BNL-29768

Conf-810376--5

Lecture
Presented at
XVIth Rencontre De Moriond
Elementary Particle Physics Meetings
Les Arcs, Savoie, France
March 15-27, 1981
(to be published in proceedings)

QCD, OZI, AND EVIDENCE FOR GLUEBALLS

S.J. Lindenbaum
Brookhaven National Laboratory, Upton, New York 11973*
and
City College of New York, New York, New York 10031+

The characteristics expected from low Q^2-QCD for the behavior of glueballs and the OZI rule will be discussed.

The reaction $\pi^- p \to \phi\phi n$ represents an OZI forbidden (hairpin) diagram. It has been observed at the Brookhaven National Laboratory multiparticle spectrometer by the Brookhaven National Laboratory/City College of New York group. The author has shown that the expected OZI suppression is essentially entirely absent and in fact the Isobar Model which does not contain OZI suppression quantitatively explains the observed results. A general evaluation of the special characteristics of the data compared to other related reactions plus the foregoing facts leads the author to conclude that the intervention of glueball resonances is the likely explanation in the context of QCD. Other explanations are shown to be improbable. In particular the hypothesis that decay of a radial excitation of the η' is responsible for lack of OZI suppression is ruled out. Planned experiments with the purpose of explicitly discovering glueballs will be discussed. The OZI rule peculiarities such as violation of crossing symmetry and unitarity are attributed to color confinement.

* Research carried out under the auspices of the United States Department of Energy under Contract No. DE-AC02-76CH00016.

+ Work supported by the National Science Foundation.

Introduction

QCD[1,2] is a non-abelian gauge theory with the following novel characteristics.

I. The gauge bosons are an octet of colored vector massless gluons which self interact.

II. Asymptotic Freedom.

Gluon-gluon and multigluon interactions occur with the same coupling constant as the quark gluon interaction and these interactions become stronger as the energy decreases (and weaker as the energy increases).

Characteristics I and II follow from the nature of the non-abelian gauge theory and do not occur in an abelian gauge theory such as QED.

III. In addition, QCD has color confinement.

Considering the properties of gluon-gluon interactions, which become stronger as the energy becomes lower, and the characteristic of confinement, I believe one would almost inescapably be led to expect to find multigluon glueball resonances at low energies. Glueballs would be the only hadrons in a pure Yame Mills Theory[3] where $SU_c(3)$ was a local symmetry.

In a strict sense there is no gauge invariant separation into 2g, 3g, etc. due to the self couplings of gluons. Any number of gluons can couple together and transform into a different number. However, a gauge invariant description is possible.[4] Nevertheless classification by number of gluons is physically appealing and may well be meaningful judging by the success of quark model classifications. It should be noted that simple confinement arguments, the bag model, the quark potential model, and lattice calculations all predict glueball resonances.[5-12]

Therefore, if glueball resonances are not found, it would be difficult not to conclude QCD is in trouble. On the other hand discovery of real glueballs would be a great triumph for QCD.

Expected Characteristics of Glueball States

Glueball states[5-8] are color singlets with I = 0, B = 0 (u,d,s,c,b = 0), J^{PC} can be either non-exotic or exotic. Manifestly exotic J^{PC} combinations occur naturally in 2g and 3g combinations in contrast to the quark case where for example to obtain an exotic J^{PC} one must go from a $q\bar{q}$ (2 quark state) to a $q\bar{q}q\bar{q}$ (four quark state). So if the J^{PC} is not exotic they could appear as additional singlets resembling the I = 0 singlet of an SU(3) nonet. These states could mix with ordinary $q\bar{q}$ mesons. Exotic J^{PC} states can only mix with the exotic quark sector which appears to be heavily suppressed since no one has discovered an explicitly exotic state. Therefore exotic glueballs which are referred to as oddballs[7] would be purer glueballs.

Glueball Widths

One would expect the full width Γ of a glueball to be reduced compared to the typical hadronic width by $\sim \sqrt{\text{OZI}}$ suppression factor for vector gluons since you go from $q\bar{q}$ to glue only once. The OZI suppression factor was determined for vector particles which require a three gluon intermediate state. For a scalar glueball only two gluons would be involved in the intermediate state, therefore one could expect that the typical hadronic width would be reduced by $\sim \sqrt[3]{\text{OZI}}$ suppression factor. Taking the OZI suppression factor of $\sim$ 100 and a typical hadronic width $\sim$ 200 MeV a ballpark figure for the expected glueball widths would be $\Gamma \gtrsim 30 \pm 10$ MeV.

Carlson, Coyne, Fishbane, Gross and Meshkov[7] use perturbation theory calculations to conclude that $J = 0$ and $J = 1$ 2g glueballs are as narrow as a few MeV or less and the exotics have $\Gamma \sim 1$ MeV. The use of perturbation theory in decays of resonant states may not be accurate. Of course 3g glueballs could be wider, with typical widths like those given above.

Glueball Masses

Estimates of glueball masses are somewhat uncertain and to say the least obviously difficult to make. An order of magnitude estimate of the ground states of the glueball spectrum can be made by using a confined size $\sim$ 1 fermi and the uncertainty principle.

This leads to the result that the average energy of a gluon is

$$E_{gluon} > \sqrt{3}/R \sim 350 \text{ MeV}.$$

Hence crudely speaking the 2g spectrum would have $M \gtrsim 700$ MeV. The 3g spectrum would have $M \gtrsim 1$ GeV.

The latest bag model prediction for glueball masses have been made by Donoghue et al.[9] and the mass estimates for the states considered for 2g range from about 0.96 GeV (for 0^{++} and 2^{++}) to 1.3 GeV (for 0^{-+} and 2^{-+}). For 3g ground states (0^{++}, 1^{+-}, 3^{+-}) the mass is estimated to be 1.45 GeV with numerous other states at 1.8 GeV.

Quark potential models can be used to estimate the glueball mass. The coefficient of the term br which is linear in r is multiplied by a factor proportional to $(\text{charge})^2$. Thus going from quark anti-quark to gluon-gluon states the linear term in the potential should be multiplied by $(3/2)^2$. Thus the two gluon glueball masses should be the quark model masses multiplied by a factor of 3/2 and hence could be expected to have their ground states in the range of 1-2 GeV. Fishbane[8] (this paper is a good review reference) uses the fact that the ideas of quantum mechanics and better understanding of low energy tests of QCD such as charmonium decays require an effective mass for the gluons which has been shown by Parisi and Petronzio[10] to be O (0.8 GeV). Thus Fishbane[8] estimates that the low lying two gluon states are at O(1.6 GeV) while the three gluon spectrum begins at O(2.4 GeV). This approach may well be one of the better ways of estimating glueball masses.

Lattice Gauge Theories,[11)] String Models,[12)] and Effective Lagrangian Models[13)] all give glueball ground state estimates of 1-2 GeV. Of course, based on experience with quark spectroscopy the excited state masses should be expected to be roughly a factor of two higher than those of the ground states.

