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# ERROR LOCALIZATION USING MODE SHAPES - AN APPLICATION TO A TWO LINK ROBOT ARM

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## ABSTRACT

A technique to localize errors between two modal models is presented. Mode shape differences are calculated from each model. A global comparison of the ratios of these corresponding differences is used to identify the physical locations on the structure where stiffness differences exist between the two models. Some of the strengths and limitations of the technique are illustrated using the mode shapes of two similar finite element models with a known stiffness difference. The technique is then applied to a two link robot arm for which a finite element model exists and a modal test has been conducted. The results of the error localization aid the selection of physical parameters to be updated in the finite element model. Sensitivity methods are used to correlate the finite element model to the modal test. The results of the correlation are presented.

## NOMENCLATURE

| | |
|---|---|
| FE | = Finite Element |
| STRECH | = (S)tructural (T)ranslation and (R)otation (E)rror (CH)ecking Technique |
| $u^c, v^c, w^c$ | = Orthogonal translation displacements in comparison model |
| $u^d, v^d, w^d$ | = Orthogonal translation displacements in desired model |
| $\theta_u, \theta_v, \theta_z$ | = Orthogonal rotation displacements abut u,v,w axes |

$u_{12}$ = Difference in displacement in u direction between grids 1 and 2
$S_t$ = Translation STRECH ratio
$S_r$ = Rotation STRECH ratio
$l_u, l_v, l_w$ = Distance between two grids in the u, v and w directions
link = The geometric region between two adjacent grid points in a model
I = Area moment of inertia
MAC = Modal assurance criterion

**INTRODUCTION**

The desire to localize and identify errors in finite element (FE) models is understood by every FE method analyst. Many investigators have worked on error localization techniques. A summary and comparison of some techniques was presented by Gysin in reference [1]. Imregun and Visser [2] have an excellent review with observations on the advantages and disadvantages of error localization and model updating techniques. Optimization techniques utilizing design sensitivities [3,4] are especially robust in correlating the FE model with modal test frequencies. One great difficulty with design sensitivity techniques is the proper selection of physical parameters to change when adjusting the FE model. Generally the problem is underdetermined, that is, there are many more potential parameters to adjust than there are modes to compare. Because of this it is possible for the analyst to adjust parameters that are not originally in error and still match the modal test frequencies. Although theoretically possible, it is much more difficult to adjust the "wrong" parameters and match the test modal frequencies AND mode shapes. This is particularly true if the form of the FE model is basically correct and some plausible physical limits are applied to the values of the parameters themselves. Typically the parameters adjusted are those to which the modal frequencies are most sensitive [2,4]. These may not be the physical parameters which are truly in error. If the errors in the model can be localized *physically*, the analyst has a much better chance of selecting the parameters which truly need to be adjusted in the optimization process. Imregun [2] enumerates the difficulties of many of the error localization techniques. Several techniques localize an element of the stiffness or mass matrix that is in error, but cannot address the physical parameters that were modeled in the FE formulation that led to the error. Most techniques require a one to one correspondence between the modal test and FE model degrees of freedom. Often this requires a reduction of the FE matrices or an expansion of the modal test model. Techniques that reduce the FE matrices to the experimental degrees of freedom lose their physical meaning and may lose accuracy. Often the master degrees of freedom that are best associated with high inertias in the FE model are not accessible for a modal test measurement. Techniques that attempt to expand from the modal test degrees of freedom are also difficult to relate to the physical parameters of the FE model, and are extremely sensitive to errors in the experimental shapes.

The concept presented here eliminates most of these problems. No matrices are required. The physical meaning is maintained. The limitations as to where the master locations should be placed on the structure are greatly relaxed. The greatest disadvantage of the design sensitivity method is also theoretically eliminated. That is, the technique leads to identification of the physical parameters that are in error, so they may be chosen as the design variables. The technique has the acronym STRECH (Structural Translation and Rotation Error CHecking). Its output can be immediately interpreted to locate the most significant stiffness differences between two modal models. It identifies the physical area of the structure where the error appears. With a little engineering judgment, the FEM analyst can usually determine the most uncertain physical parameter(s) in the localized area. The two basic requirements for this technique are an accurate set of mode shapes from the modal test and an accurate inertial representation of the modal test structure by the FE mass matrix. Although the term "modal model" is used in this paper, a full modal model (i.e. damping and modal mass) is not required. Neither reduction nor expansion of matrices is required. The test locations can be selected at certain FE model grid points or they can be added to the FE model with multi-point constraints.

