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From Fundamental Fields to Constituent Quarks and Nucleon Form Factors ?

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ABSTRACT

Constituent-quark models formulated in the frame work of nonrelativistic quantum mechanics have been succcesful in accounting for the mass spectra of mesons and baryons. Applications to elastic electron scattering require relativistic dynamics. Relativistic quantum mechanics of constituent quarks can be formulated by constructing a suitable unitary representation of the Poincaré group on the three-quark Hilbert space. The mass and spin operators of this representation specify the relativistic model dynamics. The dynamics of fundamental quark fields, on the other hand, is specifield by a Euclidean functional integral. In this paper I show how the dynamics of the fundamental fields can be related in principle to the Hamiltonian dynamics of quark particles through the properties of the Wightman functions.

1. Introduction

Nonrelativistic constituent-quark models have been successful in accounting for mass spectra of hadrons.[1-3] The wave functions of these models are functions of relative coordinates, spin, flavor and color variables. They are eigenfunctions of a mass operator (internal Hamiltonian) and the total spin, $\vec{j}^{\,2}$. The relative coordinates are related by Fourier transform to internal momenta. These wave functions are invariant under translations and independent of the total momentum. They are thus frame independent. Eigenfunctions of the energy and momentum are required for the calculation of the electromagnetic properties of such models. Relativity requires that current matrix elements must satisfy covariance conditions, which for spin-zero particles take the form

$$\Lambda^{\mu}{}_{\nu} \langle p'| I^{\nu}(x) |p\rangle = \langle \Lambda p'| I^{\mu}(\Lambda^{-1}x) |\Lambda p\rangle \;, \tag{1}$$

where Λ is any Lorentz transformation. In nonrelativistic quantum mechanics individual momenta of the constituents in an arbitrary frame are related to the internal momenta by Galilean boosts, and the eigenfunctions of the energy and momentum are obtained by Galilean boosts from the eigenfunctions of the internal Hamiltonian and the spin. The Lorentz covariance relations illustrated in Eq. (1) are then violated. Relativistic quantum mechanics,[4] which remedies this defect is based on the following observations.

1. *Symmetries in Quantum theory are implemented by unitary transformations of the Hilbert space of states.*
2. *In nonrelativistic quantum mechanics the Hilbert space of states is the tensor product of single-particle Hilbert spaces for both interacting and noninteracting systems. They differ by different unitary representation of the time evolution in the same space of states.*
3. *The Hilbert space of single-particle states is the same for relativistic and nonrelativistic particles.*
4. *Relativistic Particle dynamics is specified by the construction of a nontrivial unitary representation of the Poincaré group on the tensor product of single-particle Hilbert spaces, or on a direct sum of such tensor products.*
5. *Any internal Hamiltonian, which is invariant under translations, independent of the total momentum, and commutes with a spin operator, can be interpreted as the mass operator of a nontrivial unitary representation of the Poincaré group. Together with a choice of the kinematic subgroup the mass operator specifies the relativistic dynamics.*

On the basis of these observations it is possible to obtain the electromagnetic form factors of constituent quark models in a manner consistent with the requirements of Poincaré covariance.[5] This construction separates the requirements of relativistic invariance from the decisions on the degrees of freedom which, for physical reasons, ought to be represented explicitly in the model.

This paper is an attempt to illuminate the connection between the relativistic particle dynamics described above and the quantum theory of fundamental fields, which specifies the dynamics by a functional integral in Euclidean space-time.[6,7] Fundamental fields are operator valued distributions.[8] Products of the field operators applied to the physical vacuum state span the Hilbert space of the theory. The field operators transform covariantly under a unitary representation of the Poincaré group. A connection between the fundamental fields and the relativistic quantum mechanics of constituent quarks can be found if there is a unitary map of a Poincaré invariant subspace of the Hilbert space of the field theory onto the Hilbert space of the constituent-quark. The existence and the properties of such a map depend on the properties of the Wightman functions, which are determined in principle by the Lagrangean of the field theory.[6,7] Conventional connections between quantum field theory and quantum mechanics involve either static approximations appropriate only for sufficiently massive particles, or the perturbative Fock space. Neither approach seems adequate for constituent quarks. Time ordered Green functions and other matrix elements of time ordered products of local fields are useful quantities directly related to observable scattering amplitudes. However, they do not permit the reconstruction of the Hilbert space of the field theory and provide therefore no access to relativistic quantum mechanics of constituent particles.

