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A TAPERED POLOIDAL GAP FOR THE REDUCTION OF FIELD ERRORS†

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The conventional Reversed Field Pinch is surrounded by a toroidal conducting shell that provides wall stabilization for the plasma. Usually, this shell is made of aluminum or copper, and it must have an insulated poloidal gap to permit penetrations of toroidal loop voltage to the plasma. Many magnetic field perturbations will induce toroidal shell currents with poloidal Fourier components having mode numbers $m \geq 1$. A butt-joint poloidal gap will mode convert all of these image currents into potentially damaging magnetic-field errors with large spectra of toroidal-mode numbers. Overlapping tapered gaps have been proposed and partially tested for reducing these field errors.[1,2] The mathematical basis for minimizing the $m = 1$, $n = 0$ field errors for all frequencies using tapered gaps, is presented in this paper.

The details of field errors at a poloidal gap and corresponding plasma effects will be discussed in a future paper.[3] This note describes the theoretical basis for the overlapping tapered poloidal gap that has been proposed for ZTH.

A cylindrical model is used to represent the shell. The z-axis is the minor axis of machine, θ is the poloidal angle, and r is the radial coordinate. The thin shell is of radius a and thickness T_o (away from the gap). All of the shell has resistivity η.

An external vertical field is applied to the shell at a frequency ω,

$$\underset{\sim}{B}_o = B_o(\hat{r}\sin\theta + \hat{\theta}\cos\theta)e^{-i\omega t} . \qquad (1)$$

† Work performed under the auspices of the USDOE.

From hereon the frequency term $e^{-i\omega t}$ is assumed to be present but not specified.

The thin shell approximation is used where the skin depth of the shell material is much greater than the shell thickness,

$$\left(\frac{\eta}{\mu_o \omega}\right)^{\frac{1}{2}} \gg T_o \ . \tag{2}$$

Note that this paper is written using SI units.

It is a textbook exercise to show that the total internal and external fields for a straight thin cylinder are

$$\underset{\sim}{B}_i = B_i(\hat{r} \sin\theta + \hat{\theta} \cos\theta), \qquad r \leq a - T_o/2 \tag{3}$$

and

$$\underset{\sim}{B}_e = \underset{\sim}{B}_0 + B_s(\hat{r} \sin\theta - \hat{\theta} \cos\theta) a^2/r^2, \qquad r \geq a + T_o/2 \tag{4}$$

where

$$B_i = B_o/(1 - i\omega\tau_o) \ , \tag{5}$$

$$B_s = i\omega\tau_o B_i \ , \tag{6}$$

and

$$\tau_o = \mu_o a T_o / 2\eta \ . \tag{7}$$

The objective of this work is to design an insulated poloidal gap as illustrated in Fig. 1 so that it transmits magnetic fields through the shell according to Eqs (3) and (4). For the purpose of our discussion, an overlapping gap made of two pieces, as in Fig. 1a, will be considered. The metal thickness of the taper attached to the left hand shell is given by $T_1(z) = T(z)$, while the right hand taper is defined to be symmetric, $T_2(z) = T(\ell - z)$.

Subscripts 1 and 2 denote the left and right hand tapers respectively. ZTH has been designed to have a three piece gap as in Fig. 1b, because this design is less likely to interface with the toroidal field of an RFP. In this instance, the top and bottom pieces of taper are each half of the thickness $T_1(z)$.

It is convenient to define the current in the shell in terms of

$$
\begin{aligned}
T_1(z)\underline{J}_1(\theta, z) &= \nabla \psi_1(\theta, z) \times \hat{r} \\
T_2(z)\underline{J}_2(\theta, z) &= \nabla \psi_2(\theta, z) \times \hat{r} \,.
\end{aligned}
\tag{8}
$$

Away from the gap one has

$$
\begin{aligned}
T_a J_z(\theta) &= \frac{2B_v}{\mu_o} \cos\theta \\
&= \frac{1}{a} \frac{\partial \psi}{\partial \theta} \\
&= \frac{1}{a} \psi_o \cos\theta \,.
\end{aligned}
\tag{9}
$$

