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ABSTRACT

This paper examines optimal and market-determined extraction patterns for a depletable resource available (at a cost) from many reserves of various grades. It is shown that under a general set of conditions optimal allocations can be supported by a purely competitive market. The concept of a time varying market imperfection function is introduced. Properties of this function are shown to be sufficient to determine whether specific market form will over-extract or under-extract the resource (in comparison to a competitive allocation). Finally, the intertemporal biases associated with depletion allowances, monopolies, externalities, vulnerability costs, and price regulations are analyzed by making use of the market imperfection functions associated with each market structure.

Economics of Depletable Resources:
Market Forces and Intertemporal Bias

James L. Sweeney*

Currently there is much concern over the adequacy of the natural resource base in the world. The purpose of this paper is to theoretically model the extraction patterns for a finite depletable resource and to systematically examine the directions of intertemporal bias to depletion patterns stemming from various market forces.

One possible bias to be examined lies in the competitive mechanism itself. Can socially optimal extraction patterns be supported by a competitive market in which future monetary flows from the extraction of depletable resources are discounted at the same interest rate as are monetary flows from capital investment? The answer (under appripriate convexity conditions) will be yes.

But since markets for natural resources may be far from perfect, the question remains what will be the intertemporal biases stemming from various market structures. What biases are occasioned by percentage depletion allowances? By non-internalized externalities? By monopolistic practices? By price regulation such as the well-head regulation of natural gas prices? By vulnerability costs associated with high levels of imports? Using an axiomatic model of markets under the various conditions, it will be shown that a single general criterion will be sufficient to examine each bias. This criterion will be used to establish that the first two (generally) lead to current over-extraction at the expense of future extraction options, that the third may lead to either over- or under-extraction, depending upon the rate of growth of demand, that the fourth is not determinate without further empirical work, and that the fifth leads to over use but may lead to over- or under-extraction.

There has been significant literature on the economics of depletable resources, with the first formal models proposed by Harold Hotelling [9]. This work has been carried on by O. Herfindahl [8], A. Scott [11], R. L. Gordon [7], R. G. Cummings [3], and others. In general these works model the choices of an individual firm facing given prices or facing given demand functions. While some authors [3, 8] have used quite sophisticated models of individual firms. the examination of market interactions had tended to be superficial. Recently a new body of literature examining optimal economic growth with resource constraints has developed. Works by K. Anderson [1], T. C. Koopmans [10], R. M. Solow [12], P. Garg [5], J. Stiglitz [13], P. Dasgupta and G. Heal [4], have embedded the question of optimal resource depletion into the more general question of optimal economic growth. However, these works have given little attention to the relationships between these optimal patterns and market-determined patterns.

This paper is focused upon the relationships between optimal depletion patterns and market-determined patterns. Section I examines the optimal depletion patterns for firms facing market-determined resource price trajectories and collects many firms into a competitive market model. In Section II we examine the intertemporal biases occasioned by market institutions such as depletion allowances, monopolies, externalities, price regulation, and international vulnerability. The concept of a market imperfection function is defined and shown to provide a general set of criteria for examining intertemporal biases. Finally, Section III offers a summary and conclusions.

I. Market-Determined Extraction Patterns: Pure Competition

It will be assumed that some quantity of the resource is available as a limited reserve which can be extracted (at a cost) over time. Since the stock is limited, the quantity extracted at one time will influence the amount available from this reserve at later times. Once extracted, the resource commands a market price. The reserve will be controlled by a single firm. We will assume here that the market is competitive: each firm is a price-taker and maximizes the present value of the stream of profits accruing to it.

Let us assume that the i^{th} reserve is known with certainty at time 0 to be S_0^i (where S_0^i is finite). S_0^i measures the total quantity of the desirable resource which could utlimately be recovered from the i^{th} reserve, net of any undesirable wastes.

Let $s^i(t)$ represent the rate of extraction of the resource as a function of time. Then the constraints on extraction can be written as follows:[1]

(1) $$\int_0^{\infty} s^i(t)\ dt \le S_0^i ,$$

(2) $$s^i(t) \ge 0 .$$

The extraction cost per time period, $C^i(s^i,t)$, depends upon the rate of extraction and possibly upon time.[2] The extraction cost function provides a measure of the quality of the reserve. A high cost function would characterize a low-grade reserve or a reserve accessible only at great difficulty. For example, an extremely deep-lying petroleum deposit would be characterized by a higher cost function than would a deposit closer to the surface. A coal reserve with narrow veins and a heavy overburden would be characterized by a higher cost function than would a reserve having broad veins and a light overburden.

It will be assumed that the marginal cost of extraction, $\partial C^i/\partial s^i$, is an increasing function of extraction rate (that C^i is strictly convex in s^i). Once extracted, the reserve can be sold at a price $P(t)$ per unit which may depend upon time but not upon the chosen extraction rate. The total value of depleting the resource is equal to the net earnings rate, discounted to time zero at an interest rate, $r(t)$, and integrated over all time. If a single firm controls the i^{th} reserve and chooses the extraction rates so as to maximize profit, then it implicitly solves the following constrained maximization problem:

$$
(F)\quad \begin{cases} \text{Maximize} & \displaystyle\int_0^\infty [P(t)s^i(t) - C^i(s^i(t), t)]\, D(t)dt \\ \text{under} & s^i(t) \geq 0\ , \\ & \displaystyle\int_0^\infty s^i(t)dt \leq S_0^i\ . \end{cases}
$$

where $D(t)$ is the discount factor.[3]

$$
(3)\qquad D(t) = e^{-\int_0^t r(\tau)d\tau}
$$

The solution to problem (F) will consist of an optimal path of extraction over time, $s^{i*}(t)$, a cost over time, and a value of the reserve.

Problem (F) can be solved by use of the Kuhn-Tucker Theorem. Let λ^i be a (non-negative) Lagrange multiplier; then the Lagrangian for problem (F) can be written as follows:

$$
(4)\qquad \begin{aligned} \mathcal{L}^i = & \int_0^\infty [P(t)s^i(t) - C^i(s^i(t), t)]\, D(t)dt \\ & + \lambda^i[S_0^i - \int_0^\infty s^i(t)\, dt]\ . \end{aligned}
$$

By the Kuhn-Tucker Theorem, if $s^{i*}(t)$ is the optimal depletion path, $s^{i*}(t)$ must maximize $\mathcal{L}^i$ over all non-negative extraction paths. The Lagrange multiplier is chosen so that:

(5) $$\lambda^i \geq 0 \; ; \quad \text{and} \quad \left[\int_0^\infty s^{i*}(t)dt - S_0^i\right] \lambda^i = 0 \; .$$

Thus, either constraint (1) must be binding or λ^i must equal zero.