Table I (based on Coyne, Fishbane and Meshkov[7)]) shows J^{PC} values through spins of 3 which are allowed (√) or forbidden (X) for $q\bar{q}$ (quark model states) 2g glueballs and 3g (glueballs). Exotic states are indicated by arrows on the side of the table. If the arrow is solid, the J^{PC} is exotic in the quark model, but allowed only for 3g glueballs which allow all J^{PC} combinations. If there is a dotted arrow under the solid arrow the state is allowed for 2g as well as 3g glueballs. Of course $q\bar{q}$, 2g as well as 3g states of the same J^{PC} all mix to a certain extent. However, quark model classification successes, and general considerations, lead one to expect that to a reasonable extent these classifications may hold.

Therefore, if one discovered a flavorless boson state with I = 0 and a relatively narrow width it would make it an excellent candidate for a glueball. If it should turn out to have an exotic J^{PC}, the argument would be even more compelling.

How Would One Find Glueballs?

We have discovered many $q\bar{q}$ meson states and qqq baryon states, but to date no glueball states. From the success of the quark model, it is clear that the hadronization process leads to dominance of particle states built predominantly of q and $\bar{q}$. Therefore, starting from quark-built states, how can we produce glueballs.

It is obvious that the best way is to find a process where there is an intermediate state which contains 2 or more gluons with a variable total effective mass and no quarks (or anti-quarks). Thus the gluons can resonante and form a glueball which is a color singlet if one exists within their effective mass range. Thus an OZI forbidden process with a variable mass for the glue is an excellent candidate for a reaction to make glueballs enhanced relative to the normal OZI allowed hadronic background which is suppressed.

OZI Within the Context of QCD

The OZI rule[14,15)] has found a natural explanation in the context of QCD.[16,17)] The connected diagrams for OZI allowed processes allow a series of one-gluon exchanges without violating color confinement; on the other hand the disjoint or

TABLE I

List of states (J^{PC}) for quark model and glueballs. ✓(X) indicates allowed (disallowed). 2g (3g) correspond to glueball states formed from 2 (3) gluons. Solid arrow (→) indicates quark model exotics in 3g states. Dotted arrow (⇢) indicates exotics in 2g states.

State	$q\bar{q}$	2g	3g
0^{++}	✓	✓	✓
$\xrightarrow{3g}$ 0^{+-}	X	X	✓
0^{-+}	[illegible]	✓	✓
$\xrightarrow{3g}$ 0^{--}	X	X	✓
1^{++}	✓	✓	✓
1^{+-}	✓	X	✓
$\xrightarrow{3g}$ ⇢ 1^{-+}	X	✓	✓
1^{--}	✓	X	✓
2^{++}	✓	✓	✓
$\xrightarrow{3g}$ 2^{+-}	X	X	✓
2^{-+}	✓	✓	✓
2^{--}	✓	X	✓
3^{++}	✓	✓	✓
3^{+-}	✓	X	✓
$\xrightarrow{3g}$ ⇢ 3^{-+}	X	✓	✓
3^{--}	✓	X	✓

hairpin diagrams requires 3 gluon exchange since the ϕ is a vector (2 gluon exchange would be permissible if the ϕ were a pseudoscalar). Therefore, the OZI suppression is attributable to the factor g^6 involved in going from quarks to quarks via 3 gluon exchange.

However, it has been shown[2,18-20)] the two OZI allowed processes can be used to eliminate the OZI forbidden diagram and replace it with a sequence of two OZI allowed steps. For example $\phi \to \pi^- + \rho^+$ is an OZI forbidden process. However, $\phi \to K^-K^+ \to \pi^-\rho^+$ is a two step process, each of which is OZI allowed. Thus the OZI rule appears to violate crossing symmetry and unitarity.[2,18-20)] Furthermore, this is a quite general phenomenon occurring in production as well as decay.

For example the typical OZI suppression factor $\sim$ 100 has been found[14,15] in the reaction $\pi^- p \to \phi n$. However, this reaction can occur via a two-step process,[17,19] each step of which is allowed, namely, $\pi^- p \to K^+K^- n \to \phi n$.

I have pointed out[18,19] that what is unique here is that by the two steps we have converted a process which requires 3 gluon exchange into one which can go by a series of single gluon exchanges. Since the OZI rule works well, this evidently is not allowed, although it cannot be explicitly demonstrated by calculation, due to the well-known difficulties with strong interaction calculations. Thus, the crossing symmetry and unitarity arguments made without taking into account this QCD fact of life are probably only applicable when one does not change the nature of a gluon exchange. I view this as an added requirement due to color confinement. Thus the OZI rule can be considered reasonable within the context of QCD. Furthermore, it works.

J/ψ Radiative Decay

One suggested way[6,7)] of finding glueballs in an OZI forbidden process is in investigating radiative J/ψ decay.[21] Perturbation theory treatments indicate that the important diagram is that shown in Fig. 1. One would expect to find evidence for the glueball by plotting dN/dE as a function of γ-ray energy as shown. The rationale for Fig. 1 allowing enhanced production of glueballs is that the γ can have a variable energy so that the effective mass of the two gluons could sweep over a range of masses and thus if there is a gg resonance corresponding to a glueball, one would favor forming it. Thus the plot of dN/dE vs. E (Fig. 1) might show structure corresponding to glueballs.

In regard to the radiative decay of the J/ψ it has been considered[6,7,21,22)] that the E observed in $J/\psi \to \gamma + E(1420)$ may be a glueball. There is considerable controversy and a lack of convincing evidence about this particular interpretation for this low statistics experiment. We will not discuss it in this paper since it has been discussed by other papers in this conference and elsewhere.[6,7,21,22)]

The OZI Forbidden $\pi^- p \to \phi\phi n$

Another way suggested by the author[19,20] some time ago is shown by the diagram in Fig. 2.

The $\phi\phi$ system forms a hairpin diagram disjoint from the rest of the quark line diagram, thus it is OZI suppressed. In the context of QCD it proceeds via exchange of 2 gluons (if $\phi\phi$ is a scalar system) or 3 gluons if $\phi\phi$ is a vector system. The key point is to have a multi-gluon intermediate state of variable mass so that if glueball resonances exist they can be produced enhanced relative to other states. This experiment was performed some time ago by the BNL/CCNY collaboration.[23,24]

The quark line diagrams relevant to the BNL/CCNY experiments are shown in Fig. 3. Incident π^- mesons of 22.6 GeV/c interacted with protons to produce the reactions:

a) $\pi^- p \to K^+K^-K^+K^- n$

b) $\pi^- p \to \phi K^+K^- n$

c) $\pi^- p \to \phi\phi n$.