In Sec. 2. I briefly review the relativistic quantum mechanics of constituent quarks. For the general background and more details see refs. 4 and 5. In Sec. 3 I attempt a formal sketch of the relation to quantum field theory.

2. Relativistic Dynamics of Constituent Quarks

The bound-state wave function of a nucleon,

$$\psi_{m_N, j=\frac{1}{2}}(\vec{q}_1, \vec{q}_2, \vec{q}_3, \mu_1, \mu_2, \mu_3, \tau_1, \tau_2, \tau_3), \tag{2}$$

consisting of three constituent quarks is a symmetric rotationally invariant function of the internal momenta $\vec{q}_i$, subject to the constraint $\vec{q}_1 + \vec{q}_2 + \vec{q}_3 = 0$, spin variables, $\mu_i = \pm\frac{1}{2}$, and the isospin variables, $\tau_i = \pm\frac{1}{2}$. The total spin operator acting on these functions is

$$\vec{j} = \sum_{i=1}^{3} (\vec{y}_i \times \vec{q}_i + \vec{s}_i) \quad . \tag{3}$$

By definition the operators $\vec{y}_i$ are canonically conjugate to the internal momenta $\vec{q}_i$, and they commute with the operators $\vec{s}_i$ associated with the spin variables μ_i.

The four-momentum operator, P is determined by the mass operator and three kinematic components. The choice of three kinematic components of the four-momentum implies the choice of the kinematic subgroup of the Poincaré group and thus the "form of dynamics".[9] Let $n := \{-1, \vec{n}\}$ be a null vector $n^2 = 0$. If the kinematic subgroup leaves the null-plane $x^+ := n \cdot x = 0$ invariant, the null-plane momentum,

$$\mathbf{P} := \{P^+ := n \cdot P, \vec{P}_\perp := \vec{P} - \vec{n} \cdot \vec{P}\} \ , \tag{4}$$

is covariant under the kinematic boosts,

$$P^+ \to e^a P^+ \ , \qquad \vec{P}_\perp \to \vec{P}_\perp + \vec{b}_\perp P^+ \tag{5}$$

specified by the parameters $-\infty < a < \infty$ and $\vec{b}_\perp$.

States in the three quark Hilbert space are thus represented by functions of the kinematic components of the total four-momentum, the internal momenta, spin, and flavor variables, $\Psi(\mathbf{P}, \vec{q}_1, \vec{q}_2, \vec{q}_3, \mu_1, \mu_2, \mu_3, \tau_1, \tau_2, \tau_3)$ with the scalar product

$$\begin{aligned} \|\Psi\|^2 = & \int d^3\mathbf{P} \int d^3q_1 \int d^3q_2 \int d^3q_3 \; \delta(\vec{q}_1 + \vec{q}_2 + \vec{q}_3) \\ & \times \sum_{\mu_1,\mu_2,\mu_3} \sum_{\tau_1,\tau_2,\tau_3} |\Psi(\mathbf{P}, \vec{q}_1, \vec{q}_2, \vec{q}_3, \mu_1, \mu_2, \mu_3, \tau_1, \tau_2, \tau_3)|^2 \ , \end{aligned} \tag{6}$$

where

$$d^3\mathbf{P} := d^2P_\perp \theta(p^+) dP^+/2P^+ \tag{7}$$

is invariant under the kinematic Lorentz transformations. By definition the spin operator (3) commutes with the null-plane boosts (5). The ten generators of the Poincaré group are determined by the mass operator, the three components of the total spin, the three kinematic components of the four-momentum, and the three generators of the null-plane boosts. An eigenfunction of the four-momentum is constructed as a simultaneous eigenfunction of the mass and the kinematic components of the four-momentum,

$$\begin{aligned} |p_N, j\rangle \; \rightarrow & \; \Psi_{p_N,j}(\mathbf{P}, \vec{q}_1, \vec{q}_2, \vec{q}_3, \mu_1, \mu_2, \mu_3, \tau_1, \tau_2, \tau_3) \\ & = \psi_{m_N,j}(\vec{q}_1, \vec{q}_2, \vec{q}_3, \mu_1, \mu_2, \mu_3, \tau_1, \tau_2, \tau_3)\delta(\mathbf{P} - \mathbf{p}_N)p_N^+ \ . \end{aligned} \tag{8}$$