Maximizing $\mathcal{L}^i$ gives the following necessary condition for $s^{i*}(t)$:

(6) $$[P(t) - MC^i(s^{i*}(t), t)] \; D(t) - \lambda^i \begin{cases} = 0 & \text{for } s^{i*}(t) > 0 \\ \leq 0 & \text{for } s^{i*}(t) = 0 \; , \end{cases}$$

where MC^i is the marginal extraction cost, $\frac{\partial C^i}{\partial s^i}$. This equation can be rewritten slightly:

$$P(t) = MC^i(s^{i*}(t), t) + \lambda^i/D(t) \qquad \text{for } s^{i*}(t) > 0 \; ,$$

where λ^i is independent of time. If constraint (1) is binding, then $\lambda^i > 0$; price and marginal extraction cost will <u>not</u> be equal; marginal cost must be <u>lower</u> than price when the resource is optimally extracted. The term $\lambda^i/D(t)$ represents the <u>opportunity cost</u> of using the limited resource at time t rather than at an alternative time; λ^i is the present value of the opportunity cost discounted to time zero. Equation (6) implies that for an extraction path to be optimal, at each time the marginal cost of extracting an additional unit plus the additional opportunity cost of the extraction must equal the price of the extracted resource. Thus, Eq. (6) expresses a marginal cost rule extended to the cose of a finite stock of depletable resource.

It should be noted that the higher the cost functions, or the greater the initial stock of the resource, the smaller will be λ^i. Therefore, the greater is S_0^i (for given cost functions), the greater will be the extraction rate at every time, and the smaller will be the difference between price and marginal cost at each time.[4] For a given total reserve, different cost functions lead to different temporal patterns of extraction, though not necessarily different total quantities extracted. It should be noted that, for a given reserve, extraction rates may initially be zero, may increase to a peak, and may finally return to zero when the entire stock is depleted.

If $C^i(s^i, t)$ is convex in s^i (marginal extraction cost is a nondecreasing function of extraction rate), Eqs. (1), (2), (5), and (6) are sufficient conditions for $s^{i*}(t)$ to solve problem (F). This is stated in Proposition 1 and is proved in Arrow and Kurz [2].

<u>Proposition 1</u>. Let $C^i(s^i, t)$ be convex in s^i and suppose that a depletion path $s^{i*}(t)$ satisfying Eqs. (1), (2), (5), and (6) exists. Then this path solves Problem (F). Furthermore, if $C^i(s^i, t)$ is strictly convex in s^i, then $s^{i*}(t)$ is the unique optimal path.

Note that nothing stated so far guarantees the existence of an optimal path. In particular, for time independent extraction cost functions, no optimal path can be expected to exist if prices increase too quickly in the limit as time approaches infinity [if $\dot{P}/P > r(t)$].[5] However, if prices do not increase too quickly, then the existence of an optimal path can be assured as long as the cost function is continuous in the extraction rate and is non-decreasing in time. If the cost function is discontinuous or decreasing in time, then the existence conditions are more complicated and need not be discussed here.

Models of individual firms can be placed in a market context in which the prices are determined through an interaction of supply and demand. Assume that there exist many finite reserves, each controlled by a single firm (although each firm may control many reserves). These reserves may be of differing qualities and differing magnitudes. Then $S_0^1, S_0^2, \ldots, S_0^n$ will represent the initial reserves. The holder of each reserve faces an extraction cost function $C^i(s^i, t)$, and chooses an optimal depletion path $s^{i*}(t)$, by solving problem (F).

The optimal depletion paths of individual firms lead to a market depletion path Q(t), which represents the supply of the resource at time t:

$$Q(t) = \sum_i s^{i*}(t) \quad (7)$$

There exists a (possibly time-varying) demand curve for the resource, which determines the resource price trajectory when $Q(t)$ is given. Thus, if $P(Q, t)$ represents the demand price function, the price trajectory can be determined by:

(8) $$P(t) = P(Q(t), t).$$

Equations (7) and (8) plus problem (F), when simultaneously solved, represent the workings of the purely competitive market if $r(t)$ is given and the demand price function is given.

If a competitive allocation exists, it must satisfy equations (7) and (8) and each firm's choice must solve problem (F). Conversely, any allocation which satisfies these conditions will be said to be a competitive equilibrium.

It is useful to examine the properties of the competitive equilibrium in three cases of increasing complexity. First, a case of zero extraction costs is examined. Second is a case of constant marginal costs from a given reserve but with reserves of different costs. Finally is a case of increasing marginal costs from a given reserve with reserves having different costs. In each case the interest rate will be assumed constant over time.

Case 1, that of zero extraction costs, has been discussed previously in the literature. The marginal conditions from Eq. (6) are as follows:

$$P(t) = \lambda/D(t) = \lambda e^{rt}.$$

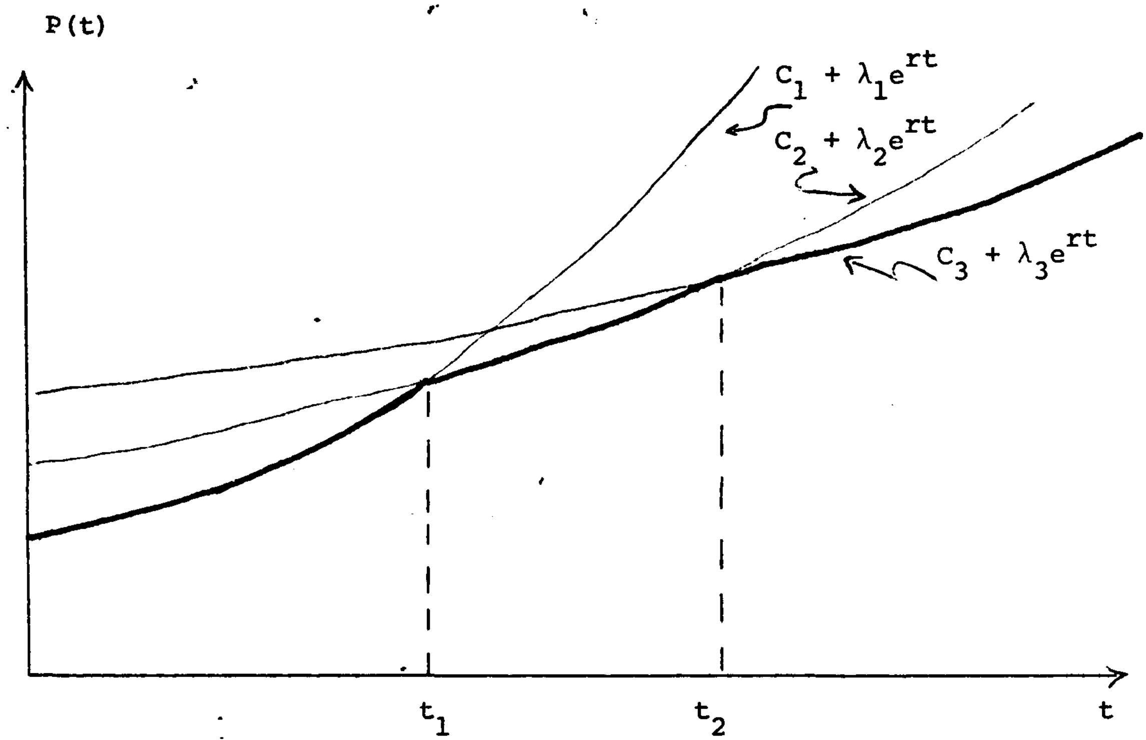
In this case each firm is indifferent to the time and the rate of extraction. Extraction rates are chosen so that the price rises at the interest rate.

In Case 2 there are many reserves each having constant marginal extraction costs, but with different reserves characterized by different costs. Thus, reserve i has a constant marginal cost equal to C_i, and an initial quantity S_0^i. They are numbered so that $C_1 < C_2 < C_3$, and so on. In this case the marginal condition for each reserve becomes:

$$P(t) \begin{cases} = C_i + \lambda_i e^{rt} & \text{for } s_i > 0 \\ \leq C_i + \lambda_i e^{rt} & \text{for } s_i = 0. \end{cases}$$

Firms with high costs will have low shadow prices and vice versa. The price trajectory will be an envelope of curves of the form $C_i + \lambda_i e^{rt}$, as is illustrated in Figure 1. In this figure, the lowest cost reserve will be extracted up until time t_1, the next lowest cost reserve will be extracted from time t_1 to t_2, and so on. Reserves are extracted strictly in order of increasing costs; the next most costly begins to be extracted the instant a given reserve is totally depleted. The depletion times and rates are based upon the demand curves. The price now increases at a rate strictly less than the interest rate.

A third case has increasingly marginal costs of extracting from a given reserve. Various reserves have different costs but in this special



$$C_1 < C_2 < C_3$$

$$\lambda_1 > \lambda_2 > \lambda_3$$

Figure 1

Price paths under constant marginal extraction costs. The heavy line represents the competitive equilibrium price trajectory.

case each cost function varies from every other one by a single scale parameter. Marginal cost functions are assumed to be:

$$MC_i(s_i) = \alpha_i \, MC(s_i) ,$$

where $\alpha_1 < \alpha_2 < \alpha_3$ and so on.

Here the marginal conditions become:

$$P(t) - \alpha_i \, MC(s_i) \begin{cases} = \lambda_i e^{rt} & \text{for } s_i > 0 \\ \leq \lambda_i e^{rt} & \text{for } s_i = 0. \end{cases}$$

In this solution, many reserves are extracted simultaneously. Furthermore, the extraction rate from a given reserve depends on S_i^0 as well as upon α_i. For two reserves, i and j, with $\alpha_i = \alpha_j$ if $S_i^0 > S_j^0$, then $\lambda_i < \lambda_j$. The more of a given reserve that exists (all else equal), the lower will be the shadow price, the higher will be the extraction rate, and the higher will be the marginal extraction cost at the optimal extraction rate. Note that in general it will not be true that marginal extraction costs at the optimal rates will be equal for all reserves.

For two reserves of identical magnitudes but of different costs, the lower cost reserve will be generally extracted first, but overlaps can be expected to occur. This is illustrated in Figure 2. Here the cheaper reserve is extracted more rapidly than is the more costly one, but for a period of time both are simultaneously being extracted.

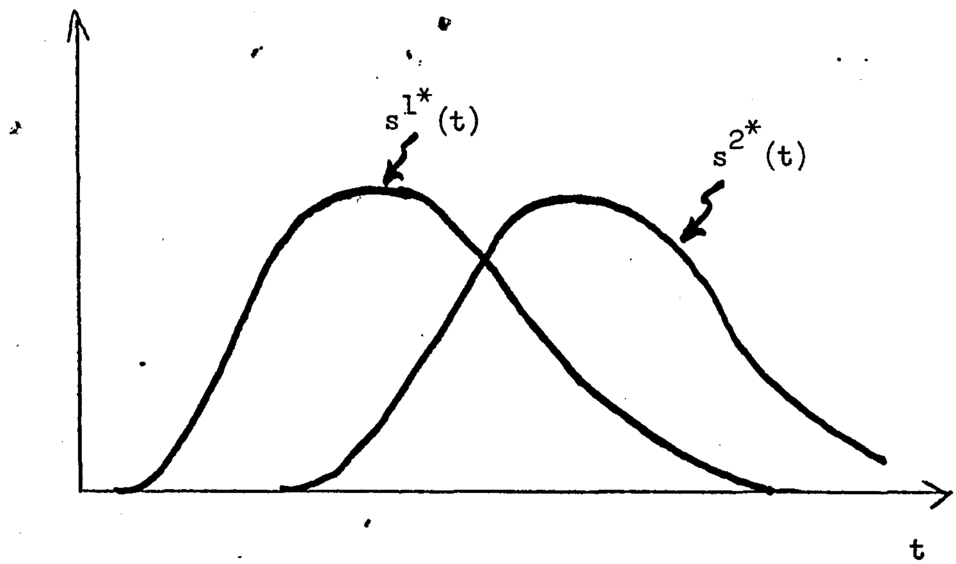


Figure 2.

Depletion patterns for two reserves.

$s_0^1 = s_0^2 \ , \ \alpha_1 < \alpha_2$

The resource price new may increase faster than or slower than the interest rate, depending upon changes in the demand functions over time. Of course, above ground storage possibilities limit the maximum growth rate of the resource price.