Figs. 3a and 3b are rearrangement diagrams that are expected to have the allowed order of cross section. An explicit check of the assumption that rearrangement diagrams have the OZI allowed order of cross section is obtained by comparing $\pi^- p \to K^+K^-\phi\pi^- p$ corresponding to a planar diagram from a CERN experiment at comparable energies with the BNL $\pi^- p \to K^+K^-\phi n$ which corresponds to a rearrangement diagram. The cross sections obtained for the two reactions are $\sigma \sim .4\ \mu b$ and $\sigma \sim .3\ \mu b$ respectively which shows there is no essential difference in cross section.[18,19,20]

A scatter plot of the mass of one K^+K^- pair, chosen in a random way, on the x-axis versus the mass of the second K^+K^- pair plotted on the y-axis is shown in Fig. 4. Thus each event is plotted twice. An enhancement over background is seen in each of the two ϕ mass bands. An enormous enhancement over background is seen where the two ϕ mass bands cross. This region (i.e. the $\phi\phi$ region) appears as a very densely populated area which is almost black. When corrections for resolution and double counting are made, the peak intensity in the $\phi\phi$ region is approximately 1500 times the adjoining background level. This clearly implies that the OZI suppression is apparently not working since if OZI suppression were perfect there would be no $\phi\phi$ events at all and thus no enhancement in the $\phi\phi$ region.

To see more quantitatively what is happening we plot in Fig. 5 the mass spectrum of K^+K^- pairs from the reaction $\pi^- p \to K^+K^-K^+K^- n$. Four K^+K^- combinations are plotted for each event. In addition, the shaded curve is the mass spectrum of like sign K pairs, which can be used as an indication of the extra background due to multiple combinations. A very clear ϕ signal is obtained corresponding to a peak to background ratio of about 4:1. We then correct by a factor of two to allow for multiple combinations which gives 8:1. We then correct for our mass resolution which is about three times the ϕ mass width so the true ϕ peak to background ratio is $\sim$ 25. Hence we see a large enhancement factor $\sim$ 25 at the ϕ mass.

Reaction (c) corresponds to Fig. 3c and is an OZI forbidden reaction. To study reaction (c) we select events with a K^+K^- pair in the ϕ mass band and plot the effective mass of the other K^+K^- pair, and obtain the mass spectra shown in Fig. 6 which exhibits a huge ϕ signal corresponding to reaction (c). The peak to background ratio is about 20:1. When corrected for the resolution, the true peak to background ratio is about 60:1. If the OZI suppression were 100%, no signal would be seen in Fig. 4c. We therefore have the unusual situation that the forbidden reaction (c) produces a higher ϕ enhancement over K^+K^- background than the allowed reaction (b), although equal enhancements cannot be ruled out because of the possible contamination of the data with $\bar{p}p$ pairs, and the errors in corrections for combinational problems.

Figure 7 shows the mass spectra for the effective masses (ϕK^+K^-) and $\phi\phi$ for our published spectrum[23] of $\approx$ 100 $\phi\phi$ events. The ϕK^+K^- mass spectrum shows a broad distribution which occupies most of the available phase space while the $\phi\phi$ mass spectrum has a low effective mass peak and is restricted to relatively low energies. Thus the $\phi\phi$ spectrum has a distinctly different character than the ϕK^+K^-. Furthermore there is some indication that there may possibly be some structure in the neighborhood of 2.4 GeV. If we integrate the two spectra we get a ratio of reaction b to reaction c of less than five which is of the order of the value for background to resonance ratios in allowed reactions.*

Hence we have clear evidence that there is no OZI suppression[24] (perhaps even an enhancement) in our K^+K^- effective mass studies and we find even in comparing total cross sections we are consistent with normal resonance behavior for the second K^+K^- pair (after the first has made a ϕ) with no evidence for OZI suppression for creation of a second ϕ. Therefore we conclude that OZI suppression is absent in these processes.

I shall now proceed to treat the observations from the point of view of the isobar model[25] which we proposed over 2 decades ago. The isobar model never heard of OZI suppression and ignores it. We will shortly find quantitative agreement of the present observations with these isobar model calculations. The agreement will depend on final state K^+K^- resonant interactions built into the isobar model treatment. I have shown[18,19,20] that such final state interactions

---

* For example C. Baltay <u>et al</u>., Phys. Rev. Letts. <u>40</u>, 87 (1978) find that production accounts for about 5% of the $\pi^+\pi^-\pi^0$ spectrum in the reaction $\pi^-p \rightarrow \Delta^{++}\pi^+\pi^-\pi^0$ at 15 GeV/c. Resonance production is typically 1/10 of a reaction.

completely defeat the OZI rule and thus we will show that the OZI rule is inoperative in these reactions.

Let us now calculate the enhancement factor expected from the point of view of the isobar model where the ϕ isobar formation probability is related to the K^+K^- resonant scattering to form a ϕ. We consider the ϕ as an $L = 1$, $I = 0$ resonance of the K^+K^- scattering which decays back into K^+K^- with a branching ratio of 0.47 and into $K^0\bar{K}^0$ with a BR of 0.35. Off resonance we assume the K^+K^- scattering cross section is estimable by using the additive quark model (AQM) to determine $\sigma(K^+K^-) \approx 2/3\ \sigma(K\text{-}N) \approx 8$ mb (non-resonance). The maximum cross section for K^+K^- scattering at the resonance can then be estimated to be:

$$\sigma(K^+K^- \rightarrow K^+K^-) \approx 12\pi\lambda\!\!\!^{-2} \times \underset{\substack{\uparrow \\ T = 0}}{1/2} \times \underset{\text{factor}}{\text{Inelasticity}} \times \text{effective BR} \approx 200 \text{ mb}$$

The estimated enhancement factor at the peak =

$$\frac{200 \text{ mb (peak resonant cross section)}}{8 \text{ mb (non-resonant cross section)}} \approx 25.$$

Thus we predict an enhancement factor over background ≈ 25 for the ϕ peak over background. This is indeed what we find for the first ϕ.

For the second ϕ employing the isobar model we would expect the same enhancement factor and as stated observe ≈ 60. This treatment from the isobar model viewpoint has no OZI suppression in it and thus the fact that the enhancement found for the second ϕ peak is if anything greater than the first, but at least comparable and thus in general agreement with the isobar model shows clearly that the OZI suppression is effectively absent.[19,20)]

Glueballs and Failure of OZI

I would give as the most likely explanation for this complete failure of the OZI suppression in $\pi^- p \rightarrow \phi\phi n$ the following:

In an OZI forbidden reaction the intermediate state which connects the two disconnected parts of the diagram is a collection of gluons. If a variable mass of this intermediate state is allowed by the reaction (as in $\phi\phi$) and some part of the mass region covers existing glueball resonances, the resonances may lead to effectively strong coupling and the OZI rule may be defeated. In essence we would be looking at a diagram where exchange of glueballs would occur. This is obviously a very good way to look for glueballs and the complete failure of the OZI rule and different shape of the $\phi\phi$ and ϕK^+K^- effective mass spectra indicates that we may well have seen them in this reaction. It is interesting to note that the peak of the $\phi\phi$ spectrum is at about 2.4 GeV which is the number deduced for the 3g ground state.[8)]

In the above treatment we have assumed that the $\phi\phi$ state comes from either a glue system or from a quark system involving only $s\bar{s}$ quarks (i.e. a singlet of an ideally mixed nonet) so that we truly can represent the process as OZI forbidden (disconnected) diagram. However one could consider for example $\pi^- p \rightarrow \eta'^* + n$ where for example the η'^* is the singlet in a nonet which is a radial excitation of the lowest mass 0^- nonet. The η' in the above could be far from ideally mixed and thus could decay via $\phi\phi$ and circumvent the OZI suppression of this process.

