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SUPERCONDUCTING TRAVELING WAVE ACCELERATORS*

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1. Introduction

This note considers the applicability of superconductivity to traveling wave accelerators.[1] Unlike CW operation of a superconducting standing wave or circulating wave[2] accelerator section, which requires improvement factors (superconductor conductivity divided by copper conductivity) of about 10^6 in order to be of practical use, a SUperconducting TRaveling wave Accelerator, SUTRA, operating in the pulsed mode requires improvement factors as low as about 10^3, which are attainable with niobium or lead at 4.2K, the temperature of liquid helium at atmospheric pressure. Changing from a copper traveling wave accelerator to SUTRA achieves the following. (1) For a given gradient SUTRA reduces the peak and average power requirements typically by a factor of 2. (2) SUTRA reduces the peak power still further because it enables us to increase the filling time and thus trade pulse width for gradient. If we lower the group velocity of a copper section and thus increase its fill time above its optimum value, the input gradient will increase but the average gradient will not. With SUTRA, we can

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increase the fill time to much higher values than the optimum fill time of a copper section and not decrease the average gradient. The SUTRA optimum fill time is determined by the ratio of refrigeration factor to improvement factor, and is considerably higher than the copper section optimum fill time. Therefore much lower peak powers are needed to obtain a given gradient. This effect is equivalent to pulse compression without external energy storage. (3) SUTRA makes possible a reasonably long section at higher frequencies. (4) SUTRA makes possible recirculation without additional rf average power.

We will illustrate the advantages of SUTRA by applying it to two specific TW accelerators: to CEBAF[3] proposed by the Southeastern Universities Reseach Association and to a SLAC upgrading. We will show that in both cases SUTRA reduces the peak and average rf power and the AC power required to maintain a given gradient.

2. Section Voltage and Efficiencies

Because the usual expression for section voltage is not practical for both copper and superconducting traveling wave section and for both wide pulse and single bunch operation we will derive alternate expression which are. The unloaded section voltage and the unloaded average gradient are

$$V_o = \sqrt{\eta_s s P_o T_f L} \quad , \quad E_o = \sqrt{\frac{\eta_s s P_o T_f}{L}} \quad . \tag{2.1}$$

Here

η_s = section efficiency

s = elastance per unit length (MΩ/μs/m)

P_o = Power input into the section (MW)

T_f = section fill time (μs)

L = section length (m)

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The section efficiency is the beam voltage divided by the beam voltage of a lossless uniform section. It is a function of fill time and the expression for it is derived in Appendix A.

The elastance per unit length is the square of the electric field divided by the energy stored per unit length. It is analogous to the shunt resistance which is the square of the electric field divided by the power dissipated per unit length. The rationale for using efficiencies and elastance per unit length is given in Appendix C. For a disk-loaded structure the elastance can be approximated by[4]

$$s = 87(M\Omega/\mu s\ m) - 3(M\Omega/m^2)v_{ga} \qquad (2.2)$$

Here $v_{ga} = L/T_f$ = average group velocity along the section in m/μs.

For a constant impedance and a constant gradient section, respectively

$$\eta_s = \frac{(1-e^{-\tau})^2}{\tau^2} \quad , \qquad \eta_s = \frac{1-e^{-2\tau}}{2\tau} \quad . \qquad (2.3)$$

$$T_a = \frac{Q}{\omega} \quad , \qquad \tau = \frac{T_f}{2T_a}$$

Q = stored energy per unit length divided by power dissipated per unit length per radian ω = operating radian frequency, τ = section attenuation in nepers.

For a constant inpedance section for a given peak power per unit length, E_a is maximum with respect to T_f when $T_f = 2.51T_a$ ($\tau = 1.257$).

For a section with linearly variable group velocity

$$\eta_s = \frac{[(1+g)^{x_1} - 1]^2}{g\ell n(1+g)x_1^2} \qquad (2.4)$$

$$g = \frac{v_{gL} - v_{go}}{v_{go}} \quad , \qquad x_1 = 0.5 - \frac{\tau}{\ell n(1+g)} \quad . \qquad (2.5)$$

v_{go}, v_{gL} = group velocities at section input and output respectively.

The expression for the self induced beam voltage and the beam induced average gradient is

$$V_b = \frac{\eta_i s I_o T_f L}{4} \quad , \quad E_b = \frac{\eta_i s I_o T_f}{4} \tag{2.6}$$

$$\eta_i = \frac{4T_a}{T_f}\left[\frac{1}{1+v_g' T_a}\right]\left[1 - \frac{(1+g)^{x_2} - 1}{g x_2}\right] \tag{2.7}$$

$$v_g' = \frac{dv_g}{dz} \quad , \quad x_2 = 0.5\left(1 - \frac{1}{v_g' T_a}\right) \quad . \tag{2.8}$$

Here I_o is the beam current through the section and η_i is the beam energy to rf energy conversion efficiency in the absence of rf input. The expression for η_i is derived in Appendix B.

For a constant impedance and a constant gradient section, respectively

$$\eta_i = 2\left[\frac{1}{\tau} - \frac{1-e^{-\tau}}{\tau^2}\right] \quad , \quad \eta_i = \frac{1}{\tau} - \frac{2e^{-2\tau}}{1-e^{-2\tau}} \quad . \tag{2.9}$$

The loaded section voltage $V_s - V_b$ is

$$V = \sqrt{\eta_s P_o T_f s L} - \frac{\eta_i s I_o T_f L}{4} = \sqrt{\eta_s P_o T_f s L}\left[1 - \frac{\eta_i I_o}{4}\sqrt{\frac{s T_f L}{\eta_s P_o}}\right] \quad . \tag{2.10}$$

The loaded average gradient is

$$V/L = E = E_a - E_b \tag{2.11}$$

$$E = \sqrt{\frac{\eta_s P_o s T_f}{L}} - \frac{\eta_i s I_o T_f}{4} = \sqrt{\frac{\eta_s P_o s T_f}{L}}\left[1 - \frac{\eta_i I_o}{4}\sqrt{\frac{s T_f L}{\eta_s P_o}}\right] \tag{2.12}$$

Whenever possible the symbols used in this note are identical to those used by P. Wilson in reference 4.

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3. RF Peak, RF Average and AC Power Requirements

The accelerating gradient and the number of sections required for a given site voltage V_t and a given site length L_t, are respectively

$$\bar{E} = \frac{V_t}{L_t}, \quad N = \frac{L_t}{L} \quad . \tag{3.1}$$

The gradient is chosen by balancing the cost of extra length and the operating cost. The length decreases but the efficiency and hence the operating cost increases as the gradient increases.

From equations (2.1) and (2.11) we have the required peak power

$$P_o = \frac{L(\bar{E} + \bar{E}_b)^2}{\eta_s s T_f} \tag{3.2}$$

The section input pulse energy is $U_a = P_o T_k$, $T_k = T_f + T_b$. Here T_k is the klystron pulse width and T_b is the beam pulse width. The average rf power into the section is

$$P_{ao} = f_r P_o (T_f + T_b) = f_r \frac{(T_f + T_b)\, L \left(\bar{E} + \frac{\eta_i s I_o T_f}{4}\right)^2}{T_f \eta_s s} \quad . \tag{3.3}$$

Here f_r is the pulse repetition rate.