The competitive solution is particularly interesting in that it corresponds to the socially optimal allocation obtainable from the corresponding optimal growth problem. Assume that the economy attempts to maximize a utilitarian objective function, W, which equals the integral of utility, $U(Cn , t)$, over time, where Cn is the rate of consumption of output. Output is produced using the resource, capital, and labor, through a neoclassical production function, $f(K, Q, t)$, where K is the capital input. Output can be consumed, used to extract resources, or applied to capital formation. The entire problem can be stated as:

$$
(S)\quad \begin{cases}
\text{Max } W = \int_0^\infty u(Cn, t)\,dt, \\
\text{under} \\
\dot{K} = f(K, Q, t) - Cn - \sum_i C^i(s^i, t) \\
Q = \sum_i s^i \\
\int_0^\infty s^i(t)\,dt \le S_0^i, \quad \text{for all } i \\
s^i(t) \ge 0 \quad \text{for all } i.
\end{cases}
$$

Then the following Proposition can be proved (proof available from the author):

Proposition 2. Assume that a socially optimal allocation derived as a solution to Problem (S) exists, and suppose that C^i is a convex function of s^i for each i. Then if savings/consumption trajectories are chosen optimally, the socially optimal allocation can be supported as a competitive allocation.

Proposition 2 implies several important results. First, if savings patterns are optimal, a competitive allocation can be used to obtain a socially optimal allocation. Second, if a decentralized mechanism does not satisfy the competitive conditions, it cannot lead to societal optimality. In particular, this proposition implies that societal optimality can be obtained only if monetary flows from resource extraction are discounted in precisely the same manner, using the same interest rates, as are monetary flows from capital equipment. Furthermore, this proposition provides a bench-mark against which to evaluate specific non-purely-competitive markets, for it implies that market phenomena which cause extraction patterns to diverge from the purely competitive patterns lead to non-socially optimal patterns. Questions of the biases caused by various institutional mechanisms can be addressed by comparing allocations they generated to those generated by the competitive allocation. This problem will be addressed in subsequent sections.

II. Market-Determined Extraction Patterns: Non-Pure Competition

In order to examine the influence on extraction patterns of various economic forces we will assume that the specific influences to be examined do not change the interest rate over time, nor do they change demand functions or cost functions other than in ways to be specified. All forces will be examined by comparing their depletion patterns to those obtained under the competitive regime.

It will be demonstrated that intertemporal biases associated with various institutions can be examined through the use of a market imperfection function, $g(Q', t)$, which characterizes the institution. For several institutions it will be demonstrated that this scalar function can easily be defined and that properties of the function can be determined. Second, it will be shown that several simple properties of the market imperfection function are sufficient to determine the directions of intertemporal bias. That is, these properties, relating to the sign of the market imperfection function, and to its growth rate (in comparison to the interest rate), are sufficient conditions for determining directions of bias. Thus the market imperfection function allows a generalization of specific results obtained elsewhere by Stiglitz [14] and by Weinstein and Zeckhauser [16].

Consider several market institutions. The first is the percentage depletion allowance for the extraction of mineral resources. This tax law provides that a fraction, β, of the revenue from extracting raw materials be exempt from corporate income tax.[6] If there were no depletion allowance and if the firm faced a tax rate of T, then the after-tax profit at time t would equal $[1-T]\ [P(t),t)s^i(t) - C^i(s^i(t),t)]$. In this case, the optimal depletion paths would not be influenced by choice of T, for $0 < T < 1$. With a depletion allowance, after-tax profit would equal

$$[1-T]\left[\left(1+\frac{\beta T}{1-T}\right)P(Q(t),t)s^i(t) - C^i(s^i(t),t)\right].$$

Defining $\alpha = \frac{\beta T}{1-T}$, the percentage depletion allowance has the effect of increasing the apparent price facing the firm from $P(Q(t),t)$ to $(1+\alpha)P(Q(t),t)$. For example, for a firm facing a 50% corporate tax rate, the depletion allowance for petroleum of 22% increases the apparent price of its output by 22%.

Under a depletion allowance regime, the revenue for a firm solving Problem (F) is changed from $P(Q(t),t)s^i(t)$ to $(1+\alpha)P(Q(t),t)s^i(t)$. All other conditions remain unchanged. Therefore the necessary conditions for optimality expressed in Eq. (6) then become:

$$(9)\quad [(1+\alpha)P(Q(t),t) - MC^i(s^{i*}(t),t)]D(t) - \lambda^i \begin{cases} = 0 & \text{for } s^{i*}(t) > 0\ , \\ \leq 0 & \text{for } s^{i*}(t) = 0\ . \end{cases}$$

In this equation λ^1, $s^1(t)$, $Q(t)$, and hence $P(t)$, will all change in response to the depletion allowance.

Extraction paths occurring under a depletion allowance regime can be compared to those occurring under a competitive regime by substituting Equation (9) for Equation (6), while retaining all other equations describing the competitive regime. Eqs. (6) and (9) can be written more simply by suppressing the explicit time dependency of the solutions. The original variables will be denoted as Q, s^i, and λ^i, while the variables under depletion allowances will be denoted as Q', $s^{i'}$, and $\lambda^{i'}$. Furthermore, Eq. (9) can be expressed in a manner which will help to underline the explicit influence of the depletion allowance and to underline the relationships among the various biases to be examined:

(10) $$P(Q) - MC^i(s^i) - \lambda^i/D(t) \begin{cases} = 0 & \text{for } s^i > 0 , \\ \leq 0 & \text{for } s^i = 0 , \end{cases}$$

(11) $$g(Q',t) + P(Q') - MC^i(s^{i'}) - \lambda^{i'}/D(t) \begin{cases} = 0 & \text{for } s^{i'} > 0 , \\ \leq 0 & \text{for } s^{i'} = 0 , \end{cases}$$

where

(12) $$g(Q',t) = \alpha P(Q') > 0.$$

Here $g(Q',t)$ expresses the direct influence of the depletion allowance on the individual firms. This function provides a measure of market imperfection and will be denoted as the market imperfection function under a depletion allowance regime.