It is assumed that the two (or three) tapers are thin compared to the shell radius and that they are very close together but electrically insulated from one another. Therefore, it is assumed that the shell currents may be superimposed and taken as one. This assumption implies

$$
\psi_1(\theta, z) + \psi_2(\theta, z) = \psi_o \sin\theta \,. \tag{10}
$$

It is also assumed that radial magnetic field that passes through one taper will also go through the other

$$
B_{1r} = B_{2r} = B_r \tag{11}
$$

At the surface of each taper the tangential electric field is

$$
E_{1,2t} = \eta J_{1,2} \,, \tag{12}
$$

and Faraday's law relates these to the radial magnetic field

$$
\nabla \times E_{1,2t} = \frac{\partial B_r}{\partial t} \,. \tag{13}
$$

It is possible to choose a variety of ψ_1 and ψ_2 functions that satisfy Eqs. (9) and (10) and solve for $T(z)$. An obvious but not necessary choice of current distributions is

$$
\begin{aligned}
\psi_1 &= \psi_o \cos^2 kz \sin\theta \\
\psi_2 &= \psi_o \sin^2 kz \sin\theta
\end{aligned}
\tag{14}
$$

where $k \equiv \pi/2\ell$

Upon solving Eq. (13) with $T(z)$ as the dependent variable one gets

$$
(ka)^2 \frac{\partial}{\partial(kz)} \left[\left(\frac{T_o}{T} \right) \sin(2kz) \right] + \left(\frac{T_o}{T} \right) \cos^2 kz - 1 = 0 \ . \tag{15}
$$

Careful analysis of the boundary conditions on Eq. (15) gives

$$
\frac{T(o)}{T_o} = 1 + \frac{\pi^2 a^2}{2\ell^2} \tag{16}
$$

and

$$
T(\ell) = 0 \ . \tag{17}
$$

Eq. (15) has been solved numerically, and the results are presented in Fig. 2. Clearly, the length of the tapered pieces is vital to determination of the shape and thickness of the tapers. When the tapers are as long as a shell diameter, their maximum thickness is $\sim 2.25\ T_o$, but if the taper length is a radius, the maximum thickness is $\sim 6\ T_o$. If the tapers are made too short and thick, the thin shell approximations could be violated, and the engineering design of the gap would become very complex.

In conclusion, if the tapered gap length is about one shell diameter, this shielded gap should provide a very uniform transmission of $m = 1$, $n = 0$ magnetic fields to the plasma. It is particularly important to note that, with the thin shell assumptions used here, this gap transmits $m = 1$, $n = 0$ fields at **all frequencies** as if the shell were a uniform unbroken cylinder. Although transmission of other m and n components is imperfect it will surely be superior to a butt joint gap.

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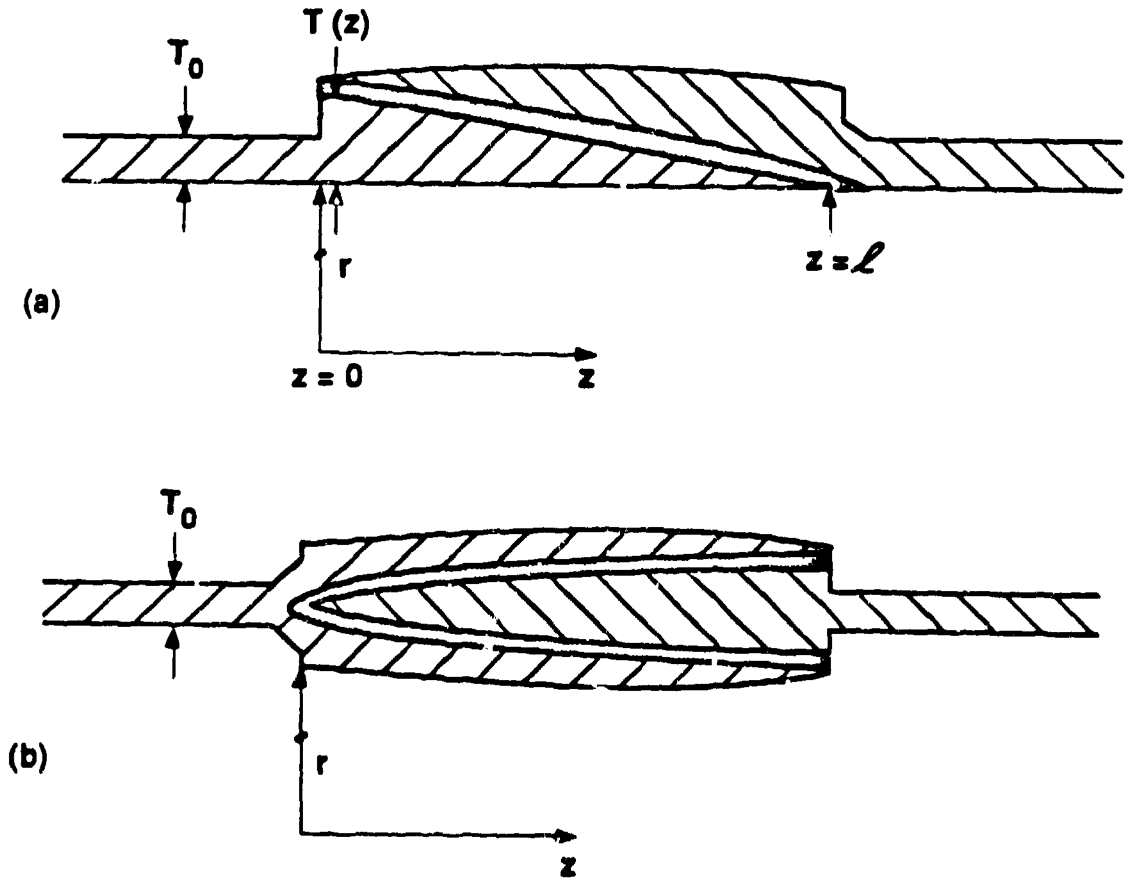