The beam energy is $U_b = \bar{E} L I_o T_b$. The efficiency of transforming sect'on rf energy into beam energy is

$$\eta_{sb} = \frac{U_b}{U_a} = \frac{\bar{E} I_o T_b L}{P_o (T_f + T_b)} = \frac{\eta_s \bar{E} s I_o T_b T_f}{(\bar{E} + \bar{E}_b)^2 (T_f + T_b)} \quad . \tag{3.4}$$

Substituting for I_o from (2.6) we obtain

$$\eta_{sb} = \frac{4\eta_s \bar{E} L \ T_b}{\eta_i (T_f + T_b,} \frac{}{E_a^2} \quad . \tag{3.5}$$

Defining $k \equiv \frac{\bar{E}}{\bar{E}_a}$, and using (2.11) $\frac{\bar{E}_b}{\bar{E}_a} = 1 - k$ we obtain

$$\eta_{sb} = \frac{4\eta_s k(1-k)T_b}{\eta_i(T_f + T_b)} \quad . \tag{3.6}$$

For a beam current pulse width of 1.6 μs, as at SLAC, this efficiency has a broad maximum when the fill time is about 0.65 μs.

The AC power into the modulator is[5]

$$P_m = \frac{P_{so}}{\eta_{ar}} \quad , \quad \eta_{ar} = \frac{0.398T_k}{T_k + 1} \quad , \qquad T_k \text{ in } \mu s \quad . \tag{3.7}$$

The efficiency of transforming AC to beam power is

$$\eta_{ab} = \frac{U_b}{U_m} = \eta_{sb}\eta_{ar} \quad . \tag{3.8}$$

The power dissipated in the section is $P_{ad} = P_{ao}(1 - e^{-2\tau})$. The total site AC power for the modulators $P_{act} = NP_m$.

4. Single Bunch Mode

We inject a single bunch of charge q at T_f. The charge-induced and accelerating fields are:[4]

$$\bar{E}_b = \frac{aq}{2} \quad ; \quad \bar{E} = \bar{E}_a - \frac{\bar{E}_b}{2} \quad . \tag{4.1}$$

Figure 1 illustrates the relationship between accelerating, unloaded and beam induced voltages and fields in the single bunch mode.

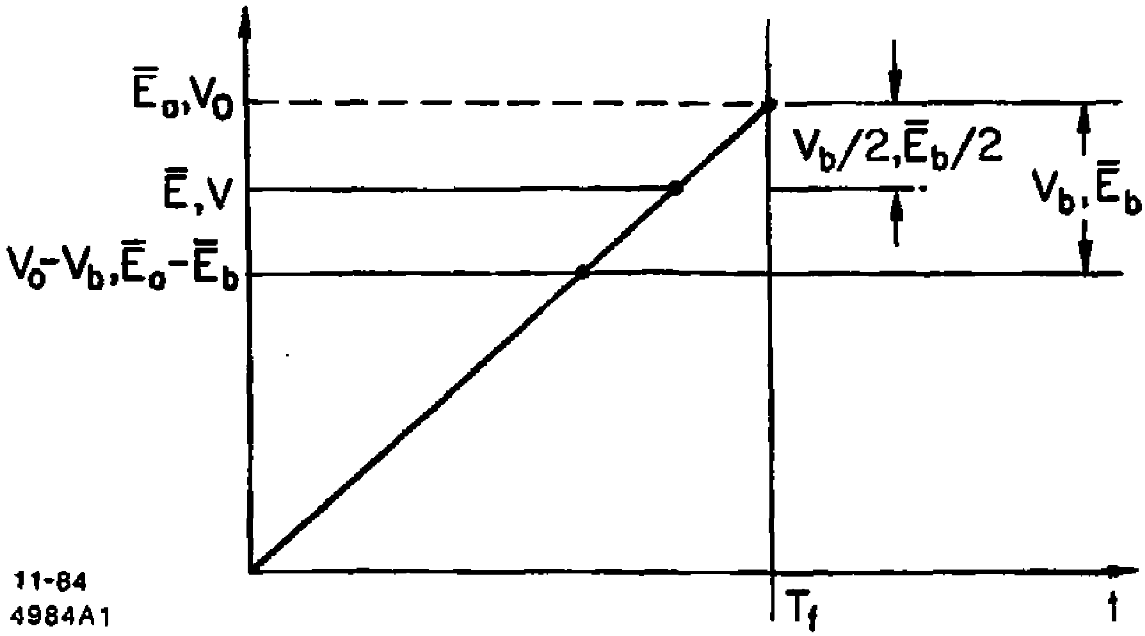

Fig. 1. Single bunch mode voltage vs time.

The pulse energy into the section is

$$P_o T_f = \frac{\bar{E}_a^2 L}{\eta_s s} = \frac{(\bar{E} + \bar{E}_b/2)^2}{\eta_s s} \quad . \tag{4.2}$$

The average power into the section is

$$P_{so} = f_r P_o T_f \quad . \tag{4.3}$$

The efficiency of transforming section energy to bunch energy is

$$\eta_b = \frac{\bar{E} L q}{\eta_s P_o T_f} = \frac{s \bar{E} q}{\bar{E}_a^2} \quad . \tag{4.4}$$

Using (4.1) and $k \equiv \bar{E}/\bar{E}_a$ we obtain

$$\eta_b = \frac{2 \bar{E} \bar{E}_b}{\bar{E}_a^2} = 4k(1-k) \quad . \tag{4.5}$$

The peak and average powers required for a given gradient and beam loading vary as the inverse of the elastance. The maximum efficiency is independent of the gradient and of the elastance. The efficiency of transforming rf input energy to bunch energy is

$$\eta_{sb} = \eta_s \eta_b = \eta_s \frac{2 \bar{E} \bar{E}_b}{\bar{E}_a^2} \quad . \tag{4.6}$$

For light loading $\bar{E} = \bar{E}_a$ and

$$\eta_{sb} = \frac{2 \eta_s \bar{E}_b}{\bar{E}} \quad . \tag{4.7}$$

For light loading the efficiency varies as the inverse of the accelerating gradient, and as the elastance. For either heavy or light single bunch beam loading the efficiency is independent of η_i, and is reduced by the beam loading enhancement factor.[4]

5. Long Pulse Superconducting Design

A lossless section has a constant gradient when it has a constant impedance, that is when the disk hole size is uniform along the section. For a superconducting section RF energy is saved by not dissipating typically three quarters of the energy entering the section before it had a chance to contribute to the accelerating field, as in the normal case. We save both average and peak power. Both η_s and η_i approach unity as the section losses approach zero, consequently for a superconducting section η_s and η_i are no longer functions of T_f. Both approach unity. Thus the previous equations apply if we substitute unity for η_s and for η_i.