Nowhere in this construction did I assume that the particles are free, or "approximately free". However, noninteracting particles live in the same Hilbert space. In that case the mass operator is

$$M_0 = \sum_i \sqrt{m^2 + \vec{q}_i^{\,2}} \ , \tag{9}$$

and the spin operator $\vec{s}_i$ is equal to the canonical spin of the ith particle for $P = \{M_0, 0, 0, 0\}$.

The "impulse approximation" for the current operators assumes that the currents can be approximated by a sum of one-body operators. In order to make this precise it is necessary to consider tensor products of single-quark Hilbert spaces. Vectors in a single-quark Hilbert space, $\mathcal{H}_1$, are represented by functions $g(\mathbf{p}, \lambda)$, with the scalar product

$$(g, g) = \int d^3\mathbf{p} \sum_\lambda |g(\mathbf{p}, \lambda)|^2 \ , \tag{10}$$

where λ is the component of the null-plane spin in the direction of $\vec{n}$. Given a mass m the unitary representation of the Poincaré group is specified as described above.

The null-plane spin operator $\vec{s}$ are invariant under null-plane boosts. Vectors in the tensor product $\mathcal{H}_3 := \mathcal{H}_1 \times \mathcal{H}_1 \times \mathcal{H}_1$ of three single-particle Hilbert spaces are thus represented by functions

$$\Psi(\mathbf{p}_1, \lambda_1, \mathbf{p}_2, \lambda_2, \mathbf{p}_3, \lambda_3) \equiv \Psi(\mathbf{P}, \xi_1, \vec{q}_{1,\perp}, \lambda_1, \xi_2, \vec{q}_{2,\perp}, \lambda_2, \xi_3, \vec{q}_{3,\perp}, \lambda_3) \; , \tag{11}$$

where

$$\mathbf{P} = \sum_i \mathbf{p}_i \; , \qquad \xi_i = \frac{p_i^+}{P^+} \; , \qquad \vec{q}_{i\perp} = \vec{p}_{i\perp} - \xi_i \vec{P}_\perp \; , \tag{12}$$

and

$$\begin{aligned} d^3\mathbf{p}_1 d^3\mathbf{p}_2 d^3\mathbf{p}_3 = & d^3\mathbf{P} \frac{d\xi_1}{\xi_1} \frac{d\xi_2}{\xi_2} \frac{d\xi_3}{\xi_3} \delta(\xi_1 + \xi_2 + \xi_3 - 1) \\ & \times d^2 q_{1\perp} d^2 q_{2\perp} d^2 q_{3\perp} \delta(\vec{q}_{1\perp} + \vec{q}_{2\perp} + \vec{q}_{3\perp}) \; . \end{aligned} \tag{13}$$

In this representation the assumption of "one-body" current operators can be stated in the obvious manner,

$$\begin{aligned} & \langle \mathbf{p}_1', \lambda_1', \mathbf{p}_2', \lambda_2', \mathbf{p}_3', \lambda_3' | \, I^\mu(x) \, | \mathbf{p}_3, \lambda_3, \mathbf{p}_2, \lambda_2, \mathbf{p}_1, \lambda_1 \rangle = \langle \mathbf{p}_1', \lambda_1' | \, I^\mu(x) \, | \mathbf{p}_1, \lambda_1 \rangle \\ & \times \delta(\mathbf{p}_2' - \mathbf{p}_2) \delta(\lambda_2' - \lambda_2) \delta(\mathbf{p}_2' - \mathbf{p}_2) \delta(\lambda_2' - \lambda_2) + \quad \text{permutations} \; . \end{aligned} \tag{14}$$

