-k_o^2 z^2/2) \sin k_o z \quad . \tag{14}$$

Then to a good approximation for $\mu_c << 1$, the z motion of a particle with initial velocity $v_o = \Omega/k_o$ and phase ϕ is

$$\frac{d^2\zeta}{d\tau^2} = -\frac{\delta}{\mu_c} \sin\zeta \sin(\tau+\phi) \exp(-\zeta^2/2) \tag{15}$$

where $\Omega t \equiv \tau$, $\zeta \equiv k_o z$, and $d\zeta/d\tau = 1$ initially. The final velocities are plotted in Fig. 2 as a function of ϕ for $\delta/\mu_c = 1$ and 2. It is seen that for $\delta/\mu_c = 1$ at least some of the particles are reflected and for $\delta/\mu_c = 2$ many of them are reflected. Now the Fourier transform of Eq. (14) gives for our definition of ε_k

$$\varepsilon_k = \frac{2\delta^2}{k_o L_1} \frac{k}{k_o} \exp\left(-\frac{1}{2} \frac{k^2+k_o^2}{k_o^2}\right) \sinh^2\left(\frac{k}{k_o}\right)$$

$$\equiv \frac{\delta^2}{k_o L} f\left(\frac{k}{k_o}\right) \quad , \tag{17}$$

if we assume there are $1/L_1$ such randomly phased wave packets per unit length. To get A in Eq. (9), we maximize $\mathcal{E}/\mu^2$ over k

$$\frac{\mathcal{E}}{\mu^2} = \left(\frac{\delta}{\mu_c}\right)^2 \frac{1}{k_o L_1} \frac{k^2}{k_o^2} f\left(\frac{k}{k_o}\right) \tag{18}$$

so $(\mathcal{E}/\mu^2)_{max} = 1.45(\delta/\mu_c)^2/k_o L$. Taking $\delta/\mu_c = 1$ from Fig. 2 and recognizing that $k_o L$ cannot be smaller than 5, we get an upper bound for A of 0.53.

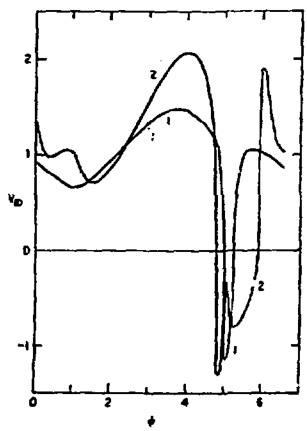

Figure 2

In summary, we have shown how the trapped cosmic rays approach a nonlinear equilibrium in which the growth of anisotropy due to the tendency to escape is balanced by quasilinear scattering by Alfvén waves while the linear growth of Alfvén waves is balanced by nonlinear damping. We have also examined conditions for scattering of cosmic rays near pitch angle of 90° by higher order nonlinear processes and have established conditions under which this can actually occur, Eqs. (12) and (13).

This work was supported by the NSF grant No. AST 76-80800, U.S. AFOSR Contract No. F 44620-75-C-0037, and U.S. DoE Contract No. EY-76-C-02-3073.

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