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By

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ABSTRACT

It is shown that the finite $k_\parallel$ linear theory of absorption predicts: first, that the Doppler effect splits the $k_\parallel = o$ resonance into two closely spaced resonances instead of the usual Gaussian broadening; and second, that although the total absorption is due to the finite size of the electron Larmor orbits, it is mainly determined by $T_\parallel$ and is only weakly dependent on $T_\perp$ via cyclotron overstability type terms. Some consequences of these unique features on plasma heating and rf current drive are also examined.

Recent success in producing high-power gyrotrons has dramatically raised the importance of electron-cyclotron resonance heating (ECRH) for tokamak research. In comparison with other supplementary heating schemes, ECRH has several major advantages: quasi-optical propagation allows for low-loss power transmission; simple antenna structures with small dimensions facilitate the engineering; penetration to the ECR zone is relatively independent of density, temperature, and conditions at the plasma edge; and, power deposition is localized near the ECR zone. The localized absorption property of ECRH permits local power deposition much higher than the ohmic value in tokamak plasmas, and hence offers a hitherto unattainable degree of control over the evolution of the electron temperature profile which may improve tokamak MHD behavior and confinement. Further applications of ECRH to tokamak problems include studies of electron energy transport, plasma initiation, enhancement of neutral beam heating, improvement of divertor action, steady-state rf current drive, etc.

In this paper we are primarily interested in discussing some of the novel and unique features of the ordinary (O) mode ECRH. We will show that the finite $k_{\parallel}$ linear theory of O-mode absorption predicts: first, that the Doppler effect splits the two-fold degenerate $k_{\parallel} = o$ resonance into two closely spaced resonances instead of the usual Gaussian broadening; and second, that although the total absorption is due to the finite size of the electron Larmor orbits, it is mainly determined by $T_{\parallel}$ and is only weakly dependent on $T_{\perp}$ via cyclotron overstability type terms. Here $\parallel$ and $\perp$ refer to directions parallel and perpendicular, respectively, to the confining magnetic field $\vec{B} = B\,\hat{i}_z$. Finally, we will examine some of the consequences of these unique O-mode absorption features on plasma heating, MHD behavior, and steady-state rf current drive in tokamaks.

After Fourier analysis in space and time, the Maxwell electromagnetic field equations yield, for plane waves of the form

$$\vec{E} = \vec{E}_0 \exp[i(\vec{k} \cdot \vec{r} - \omega t)] ,$$

$$\vec{k} \times (\vec{k} \times \vec{E}) + (\omega^2/c^2) \overleftrightarrow{D} \cdot \vec{E} = 0 \quad . \tag{1}$$

Here $\overleftrightarrow{D}(\omega,\vec{k})$ is the hot plasma dielectric tensor.[1-2] The condition for a nontrivial solution of Eq. (1) is obtained by setting the determinant of the coefficient of E_x, E_y, and E_z equal to zero. Thus knowing all the nine dielectric tensor elements D_{ij}, i, j = x, y, and z, one can get a closed form expression for the allowed wave number k as a function of ω. Then from the imaginary part of k (i.e., Im k), one obtains the optical depth τ for the mode of propagation under study.

We now consider the case of O-mode radiation ($\vec{E} \| \vec{B}$) of frequency $\omega \approx \omega_c$ = (eB/mc) propagating through a low-density (n) plasma [$\omega_p = (4\pi ne^2/m)^{1/2} < \omega_c$] nearly perpendicular to $\vec{B}$ (i.e., $k_\perp > k_\parallel$). Then Eq. (1) becomes

$$[-(c^2 k_\perp^{\,2}/\omega^2) + D_{zz}]\, E_z \approx 0 , \tag{2}$$

which gives the dispersion relation

$$\left(c^2 k_\perp{}^2/\omega^2\right) \approx D_{zz} \; . \tag{3}$$

Using the Maxwell-Boltzmann zero-order distribution, and taking the large argument expansion for the real part of the dispersion function,[1-2] the dielectric tensor element D_{zz} may be written