These operators are covariant only for free particles, where the Poincaré representation on $\mathcal{H}_1$ implies the product representation on $\mathcal{H}_3$. In general there must be two- and three-body operators in order to satisfy the requirements of Lorentz covariance and current conservation. It is possible, however, to assume without inconsistency that only one-body currents contribute to the matrix elements $\langle p_N', \lambda_N' | \, I^+(0) \, | p_N, \lambda_N \rangle$, if the orientation of the null plane is chosen such that $Q^+ \equiv p_N^{+\prime} - p_N^+$ vanishes. These matrix elements are sufficient to obtain the elastic form factors. In this context the "impulse approximation" is a possible model assumption for the current.

There remains the task of establishing the unitary equivalence of the function spaces specified by (6) - (7) and (11) - (13) respectively. This is done by considering the equivalence of the free-particle Poincaré representations in the two function spaces. I note that

$$M_0^2 = \sum_i \frac{\vec{q}_\perp^{\,2} + m^2}{\xi_i} = \left[\sum_i \sqrt{m^2 + \vec{q}_i^{\,2}} \right]^2 \; , \tag{15}$$

where

$$\vec{n} \cdot \vec{q}_i := \frac{1}{2} \left[M_0 \xi_i - \frac{m^2 + \vec{q}_{i\perp}^{\,2}}{M_0 \xi_i} \right] \; . \tag{16}$$

The canonical spin operators $\vec{\mathfrak{s}}_i$ are related to the null-plane spins $\vec{s}_i$ by Melosh rotations[10,11]

$$\vec{\bar{s}}_i = \mathcal{R}_M \vec{s}_i \ . \tag{17}$$

The matrix representation of $\mathcal{R}_M$,

$$\mathcal{D}^{\frac{1}{2}}_{\mu,\lambda}(\mathcal{R}_M) \equiv \left[\frac{m + \xi M_0 - i\vec{\sigma}\cdot(\vec{n}\times\vec{q}_\perp)}{\{(m+\xi M_0)^2 + \vec{q}_\perp^{\,2}\}^{\frac{1}{2}}}\right]_{\mu,\lambda} , \tag{18}$$

gives the relation between the spin variables μ and λ.

Thus the functions $\Psi(\mathbf{P}, \vec{q}_1, \vec{q}_2, \vec{q}_3, \mu_1, \mu_2, \mu_3)$ represent vectors in the tensor product of three single-particle Hilbert spaces. As is the case in nonrelativistic quantum mechanics states of interacting and noninteracting particles are represented by functions in the same space. The interaction dependence of the dynamic Poincaré generators enters only through the mass operator. For interacting particles one could, of course, introduce an interaction dependence into the relations between the functions (6) - (7) and (11) - (13). However, that would be inconsistent with the physical assumptions underlying the impulse approximation which imply that the one-body current operator should be the current of a free particle of mass m, oblivious of the presence of other particles. As such it should not depend on the interactions, or on the mass of the nucleon. The constituent-quark mass appears in this picture as a parameter in the model assumptions specifying the quark current.

3. From Fundamental Fields to Constituent Quarks

In order to establish a relation of the quantum mechanics of constituent quarks described in the previous Section to the dynamics of fundamental fields I will start with certain axiomatic assumptions.[8] The objective is to find an isometric map of the quantum mechanical Hilbert space into a Poincare invariant subspace of the Hilbert space $\mathcal{H}$ of the field theory.[13] This map should provide a relation between the field dynamics specified by the Lagrangean and the dynamics specified by the unitary representation of the Poincaré group in the quantum mechanical Hilbert space.