A second influence is the market structure of the extractive industry. In particular, we can compare a competitive regime to one in which the entire extractive industry is monopolized. In this case, one firm makes all extractive decisions. Therefore, Problem (F) becomes:

$$
\text{(M)} \quad \begin{cases} \text{Max} \sum_i V^i = \int_0^\infty \sum_i [P(Q(t),t)s^i(t) - C^i(s^i(t),t)]D(t)\,dt \\ \qquad\qquad \text{under} \quad s^i(t) \geq 0 \quad \text{for all } i, \\ \qquad\qquad\qquad \int_0^\infty s^i(t) \leq S_0^i \quad \text{for all } i, \\ \qquad\qquad\qquad Q(t) = \sum_i s^i(t). \end{cases}
$$

Problem (M) leads to necessary conditions analogous to those of the competitive regime:

$$
(13) \qquad MR(Q') - MC^i(s^{i'}) - \lambda^{i'}/D(t) \begin{cases} = 0 & \text{for } s^{i'} > 0 , \\ \leq 0 & \text{for } s^{i'} = 0 , \end{cases}
$$

where Q', $s^{i'}$, and $\lambda^{i'}$ are now interpreted to be the monopolistic equivalents of Q, S^i, and λ^i, and $MR(Q')$ is the marginal revenue of resource extraction. Equation (13) can be expressed in the form of Eq. (11) where now,

$$
(14) \qquad g(Q',t) = MR(Q') - P(Q') < 0.
$$

The scalar function $g(Q',t)$ in Eq. (14) is the market imperfection function for a monopolistic industry.

A third influence is that of externalities associated with the extraction or the use of the depletable resource. For example, atmospheric residuals may be an externality associated with the use of petroleum products. Assume now that there is some monetary social cost of externalities associated with the use of a resource and this cost is denoted by $E(Q)$. Assume that in the competitive regime pollution taxes are chosen to be equal to the marginal cost of the externality. Then we can compare the situation of non-internalized externalities to the situation which would occur if externalities were internalized. The non-internalized situation leads to marginal conditions which can be written in the form of Eq. (11), except now we have a market imperfection function $g(Q', t)$ such that:

$$g(Q',t) = \frac{\partial E(Q')}{\partial Q'} > 0 \; . \tag{15}$$

In the case of no internalization the price facing the seller of the resource is greater than the social marginal productivity of the resource by a quantity equal to the marginal pollution cost, and this difference over time defines the market imperfection function with externalities.

A fourth influence is price regulation in the resource industry. We will assume that prices are limited to not exceed $\overline{P}(t)$, an exogenously given function of time and that this is a binding constraint. That is, the demand for the resource at any time is assumed to be greater than the quantity supplied, and actual sales are limited by the supply decisions of firms. In this case, the necessary conditions for optimality for a firm are given as:

$$(16) \qquad \bar{P}(t) - MC^i(s^{i'}) - \lambda^{i'}/D(t) \begin{cases} = 0 & \text{for } s^{i'} > 0 , \\ \leq 0 & \text{for } s^{i'} = 0 . \end{cases}$$

This equation can be written in the form of Eq. (11) with a market imperfection function equal to the difference between controlled prices and market clearing prices:

$$(17) \qquad g(Q',t) = \bar{P}(t) - P(Q') < 0.$$

These four economic influences have a similar mathematical structure--in each case a market imperfection function can be defined. For depletion allowances and externalities the market imperfection is positive, while this function is negative for price control and for monopolistic structure. This difference will be crucial for determining the patterns of intertemporal bias.

The sign of the market imperfection function will not be sufficient to determine patterns of intertemporal bias. In particular, the time patterns of this function must also be examined. It will be necessary to distinguish between the cases in which the market-determined (absolute) value of g rises (on the average) more quickly than the interest rate from those cases in which this does not occur. More precisely, we can state three alternative conditions:

Condition 1 (Normal Change): For all $t > 0$,

$$\frac{D(t)g(Q'(t),t)}{g(Q'(0),0)} < 1.$$

Condition 2 (Exponential Change): For all $t > 0$,

$$\frac{D(t)g(Q'(t),t)}{g(Q'(0),0)} = 1.$$

Condition 3 (Rapid Change): For all $t > 0$,

$$\frac{D(t)g(Q'(t),t)}{g(Q'(0),0)} > 1.$$

Finally, it will be important to distinguish between those cases in which constraint (1) is binding for all reserves and those cases for which it may not be binding for some. In the former case, that of ultimate depletion of all resources, stronger results can be obtained than in the latter case, that of partial depletion of some resources, ultimate depletion of others. In order to facilitate discussion the following case can be defined:

Definition: Ultimate depletion will be said to occur under the competitive regime if $\int_0^\infty s^i(t)dt = S_0^i$ for all i, and under the alternative regime if $\int_0^\infty s^{i'}(t)dt = S_0^i$ for all i.

It will be shown that under conditions of ultimate depletion in both regimes, if the sign of the market imperfection function is known and if condition 1, 2, or 3 is satisfied, then whether the competitive regime or the alternative regime leads to a more rapid initial depletion of resources can be determined unequivocally. If ultimate depletion does occur, then the direction of bias can be determined under condition 1 or 2, but not necessarily under condition 3.

For the depletion allowance case the normal change condition would imply that prices do not rise (on the average) more quickly than the interest rate. For externalities, if the marginal non-internalized externality cost never rises as quickly as the interest rate, condition 1 holds, while if it always rises more quickly, then condition 3 holds. For monopolies and for price control, these conditions are similarly related to the rates of change of the difference between price and marginal revenue and the difference between market equilibrium price and controlled price.

One final assumption will be made:

Assumption (Convexity, Continuity): The marginal extraction cost (MC^i) is a continuous, non-decreasing function of extration rate (s^i) for each firm.

We can now state the following theorem. The proof appears in an appendix.

Theorem 1. Suppose that the convexity, continuity assumption holds and that either $Q'(0) > 0$ or $Q(0) > 0$. Let unprimed variables refer to outcomes under the competitive regime, while primed variables refer to outcomes under the alternative regime.

a) Suppose that condition 1 holds;

$$\text{if } g(Q'(0)) \begin{Bmatrix} > \\ < \end{Bmatrix} 0, \text{ then } Q'(0) \begin{Bmatrix} > \\ < \end{Bmatrix} Q(0)$$

$$\text{and } P(Q'(0)) \begin{Bmatrix} < \\ > \end{Bmatrix} P(Q(0)).$$

b) Suppose that condition 2 holds;

i) if ultimate depletion occurs under each regime,

$$Q'(0) = Q(0) \text{ and } P(Q'(0)) = P(Q(0)).$$

ii) if ultimate depletion fails to occur under one regime,

$$\text{if } g(Q'(0)) \begin{Bmatrix} > \\ < \end{Bmatrix} 0, \text{ then } Q'(0) \begin{Bmatrix} > \\ < \end{Bmatrix} Q(0)$$

$$\text{and } P(Q'(0)) \begin{Bmatrix} < \\ > \end{Bmatrix} P(Q(0)).$$

c) Suppose that condition 3 holds <u>and</u> ultimate depletion occurs under both regimes:

$$\text{if } g(Q'(0)) \begin{Bmatrix} > \\ < \end{Bmatrix} 0, \text{ then } Q'(0) \begin{Bmatrix} < \\ > \end{Bmatrix} Q(0)$$

$$\text{and } P(Q'(0)) \begin{Bmatrix} > \\ < \end{Bmatrix} P(Q(0)).$$

d) Suppose that $g(Q'(0)) = 0$;

$$\text{if} \quad g(Q'(t)) \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \quad \text{for all} \quad t > 0, \quad \text{then} \quad Q'(0)) \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} Q(0)$$

$$\text{and} \quad P(Q'(0)) \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} P(Q(0)).$$