$$D_{zz} = D_{zz}^{(c)} + \Delta_{zz} = \left[1 + \omega_p^2\, \chi_0'\right] + \Delta_{zz} \; , \tag{4}$$

where

$$\Delta_{zz} = \lambda\omega_p^2 \left(-\chi_0' + \left[\frac{\omega - \omega_c}{2\omega}\right] \left[1 - \frac{\omega_c}{\omega} \left\{1 - \frac{T_\parallel}{T_\perp}\right\}\right] \chi_{-1}' \right.$$

$$\left. + \left[\frac{\omega + \omega_c}{2\omega}\right] \left[1 + \frac{\omega_c}{\omega} \left\{ 1 - \frac{T_\parallel}{T_\perp} \right\}\right] \chi_1' \right) , \tag{5}$$

$$\chi_\ell(\omega) = P\,\frac{1}{\omega + \ell\omega_c} - \frac{i\pi^{1/2}}{|k_\parallel|(2\kappa T_\parallel/m)^{1/2}} \; \exp\left[-\frac{m}{2\kappa T_\parallel} \left(\frac{\omega + \ell\omega_c}{k_\parallel}\right)^2 \right] \; . \tag{6}$$

The prime denotes differentiation with respect to the argument ω, $\lambda = (k_\perp^2\, \kappa T_\perp/\omega_c^2 m)$, and P denotes the principal value. In Eq. (4), $D_{zz}^{(c)}$ is the cold plasma dielectric tensor element appropriate to the retarded boundary conditions, and Δ_{zz} is the resonant hot-plasma contribution. From Eq. (6), it is relatively easy to see that in the limit $|k_\parallel| \to 0$, $\mathrm{Im}\ \chi_\ell(\omega) \to -\pi\delta(\omega + \ell\omega_c)$, and

$$\mathrm{Im}[(\omega + \ell\omega_c)\ \chi_\ell'(\omega)] \to -\pi(\omega + \ell\omega_c)\ \delta'\ (\omega + \ell\omega_c) = \pi\ \delta(\omega + \ell\omega_c)$$

since the Dirac δ-function satisfies the relation $x\ \delta'(x) = -\delta(x)$. Thus, the real and imaginary parts of D_{zz} are, of course, related to each other through the well-known Kramers-Kronig relations[3] as a consequence of the laws of causality (i.e., the effect should not precede the cause).

From Eqs. (3) to (6), writing $k_\perp = \mathrm{Re}\ k_\perp + i\ \mathrm{Im}\ k_\perp$ and assuming that $\mathrm{Re}\ k_\perp > \mathrm{Im}\ k_\perp$, we obtain

$$\left(\frac{c\ \mathrm{Re}\ k_\perp}{\omega}\right)^2 = \mathrm{Re}\ D_{zz} = 1 - \frac{\omega_p^2}{\omega^2}\left(1 + p\ \frac{k_\perp^2\ \kappa T_\parallel/m}{\omega^2 - \omega_c^2}\right) \approx 1 - \frac{\omega_p^2}{\omega^2}\ , \tag{7}$$

and

$$2\mathrm{Im}\ k_\perp = \left(\frac{\omega^2 \mathrm{Im}\ D_{zz}}{c^2\ \mathrm{Re}\ k_\perp}\right) \approx \left(1 - \frac{\omega_p^2}{\omega^2}\right)^{1/2}\left(\frac{\kappa T_\perp \omega^2 \omega_p^2}{2\ mc^3 \omega_c^2}\right)$$

$$\left(\left[1 - \frac{\omega_c}{\omega}\left\{1 - \frac{T_\parallel}{T_\perp}\right\}\right](\omega - \omega_c)\ \mathrm{Im}\ \chi_{-1}' + \left[1 + \frac{\omega_c}{\omega}\left\{1 - \frac{T_\parallel}{T_\perp}\right\}\right](\omega + \omega_c)\mathrm{Im}\ \chi_1'\right)\ . \tag{8}$$