This technique might also be applied in the health monitoring field as well. An original modal test on a structure could be used to compare with a later modal test to locate "soft" spots that would indicate that structural damage had occurred since the original modal test. However, this application is not addressed further in this paper.

## STRECH CONCEPT

For the fundamental mode of an n degree of freedom system, the stretch in the connection between two adjacent grid points changes inversely to the connection stiffness when compared to the stretch of other unchanged connections on the structure. This may not be true for certain stiffness changes for high order modes, but if care is used, this observation may be extended past the fundamental mode to other low order modes. To illustrate the simplicity of the basic concept, consider the displacements of two adjacent grids in a modal model. Only one displacement coordinate will be addressed initially. If $u_1$ is the displacement of the first grid point and $u_2$ is the displacement of the second grid point, then $u_{12}$ is defined as the difference.

$$u_{12} = u_2 = u_1 \qquad (1)$$

Two modal models exist for the same system. One model is the desired model (denoted superscript d) and the other is a comparison model (denoted superscript c). The desired model is an accurate modal test model. The comparison model is the uncorrelated FE model. For a system with only one displacement coordinate, the translation STRECH ratio is calculated for each mode as

$$S_t = \frac{u_{12}^{c}}{u_{12}^{d}} \qquad (2)$$

This simple concept can be extended to three dimensions in the cartesian coordinate system. If u, v and w define the orthogonal translational displacements for a grid point then

$$S_t = \sqrt{\frac{\left(u_{12}^{c}\right)^2 + \left(v_{12}^{c}\right)^2 + \left(w_{12}^{c}\right)^2}{\left(u_{12}^{d}\right)^2 + \left(v_{12}^{d}\right)^2 + \left(w_{12}^{d}\right)^2}} \qquad (3)$$

If there are rotations in the structure, $u_{12}$ may be large due to the rotation at grid point 1 and the lever arm to grid point 2. Since a relationship between the stiffnesses and the elastic displacement is desired, a formulation that removes the lever arm displacements due to rotations is desired. A better quantification of the translational STRECH ratio for this purpose includes the rotational degree of freedom at grid 1. Let $l_u$, $l_v$, and $l_w$ represent the lengths in the u, v and w directions between grids 1 and 2. Let $\theta_u$, $\theta_v$ and $\theta_w$ be the rotational displacements about the u, v and w axes. Considering this enhancement for lateral bending in a plane, equation (2) becomes

$$S_t = \frac{u_{12}{}^{c} - \theta_{v1}{}^{c}\left(l_w\right)}{u_{12}{}^{d} - \theta_{v1}{}^{d}\left(l_w\right)} \qquad (4)$$

Figure 1 illustrates a case in which equation (4) captures the elastic deflection ratios better than equation (2) for a case where the rotational displacement at grid 1 is not zero.