Figure 1. Longitudinal cross section of: (a) a double tapered gap and (b) a triple tapered gap.

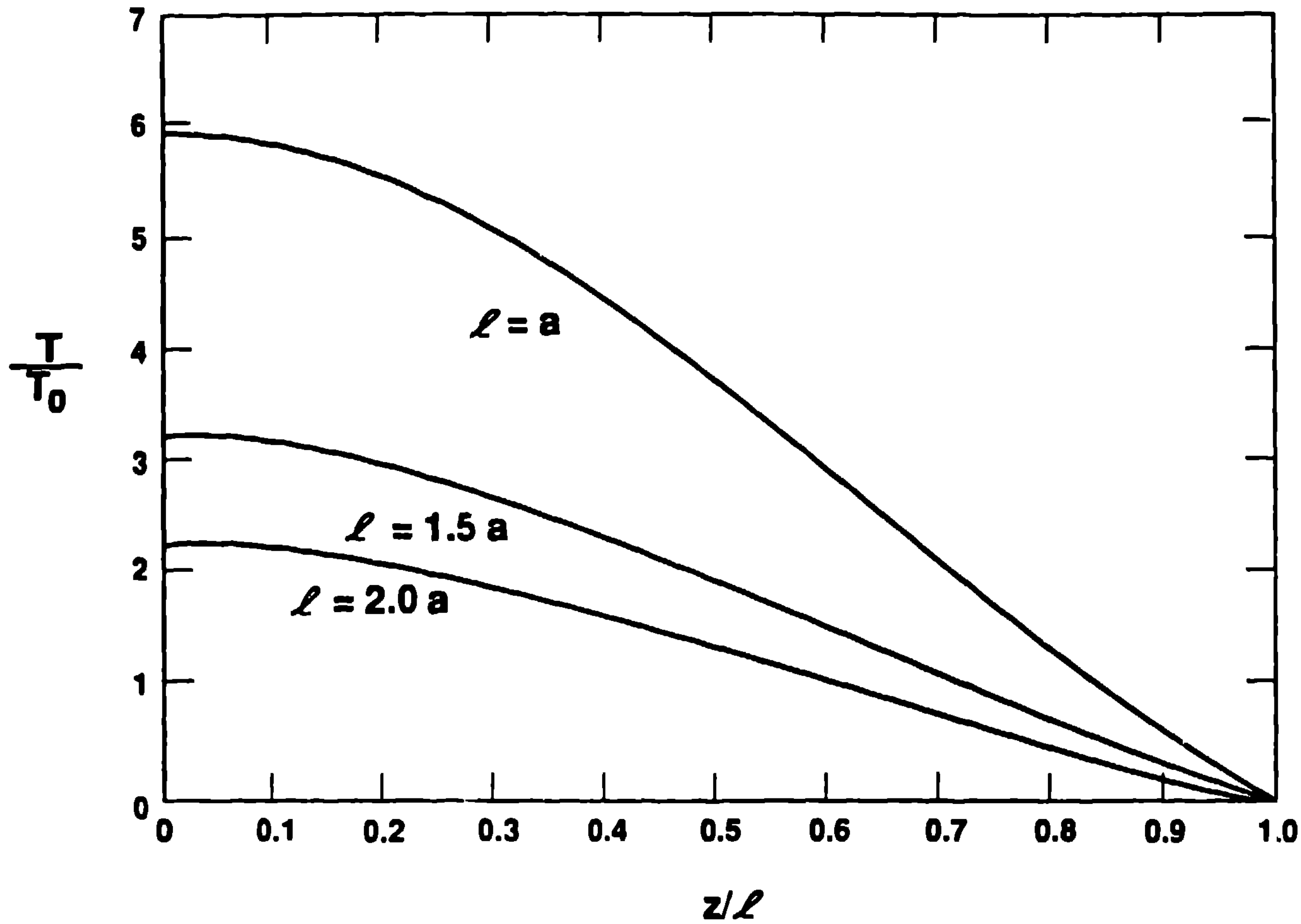

Figure 2. Computed taper thickness as a function of toroidal coordinate, z, and overall gap length ℓ/a.