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Calculation of longitudinal and transverse wake-field effects in dielectric structures

W. Gai

Argonne National Lab, Argonne, IL 60439

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The electro-magnetic radiation of a charged particle passing through a dielectric structure [1] [2] has many applications to accelerator physics. Recently a new acceleration scheme, called the dielectric wake field accelerator, has been proposed[3]. It also can be used as a pick up system for a storage ring because of its slow wave characteristics. In order to study these effects in detail, in this paper we will calculated the wake field effects produced in a dielectric structure by a charged particle. First we will give a general expression of the wake field for any value of $\beta = v/c$,then discuss the implication of the results under the limit of $\beta \to 1$. The results indicate that the longitudinal wake field generated is very strong but the transverse wake field goes to zero under the limit of particle velocity v $\to c$, even when the particle position is off-center. The result of this is fascinating, and of great interest to the accelerator physics because it implies absence of the beam break up (BBU) mode[4] in this type of structure, unlike the disk-loaded structures which are commonly used in accelerators. This particular feature of the dielectric cavity could have important applications to the future linear colliders.

Consider the system of a metallic tube with inner radius a which is partially filled with isotropic material with dielectric constant ϵ, and a hole of radius b at the center of the tube which allows charged particles pass through. A particle with charged $-e$ moves at velocity v along a line parallel to the axis of the tube at a distance r_0 as shown in figure 1. Due to the presence of the dielectric material, the Cherenkov radiation conditions will be satisfied and this particle will generate wake fields behind it. The wake fields produced by the motion of a charged particle are given by the Maxwell equations, (which are always the truth)

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

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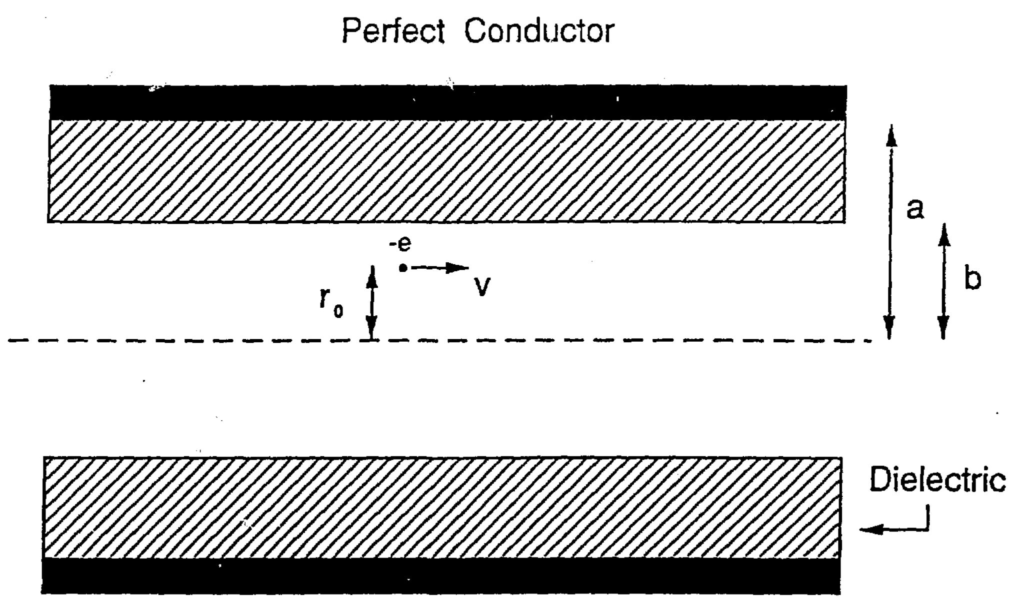

Figure 1. Schematic diagram of the dielectric wake field structure.

$$\begin{aligned}\nabla\cdot\mathbf{B} &= 0\\ \nabla\cdot\mathbf{D} &= 4\pi\rho\\ \nabla\times\mathbf{H} &= \frac{1}{c}\frac{\partial\mathbf{D}}{\partial t}+\frac{4\pi}{c}\mathbf{j}\\ \mathbf{D} &= \epsilon\mathbf{E}\\ \mathbf{B} &= \mu\mathbf{H}\end{aligned}\tag{1}$$