Let $\psi(x)$ be the fundamental quark field. There is a unitary representation of the Poincaré group in the Hilbert space $\mathcal{H}$ of the field theory such that the vacuum state Ω is invariant,

$$U(\Lambda, a)\Omega = \Omega \ , \tag{19}$$

and the field satisfies the covariance relations,

$$U^\dagger(\Lambda, a)\psi(x)U(\Lambda, a) = S(\Lambda)\psi(\Lambda^{-1}x + a) \ , \tag{20}$$

where the matrix $S(\Lambda)$ is the Dirac spinor representation of the Lorentz transformation Λ. I am suppressing color and flavor quantum numbers. Products of fields folded with Schwartz functions [infinitely differentiable functions that decrease faster than any power],[12] $h \in \mathcal{S}(\Re^4)$, applied to the vacuum state generate state vectors in the Hilbert space $\mathcal{H}$, *e.g.*

$$\Psi = \bar{\psi}(h_1)\bar{\psi}(h_2)\bar{\psi}(h_3)\Omega \ , \tag{21}$$

where

$$\bar{\psi}(h) := \int d^4x \bar{\psi}(x)h(x) \ . \tag{22}$$

Linear combinations of such states span a subspace of baryon number one. The scalar product of these vectors implies a scalar product for functions $h(x_3, x_2, x_1)$ of three space-time points,

$$(h, h) = \int d^4x'_1 \cdots \int d^4x_1 \cdots \bar{h}(x'_1, \ldots)\mathcal{W}(\ldots, x'_1; x_1, \ldots)h(\ldots, x_1) \geq 0 \ , \tag{23}$$

where $\mathcal{W}$ is the Wightman function,

$$\mathcal{W}(x'_3, x'_2, x'_1; x_1, x_2, x_3) := \left(\Omega, \psi(x'_3)\psi(x'_2)\psi(x'_1)\bar{\psi}(x_1)\bar{\psi}(x_2)\bar{\psi}(x_3)\Omega\right) \ . \tag{24}$$

Since the scalar product is not strictly positive the correspondence between the functions and the states is many to one. It follows from the invariance of the vacuum (19) and the covariance of the field operators (20) that the scalar product (23) is invariant under the transformations

$$h(x_1, \ldots) \rightarrow S(\Lambda^{-1}) \cdots h(\Lambda(x_1 - a), \ldots) \ . \tag{25}$$

We have thus a representation of states by equivalence classes of manifestly covariant functions with a nontrivial scalar product specified by the Wightman functions, which specify the dynamics of the field theory.

In order to to establish the connection to quantum mechanics I will generalize to spinor fields results obtained by Schlieder and Seiler[13] for scalar fields. The objective is to obtain one-to-one mapping to a subspace of $\mathcal{H}$ of functions restricted to the null plane $x^+ = 0$. Let $\mathbf{x}$ be any point on the null plane $x^+ = 0$, and let $f(\mathbf{x})$ be the Fourier transform of

$$f(\mathbf{p}) := u(\mathbf{p}, m)g(\mathbf{p}) \ , \tag{26}$$

where

$$g(\mathbf{p}) = \begin{pmatrix} g(p^+, \vec{p}_\perp, \frac{1}{2}) \\ g(p^+, \vec{p}_\perp, -\frac{1}{2}) \end{pmatrix} \ , \tag{27}$$

and the null-plane spinor amplitudes are

$$u(\mathbf{p},m) := \frac{\alpha_\perp \cdot p_\perp + \beta m + p^+}{\sqrt{mp^+}} \frac{1+\alpha_3}{2} \frac{1+\beta}{2} = \frac{m - \gamma \cdot p}{2\sqrt{mp^+}} \gamma^+ \frac{1+\beta}{2} , \tag{28}$$

where $\gamma^\mu := \beta\alpha^\mu$, $\alpha^0 := 1$. Note that no p^- appears on the right hand side of Eq. (28) because $\gamma^{+2} = 0$. Here the functions $g(p^+, \vec{p}_\perp, \pm\frac{1}{2})$ are Schwartz functions in the space $\hat{S}(\Re^3)$ defined in ref. 13. These functions are Schwartz functions that vanish for $p^+ = 0$. [See Eq. (2.4) of ref. 13] They are a dense set in Hilbert space $\mathcal{H}_1$, with the scalar product (10),

$$(f,f) := \int \frac{dp^+}{2p^+} \int d^2p_\perp \theta(p^+) \bar{f}(\mathbf{p}) f(\mathbf{p}) = (g,g) . \tag{29}$$