From (3.2) and (3.3) we obtain the peak and average powers per unit length

$$p_o = \frac{P_o}{L} = \frac{(\bar{E} + \bar{E}_b)^2}{sT_f} \tag{5.1}$$

$$p_{ao} = \frac{P_{ao}}{L} = f_r \frac{(T_b + T_f)(\bar{E} + \bar{E}_b)^2}{T_f s} \quad . \tag{5.2}$$

The rf energy to beam energy conversion efficiency of a SC section becomes

$$\eta_b = \frac{\bar{E} I_o s T_f T_b}{(\bar{E} + sI_o T_f/4)^2 (T_f + T_b)} = \frac{4k(1-k)T_b}{T_f + T_b} \quad . \tag{5.3}$$

For a given loaded gradient, beam current, and beam pulse width the efficiency as a function of fill time has a maximum at

$$T_f = \frac{T_b}{4}\left[\sqrt{1 + \frac{32\bar{E}}{sI_o T_b}} - 1\right] \quad . \tag{5.4}$$

With T_b much larger than T_f, the efficiency has a maximum of unity at $k = 1/2$, $\bar{E}_b = \bar{E}$, the no-load gradient $\bar{E}_a = 2\bar{E}$, and $T_f = 4\bar{E}/sI_o$. One can approach 100% efficiency with a copper section also but the section has to be very short and therefore either the number of sources have to be very large or the peak power very high. With SUTRA the beam induced output power can be extremely high.

In addition to the ac modulator power we also require ac power for refrigeration. Thus part of the AC power saved is offset by the AC power needed to keep the section cool. Maximum power is required at no-load when the dissipation is maximum. At no load the average power entering the section is P_{ao} given by (5.2). The power dissipated in the section is

$$P_{ad} = P_{ao} 2\tau / I_f . \tag{5.5}$$

Here τ is the attenuation of a copper section and I_f is the improvement factor. Part of the input power P_{ao} is absorbed by the beam. Therefore to obtain the loaded P_{ad} we multiply P_{ad} by the square of the ratio of the no-load to loaded gradient. The refrigeration power is $P_r = R_f P_{ad}$. Here R_f is the refrigeration factor. The AC to beam energy conversion efficiency of a SC section is

$$\eta_{ab} = \frac{P_b}{P_m + P_r} \tag{5.6}$$

Figure 2a shows a plot of RF to beam energy conversion efficiency as a function of filling time, for a copper section, for a 6m and for a 8 m SC section. We used the CEBAF parameters $\bar{E} = 18.5 MV/m$, $I_o = 0.2A$, $s = 76.4 M\Omega/\mu s\ m$, $T_b = 1.2\mu s$. The reason why the optimum T_f and the efficiency varies somewhat with length is that the group velocity and hence the elastance varies with length. Figure 2b is the same as Figure 2a except we replaced RF with AC efficiency. We assumed a refrigeration factor of 400 and an improvement factor of 4000. The AC efficiency peaks at a somewhat higher filling time than the RF efficiency. Figures 3a, 3b, 3c and 3d show plots of klystron peak and average powers as a function of filling time for a 3m copper, a 3m SC, a 6m SC and a 9m SC section. Here also we used the CEBAF parameters. The length determines the number of required sources. We chose the maximum length for which the klystron average and peak power rating are not exceeded. Because we kept the beam pulse energy constant, the klystron average power is proportional to the reciprocal of the rf to beam energy conversion efficiency and nearly

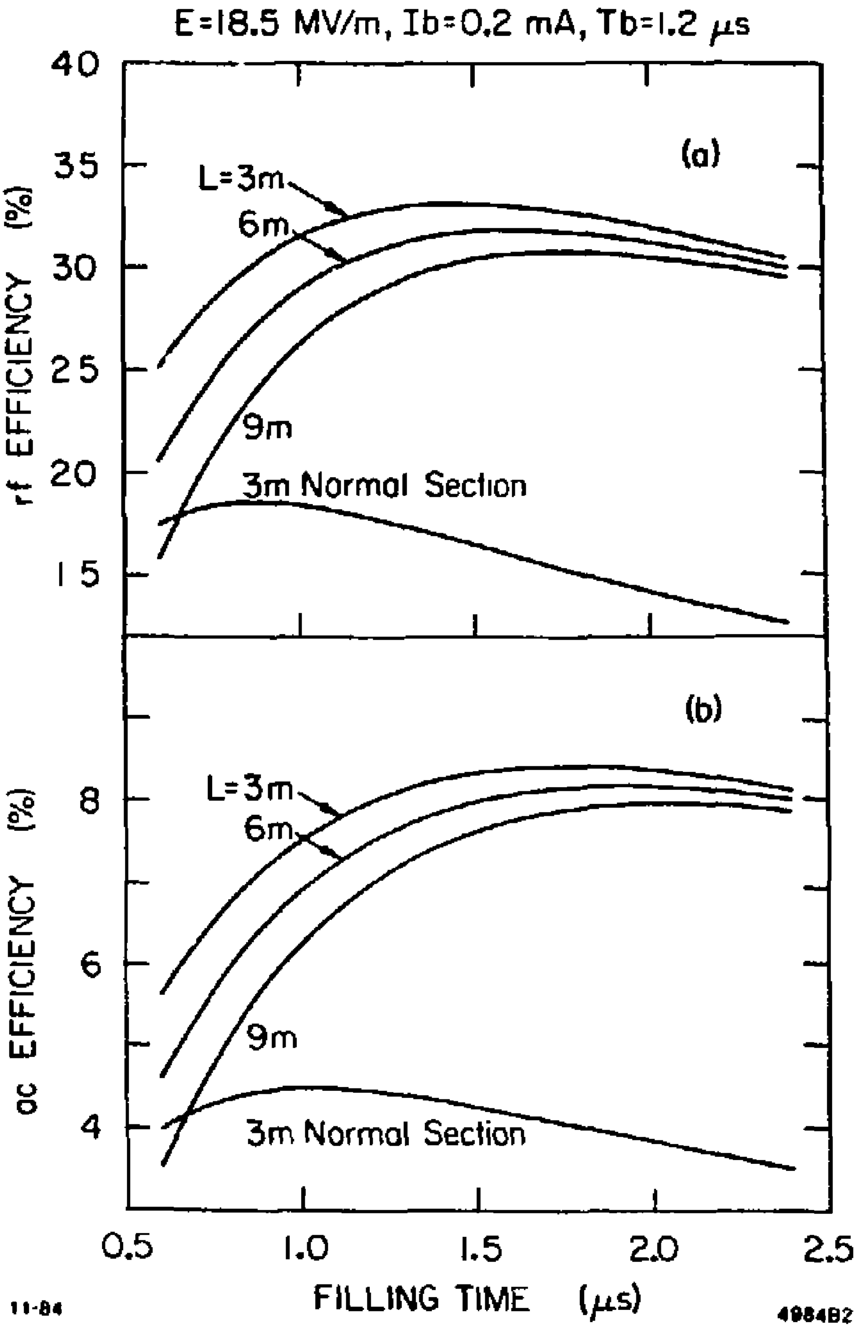

Fig. 2. AC and rf to beam energy conversion efficiency.