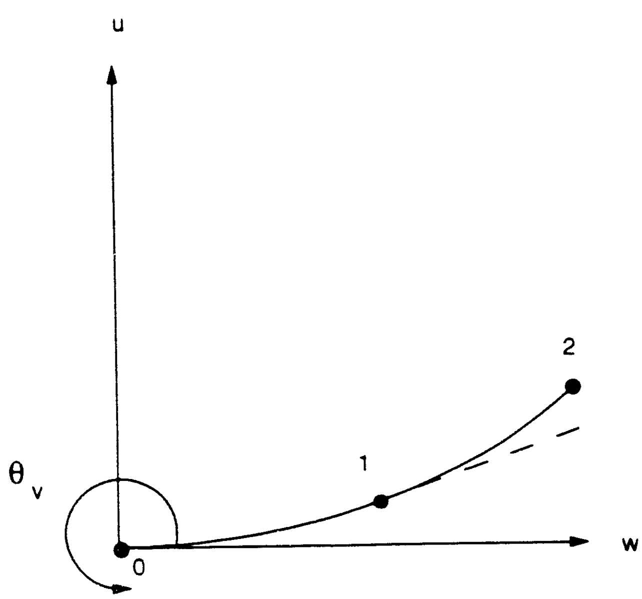


**FIGURE 1 - Beam Element in One Plane**

The extension to three dimensions in the cartesian coordinate system is

$$S_t = \sqrt{\frac{A^2 + B^2 + C^2}{D^2 + E^2 + F^2}} \qquad (5)$$

where

$$A = u_{12}^{c} - \theta_{v1}^{c} \left(l_w\right)$$

$$B = v_{12}^{c} - \theta_{w1}^{c} \left(l_u\right)$$

$$C = w_{12}^{c} - \theta_{u1}^{c} \left(l_v\right)$$

$$D = u_{12}^{d} - \theta_{v1}^{d} \left(l_w\right)$$

$$E = v_{12}^{d} - \theta_{w1}^{d} \left(l_u\right)$$

$$F = w_{12}^{d} - \theta_{u1}^{d} \left(l_v\right)$$

Equation (5) is a more generalized translation STRECH ratio. If the rotational displacements are removed it collapses to equation (3). A rotational STRECH ratio can be formed that is analogous to equation (3) using only the rotational degrees of freedom.

$$S_r = \sqrt{\frac{\left(\theta_{u12}^{c}\right)^2 + \left(\theta_{v12}^{c}\right)^2 + \left(\theta_{w12}^{c}\right)^2}{\left(\theta_{u12}^{d}\right)^2 + \left(\theta_{v12}^{d}\right) + \left(\theta_{w12}^{d}\right)^2}} \qquad (5)$$

**APPLICATION OF STRECH RATIOS**

In preparation to apply this technique, the user specifies a list of adjacent grids (following the load path from one end of the structure to the other) for which the STRECH ratios are to be calculated. Let each ratio be associated with a "link" between grid a and grid b. After the STRECH ratios are calculated how are the results to be used? Consider an example using the first mode of the structure. Suppose the modal frequency of the comparison modal model is lower than the frequency of the desired model. Then it is obvious that some stiffness value in the comparison model is too low. The analyst then looks for the largest numerical STRECH ratio which will locate the link where the comparison model is stretching <u>more</u> than the desired model. Engineering judgment is used to determine the physical properties that are most uncertain in the area of the soft link(s). These are then included in the set of design parameters to update the model. A similar process is followed using other low order modes. (Caution should be used here. This technique is not universally applicable for high order modes. It may be used successfully by application to only a portion of the structure that is exhibiting a fundamental type modal deformation, i.e. no inflections in shape). The desired end result is for the comparison model to approach the desired model in both modal frequencies and shapes. MAC or orthogonality calculations may be used to quantify the improvement in the mode shapes.

In the next section, equations (3) and (5) will be applied to a FE model that represents a cantilevered robot arm.

**ANALYTICAL STUDY**

Figure 2 shows the top view of a flexible two link robot arm and the corresponding grid points of a FE model. Also shown are the first four mode shapes.

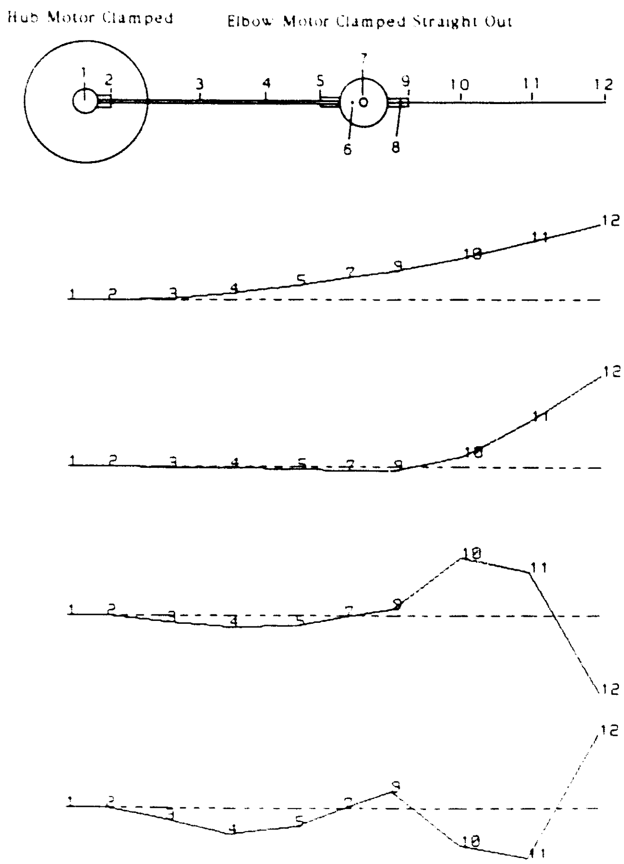


**FIGURE 2 - Schematic Beam Model and First Four Modes**

The actual hardware was designed as a research and development project for control of flexible structures [5]. The overall length is 1.3 meters. The hub motor is fixed for the analysis and therefore the arm is cantilevered. The elbow motor is locked. Only motion in the horizontal plane is considered. Links 2-5 and 9-12 are the controlling stiffnesses, being many orders of magnitude softer than links 1-2 and