From Eq. (6), we get

$$\text{Im}\ \chi'_\ell(\omega) = \frac{2\pi^{1/2}(\omega + \ell\omega_c)}{|k_\parallel| k_\parallel^2 (2\kappa T_\parallel/m)^{3/2}} \exp\left[-\frac{m}{2\kappa T_\parallel}\left(\frac{\omega + \ell\omega_c}{k_\parallel}\right)^2\right] . \quad (9)$$

It is interesting and physically instructive to note that in the limit $|k_\parallel| \to 0$, Eq. (8) becomes

$$2\text{Im}\ k_\perp \approx \left(1 - \frac{\omega_p^2}{\omega^2}\right)^{1/2} \left(\frac{\pi\kappa T_\parallel \omega^2 \omega_p^2}{2mc^3 \omega_c^2}\right) \left[\delta(\omega - \omega_c) + \delta(\omega + \omega_c)\right] . \quad (10)$$

The absorption coefficient per unit path length $\alpha = 2\ \text{Im}k \approx 2\ \text{Im}\ k_\perp$ for $k_\perp >> k_\parallel$. Thus from Eqs. (8) and (9), it is seen that for finite $k_\parallel$ there are absorption resonances having maximum values at $|\omega| \approx |\omega_c| \pm k_\parallel (2\kappa T_\parallel/m)^{1/2}$ and that the absorption approaches zero as $\omega \to \pm\ \omega_c$ and also for $|\omega \pm \omega_c| \to \infty$. That is, for finite $k_\parallel$ the Doppler effect splits the two-fold degenerate $k_\parallel = 0$ resonance of Eq. (10) into two closely spaced resonances of Eq. (8) instead of the usually expected Gaussian broadening. This splitting is illustrated in Fig. 1 for three different angles of propagation. The frequency integral of these curves is independent of θ and yields a value of τ of 3.8 and is the same value obtained from Eq. (10) with R=130 cm. It is also seen from Eqs. (8) to (10) that although this absorption is due to the finite size of the electron Larmor orbits [i.e., note the appearance of λ in Eq. (5)], it is mainly determined by $T_\parallel$ [see Eq. (10)] and is only weakly dependent on $T_\perp$ via cyclotron overstability type terms [see Eq. (8)].

The optical depth τ is given by

$$\tau = \int 2 \, \mathrm{Im}\, k_\perp \, |dR| \ , \tag{11}$$

and the power P_a absorbed on a single transit of the absorption region is $P_a \approx P_o \, [1 - \exp(-\tau)]$, where P_o is the incident microwave power. If the wall reflection coefficient r is large, then the fractional absorbed power $F \approx [1 - \exp(-\tau)]/[1 - r \exp(-\tau)]$, and multiple transits will enhance the ECR absorption considerably. For tokamak plasmas the confining magnetic field $B \propto R^{-1}$, where R is the major radius of the torus, and $dR = -(R/\omega_c)d\omega_c$. Then from Eqs. (10) and (11), for near perpendicular propagation of the ordinary wave through the toroidal plasma, we get

$$\tau \approx \left(1 - \omega_p^2/\omega^2\right)^{1/2} \left(\pi R \omega_p^2 \kappa T_\parallel / 2\omega \, mc^3\right) . \tag{12}$$

This result of Eq. (12) agrees with those of Fidone et al.,[4] and Antonson and Manheimer,[5] for O-mode with $\omega \approx \omega_c$ when the plasma has no pressure anistropy (i.e., when $T_\perp = T_\parallel = T$), and is also valid for anisotropic plasma when $k_\parallel \approx 0$. Equation (12) was verified in a wave propagation experiment in the PLT.[6] It is seen from Eq. (8) that the cyclotron overstability terms $[1 \pm (\omega_c/\omega) \{1 - (T_\parallel/T_\perp)\}]$ tends to one as $T_\perp \approx T_\parallel$. Hence the result of Eq. (12) is valid also for an isotropic plasma when $k_\parallel$ is small but finite.