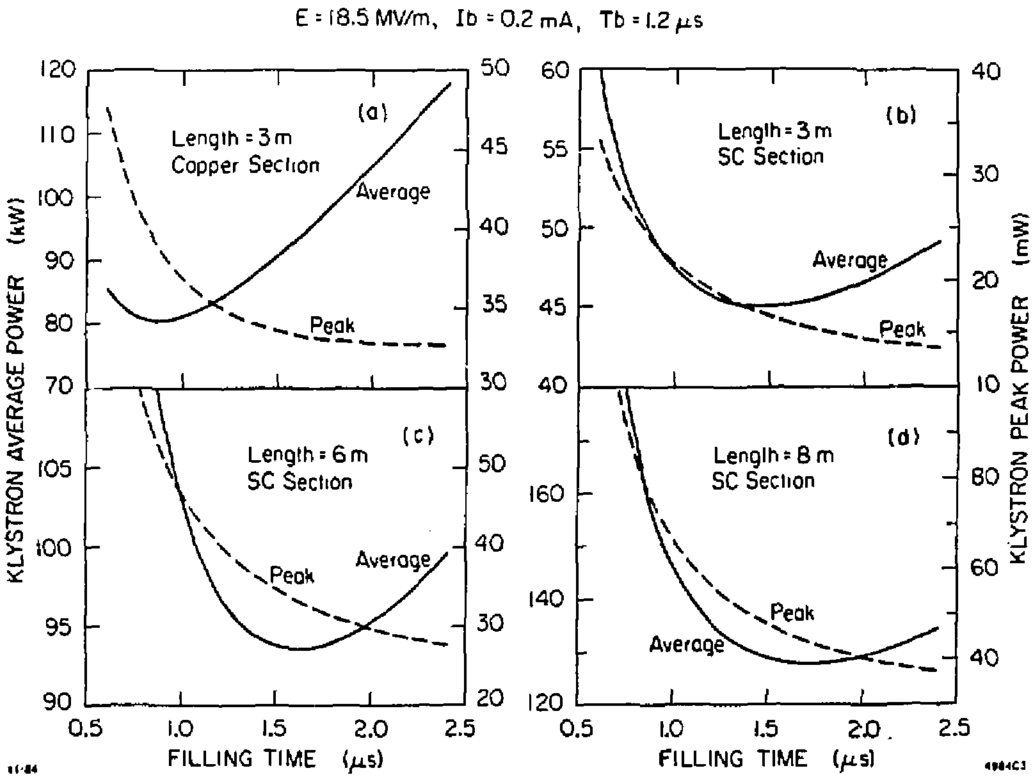

Fig. 3. Klystron average and klystron peak powers vs fill time.

proportional to the required AC power. The peak and average powers per unit length do not depend on length. We see that superconductivity reduces the rf and AC power requirements to reach 18.5 MV gradient by about a factor of 2, and that we can trade peak for average power.

6. Single Bunch Superconducting Design

For the single bunch mode

$$P_{ao} = f_r P_o T_f = f_r \bar{E}_a^2 L/s \tag{6.1}$$

$$\eta_b = \frac{2\bar{E}\bar{E}_b}{\bar{E}_a^2} = 4k(1-k) \tag{6.2}$$

The maximum efficiency of unity is obtained when $k = 1/2$ at $\bar{E} = \bar{E}_a/2$ and $\bar{E}_b = \bar{E}_a$. All the energy in the section which is also the input energy is transferred to the beam, at least theoretically. But it may cause unacceptable energy spread within the bunch.

For lightly loaded single bunch mode

$$\eta_b = \frac{2\bar{E}_b}{\bar{E}} = \frac{2(1-k)}{k} \tag{6.3}$$

In a lightly loaded superconducting TW section the rf energy loss is essentially zero, and all the input rf energy is available at the output end and can be used again to drive a second parallel accelerator section. We recognize two possible modes of operation:

(i) A second bunch injected a fill time later into a second accelerator.

(ii) The original bunch is returned to the input and injected into the second accelerator. The fill time must equal the round trip time of the bunch as illustrated in Figure 4. We do not need another accelerator if we use high power rf switches.

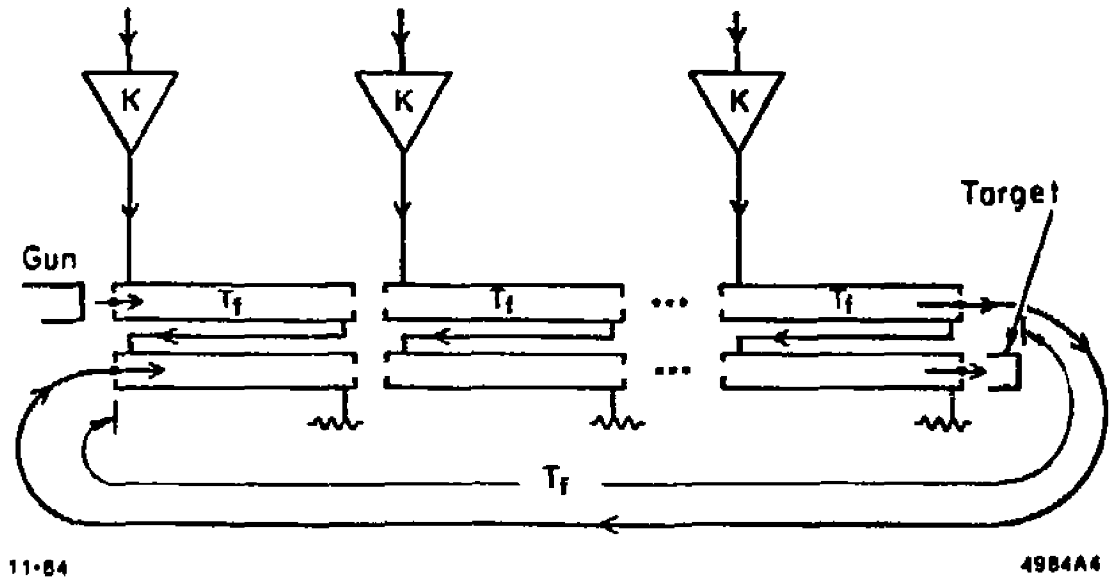

Fig. 4. Double pass recirculation scheme.

Recirculation for the same beam voltage reduces rf and ac energy by a factor of 4. The refrigeration power is doubled under our assumptions, but as it is a small fraction of modulator power, the net increase in ac power is small.

7. Short Pulse Superconducting Design

In the preceeding section we used the steady state values, that is the values at and after a filling time, for both no load and beam induced voltages. Another pulse operation mode is possible, the short pulse mode, where we inject the beam at $t = T_f - T_b$ with T_b less then T_f when the loaded voltage is $V = kV_o$. We force the beam-induced voltage to equal the change in the unloaded voltage during T_b so that the beam voltage at the beginning and end of the pulse are equal. This is illustrated in Figure 5. Thus

$$V_b = V_o - V = V_o(1-k) \quad ; \qquad \bar{E}_b = \bar{E}_a - \bar{E} = \bar{E}_a(1+k) \tag{7.1}$$