Theorem 1 is summarized in Table 1, which shows the signs of $Q'(0) - Q(0)$ for different assumptions about $g(0)$, about the growth rate of g, and about whether all resources are ultimately depleted.

Theorem 1 is applicable for arbitrary choice of time origin. In particular, $t = 0$ can always refer to the current time. Thus, the theorem has more generality than may be apparent at first. It provides sufficient conditions to determine whether the current market extraction rate from the given current resource stocks will be greater (or smaller) under the alternative regime than under the competitive regime. Of course, the resource stocks remaining at any time influence the extraction rates at that time and these stocks are determined by past extraction decisions. Hence, the theorem does not allow us to predict the relationship between $Q'(t)$ and $Q(t)$ for all future t for an exogenously determined resource stock at $t = 0$, but an endogenously determined stock at future times. However, it follows immediately that, in the case of ultimate depletion, $Q'(0) > Q(0)$ implies that the reverse inequality must hold at some future time.

Table 1

Signs of Q'(0) - Q(0) [and of P(Q(0)) - P(Q'(0))] for different values of the market imperfection function, g(0), and of the growth rate of g. Columns labeled ult. dep. apply to the situation in which all reserves are ultimately depleted; columns labeled no ult. dep. apply to situations in which at least one reserve is not ultimately depleted.

	$g(0) > 0$	$g(0) > 0$	$g(0) < 0$	$g(0) < 0$
	Ult. Dep.	No Ult. Dep.	Ult. Dep.	No Ult. Dep.
$\frac{\dot{g}}{g} < r$	+	+	-	-
$\frac{\dot{g}}{g} = r$	0	+	0	-
$\frac{\dot{g}}{g} > r$	-	?	+	?

For example, assume that condition 1 (normal change) holds for depletion allowances. Then it can be shown that Theorem 1 implies that for any given resource stocks, the market depletion rate will be higher under a depletion allowance regime than under a competitive regime. For any time after the initial point the resource stocks which actually exist will always be lower under a depletion allowance regime than under a competitive regime, since actual stocks equal original stocks minus total quantities extracted. The lower are actual stocks at a given time, the lower will be the extraction rates. At early times, the first phenomenon will dominate; depletion allowances will lead to greater quantities of the resource supplied. At later times, the second phenomenon will dominate; a decrease in actual reserves will lead to lower quantities of the resource extracted under the depletion allowance regime. Thus, depletion allowances will lead to over-use of resources in initial years at the cost of eventual lower rates of extraction in later years. This pattern is illustrated in Figure 3.

In using Theorem 1 it is necessary to know the sign of g and its (relative) magnitude over time <u>given</u> the extraction pattern which <u>actually occurs</u> under the alternative regime. The theorem is also valid when stated in terms of the extraction pattern which <u>actually occurs</u> under the competitive regime. Yet for many policy purposes we would like to evalu-

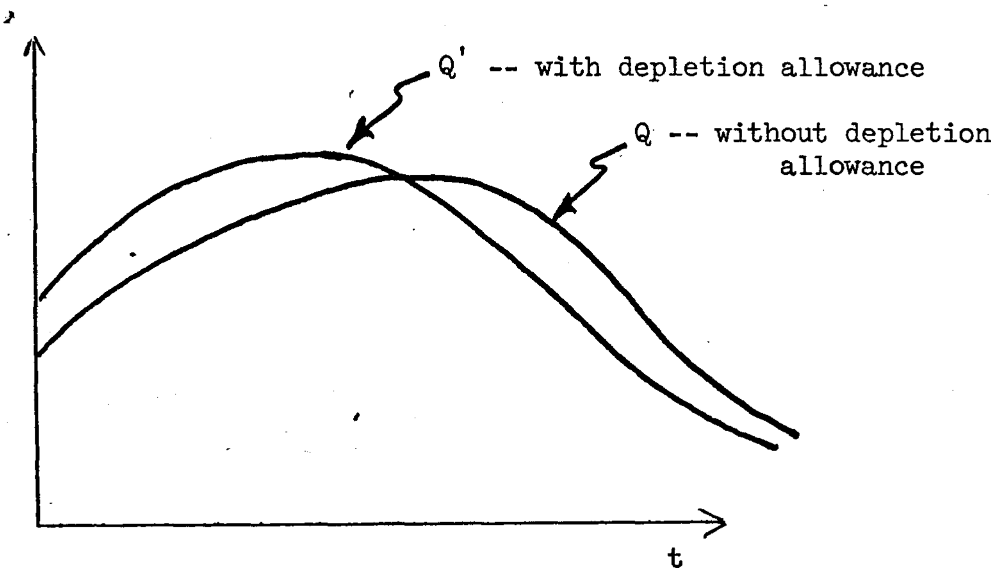


Figure 3

Depletion Paths with and without a Percentage Depletion Allowance ate market biases without explicitly solving for the actual trajectory which would occur either with or without the given influence. For this latter purpose, the following Lemma is stated without proof.

Lemma 1. Suppose that no cost functions are increasing over time, that there are positive extraction costs, and that the following conditions hold for all Q,t:

$$\frac{\partial}{\partial Q}[g(Q,t) + P(Q,t)] < 0 , \tag{18}$$

$$\frac{\partial}{\partial t}[g(Q,t) + P(Q,t)] \leq [g(Q,t) + P(Q,t)]\, r(t). \tag{19}$$

Then

$$[g(Q'(t),t) + P(Q'(t),t)]\, D(t) < [g(Q'(0),0) + P(Q'(0),0)] \quad \text{for all } t > 0. \tag{20}$$

Given Theorem 1 and Lemma 1, the following corollaries are readily established.