This decay from another state which is not $s\bar{s}$ and is heavily mixed with ordinary quarks and thus circumvents OZI suppression could in principle occur. I deem this to be an improbable explanation of what we observe, since there would in general be no correlation between the enhancement factor we observed in producing the second ϕ over the ϕK^+K^- compared to the enhancement factor in producing the first ϕ. We are dealing with a very small cross section ($\sim$ 23 nb) thus the chance that an uncorrelated number of $\phi\phi$ events would give us within errors just the huge ($\sim$ 25) enhancement factor we expect for the isobar model treatment with the absence of OZI suppression is in my opinion small.

Furthermore the η'^* (a radial excitation) of the η is considered the most likely candidate for production of a singlet in a far from ideally mixed nonet. Let us assume our $\phi\phi$ spectrum is due to decays of η'^*. In the case of a pseudoscalar like the η' it has been shown[26,27] that the angle χ between the two planes formed by the decays $\phi \rightarrow K^+K^-$ has the angular distribution $1 - \cos 2\chi$. Figure 8 shows the comparison of the χ distribution from our data and that expected from a pseudoscalar.

The angular distribution of our events is such that we obtain a χ^2 of 588 for 4 degrees of freedom which clearly rules out ($\sim 20\sigma$) the possibility that our $\phi\phi$ spectrum comes from decay of an η'^*.

It is obvious that a higher statistics experiment which would allow any visible structure in the $\phi\phi$ spectrum to be definitely established and allow a definite determination of the J^P of the states in the $\phi\phi$ spectrum is clearly required and we plan to do such an experiment shortly.

Figure 9 shows the observed $\phi\phi$ spectrum with the additional (unpublished) events we have obtained by further analysis. Approximately 170 $\phi\phi$ events are plotted in 20 MeV mass bins which is $\sim$ our full width half maximum resolution. This spectrum may well contain glueballs. Although there is some indication (not statistically significant) of possible structure, it is clear we need much more statistics, to form a conclusion.

The BNL/CCNY collaboration has for some time been preparing for a new experiment with MPS II. In MPS II the spark chambers of MPS I are being replaced by narrow gap high space and time resolution drift chambers. This should allow us to obtain $\sim$ 20 times more statistics and thus gather $\sim$ 3,000-4,000 $\phi\phi$ events in the $\pi^- p \rightarrow \phi\phi n$.

We expect that with this number of events we can make a significant observation of possible structure in the spectrum and also perform an effective partial wave analysis to determine whether there are resonant states and their J^P. Therefore we hope to be able to demonstrate the existence of glueballs.

Conclusions

1. The most likely explanation in the context of QCD for our observed failure of the OZI rule in $\pi^- p \to \phi\phi n$ is the intervention of glueball resonances. Therefore we believe the $\phi\phi$ system in this experiment is an excellent place to search for glueballs and their effects may well already have been seen.

2. The possibility that our $\phi\phi$ spectrum comes from the decay of a non-ideally mixed nonet particle and thus circumvents the OZI rule is considered improbable. In particular a radial excitation of the η' which is considered to be the most likely candidate for such a particle is ruled out by our angular distribution.

3. Approximately 170 $\pi^- p \to \phi\phi n$ events have been observed in the previous experiment. The BNL/CCNY collaboration for some time has been planning a second experiment to be done in the near future which is expected to provide $\sim$ 20 times more data. With this amount of data we expect to be able to make significant observations of possible visible structure in the $\phi\phi$ spectrum and to do a significant partial wave analysis which would allow an identification of resonances and their J^P. C is known to be + for $\phi\phi$ and I is known to be 0. Hence we expect that all the quantum numbers can be determined. Thus there is an excellent possibility for the explicit discovery of glueballs especially if J^{PC} is exotic.

References

1. H. Fritzch and M. Gell-Mann, XVI Int. Conf. on High Energy Physics, Chicago-Batavia 1972, Vol. 2, p. 135; H. Fritzch, M. Gell-Mann and H. Leutwyler Phys. Lett. 47B, 365 (1973); S. Weinberg, Phys. Rev. Lett. 31, 494 (1973); Phys. Rev. D8, 4482 (1973); D.J. Gross and F. Wilczek, ibid, 3633 (1973).

2. For a review, see W. Marciano and H. Pagels, Physics Reports Vol. 36, 137 (1978).

3. C.N. Yang and R.L. Mills, Phys. Rev. 96, 191 (1954).

4. J.D. Bjorken, SLAC-PUB-2366 (August 79); SLAC Summer Institute on Particle Physics (1979).

5. a) Fritzch and Minkowski, Nuovo Cimento 30A, 393 (1975); b) R.P. Freund and Y. Nambu, Phys. Rev. Lett. 34, 1645 (1975); c) Kogut, Sinclair and Susskind, Nucl. Phys. B114, 199 (1975); d) D. Robson, Nucl. Phys. B130, 328 (1977).

6. J. Donoghue, Proc. VI Int. Conf. on Experimental Meson Spectroscopy, Brookhaven National Laboratory, April 25-26, 1980, AIP Conf. Proc., No. 67, Subseries on Particles and Fields, No. 21, Eds. S.U. Chung and S.J. Lindenbaum, pg. 104.

7. a) J. Coyne, P. Fishbane, and S. Meshkov, Phys. Lett. 91B, 259 (1980); b) Carlson, Coyne, Fishbane, Gross and Meshkov, Phys. Lett. 98B, 110 (1981); also William and Mary Preprint.

8. P. Fishbane, Glueballs, A Little Review, Talk presented at the 1981 ORBIS Scientiae, Ft. Lauderdale, Fla.

9. J. Donoghue, K. Johnson, B. Li, Phys. Lett. B (to be published).

10. G. Parisi and R. Petronzio, Phys. Lett. 94B, 51 (1980).

11. J. Kogut, D. Sinclair and L. Susskind, Nucl. Phys. B114, 199 (1976); B. Borg, Phys. Lett. 97B, 401 (1980).

12. B. Skagerstam and A. Stern, Phys. Lett. 97B, 405 (1980).

13. M. Cornwall, Deeper Pathways in High Energy Physics, A. Perlmutter Ed., Plenum Press, N.Y. (1977); see also Ref. 8 for latest estimate.