5-9. The dominant inertia is concentrated in the elbow motor and its brackets. The arm is very flexible as illustrated by the fact that the first modal frequency is about 2 Hz. There are four modes below 35 Hz. A baseline FE model was chosen as the "desired" model. Three separate changes in the bending stiffness (area moment of inertia, I) of specific beams were made to create the "comparison" models. The STRECH ratios were analyzed to see how they correlated with the stiffness changes. Equation (5) was applied in two ways during this process. First, the exact rotations provided by the FE models were used. Second, an approximation of the rotations was used which averaged the slopes of the adjacent flexible links. Finally, only translational displacements were used (equation 3).

Three arbitrary modifications of the comparison FE model were selected. The first (Mod 1) was to reduce I for links 2-5 by five percent. The second (Mod 2) was to reduce I for link 3-4 by 20 percent. The third (Mod 3) was to increase I for links 9-10 by 10 percent. The results are listed in Table 1. The interpretation of a "good" result is that the changed links were identified. "Fair" is used only for MOD 1 indicating that one of the three soft links was identified. A "bad" result indicates that an unchanged link in another part of the model was identified incorrectly.

**TABLE 1**
**STRECH Identification of Differences**

| Mode# | Exact Rotat. | Approx. Rotat. | No Rotat. | % Freq Change |
|---|---|---|---|---|
| | **MOD 1 - Soften Links 2-5 5%** | | | |
| 1 | Good | Good | Good | -2.5 |
| 2 | Fair | Fair | Fair | -0.1 |
| 3 | Good | Good | Good | -1.0 |
| 4 | Bad | Bad | Bad | -1.5 |
| | **MOD 2 - Soften Link 3-4 20%** | | | |
| 1 | Good | Bad | Bad | -3.4 |
| 2 | Good | Bad | Bad | -0.1 |
| 3 | Good | Bad | Bad | -0.4 |
| 4 | Bad | Bad | Bad | -0.9 |
| | **MOD 3 - Stiffen Link 9-10 10%** | | | |
| 1 | Good | Good | Good | 0.1 |
| 2 | Good | Good | Good | 3.6 |
| 3 | Good | Good | Good | 0.8 |
| 4 | Bad | Bad | Bad | 0.7 |

Several guidelines resulted from the analyses which showed some of the strengths and weaknesses of the technique. The technique was implemented visually by a plot of the actual mode shape in which the links were color coded to show whether the link was comparatively stiff, soft or similar. Since this paper must be printed in black and white, the results must be explained and tabulated rather than plotted. The general results show that the STRECH ratio gives considerable insight to differences in the controlling stiffnesses for modes in which the modal frequency is sensitive to the error.

First, a limitation of the STRECH ratio was observed. In nearly every case, for the almost rigid links between 1-2 and 5-9, the STRECH ratio indicated a very stiff or very soft area when in actuality they were exactly the same in the comparison and desired models. This brings forth the first guideline in applying the technique. The link must be somewhat exercised (i.e. there must be measurable elastic displacement between the two grids). Otherwise computational noise (or in the more practical case, noise on the experimental mode shapes) will yield an inaccurate STRECH ratio. Links which have little elastic displacement (essentially rigid links or links not exercised in the mode) should be removed from the analysis. The results in Table 1 have utilized this guideline. A quick look at mode shape two of Figure 1 shows that only links 9-12 should be compared in a practical application since links 1-9 are not exercised.

A second very important result is observed for mode four. Table 1 shows that for all three modifications the STRECH ratios from mode 4 gave wrong results. The links are exercised, but obviously the technique failed. There are two possible causes. One cause could be that this mode can no longer be considered a low order mode. Remember that the basis for the concept is an observation of the fundamental mode of a structure. It is known that high order modes cannot be used in this formulation. Mode four may cross the unknown line between low order and high order modes for these particular stiffness changes. The second possible cause is that mode four has a particularly strong inflection point in link 9-10. Neither equation (3) or (5) adequately captures the elastic displacement differences where there is a large inflection point. Equation (5) assumes a continuously increasing difference in the displacements when moving from grid 1 to 2, which is not the case when an inflection point is present.