Here we introduce a scalar and a vector potential φ and $\mathbf{A}$, are defined as

$$\mathbf{E}=-\nabla\varphi-\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}$$

$$\mathbf{B}=\nabla\times\mathbf{A}\tag{2}$$

we choose the Lorentz condition,

$$\nabla\cdot\mathbf{A}+\frac{1}{c}\frac{\partial\varphi}{\partial t}=0\tag{3}$$

The Maxwell equations can then be transformed to two uncoupled inhomogeneous wave equations, one for φ, one for $\mathbf{A}$,

$$\nabla^2\varphi-\frac{\epsilon\mu}{c^2}\frac{\partial^2\varphi}{\partial^2 t}=-\frac{4\pi\rho}{\epsilon}$$

$$\nabla^2\mathbf{A}-\frac{\epsilon\mu}{c}\frac{\partial^2\mathbf{A}}{\partial^2 t}=-4\pi\mu\mathbf{j}\tag{4}$$

For a charged particle moving at velocity $\mathbf{v}$ in the $\mathbf{z}$ direction, the charge and current densities are

$$\begin{aligned}\rho &= -e\delta(\mathbf{r}-\mathbf{r}_0)\delta(z-vt)\\ \mathbf{j} &= \mathbf{v}\rho\end{aligned}\tag{5}$$

From symmetry, it is easy to show that

$$\mathbf{A}=\frac{\mathbf{v}}{c}\epsilon\mu\varphi\tag{6}$$

so as long as we solve for φ, we will have a complete solution for the electromagnetic fields.

First we solve for the potential in the vacuum hole. In this case $\epsilon = 1$ and $\mu = 1$. Expanding the δ functions

$$\delta(\mathbf{r}-\mathbf{r_0}) = \frac{\delta(r-r_0)}{r}\delta(\theta) = \frac{1}{2\pi}\sum_{m=-\infty}^{\infty} e^{im\theta}\int_0^\infty J_m(kr)J_m(kr_0)kdk \quad (7)$$

$$\delta(z-vt) = \frac{1}{2\pi v}\int_{-\infty}^{\infty} e^{i\omega/v(z-vt)}d\omega \quad (8)$$

the charge density $\rho(r,\theta,z,t)$ can be written as

$$\rho(r,\theta,z,t) = \frac{e}{(2\pi)^2 v}\sum_m \int\int \rho_m(r,\theta,z,t,k,\omega)kdkd\omega \quad (9)$$

it is easy to show that ρ_m satisfies the wave equation (4)

$$\nabla^2\rho_m - \frac{1}{c^2}\frac{\partial^2\rho_m}{\partial^2 t} = \lambda\rho_m \quad (10)$$

i.e., it is an eigenfunction of the Dalembertian operator with eigenvalue $\lambda = -(k^2+(\omega/c)^2(1-\beta^2)$. It is straightforward to show that the inhomogeneous solution of the wave equation is

$$\varphi(\mathbf{r},z,t) = \frac{e}{\pi v}\sum_{m=-\infty}^{\infty} e^{im\theta}\int_0^\infty\int_{-\infty}^{\infty}\frac{J_m(kr)J_m(kr_0)e^{i\omega/v(z-vt)}}{\epsilon[k^2+(\omega/c)^2(1-\beta^2)]}d\omega kdk$$
$$= \frac{e}{\pi v}\sum_m e^{im\theta}\int e^{i\omega/v(z-vt)}\varphi_{m,inhomogeneous}(r,\omega)d\omega \quad (11)$$

where

$$\varphi_{m,inhomogeneous}(r,\omega) = K_m(\omega/c\sqrt{1-\beta^2}r_>)I_m(\omega/c\sqrt{1-\beta^2}r_<) \quad (12)$$

In order to form a complete set of solutions which satisfy all the boundary conditions, we have to have homogeneous solutions of equation (4) which can be found easily to be

$$\varphi_{m,hom} = \alpha_m I_m(\omega/c\sqrt{1-\beta^2}r_0)I_m(\omega/c\sqrt{1-\beta^2}r) \quad (13)$$

The complete solution for $r < b$ is

$$\varphi_{m,v} = \varphi_{m,inhomogeneous} + \varphi_{m,hom} \tag{14}$$

As we can see here that the central problem is finding α_m. Once α_m is found, the fields **E** and **B** inside the vacuum channel $r < b$ can be calculated from (2).