The mass m is a parameter in the definition of $f(\mathbf{x})$, unrelated to the field $\psi(x)$. If the operators

$$\bar{\psi}(f) := \int d^3\mathbf{x} \bar{\psi}(\mathbf{x}) f(\mathbf{x}) . \tag{30}$$

exist their products may provide the desired mapping from $\mathcal{H}_3$ into $\mathcal{H}$. It is necessary that the norm $\|\bar{\psi}(f)\Omega\|$ be finite and nonzero for all g. This norm depends on the two-point Wightman function, $\mathcal{W}_2(x,x') := (\Omega, \psi(x)\bar{\psi}(x')\Omega)$,

$$\|\bar{\psi}(f)\Omega\|^2 = \int d^3\mathbf{x} \int d^3\mathbf{x}' \bar{f}(\mathbf{x}) \mathcal{W}_2(\mathbf{x},\mathbf{x}') f(\mathbf{x}') . \tag{31}$$

In order to show that the norm is finite and nonvanishing I assume that $\mathcal{W}_2$ has the Lehmann representation[14]

$$\mathcal{W}_2(x,x') = \frac{1}{(2\pi)^3} \int d^4p \int_0^\infty d\kappa^2 \theta(p^0) \delta(p^2+\kappa^2) [A(\kappa^2) - B(\kappa^2)\gamma p] e^{ip\cdot(x-x')} , \tag{32}$$

It follows from Eq. (32) that the norm,

$$\|\bar{\psi}(f)\Omega\|^2 = \int d^3\mathbf{p} \sum_\lambda |g(\mathbf{p},\lambda)|^2 \int d\kappa^2 [A(\kappa^2) + mB(\kappa^2)] , \tag{33}$$

is finite and positive for all non vanishing functions g if the integrals over κ^2 are finite,

$$\int d\kappa^2 A(\kappa^2) < \infty \qquad \text{and} \qquad \int d\kappa^2 B(\kappa^2) < \infty . \tag{34}$$

Let $f_1(\mathbf{x}_1)$ and $f_2(\mathbf{x}_2)$ be two functions with support on different null planes. In general the scalar product,

$$(\bar{\psi}(f_1)\Omega, \bar{\psi}(f_2)\Omega) == \int d^3\mathbf{x}_1 \int d^3\mathbf{x}'_2 \bar{f}_1(\mathbf{x}_1)\mathcal{W}_2(\mathbf{x}_1, \mathbf{x}_2) f_2(\mathbf{x}_2) \ , . \tag{35}$$

does not vanish. It is easy to verify with Eq.(32) that for any f_1 on the first null plane there exists an f_2 on the second null plane such that the scalar product (35) differs from zero. It follows that functions with support on a different null planes generate the same space of states. This space is therefore Poincaré invariant.

For a free field of mass m the functions $A(\kappa^2)$ and $B(\kappa^2)$ would be proportional to the delta function, $\delta(\kappa^2 - m^2)$,

$$A(\kappa^2) = mB(\kappa^2) = m\,\delta(\kappa^2 - m^2), \tag{36}$$

which yields the free-field Wightman function,

$$\mathcal{W}_{2f}(x - x') = \frac{1}{(2\pi)^3} \int d^4p\, \theta(p^0)\delta(p^2 + m^2)[m - \gamma \cdot p] e^{ip\cdot(x-x')} \ . \tag{37}$$

For confined quarks there is no delta-function singularity in $A(\kappa^2)$ and $B(\kappa^2)$.

The scalar product of states Ψ defined by

$$\Psi := \int d^3\mathbf{x}_1 \int d^3\mathbf{x}_2 \int d^3\mathbf{x}_3\, \bar{\psi}(\mathbf{x}_3)\bar{\psi}(\mathbf{x}_2)\bar{\psi}(\mathbf{x}_1)\Omega f(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) \ , \tag{38}$$

implies for the functions f the scalar product

$$\begin{aligned}(f, f) := &\int d^3\mathbf{x}_1 \int d^3\mathbf{x}_2 \int d^3\mathbf{x}_3 \int d^3\mathbf{x}'_1 \int d^3\mathbf{x}'_2 \int d^3\mathbf{x}'_3 \\ &\times \bar{f}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)\mathcal{W}(x_3, x_2, x_1; \mathbf{x}'_1, \mathbf{x}'_2, \mathbf{x}'_3) f(\mathbf{x}'_3, \mathbf{x}'_2, \mathbf{x}'_1) \ ,\end{aligned} \tag{39}$$