The comparisons which utilized the FE rotations gave the best results. Very accurate rotations were required to locate the stiffness error in Mod 2. The comparisons utilizing approximate rotations were better than those using no rotations. This was apparent when examining the actual magnitude of the STRECH ratios. If a reasonable scheme to approximate rotations for two models can be devised, the payoff in more accurate STRECH ratios is probably worth the effort. However, in both Mod 1 and Mod 3 the results with no rotations correctly identified the changed links. More applications are needed to define guidelines as to what structures require rotations to locate errors.

An interesting feature of the STRECH analysis is that it appears to be sensitive to the differences in stiffnesses that are important to the particular mode being analyzed and insensitive to differences in stiffnesses that are not so important. The FE analyst can take advantage of this to immediately focus on the important differences and neglect the others.

## CORRELATION OF FE MODEL USING MODAL TEST RESULTS

A modal test was performed on the two link robot arm in the configuration described previously. The purpose of the test was to provide data to correlate an existing FE model. Obviously, the test mode shapes must be accurate for the STRECH technique to give useful information to the FE analyst. Frequency response functions synthesized from the extracted modal parameters are overlaid on the original measured data in Figure 3.

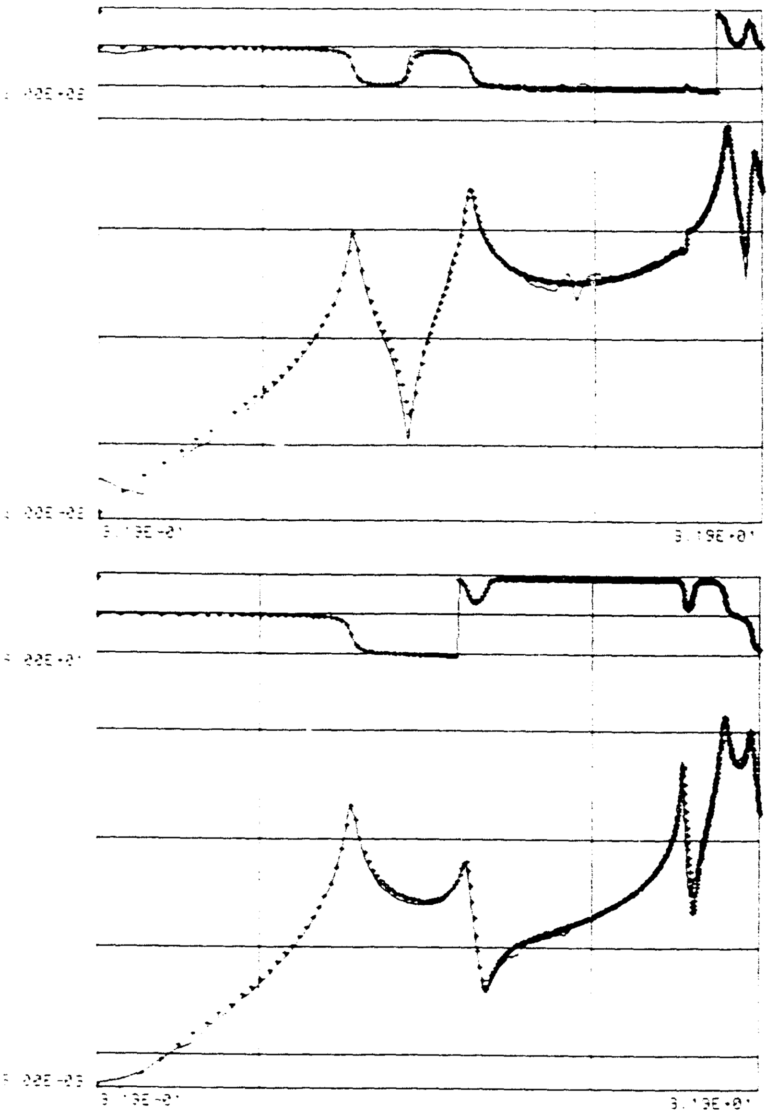

**FIGURE 3 - Synthesized FRF's Overlaid With Measured FRF's**

It can be seen that the synthesized functions (which assumed real modes) fit the data well, so there is little error in the modal extraction routine. Examination of the test mode shapes revealed an obvious inaccuracy in mode 1 indicating measurement error in the data at grid 4 for the first mode. Unfortunately this eliminated any information from mode 1 about links 3-4 and 4-5. The author judged modes 3 and 4 to be the best test shapes. However, from the previously mentioned analyses, it was evident that mode 4 should be used with caution. Modes 2 and 3 were used most confidently for the STRECH ratio comparisons.