For $k_\parallel \approx 0$, the wave absorption is localized near the resonance zone at $R \approx R_c$ where $\omega \approx \omega_c$. However, when $k_\parallel \neq 0$, there exist two closely spaced resonant layers centered at R_1 and R_2, respectively, such that $\omega = \omega_c(R_1) + k_\parallel(2\kappa T_\parallel/m)^{1/2}$ and $\omega = \omega_c(R_2) - k_\parallel(2\kappa T_\parallel/m)^{1/2}$, and there is no absorption at $R_c = (R_1 + R_2)/2$. Thus, $|R_1 - R_2| \approx R_c\ [2k_\parallel(2\kappa T_\parallel/m)^{1/2}/\omega_c]$ and is linearly proportional to $k_\parallel$. This double resonant layer might prove beneficial in suppressing plasma MHD instabilities with certain wavelengths; e.g., for $|R_1 - R_2| \approx p\lambda$ or $(p + 1/2)\lambda$ where p is an integer. Further, in the resonant layer R_1 the wave energy and momentum are transferred to co-moving electrons (i.e., to electrons with z-velocities v_z which are parallel to $\vec{B} = B\,\hat{i}_z$); while in the resonant layer R_2 the wave energy and momentum are transferred to counter-moving electrons (i.e., to electrons with v_z which are antiparallel to $\vec{B}$). This means that in ECR current drive experiments, the induced steady-state current will flow in opposite directions on either side of R_c (i.e., the current at R_1 will be antiparallel to the current at R_2), and there is no rf-induced current at R_c. Thus it appears with finite $k_\parallel$ O-mode ECRH one can control not only the electron pressure profile but also the current profile which may improve tokamak MHD behavior and confinement.

When $T_\perp \neq T_\parallel$ it is extremely difficult to obtain an analytic expression for τ from Eqs. (8) and (11). However, since the dominant absorption occurs only near the resonant layers R_1 and R_2, one can show that for the R_1 layer

$$\tau(R_1) \approx (\tau/2)\ \left[1 + (k_\parallel/\omega_c)\ (2\kappa T_\parallel/m)^{1/2}\ \{(T_\perp/T_\parallel) - 1\}\right] \quad ,$$

and for the R_2 layer

$$\tau(R_2) \approx (\tau/2)\ \left[1 - (k_\parallel/\omega_c)\ (2\kappa T_\parallel/m)^{1/2}\ \{(T_\perp/T_\parallel) - 1\}\right] .$$

Thus, if the first absorption layer is optically thick [i.e., $\tau(R_1$ or $R_2) >> 2$], the wave energy never reaches the second layer. In interpreting future ECRH experiments one must bear this point in mind.

The theory upon which O-mode ECRH experiments in toroidal plasmas are currently based is the linear theory for hot plasmas. For $T_\perp = T_\parallel$, it is shown elsewhere[4] that the result of Eq. (12) is unaltered even if one takes account of the broadening due to the relativistic mass variation when $k_\parallel \approx 0$. Since the relativistic effect broadens the resonance only towards lower values of ω_c, the Doppler splitting will always occur even for small $k_\parallel$. But, when

$$(k_\parallel/k) \leq [\kappa(2T_\perp + T_\parallel)]/[3(mc^2)^{1/2}\ (2\kappa T_\parallel)^{1/2}] ,$$

$\tau(R_1)$ will differ significantly from $\tau(R_2)$ keeping $\tau(R_1) + \tau\ (R_2) = \tau$. We have not taken this effect into account since the gross features presented here are always there even for very small $k_\parallel$. For very small $k_\parallel$ one must take account of the relativistic broadening in evaluating $\tau(R_1)$ and $\tau(R_2)$. In this paper we have examined the results of a finite $k_\parallel$ linear theory as a first step toward prescribing the conditions needed for efficient application of ECRH in tokamak plasmas. However, it should be noted that the quasilinear and nonlinear theories must be developed in conjunction with experiments to determine fully the effectiveness of ECR for the heating regimes of interest and for steady-state rf current drive in tokamaks.