Note that short pulse beam loading compensation is different than staggering the times at which rf power is fed to successive sections used when the beam is injected one fill time after rf turn on. With SUTRA we can use long short pulses, because the filling times can be long. The rf to beam energy conversion efficiency

$$\eta_b = \frac{VI_oT_b}{P_oT_f} \tag{7.2}$$

is obtained as follows. For $t \ll T_f$, the beam induced gradient is [4]

$$\bar{E}_b = V_b/L = \frac{sI_oT_b}{2} \tag{7.3}$$

$$I_oT_b = \frac{2\bar{E}_b}{s} = \frac{2(1-k)\bar{E}_a}{s} \tag{7.4}$$

$$P_oT_f = \frac{\bar{E}_a^2 L}{s}, \quad V = k\bar{E}_aL \tag{7.5}$$

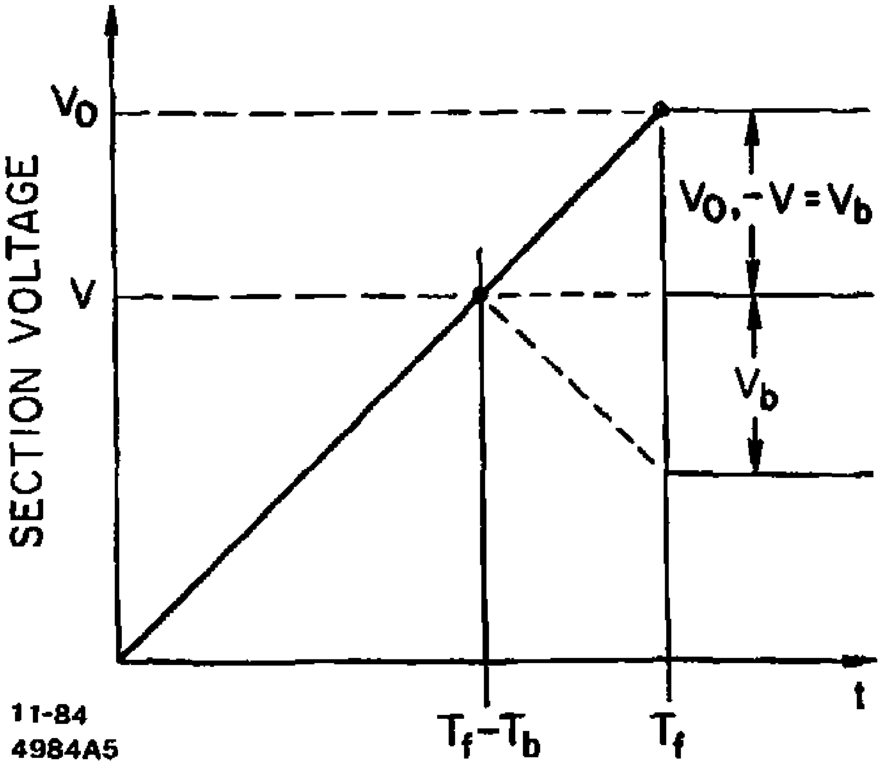

Fig. 5. Short pulse mode section voltage vs time.

$$\eta_b = 2k(1-k). \tag{7.6}$$

The maximum efficiency, which occurs when k=1/2, is 50%. If I_b, T_b and $\bar{E}$ are given. Then

$$\bar{E}_b = \frac{sI_oT_b}{2}, \quad k = \frac{1}{\bar{E}_b/\bar{E}+1}, \quad T_f = \frac{T_b}{1-k} \quad . \tag{7.7}$$

Take for example the CEBAF parameters

$$\bar{E} = 18.5MV/m \ , \quad I_o = 0.2mA \ , \quad T_b = 1.2\mu s. \tag{7.8}$$

which yield $\bar{E}_b = 9.17$, $\bar{E}_b/\bar{E} = .5$, $k = 2/3$, $T_f = 3.6$ and $\eta_b = 0.444$.

The required peak power is

$$P_o = \frac{V_o^2}{sT_fL} = \frac{\bar{E}^2L}{k^2sT_f} \quad . \tag{7.9}$$

The required average power is

$$P_{ao} = P_oT_ff_r = \bar{E}^2Lf_r/k^2s \quad . \tag{7.10}$$

The efficiency of short pulse operation is greater than medium pulse operation, that is when the beam is injected one fill time after the rf has been turned on and is not much longer than the fill time, because in the latter case all the energy it took to fill the section is wasted. After the section has been filled, the input power, except for the power exiting the section, is transferred to the beam. We can choose a beam loading that make the section output power zero and hence a filling time after the beam has been turned on all the rf power is transferred to the beam. The single bunch efficiency as the steady state efficiency approaches 100% with SUTRA.

8. CEBAF Design

The CEBAF beam voltage is 4160 MV, the beam current is 0.2 mA. and the beam pulse width is 1.2μs. It uses a SLAC section whose parameters are: $s = 76.4 M\Omega/\mu s\ m$, $Q = 12900$, $T_f = 0.82\ \mu s$, $L = 3m$. It has effectively 75 sections hence the accelerating gradient is 18.5 MV/m. The attenuation of the waveguide that connects the klystron to the section input is 0.5 db so that the klystron output power $P_k = 1.122\, P_o$. The pulse repetition frequency is 1kHz. Substituting these given values in the appropriate expressions we obtain the standard design whose parameters are listed in the first column of Table 1.

Table 1. COMPARISON of CEBAF SYSTEMS

$V_t = 4.160GV$, $I_b = 0.2mA T_b = 1.2\mu s$, $E_a = 18.5MV/m$

System	L=3 Standard	L=3 SC	L=9 SC	L=3 SC SP	L=3 RC
L_t (m)	225	225	225	225	120
N	75	75	25	25	40
T_f (μs)	.82	2.0	2.4	3.6	.82
v_g (m/μs)	3.66	1.5	3.75	2.5	3.66
P_k (MW)	40	14	42	28	40
T_k (μs)	2.02	3.2	3.6	3.6	3.22
P_{ak} (kW)	80.	48	152	100	129
P_{ad} (1/m)	8 kW	5 W	6 W	6 W	12kW
P_r (kW)	—	5.5	20	20	-
P_m (kW)	303	155	487	327	424
P_{ac} (kW)	303	161	442	347	424
P_{act} (MW)	23	12	12	8.7	17

The parameter of the first SUTRA design are listed in the second column of Table 1. We see that SUTRA reduces the required peak and average klystron powers and ac line power. The reduction of peak power enables us to reduce the number of klystrons by splitting the power so that one klystron feeds several sections. The second SUTRA design, whose parameters are listed in the third column of Table 1, reduces the number of klystrons by one third, but increases the required klystron average power. Both designs halve the AC line power.

If we accept additional reduction in AC efficiency and are not limited by klystron average power and by klystron pulse width, then we can increase the section length further, and further reduce the number of klystrons. A long section need not be a continuous disk loaded structure. It can be made up of several shorter sections connected in tandem with a smooth waveguide. Another alternative is to keep the section length but reduce the peak power even further by decreasing the group velocity, further subdivide a klystron output and again further reduce the number of klystrons.

The third SUTRA design operates in the short pulse mode. Its parameters are listed the fourth column of Table 1. It reduces the rf peak, rf average and ac power requirements below that of the second SUTRA design.

Because of the high cost of a klystron and its modulator, the proposed CEBAF design recirculates the beam in order to reduce the required number of klystrons. The parameters of this proposed design are listed in the the last column of Table 1. Recirculation also converts average power to gradient by increasing the klystron pulse width. This saves 35 klystrons and 105m of site length. It has the same peak and average klystron powers as the second SC design. Both recirculation and SUTRA reduce the number of klystrons and the site ac power. The question is which does it better and cheaper. We will list the difference in the requirements of the recirculation and the L = 9m SUTRA designs. Recirculation requires 12 additional modulator-klystrons which is about 3M$ capital costs, about 5MW of additional ac power which corresponds to about 20M$ operating costs for a period of 10 years, a system for removing 12MW/m and stabilizing the section temperature, and, of course, the cost of recirculating the beam. SUTRA requires 28 60-watt refrigerators at 0.1M$ each, 40 cryostats at 0.01M$ each, 105 m additional accelerator sections at 0.01M$/m, a total of 3M$, and the extra cost of the superconducting sections. Also it requires an additional 126 meters of accelerator housing. Even if the costs were to balance we would still be ahead with the superconducting system because recirculation limits the operating flexibility.