The existing FE model was a simple beam and mass model that was being utilized in a control model to simulate the flexible body dynamics of the robot. Its basic form was not to be changed, i.e. the number of grids and beam elements were to remain the same. The STRECH technique was applied to the first four bending modes. The accelerometer locations coincided with the FE grid points.

Table 2 shows the indications of the links that were too stiff using translation STRECH ratios for each mode. Equation (5) was used. Refer to Figure 2 for location of the numbered links.

**TABLE 2 - STIFF LINKS**

| MODE # | LINK #'s |
|---|---|
| 1 | 2-3 |
| 2 | 9-10 |
| 3 | 2-3, 9-10 |
| 4 | 2-3, 4-5, 9-10 |

Since the bending stiffness was easy to accurately determine for the flexible members, there had to be a good physical justification for the lower stiffness values in the experimental mode shapes. The explanation is that there are clamped joints at points 2, 5 and 9. Therefore, the identification process involved reducing I for links 2-3, 4-5 and 9-10. The most reliable results from the STRECH calculations were to soften only links 2-3 and 9-10. (Only mode 4 indicated that link 4-5 was too stiff, and the validity of mode 4 for STRECH calculations was questioned earlier). However, the clamped joint at grid 5 was the same type as used at grids 2 and 9, so engineering judgment dictated that the result from mode 4 be included.

The control designers desired the FE model to be very accurate with respect to the lowest natural frequencies. Since three stiffnesses were to be adjusted, theoretically it was possible to match the first three modal frequencies. The sensitivities of the modal frequencies with respect to a small change in each of the three area moments of inertia were calculated. These were used in a system of three equations to calculate the change in each area moment of inertia required to match the first three modal frequencies. Table 3 gives the modal frequencies of the test and FE models before and after this correlation. Also, the Modal Assurance Criterion (MAC) values are given. It can be seen that the correlation was able to match the first three modal frequencies extremely well and significantly reduce the error in the fourth modal frequency from 9.1 to 3.5 percent. The high MAC values are about the same for each case.

**TABLE 3 - CORRELATION RESULTS**

| Test | Uncorrelated FEM | Correlated FEM |
|---|---|---|
| | frequencies - Hz | |
| 1.94 | 2.17 | 1.94 |
| 5.23 | 5.53 | 5.23 |
| 26.97 | 30.20 | 26.97 |
| 33.68 | 36.73 | 34.86 |
| | Test/FEM MAC values | |
| | .9964 | .9972 |
| | .9981 | .9988 |
| | .9897 | .9858 |
| | .9880 | .9840 |

Several arbitrary changes were attempted to achieve a better correlation for the fourth mode. Interestingly, in almost every case the frequency could be matched, but the MAC values were much worse. This emphasizes the value of using mode shapes to help quantify whether the changes are realistic. It also reinforces the observation that if the physical parameters that are truly in error can be isolated, a realistic correlation using optimization with design sensitivities is quite feasible. The author believes that there was some unidentified mass error that caused the difficulty in matching all four modes. The masses of strain gauges, wiring and connectors mounted on the hardware were neglected. The STRECH technique, as mentioned earlier, does not currently apply to location of mass errors.

## CONCLUSIONS

The Structural Translation and Rotation Error CHecking (STRECH) technique has been presented and found useful in physically locating stiffness differences between two analytical modal models for low order modes of the structure. Good approximations of the rotations increase the reliability of the STRECH ratios. The technique requires accurate mode shape estimates from the two modal models being compared. STRECH was used successfully in comparing a FE model with a modal test model to locate stiffnesses that required adjustment. An optimization technique based on frequency sensitivity analysis was used to adjust the stiffnesses. It improved the correlation of the modal frequencies of a FE model to a modal test while maintaining high MAC values for the associated mode shapes.

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