The six-point Wightman function in this expression is a sum of products of two-point functions and truncated 4-point functions plus the truncated six-point function. Under suitable restrictions on the truncated Wightman functions this scalar product is strictly positive, and the space spanned by the states (38) is Poincaré invariant. The covariant Wightman functions thus specify the representation of the Poincaré group in the function space, where the scalar product is given by restriction of the Wightman function to a null plane. Since this restriction provides a nonsingular Hilbert-space measure the space is related to the quantum mechanical Hilbert space $\mathcal{H}_3$ by a similarity transform.

The fundamental Lagrangean determines the Euclidean functional integral. The moments of this functional integral (Schwinger functions) determine the Wightman functions and the properties of the field operators. These relations establish in principle a connection between the fundamental Lagrangean and the Poincaré invariant quantum mechanics of constituent quarks.

4. Conclusions

Quantum field theory and relativistic quantum mechanics are each formulated as a theory of operators acting on a Hilbert space of states with Lorentz transformations and translations implemented by unitary transformations. I have shown that the Wightman functions of the field theory can provide unitary map from a Poincaré invariant subspace of the Hilbert space of the field theory onto the quantum mechanical Hilbert space of constituent quarks. The Wightman functions thus determine the dynamics in the model Hilbert space of the constituent quarks. It should be emphasized that covariant amplitudes defined as matrix elements of time ordered products of fields do not represent vectors in any Hilbert space. The use of "light-cone variables" in the context of a Green function formalism should not be confused with the null-plane dynamics discussed in this paper.

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6. References

1. A. De Rujula, H. Georgi and S. I. Glashow, *Phys. Rev.* **D12**, 147 (1975).
2. N. Isgur and G. Karl, *Phys. Rev.* **D18**, 4186 (1978), *Phys. Rev.* **D19**, 2653 (1979), *Phys. Rev.* **D20**, 1191 (1979); C. Hayne and N. Isgur, *Phys. Rev.* **D25**, 1944 (1982); S. Capstick and N. Isgur, *Phys. Rev.* **D34**, 2809 (1986).
3. J. Carlson, J. Kogut and V. R. Pandharipande, *Phys. Rev.* **D27**, 233 (1983), *Phys. Rev.* **D28**, 2807 (1983).
4. B. D. Keister and W. N. Polyzou to be published in *Advances in Nuclear Physics*; W. N. Polyzou, *Ann. Phys.* **193**, 367 (1989) .
5. P. L. Chung and F. Coester, ANL preprint PHY-6622-TH-90.
6. J. Glimm and A. Jaffee, *Quantum Physics* (Springer-Verlag, NewYork 1987), Chapter 6.
7. E. Seiler, *Gauge Theories as a Problem of Constructive Quantum Field Theory and Statistical Mechanics* (Springer-Verlag, New York 1982).

8. N. N. Bogolubov, A. A. Logunov and I. T. Todorov, *Introduction to Axiomatic Field Theory,* (W. A. Benjamin, Reading, Mass. 1975); N. N. Bogolubov, A. A. Logunov, A.I. Oksak and I. T. Todorov, *General Principles of Quantum Field Theory* (Kluwer Academic Publishers, Boston 1990).
9. P. A. M. Dirac, *Rev. Mod. Phys.* **21**, 392 (1949).
10. H. Leutwyler, in *Current Algebra and Pheomenological Lagrange Function*, edited by G. Höhler (Springer-Verlag, Berlin/New York, 1969), p. 35.
11. H. J. Melosh, *Phys. Rev.* **D9**, 1095 (1974).
12. I. M. Gelfand and G. E. Shilov, *Generalized Functions* (Academic Press, New York 1964).
13. S. Schlieder and E. Seiler, *Commun. Math. Phys.* **25**, 62 (1972) .
14. S. S. Schweber, *An Introduction to Relativistic Quantum Field Theory* (Row, Peterson, Evanston IL 1961), See p. 672-674.

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