If we increase the peak and average powers of the first SUTRA design to that of the recirculation design, then we can keep the length of the recirculation design but we do not save ac line power. Here we compare the costs of 40 60-watt refrigerators and 40 cryostats about 4M$ to the cost of the recirculator. The higher gradient and beam energy flexibility favors this choice over recirculation.

If we double the beam pulse width and halve the repetition rate (so that the average current and duty cycle do not change) then we can have the parameters of the second SUTRA design except that we can reduce the klystron average power to 90 kW and the site ac power to 8MW. We save an additional 30M$ over 10 years of operation.

9. Upgrading SLAC

Using SUTRA to upgrade SLAC is a good example of trading pulse width for increased gradient. Instead of dividing a klystron output so that it feeds four 3m long sections, we connect the four sections in tandem so that now a klystron feeds one 12m long section. We increased the fill time from 0.82 to 3.28μs. As a result, we increased the gradient by a factor of two because we quadrupled the section input power and by an additional factor of 1.294 becase we eliminated the power loss along the section, a total increase of 2.6. We increased the gradient from 11.8MV/m to 30.5MV/m a 92GV SLAC.

We can operate in the pulse mode or in the single bunch mode. In the pulse mode the klystron pulse width is 5μs and we have a 1.7 μs beam pulse; in the single bunch mode we have a 3.3 μs klystron pulse, and therefore we can increase the repetition frequency and hence the luminosity 52%. We would have to have a switch to change the pulse forming network delay between 3.3 and 5 μs.

At 180 pps, the klystron average power is 50kW, the power dissipated by a section 12.5watts, and the refrigeration power is 5kW. Thus for less than 2% increase in ac power we increased the gradient 2.6 times. To do this with copper sections we would have had to increase the peak power 6.7 times and require 335MW klystrons.

We could do both, increase the fill time of a section and connect 4 sections in tandem. If we wish to operate only in the single bunch mode then we can increase a section fill time from 0.82 to 1.25μs so that the total fill time is 5 μs. As a result the gradient increases by an additional factor of 1.23 to 38MV/m. Increasing the fill time of each section from 0.82 to 2.5 so that the total fill time is 10μs, and using an existing 35MW, 10μs (ITT) klystron results in a 3.78 fold gradient multiplication, from 11.8 to 45MV/m, a 135GV SLAC, 270GV if we recirculate.

Are these gradients realistic? Recent tests on single port superconducting cavities at SLAC demonstrated [6] that peak magnetic fields of 750 gauss for lead plated copper cavities[7] at a temperature of 4.2K, 1150 gauss for niobium cavities at 4.2K, and also 1150 gauss for niobium-tin cavities at 10K, can be reached with 2.5μs pulse width. Using the ratio of magnetic to accelerating field of 31 gauss/(MV/m) calculated by Helm[2], these magnetic fields correspond to accelerating fields of 24MV/m for lead plated cavities, 37MV/m for niobium cavities, and niobium-tin cavities.

So far all our tests were with single port cavities, where the field increases during the pulse to a peak value at the end of the pulse when breakdown occurs. To determine the duration a given gradient can be sustained we need a traveling wave, that is, a transmission type cavity.

Figure 6 shows a single port or SW cavity and a two port transmission or

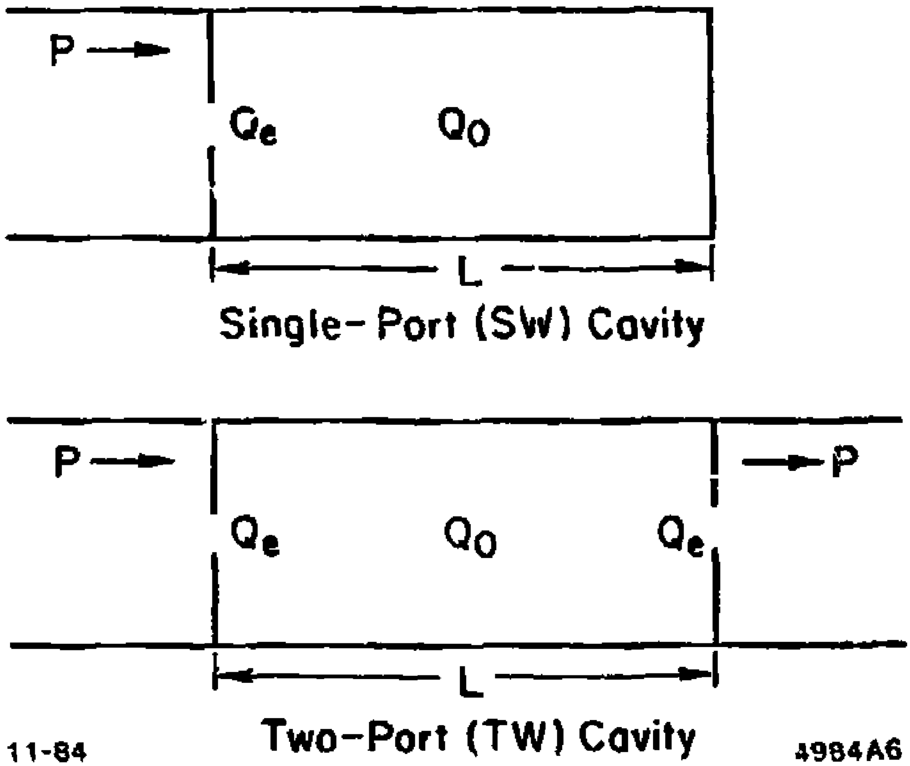

Fig. 6. Comparison of one port and two port cavities.

TW cavity. We postulate that for either cavity Q_o is greater than Q_e, and that the TW cavity input and output Q_es are identical. The energy transfer efficiency of an incident pulse of duration T_p, that is the fraction of energy stored at the end of the pulse, is

$$\eta = \alpha \frac{(1 - e^{-\tau})^2}{\tau} \tag{9.1}$$

where $\tau = T_p/T_c$ and the cavity time constant $T_c = \alpha Q_e/\omega$. This is true for either the SW or for the TW cavity. However, there are differences. For a SW cavity $\alpha = 2$, $T_c = 2Q_e/\omega$. The fraction of energy is maximum and equals 0.815 when $\tau = 1.257$. Hence $Q_e = 2.5fT_p$. For a TW cavity $\alpha = 1$, $T_c = Q_e/\omega$. The efficiency η of a TW cavity is maximum when its external Q is twice the SW external Q. Thus for a TW cavity $Q_e = 5fT_p$. Its η is half the SW η. This is because during charging of the TW cavity half of the incident energy leaks out through the output port.

The steady state energy in both cases is only slightly higher than the energy at the end of the pulse. But there is a significant difference between them. Whereas the SW cavity reflects any additional incident energy and therefore its input is mismatched, the TW cavity transmits it to a load (or to another cavity in case of tandem cavities) and therefore its input is matched. As was pointed out to me some time ago by R. Miller, a SW cavity behaves much like a TW cavity when operating in the pulse mode.