In the dielectric region $r > b$, we have , for $\epsilon\mu\beta^2 > 1$, the solution of wave equation is

$$\varphi_{md} = A_m N_m(\omega/c\sqrt{\epsilon\mu\beta^2 - 1}r) + B_m J_m(\omega/c\sqrt{\epsilon\mu\beta^2 - 1}r) \tag{15}$$

Here J_m and N_m are m-th order Bessel functions of the first and second kind. The coefficients α_m, A_m and B_m must be determined from the boundary conditions at $r = a$ and $r = b$. These boundary conditions are must chosen such that E_z and D_r continues at $r = b$ and $E_z = 0$ at $r = a$. Once we do so, all the other boundary conditions will be automatically satisfied.

From equation (3), the boundary conditions at $r = a$, and $r = b$ can be reduced to continuities of the potentials $\varphi_{m,v}$ in the vacuum and $\varphi_{m,d}$ in the dielectric channel,

$$\begin{aligned} \frac{\partial\varphi_{mv}}{\partial r} &= \epsilon\frac{\partial\varphi_{md}}{\partial r} \\ (1-\beta^2)\frac{\partial\varphi_{mv}}{\partial r}_{r=b} &= (1-\epsilon\mu\beta^2)\frac{\partial\varphi_{md}}{\partial r}_{r=b} \\ \varphi(r=a) &= 0 \end{aligned} \tag{16}$$

Defining $k \equiv \omega/c\sqrt{1-\beta^2}, s \equiv \omega/c\sqrt{\epsilon\beta^2 - 1}$, and using the above conditions(16), we have

$$\begin{aligned} A_m N_m(sa) + B_m J_m(sa) &= 0 \\ (1-\beta^2)[K_m(kb)I_m(kr_0) + \alpha_m I_m(kr_0)I_m(kb) &= (1-\epsilon\beta^2)[A_m N_m(sb) + B_m J_m(sb)] \\ I_m(kr_0)kK_m'(kb) + \alpha_m I_m(kr_0)kI_m'(kb) &= \epsilon[A_m sN_m'(sb) + B_m sJ_m'(sb)] \end{aligned}$$

where $f'(x) \equiv df(x)/dx$. Eliminating A_m and B_m, we find

$$(1-\beta^2)\frac{K_m(kb) + \alpha_m I_m(kb)}{kK_m'(kb) + \alpha_m kI_m'(kb)} = \frac{1-\epsilon\beta^2}{\epsilon s}C_1/C_2 \tag{17}$$

where

$$C_1 = N_m(sb)J_m(sa) - N_m(sa)J_m(sb)$$
$$C_2 = N_m'(sb)J_m(sa) - N_m(sa)J_m'(sb)$$

From (17), we have

$$(1-\beta^2)\alpha_m = \frac{\frac{1-\beta^2\epsilon}{\epsilon s}\frac{C_1}{C_2}kK_m'(kb) - (1-\beta^2)K_m(kb)}{I_m(kb) - \frac{1-\epsilon\beta^2}{(1-\beta^2)\epsilon s}\frac{C_1}{C_2}kI_m'(kb)} \tag{18}$$

Remembering that the longitudinal wake field can be calculated from the potential φ directly, that is

$$\begin{aligned} E_z(r,\theta,z,t) &= -\frac{2e}{\pi v^2} Re \sum_m cos(m\theta) \int_{-\infty}^{\infty} d\omega i\omega(1-\beta^2)e^{i\omega/v(z-vt)}\varphi_m(\omega,r) \\ &= \sum_m E_z^m \end{aligned} \tag{19}$$

equation (19) along with (18) gives a complete solutions for all the wake field effects in the region $r < b$ because the transverse wake fields can be calculated from the longitudinal wake fields by using the differential form of the Panofsky-Wenzel theorem[5]

$$\nabla_\perp eE_z(r,z) = \nabla_\perp W_z = \frac{\partial \mathbf{F}_\perp}{\partial z} \tag{20}$$