Corollary 1. Depletion Allowance. Suppose that the depletion allowance regime can be characterized as an alternative regime with $g(Q',t)$ determined by Eq. (12).

(a) If extraction is costless, and all resources are ultimately depleted, then the depletion allowance does not bias resource allocation: $Q'(0) = Q(0)$ and $P(Q'(0)) = P(Q(0))$.

(b) If extraction is costly, if extraction cost functions do not increase over time, and if

$$\frac{\partial P(Q)}{\partial t} \leq r\, P(Q) \quad \text{for all } Q\,,\, t\,,$$

Then whenever $Q(0) > 0$, the depletion allowance regime leads to more rapid initial extraction from given reserves than does the competitive regime: $Q'(0) > Q(0)$ and $P(Q'(0)) < P(Q(0))$.

Proof:

Part (a). By Eq. (12), $g(Q') = \alpha\, P(Q')$. In market equilibrium, prices rise at the rate of interest and therefore the market imperfection also does. Condition 2 (exponential change) is satisfied. By Theorem 1 the result follows:

Part (b). Since $\frac{\partial P(Q)}{\partial t} \leq r\, P(Q)$, and since $P(Q') + g(Q') = (1 + \alpha)\, P(Q')$, the premises of Lemma 1 are satisfied; inequality (20) must follow. Therefore, condition 1 (normal change) is satisfied. By Theorem 1, since $g(Q'(0)) > 0$, it follows immediately that $Q'(0) > Q(0)$.

■

Thus, over a broad range of demand functions, depletion allowances will bias markets in favor of current extraction at the expense of future availability of the resource and will reduce current prices. The limiting case is that of zero extraction costs; in this situation no bias occurs.

For a monopoly, the precise results will depend on the shape of the demand function and the growth rate of that function.

However, independent of the precise demand function, if all resources are depleted in finite time, a monopoly will not reach ultimate depletion as quickly as would a competitive system. Let τ^* be the ultimate depletion date under the competitive system. Then τ^* can be defined as follows:

$$\tau^* = \inf \left\{ \tau \;\middle|\; \int_0^{\tau} s^i(t)\,dt = S_0^i \quad \text{for all} \quad i \right\} . \tag{21}$$

It will be shown that under a monopolistic regime resources will not be ultimately depleted by time τ^*.

Corollary 2. Monopoly. Suppose that the monopolistic regime is described by Problem (M) and suppose that $Q'(0) > 0$.

(a) Assume that under a competitive regime all resources are ultimately depleted at time τ^*. If the demand price function is continuous in t and Q at $Q = 0$, then under a monopolistic regime, resources will not be ultimately depeleted by time τ^*.

(b) Assume that $\frac{\partial}{\partial Q}$ $MR(Q,t) < 0$,

$$\text{and} \quad \frac{\partial}{\partial t} MR(Q,t) \leq r\, MR(Q,t) .$$

Then whenever the magnitude of the demand elasticity is non-decreasing over time, the monopoly will lead to an extraction rate from a given stock of resources which is no faster than would occur with a competitive regime. Furthermore, if there are positive, non time increasing cost functions, then the monopoly regime will lead to a slower extraction rate from a given stock of resources than will the competitive regime: $Q'(0) < Q(0)$ and $P(Q'(0)) > P(Q(0))$.

Proof:

Part (a): By definition, at the instant of time before τ^*, $s^i > 0$ for some i, say for $i = j$. Therefore, by the continuity assumptions for $i = j$, the left-hand side of Eq. (10) must equal zero at time τ^*. Thus, for $i = j$ at $t = \tau^*$, Eqs. (10) and (11) can be written:

$$P(Q) - MC^j(s^j) - \lambda^j/D(\tau^*) = 0 \ ,$$

$$MR(Q') - MC^j(s^{j'}) - \lambda^{j'}/D(\tau^*) \leq 0.$$

The continuity of $P(Q,t)$ and $MC^i(s^i)$ implies that $Q(\tau^*) = 0$. Furthermore, if part (a) were not valid, then $Q'(\tau^*)$ would also equal 0. Since $P(Q,t)$ is continuous in Q at $Q = 0$, $MR(0,t) = P(0,t)$. Assuming then that part (a) is not valid, these two equations become at $t = \tau^*$:

$$P(0) - MC^j(0) - \lambda^j D(\tau^*) = 0 \ ,$$

$$P(0) - MC^j(0) - \lambda^{j'} D(\tau^*) \leq 0 \ .$$

These equations imply that $\lambda^{j'} \geq \lambda^j$. However, a proof similar to that of Theorem 1 implies that $\lambda^{i'} < \lambda^i$ for all i. Hence a contradiction; the corollary is established. ∎

<u>Part (b)</u>: Under the premises of part (b), Lemma 1 establishes that

$$MR(Q',t)\ D(t) \leq MR(Q'(0),0) \quad \text{for all } t > 0.$$

or that

$$\frac{\dot{MR}}{MR} = \frac{\dot{P}(1 - 1/\varepsilon) + P\dot{\varepsilon}/\varepsilon^2}{(1 - 1/\varepsilon)P} \leq r ,$$

where ε is the absolute value of the elasticity of demand. Now the ratio $\dot{g}/g$ can be shown to be less than r. By the definition of g (Eq. (14)):

$$\frac{\dot{g}}{g} = \frac{-\dot{P}/\varepsilon + P\dot{\varepsilon}/\varepsilon^2}{-P/\varepsilon} = \frac{\dot{P}}{P} - \frac{\dot{\varepsilon}}{\varepsilon}$$

Using the condition that $\dot{MR}$ grows less than a rate r, we obtain:

$$\frac{\dot{g}}{g} \leq r + \dot{\varepsilon} \quad [\frac{1}{1-\varepsilon}]$$

with strict inequalities holding for positive extraction costs. Now since ε is greater than unity, $\frac{\dot{g}}{g} \leq r$ whenever $\dot{\varepsilon} \geq 0$, with a strict inequality whenever extraction costs are positive. By Theorem 1, part (6) of the corollary is established, since $g(Q',0) < 0$ and $\frac{\dot{g}}{g} < r$.