To reach 50MV/m accelerating field, using the ratio of peak magnetic field to the square root of stored enengy $B_s/\sqrt{U} = 1550$ gauss/joule$^{1/2}$ obtained with SUPERFISH, 1 joule is required. With a 2.5 μs pulse, the required peak power is 0.5MW for a SW cavity and twice that for a TW cavity. But we wish to reach steady state in a time interval much less then the duration of the incident pulse. As already stated, to avoid steady state reflection we need a TW cavity. Let the time constant of the TW cavity, Q_e/ω be denoted by T_e. The steady state energy is PT_e. Let $T_e = 0.1\mu$s, hence $Q_e = 2000$ and the required peak power is 9MW. This is well within the capability of our klystron.

We can assign a group velocity to a single TW cavity. By definition $Q_e = \omega U/P$ where U is the energy stored in the cavity, and P is the power transmitted through the cavity. Substituting for U, $U = P\frac{L}{v_g}$ we obtain

$$v_g = \frac{L}{Q_e/\omega}. \tag{9.2}$$

If L is a half wavelength then $v_g = 0.5m/\mu s$.

With this low Q_e we should make the waveguide near the input and output to the cavity of superconducting material. First we should connect the input to output via a straight waveguide in order to ascertain that the adapters are matched and in order to measure the vave guide insertion loss. Only then should we connect the TW cavity. With a TW cavity we can determine the stored energy by simply measuring the output power.

10. Conclusion

We expressed both the rf induced and beam induced voltages and accelerating fields in terms of elastance and efficiency. The elastance is independent of loss and the efficiency reduces to unity for a lossless section. Thus we developed expressions for the rf and beam induced section voltages and fields and for the rf and AC powers needed to obtain a given gradient that are valid for both copper and superconducting sections. We have shown that for long pulse, short pulse and single bunch beam operation SUTRA can significantly reduce the the rf peak, rf average and ac power requirements to obtain a given gradient. We showed that SUTRA would increase the SLAC gradient from 21 to 38MV/m, and if we recirculate to effectively 76MV/m, with little increase of ac power. SUTRA with recirculation increase the rf energy to beam energy conversion efficiency by, a factor of 1.7 by eliminating section loss, a factor of 2 by eliminating the need for pulse compression, and a factor of 4 due to recirculation without additional rf average power, a total factor of about 14. Thus for a given AC power and a given number of klystrons with a given average power ratings, SUTRA increases the luminosity 14 fold.

It should be emphasized that the formidable problems of a SC accelerator operating in the CW mode do not exist with SUTRA. Maintaining frequency stability is no more difficult then with copper section. In fact it is easier. Difficult processing and extreme care are not necessary. Finally we have shown that a high gradient SUTRA is feasible using present state of the art.

Although we derived the expressions for the general case of variable group velocity along the section, we did not make use of it. The group velocity gradient can be used to reduce energy spread when operating in the short pulse or multi-bunch mode. It may be needed to reduce beam breakup threshold and therefore its effect must be considered.

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APPENDIX A

Unloaded Voltage

We will derive the rf induced unloaded voltage gained by a charged particle traversing a section with linearly varying group velocity.

The transmitted power at z is obtained from the fundamental expressions for w is the energy stored per unit length:

$$w = \frac{P}{v_g}; \quad w = \frac{Q}{\omega}\frac{dP}{dz} = T_a \frac{dP}{dz} \tag{A1}$$

We defined $T_a \equiv Q/\omega$. Thus

$$\frac{dP}{dz} = \frac{-P}{v_g T_a} \tag{A2}$$

Using the definition $s \equiv \frac{E^2}{w}$ and using $w = \frac{P}{v_g}$ we obtain the accelerating field as a function of z

$$E(z) = \sqrt{\frac{sP(z)}{v_g}} \tag{A3}$$

For a constant group velocity (constant impedance) section

$$P(z) = P_o e^{-z/v_g T_a} \quad , \quad E(z) = \sqrt{\frac{sP_o}{v_g}} e^{-z/2v_g T_a} \tag{A4}$$

$$V_o = \sqrt{\frac{sP_o}{v_g}} \int_0^L e^{-z/2v_g T_a} = \sqrt{sP_o T_f L}\left(\frac{1 - e^{-T_f/2T_a}}{T_f/2T_a}\right) \tag{A5}$$

The attenuation per unit length and the section attenuation both in neper are respectively

$$\alpha = \frac{1}{2v_g T_a} \quad , \quad \tau = \frac{T_f}{2T_a}$$

For linearly variable group velocity

$$v_g = v_{go}(1+mz) = v_{go} + v_g' z \quad \text{where} \quad v_g' \equiv \frac{dv_g}{dz} = m v_{go} \tag{A6}$$

$$\frac{dP}{P} = \frac{-dz}{T_s\, v_{go}(1+mz)} = \frac{-m\, dz}{T_s\, v_g'(1+mz)} \tag{A7}$$

whose solution is

$$P(z) = P_o\, e^{-\frac{\ell n(1+mz)}{T_s v_g'}} \tag{A8}$$

Define $a = 1/2T_s\, v_g'$. Then

$$P(z) = P_o e^{-2a\ell n(1+mz)} \tag{A9}$$

$$E(z) = \sqrt{\frac{sP}{v_g}} = \sqrt{\frac{sP_o}{v_{go}}}\ \frac{e^{-a\ell n(1+mz)}}{(1+mz)^{.5}} \tag{A10}$$

$$E(z) = E_o(1+mz)^{-.5-a}; \quad E_o = \sqrt{\frac{sP_o}{v_{go}}} \tag{A11}$$

The voltage as a function of z is

$$V(z) = \int_0^z E\, dz = E_o z\, \frac{(1+mz)^{.5-a} - 1}{(.5-a)mz} \tag{A12}$$

Let $x_{11} = .5 - a$, $z = L$ and $g \equiv mL$. Then

$$V_o = E_o L \frac{(1+g)^{x_{11}} - 1}{x_{11} g} \tag{A13}$$

Substituting for E_o and using

$$v_{go} = \frac{L}{T_f}\frac{\ell n(1+g)}{g} \tag{A14}$$

we obtain

$$V_o^2 = sP_o T_s L\ \frac{[(1+g)^{x_1} - 1]^2}{g\ell n(1+g) x_1^2}\ . \tag{A15}$$

Hence

$$\eta_s = \frac{[(1+g)^{x_1} - 1]^2}{g\ell n(1+g) x_1^2}\ . \tag{A16}$$

APPENDIX B

Beam Induced Voltage

We will derive the beam induced voltage in a section with linearly varying group velocity. From the conservation of energy and using $P = (v_g/s)E^2$, and also using $2\alpha = 1/(v_g T_a)$

$$\frac{dP}{dz} = I_o E - 2\alpha P = I_o\sqrt{\frac{s}{v_g}P} - \frac{P}{v_g T_a} \quad . \tag{B1}$$

For a lossless constant group velocity section

$$\frac{dp}{dz} = \frac{2Ev_g}{s}\frac{dE}{dz} = I_o E \tag{B2}$$

$$\frac{dE}{dz} = \frac{I_o s}{2v_g} \quad ; \qquad E = \frac{I_o s}{2v_g} z \quad ; \qquad P = \frac{I_o^2 s}{2v_g} z^2 \tag{B3}$$

$$V_b = \int_0^z E\,dz = \frac{I_o s z^2}{4v_g} = \frac{I_o s T_f L}{4} \tag{B4}$$