14. S. Okubo, Phys. Lett. 5, 165 (1963); G. Zweig, CERN Report TH 401 and 412 (1964); J. Iizuba, Prog. Theor. Phys. Suppl. 37-38, 21 (1966); J. Iizuba, K. Okuda and O. Shito, Prog. of Theoretical Physics 35, 1061 (1966).

15. S. Okubo, A Survey of the Quark Line Rule (U.R. 641, Dept. of Physics and Astronomy, University of Rochester, N.Y. 1977).

16. T. Applequist, K. Kane and M. Barnett, Ann. Rev. Nucl. Sci. 28, 387 (1978).

17. I.J. Muzinich and F.E. Paige, Phys. Rev. D 21, 1151 (1980).

18. S.J. Lindenbaum, Quark Line Diagrams, Rules and Some Recent Data, BNL 50812 (Dec. 1977).

19. S.J. Lindenbaum, Hadronic Physics of $q\bar{q}$ Light Quark Mesons, Quark Molecules and Glueballs, Lecture presented in XVIII Course: The High Energy Limit, 31 July - 11 August 1980, The International School of Subnuclear Physics, Erice (to be published in proceedings) also BNL 28498, October 1980.

20. S.J. Lindenbaum, The QZI Rule and Glueballs, Il Nuovo Cimento (in press).

21. D.L. Scharre et al., Proc. VI Int. Conf. on Experimental Meson Spectroscopy, Brookhaven National Laboratory, April 25-26, 1980, AIP Conf. Proc., No. 67, Subseries on Particles and Fields, No. 21, Eds. S.U. Chung and S.J. Lindenbaum, pg. 329; Phys. Lett. 97B, 329 (1980).

22. M. Chanowitz, Berkeley Preprint LBL-11977.

23. A. Etkin, K.J. Foley, J.H. Goldman, W.A. Love, T.W. Morris, S. Ozaki, E.D. Platner, A.C. Saulys, C.D. Wheeler, E.H. Willen, S.J. Lindenbaum, M.A. Kramer, U. Mallik, Phys. Rev. Lett. 40, 422 (1978).

24. A. Etkin, K.J. Foley, J.H. Goldman, W.A. Love, T.W. Morris, S. Ozaki, E.D. Platner, A.C. Saulys, C.D. Wheeler, E.H. Willen, S.J. Lindenbaum, M.A. Kramer and U. Mallik, Phys. Rev. Lett. 41, 784 (1978).

25. S.J. Lindenbaum and R.M. Sternheimer, Phys. Rev. 105, 1874 (1957); 106, 1107 (1957); 109, 1723 (1958); 123, 333 (1961).

26. N.P. Chang and C.T. Nelson, Phys. Rev. Lett. 40, 1617 (1978).

27. T.L. Trueman, Phys. Rev. D18, 3423 (1978).

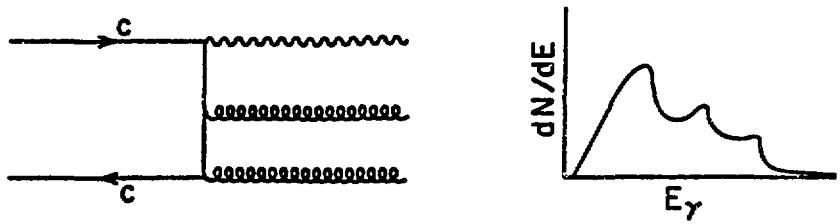

Figure 1 The dominant diagram in radiative J/ψ decay, and to the right of it a plot of dN/dE vs. E_γ.

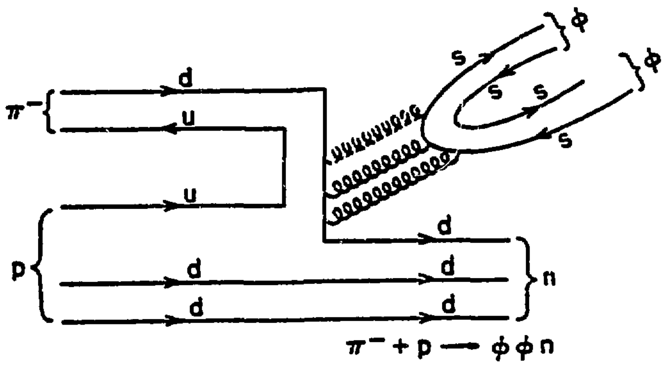

Figure 2 $\pi^- p \to \phi\phi n$ an OZI forbidden diagram where the multigluon intermediate state (which leads to $\phi\phi$ production) can have a variable mass.