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FIGURE CAPTIONS

FIG. 1 Plots of 2 Im $k_{\perp}$ of Eq. (8) as a function of ω/ω_c for three different angles of propagation: ____ is for $\theta \approx 0.5°$, _ _ _ for $\theta \approx 2°$, and ____ _ ____ for $\theta = 10.0°$, where $(\pi/2 - \theta)$ is the angle between $\vec{k}$ and $\vec{B}$. Conditions are $n = 3 \times 10^{13}$ cm^{-3}, $T_{\perp} =$ 3 keV, $T_{\parallel} = 2$ keV, and $(\omega_c/2\pi) = 9.0 \times 10^{10}$ cps. The frequency integral of these curves is independent of θ and yields a value of τ of 3.8 and is the same as the value obtained from Eq. (10) with R = 130 cm.

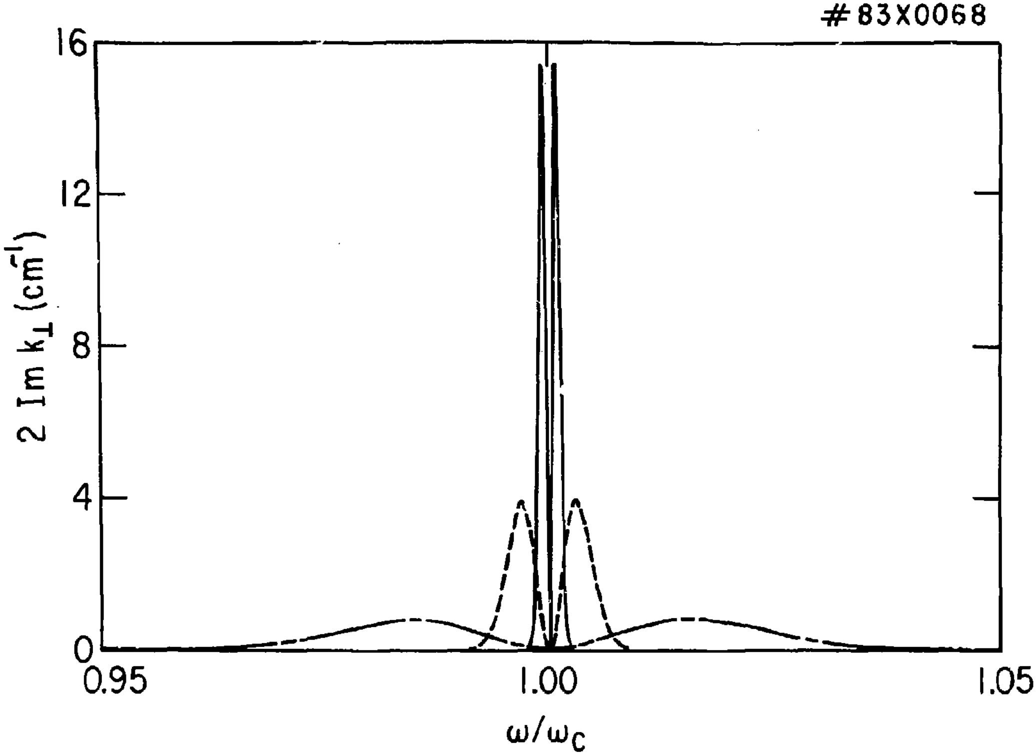

Fig. 1

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