For a lossy constant group velocity section (substitute $\sqrt{P} = y$)

$$P = I_o^2 T_a^2 s v_g (1 - e^{-z/2v_g T_a})^2 \tag{B5}$$

$$E = I_o T_a s (1 - e^{-z/2v_g T_a}) = I_o T_a s (1 - e^{-\alpha z}) \tag{B6}$$

$$V_b = I_o T_a s [z - 2v_g T_a (1 - e^{-z/2v_g T_a})] \tag{B7}$$

For a lossy variable group velocity section

$$\frac{dp}{dz} = \frac{v_{go} E}{s}\left[2(1 + mz)\frac{dE}{dz} + mE\right] \tag{B8}$$

$$\frac{dE}{dz} = \frac{\frac{I_o s}{v_g^T} - \left(1 + \frac{1}{T_a v_g^\prime}\right)E}{\frac{2}{m}(1 + mz)} \quad . \tag{B9}$$

Let $a = (I_o s)/v'_g$ and $b = -(1 + 1/(T_a v'_g)$. We separate the variables and obtain

$$\frac{dE}{a+bE} = \frac{m}{2}\,\frac{dz}{1+mz} \quad . \qquad (B10)$$

whose solution is

$$\frac{1}{b}\,\ell n\,(a+bE) = \frac{m}{2}\,\frac{1}{m}\,\ell n\,(1+mz) \;\rightarrow\; a+bE = c(1+mz)^{b/2} \quad . \qquad (B11)$$

When $z = 0$, then $E = 0$ therefore $c = a$. Thus,

$$E = \frac{I_o s}{v'_g\left(1+\frac{1}{v'_g T_a}\right)}\left[1-(1+mz)^{-\left(1+1/v'_g T_a\right)/2}\right] \quad . \qquad (B12)$$

Multiplying numerator and denominator by $v'_g T_a$, we can write Eq. (B12) as

$$E = \frac{I_o s T_a}{\left(1+v'_g T_a\right)}\left[1-(1+mz)^{-\left(1+1/v'_g T_a\right)/2}\right] \quad . \qquad (B13)$$

We integrate again and obtain the self induced beam voltage

$$V_b = \frac{I_o s T_a L}{1+v'_g T_a}\left[1-\frac{(1+g)^{x_2}-1}{g x_2}\right] \quad . \qquad (B14)$$

r $\left(1-\frac{1}{v'_g\,T_a}\right)$ $g = mL$

The beam induced vo ...vided by the beam induced voltage of a uniform and lossless section is the be... to rf energy conversion efficiency

$$\eta_i = \frac{4T_a}{T_f}\left[\frac{1}{1+v'_g T_a}\right]\left[1-\frac{(1+g)^{x_2}-1}{g x_2}\right] \qquad (B15)$$

APPENDIX C

Rationale for Using Efficiencies and Elastance per Unit Length

An astute observer will notice that the shunt resistance per unit length r was not used in the above derivation. The answer to 'why not?' is similar to the answer Laplace gave to Napoleon when asked why he omitted mention of the Deity from his *Mécanique Céleste*, to wit "I had no need for it".

Three constants are necessary and sufficient to characterize a cell of a traveling wave section. They are:

1. The elastance s. It indicates the cross-sectional area over which the rf energy is distributed and how well this rf energy is concentrated at the beam trajectory, which is, generally at the center of the guide. The elastance varies as the square of the operating frequency.

2. The group velocity v_g. It indicates the longitudinal energy density.

3. The quality factor Q. It indicates the fraction of the rf energy lost to the section walls per unit time, per unit length.

Instead of defining a constant that depends on loss, the usual shunt resistance $r = \dfrac{E^2}{dP/dz}$ I defined a constant that does not depend on loss, the elastance $s = \dfrac{E^2}{w}$. We already have a constant, Q which takes care of loss.

The reasons for deriving an alternative expressions for the rf and self induced beam voltages in terms of η and s rather then using the usual expressions which is in terms of $f(\tau)$ and r, are:

1) $f(\tau)$ goes to zero and r goes to infinity as the surface resistance R_s approaches zero. Thus the usual expressions is indeterminate when R_s is zero. The alternative expressions contain only one R_s dependent factor, η which approaches unity as R_s approaches zero. Thus the alternative expressions are more suitable for a SC section because then the surface resistance does indeed approach zero. The alternative expressions are also more suitable for single bunch operation.

2) The alternative expression for the unloaded voltage contains more information. Assume negligible loss. We know that if we double L we double the voltage. Yet the usual expression implies a $\sqrt{2}$ increase, because it does not directly show the filling time. The alternate expression however does indicate voltage doubling. If we double the group velocity the alternate expression tells us, correctly, that we increased the voltage by a factor of 2. The usual expression at best does not tell us anything, at worse it tells us that nothing happened. The usual expression does not, but the alternate expression does tell us the amount of average rf power required to attain a given voltage. The expression for V_0 we derived reduces to the expression we would have obtained had we started without losses in the first place.

The shunt resistance is derived from cavity analogy to the lumped parameter parallel RLC circuit. It is the R of the RLC circuit. To carry the analogy to its logical conclusion, we should call R/Q the shunt reactance. It is the reactance of either the shunt capacitor or of the shunt inductor: $X = 1/\omega C = \omega L$. The elastance can also be derived from the same circuit. It is C^{-1}. The shunt resistance is useful when considering average power and the elastance when considering the average power required to periodically charge the capacitor. A travelling wave section is more analogous to the latter. It is fortuitous that S follows R in the alphabet.

Presently, the elastance and shunt reactance are given in terms ω, R, and Q. There is no basic objection for calling the elastance, $\omega R/Q$ and the shunt reactance R/Q. But why denote fundamental parameters that do not depend on loss, by several parameters that do. The elastance can be derived from fundamental relations. From the energy stored per unit volume

$$\frac{1}{2}\epsilon E^2 = \frac{U}{A_e L} \quad \text{we obtain} \quad s = \frac{V^2}{UL} = \frac{E^2}{U/L} = \frac{2}{\epsilon A_e}$$

Here A_e is the effective cross-sectional area of the cell.

The expressions for the no-load and beam induced voltages in reference 5 can be converted to expressions that do not become indeterminate as the resistivity of the section material approaches zero, that is to the expressions of this note if we make the following substitutions.

$$T_a = \frac{Q}{\omega}, \quad r = sT_a, \quad r\tau = sT_f/2, \quad \alpha r = s/2v_g \quad \text{and} \quad \alpha = \frac{1}{2T_a v_g} .$$

Also, with the above substitutions, all combination of parameters that are independent of loss can be written in terms of parameters that are independent of loss. The expressions in reference 5 are not generalized to variable group velocity as are the expressions in this note.

In reference 4 and 8, P. Wilson uses for what we have defined as elastance the symbol $4k_1$ because k_1 has been defined previously by the expression $V = k_1 q$. He calls it the loss parameter. However in the present context the use of elastance as a positive structure "figure of merit" defined as $s = \frac{E^2}{w}$ is more appropriate. The elastance relates the accelerating electric field to stored energy per unit length. It is of no consequence whether the energy is due to input rf or is generated by a passing charge.

The elastance is a fundamental measurable parameter which is appropriate for both the unloaded and the beam or charge induced fields for both pulse and single bunch operation and for both copper and superconducting sections.

The symbol S for elastance is recognized by the International Standards Organization (ISO).