From our point of view, we are primarily interested in the case of wake fields generated by a charged particle moving at close the speed of light or $\beta \to 1$. In this case $kr = \omega/c\sqrt{1-\beta^2}r << 1$. For a very small x, the Bessel functions can be expanded as

$$I_m(x) = \frac{1}{\Gamma(m+1)}(x/2)^m$$

$$K_m(x) = \begin{cases} -ln(\frac{x}{2}) - 0.5772 & m = 0 \\ \frac{\Gamma(m)}{2}(\frac{2}{x})^m & m \neq 0 \end{cases}$$

For $m = 0$ mode, from (18) we find

$$(1-\beta^2)\alpha_0 = \frac{1-\beta^2\epsilon}{\epsilon s}C_1/C_2 \tag{21}$$

Substituting this into (19), we have the longitudinal wake field at (r, z_0), where z_0 is longitudinal distance behind the charged particle,

$$E_z(r, z_0) = \frac{2e}{\epsilon b} \sum_\lambda \frac{J_0(s_\lambda b)N_0(s_\lambda a) - N_0(s_\lambda b)J_0(s_\lambda a)}{\frac{d}{ds}[J_1(sb)N_0(sa) - J_0(sa)N_1(sb)]|_{s=s_\lambda}} cos(\omega_\lambda / c z_0) \qquad (22)$$

where s_λ satisfies the condition

$$N_0(s_\lambda a)J_1(s_\lambda b) - J_0(s_\lambda a)N_1(s_\lambda b) = 0$$

This result is of great interest of dielectric wake field accelerator because it can produce a very high acceleration gradient which is one of the requirements for the next generation of linear colliders. As an example, an electron bunch with 100 nC total charge and rms bunch length of 2 ps passing through a structure with $a = 0.5cm, b = 0.3cm$ and dielectric constant $\epsilon = 3$ produces a maximum accelerating gradient $E_z = 100\text{MeV/m}$. We have performed a proof of principle experiment for the dielectric wake field accelerator at Argonne National Laboratory using the AATF[7], the results shown a very good agreement between the experiments and calculations from (22). An example is shown in figure 2. Due to this strong wake field effect, it should be pointed out that one should be very careful when inserting a dielectric tubing into an accelerator section. One should notice that there is no r dependent of E_z in (22) and the consequence of this is there is no transverse focusing force exited. The transverse beam profile will not be influenced by the wake fields.

For $m \neq 0$ mode, the excited modes are determined by the poles of (18). The modes will have a β dependent, however if we take the limit of $\beta \to 1$, then (18) can be simplified

$$(1 - \beta^2)\alpha_m = \frac{2^{m-1}\Gamma(m)\Gamma(m+1)}{b^{2m}(\omega/v)^{2m}(1 - \beta^2)^{m-1}}$$

Again we substitute this into (19), and along with the Panofsky-Wenzel theorem we have