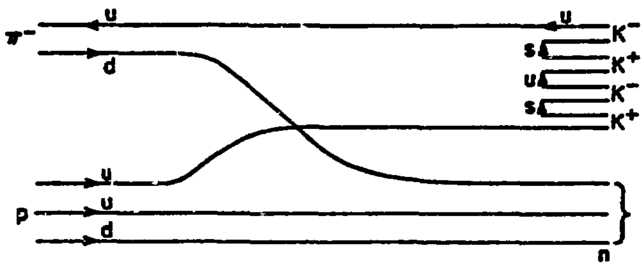

Figure 3a $\pi^- p \rightarrow K^+K^-K^+K^- n$

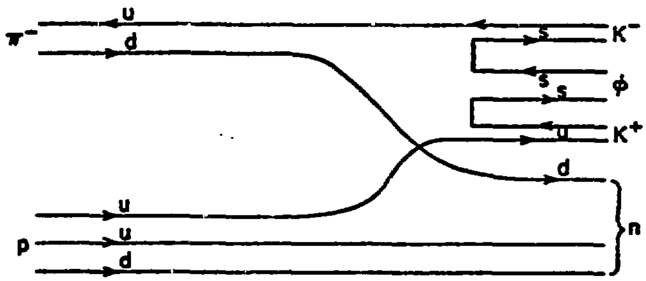

Figure 3b $\pi^- p \rightarrow \phi K^+K^- n$

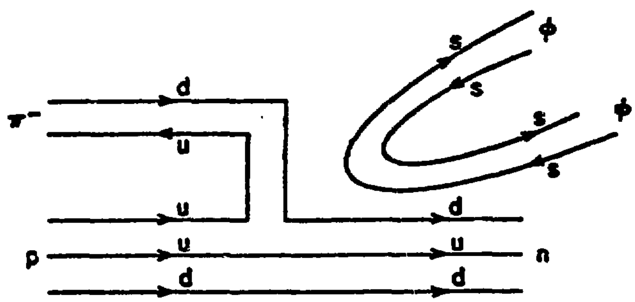

Figure 3c $\pi^- p \rightarrow \phi\phi n$

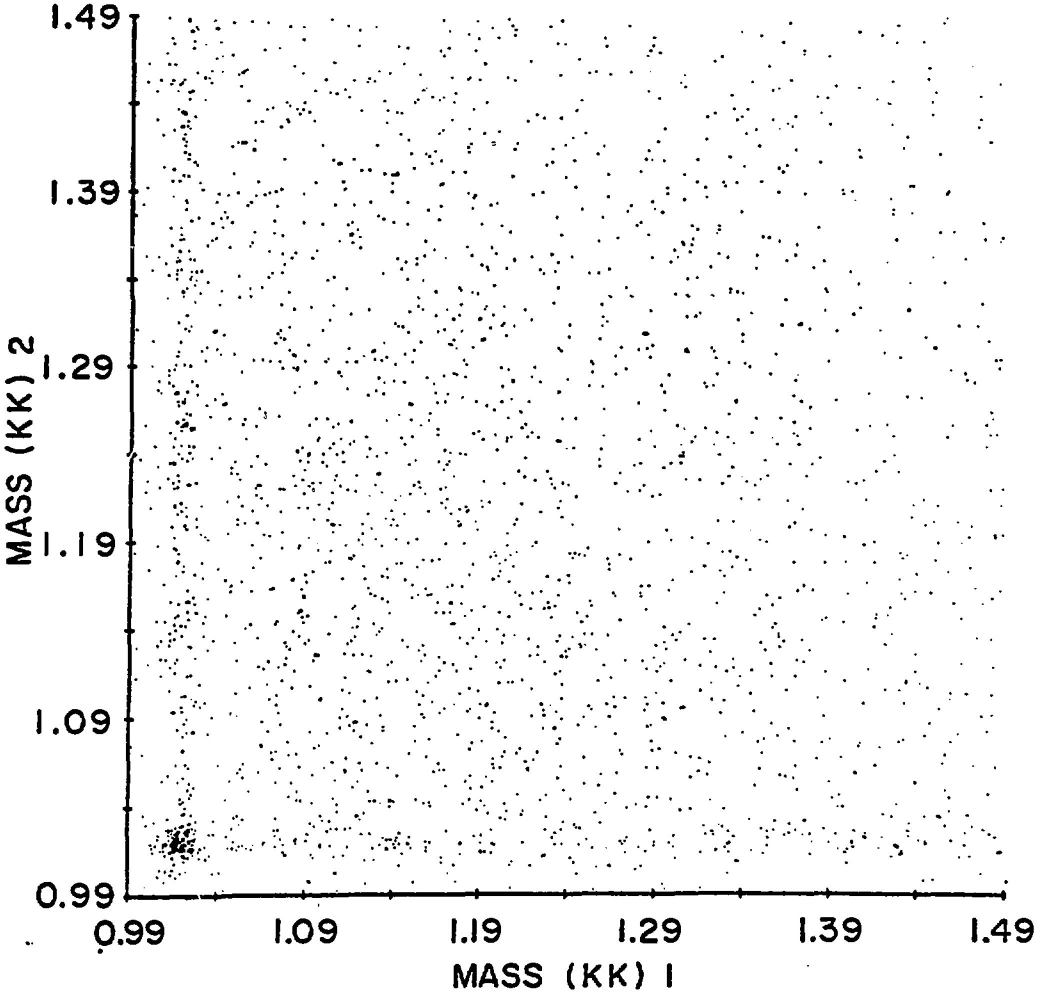

Figure 4 Scatter plot of K^+K^- effective mass for effective masses less than 1.49 GeV/c^2. Two randomly chosen mass combinations are plotted for each event. Clear bands of $\phi(1019)$ are seen with an enormous enhancement where they overlap (i.e. $\phi\phi$).

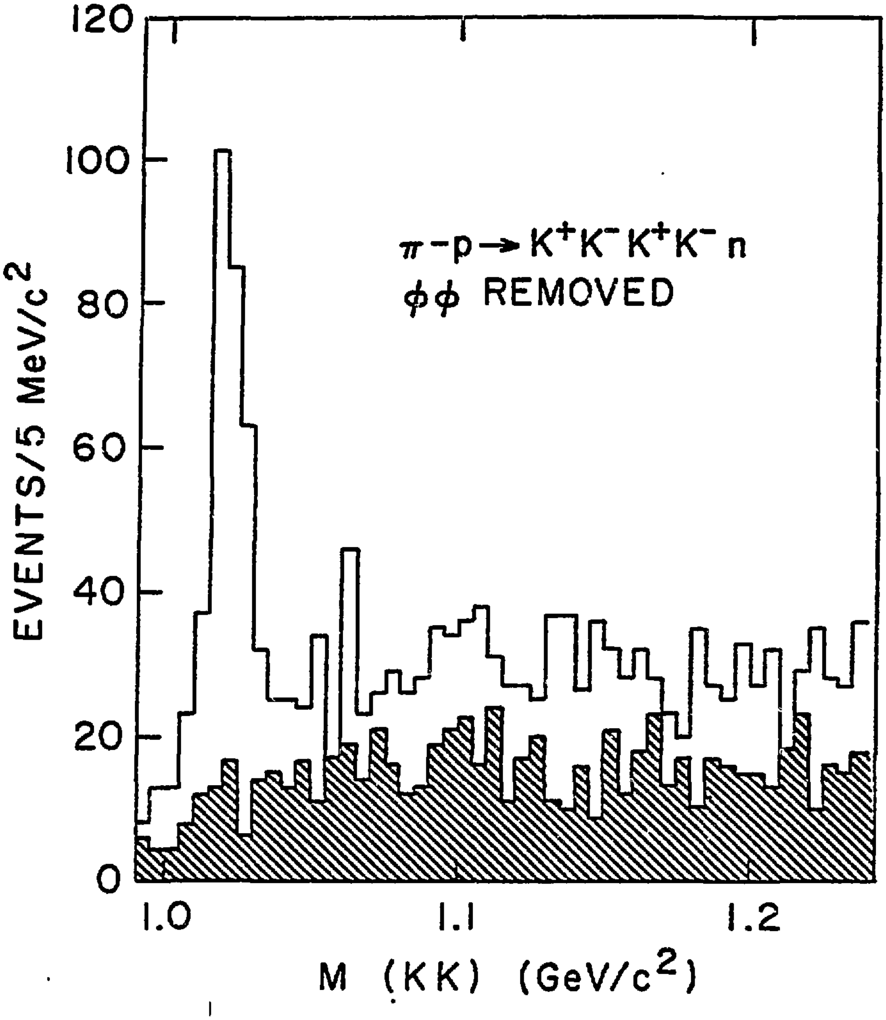

Figure 5 The effective mass spectrum of K^+K^- pairs after removing $\phi\phi$ events. The shaded histogram is the sum of like sign K pairs and is an indication of background due to multiple combinations.

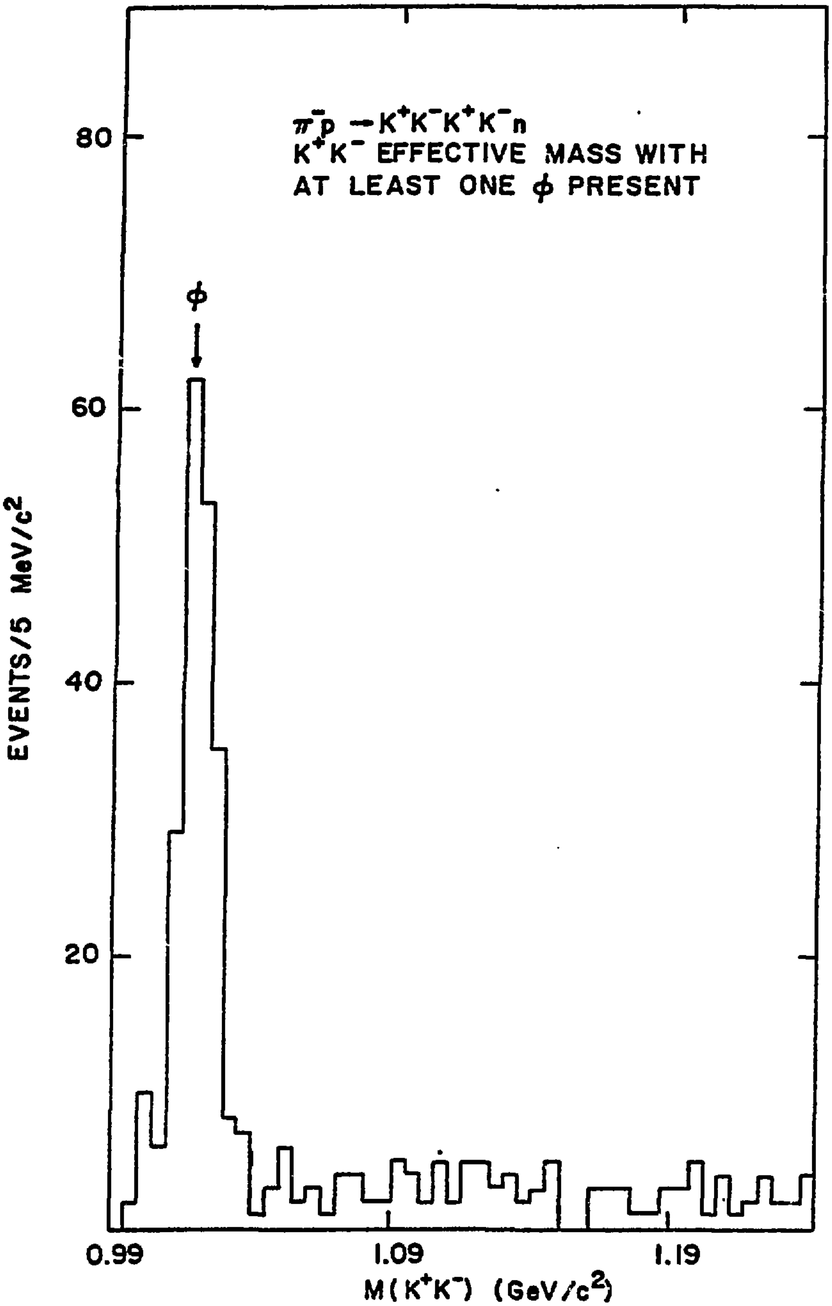

Figure 6 The effective mass of each K^+K^- pair for which the other pair was in the mass band.

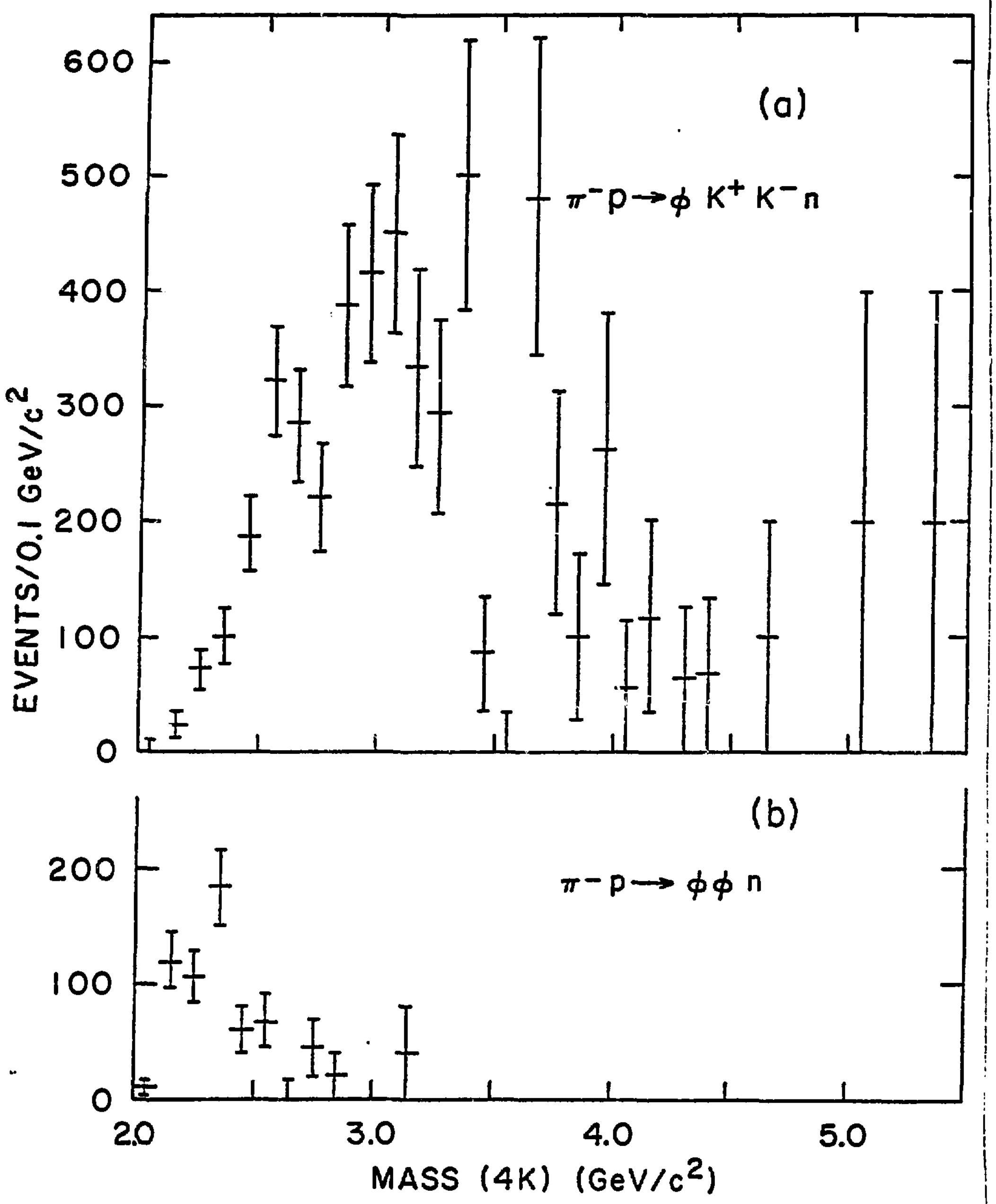

Figure 7 (a) Effective mass spectrum (corrected for acceptance) of the [ϕK^+K^-] system where the K^+K^- does not lie in the ϕ band.