$$E_z^m = 0$$

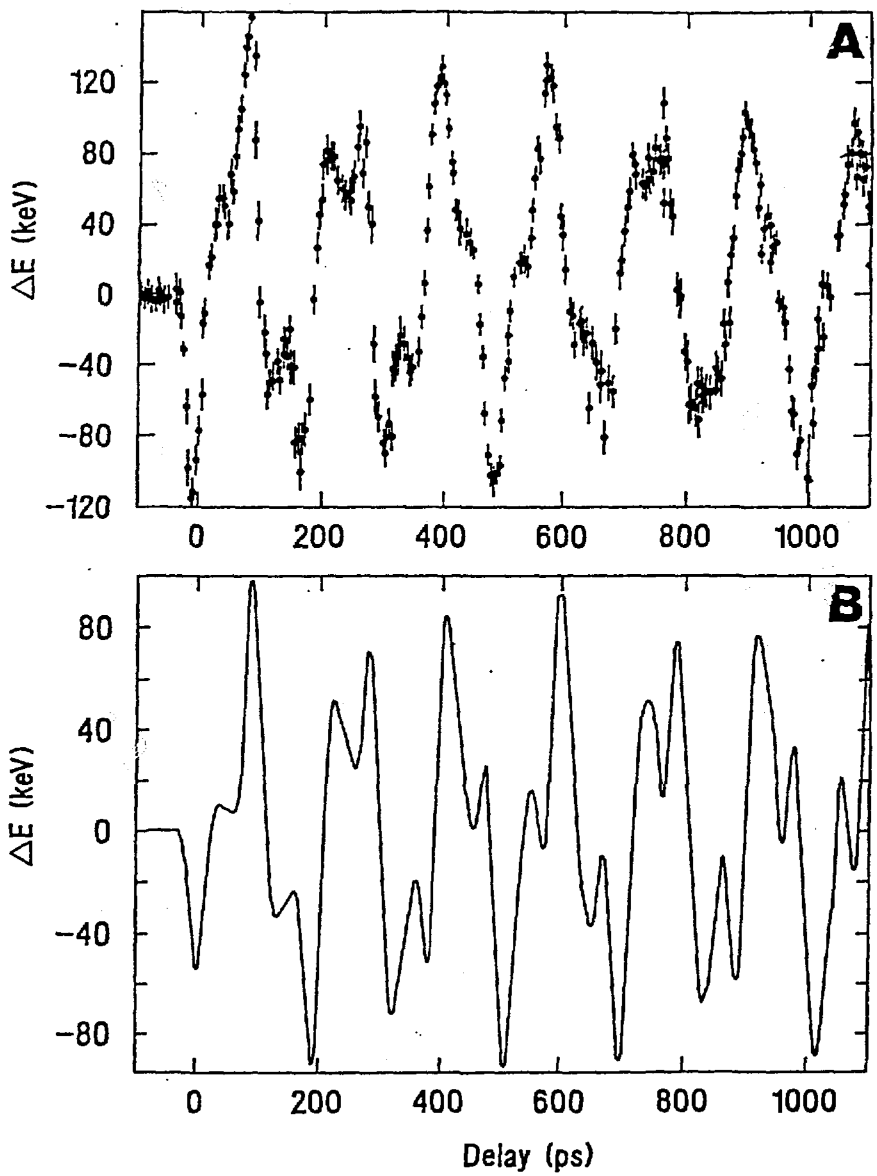

Figure 2. Wakefield acceleration in dielectric structure (steatite, dielectric constant $\varepsilon = 5.9$). A). measured and B). calculated wake potential vs delay. The experimental conditions: a = 1.27 cm, b = 0.63 cm. Length of the structure 51 cm, driver charge 2.0 nc and pulse length 23 ps (FWHM).

and

$$\mathbf{F}_{\perp}^{m} = 0$$

This is a very interesting result, as it indicates that in a dielectric wake field device only the fundamental modes ($m = 0$) can be excited. No other higher modes can be excited. One of the direct argument is that the beam excited electro-magnetic fields (when $\beta \to 1$) always have relation $B_\theta = \beta D_r$ in this type of structures, so the net radial force will be $F_r = e(1-\beta^2)E_r$ which goes to zero when $\beta \to 1$. This result concludes that there will be no beam break up mode in a dielectric wake field device, which is a source of beam instabilities in linear accelerators.

From the above discussions, we can see that the dielectric wake field device as we discussed here has two very interesting features. One is a very strong longitudinal wake field and the second is no transverse wake field produced by an ultra relativistic beam. As a wake field device, it has great advantages over other schemes because not only the focusing (unlike plasma[6]), but also the deflecting force (unlike iris disk loaded structures) have vanished. Because of the interesting properties of dielectric wake field device, particular the non-existence of deflecting mode, one can use an external RF driven dielectric structure as an accelerating cavities which will not produce any beam breakup mode[8].

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