(b) $\phi\phi$ effective mass spectrum (corrected for acceptance).

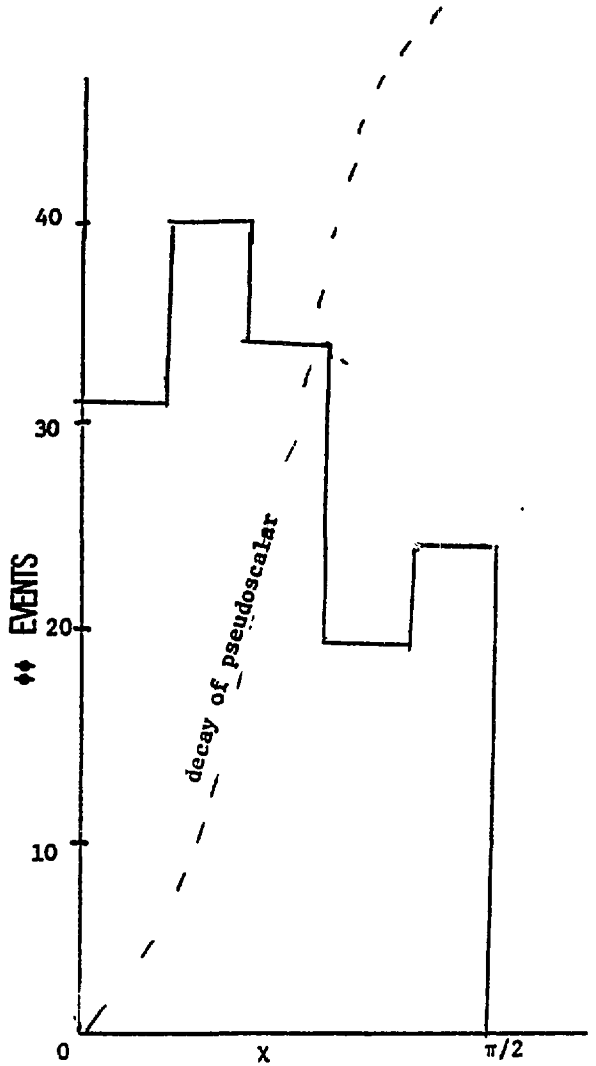

Figure 8 The distribution of $\phi\phi$ events as a function of χ, the angle between the decay planes of the two ϕ's. The dotted curve is the prediction for this distribution if the $\phi\phi$ were the decay products of a pseudoscalar state such as a radial excitation of the η'. This is clearly ruled out (see text for further details).

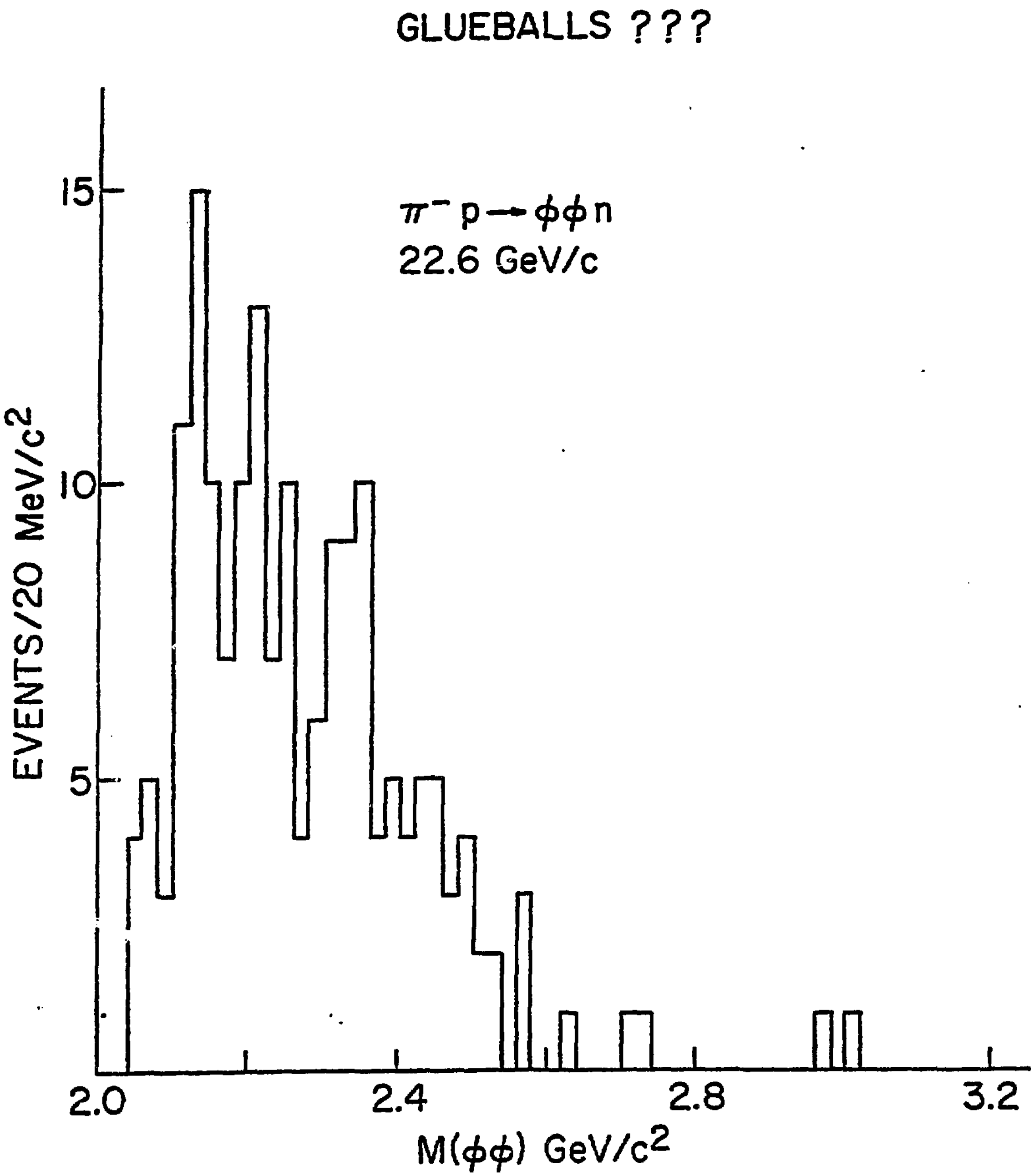

Figure 9 Observed $\phi\phi$ mass spectrum for the final sample $\sim$ 170 $\phi\phi$ events. This spectrum is uncorrected for acceptance.