∎

Several special cases of monopoly biases can be easily examined by means of Corollary 2. Two cases have been partially explored by Weinstein and Zeckhauser who use a zero-extraction cost assumption. This zero-cost assumption will be discarded here although the zero-cost case can be readily examined. First consider a <u>constant elasticity</u> demand curve:

(22) $$P(t) = \beta(t)\, Q^{-1/\varepsilon},$$

Where ε is the elasticity of demand (assumed to be greater than unity). Under this case, we can write $g(Q',t)$ as follows:

$$g(Q',t) = MR(Q') - P(Q') = \frac{-P(Q'(t),t)}{\varepsilon} < 0.$$

Now as long as $\frac{\dot{\beta}(t)}{\beta(t)} \leq r$, it follows that $\frac{\dot{g}}{g} < r$, since $\dot{\varepsilon} = 0$.

By Theorem 1 (or Corrollary 2), it follows that a monopolist facing a constant elasticity demand curve will supply resources at a lower rate from given reserves than will a competitive industry facing the same demand curves. In the special case of zero extraction costs, the price will rise precisely at the interest rate and monopoly will not lead intertemporal bias.

A second special case is of linear demand curves:

(23) $$P(t) = C(t) - \frac{Q(t)}{H(t)}, \text{ for } P,Q \geq 0.$$

In this case:

(24) $$g(Q',t) = -\frac{Q'(t)}{H(t)} = P - C(t) \leq 0.$$

Condition 1 holds as long as

(25) $$\frac{\dot{Q}'(t)}{Q'(t)} < \frac{\dot{H}(t)}{H(t)} + r.$$

Hence during those times which condition (25) holds, a monopolist facing a linear demand curve will over-conserve depletable resources. Conversely, during those times which the inequality in (25) is reversed, the monopolist will over-supply resources. In summary, unless the market determined quantity is rising very rapidly during some period of time, a monopoly (facing a linear demand curve) will under-supply the depletable resource.

Theorem 1 can also be used to examine the biases associated with not internalizing externalities associated with resource use. These results will depend critically upon the growth rate of the marginal external cost function (the pollution price):

Corollary 3: Assume that a regime of non-internalized externalities can be described as an alternative regime for which Eq. (15) is valid. Assume further that $Q'(0) > 0$ or $Q(0) > 0$.

(a) If the marginal non-internalized pollution cost never grows at a rate greater than or equal to $r(t)$, then non-internalized externalities lead to a depletion rate higher than that occurring if the pollution costs are internalized: $Q'(0) > Q(0)$, $P(Q'(0)) < P(Q(0))$.

(b) Suppose that all resources are ultimately depleted. If the marginal non-internalized pollution cost grows at a rate equal to $r(t)$ at all times before eventual depletion of the resource, then the depletion paths occurring under a regime of non-internalized externalities are identical to the paths occurring under a competitive regime: $Q'(t) = Q(t)$, $P(Q'(0)) = P(Q(0))$.

(c) Suppose that all resources are ultimately depleted. If the marginal non-internalized pollution cost grows at a rate greater than $r(t)$ at all time before ultimate depletion of the resource, then non-internalized externalities lead to a depletion rate lower than that occurring if the pollution costs are internalized: $Q'(0) < Q(0)$, $P(Q'(0)) > P(Q(0))$.

Proof:

The proof is immediate from Theorem 1 and Eq. (15). ■

While the comparative dynamics of the system depend upon the rate of growth of the pollution price, case (a) can be presumed to hold unless contrary evidence is established. Hence, generally non-internalized externalities lead to over-use of resources and too-low resource prices.

Finally, price control in the natural resource industry can lead to too rapid depletion or too slow depletion depending upon the relationship between the regulated price and the market clearing price.

Corollary 4: Assume that the price control regime can be described as an alternative regime for which Eq. (16) is valid and assume that $Q(0) > 0$ or $Q'(0) > 0$. Let $EP = P(Q'(t),t) - \overline{P}(t) \geq 0$.

(a) If $\dot{EP} < r\, EP$ for all time, then price regulation will lead to a slower rate of depletion than will occur under a competitive regime: $Q'(0) < Q(0)$.

(b) Suppose that all resources are ultimately depleted under the price control regime. If $\dot{EP} = r\, EP$ for all time, then price regulation will not change the intertemporal pattern of depletion: $Q'(0) = Q(0)$.

(c) Suppose that all resources are ultimately depleted under the price control regime. If $\dot{EP} > r\ EP$ for all time, then price regulation will lead to a faster rate of depletion than will occur under a competitive regime: $Q'(0) > Q(0)$.

Proof:

This corollary follows immediately from Theorem 1 and Eq. (16).

∎

This corollary shows that the effects of price control (such as the well-head regulation of natural gas) depend upon the changes of the "excess price" over time. Since the behavior of this variable is not obvious without empirical work, we cannot ascertain a priori the impacts of well-head regulation.

A final example is that of vulnerability costs associated with imports of vital products such as crude oil. Assume that the economy can extract the resource from many domestic locations but also can import any quantity from foreign locations at an exogenously determined price, $P_I(t)$, which may vary over time. Assume further that a vulnerability cost, $V(I,t)$, where I is the level of imports, is imposed on the domestic economy. The vulnerability cost is presumed to be an increasing function of the rate of imports. The question is how depletion patterns from domestic reserves and consumption patterns are biased from the optional patterns if the appropriate tariffs are not instituted.

This problem is a special case of the more general theory presented in this paper. For now the price path is exogenously determined and it is not necessary to examine the feedback from quantity decisions to market prices. The vulnerability cost is now simply an externality which varies over time; as such, economic efficiency requires that a tax, $T(t)$, be imposed on the activity producing the externality, that is, on imports of crude oil. That tax must be equal to the marginal vulnerability costs of additional imports:

$$(26) \qquad T(t) \doteq \frac{\partial}{\partial I} V(I,t) = > 0.$$

The efficient price path for crude oil will equal $P_I(t) + T(t)$, which is always higher than $P_I(t)$. Unless the appropriate tax is imposed, the consumption of the resource will be too high at each moment of time.

The biases on domestic supply can be examined easily. The market imperfection function for this problem equals the negative of the marginal vulnerability cost and is thus always negative. That is, <u>failure</u> to impose the tax leads to lower prices than are optimal. The intertemporal bias to domestic supply patterns depends upon the rate of growth of the marginal vulnerability cost. If the marginal vulnerability cost uses more slowly than r, then efficiency requires an accelerated rate of extraction in early years, with less savings for later years. However, if the marginal vulnerability cost rises faster than r, then the converse holds. In this case, moving toward efficiency requires a decreased supply in early years with greater reserves saved for later years. Such a case could occur if vulnerability costs were a sharply increasing function of import levels and if imports were rapidly increasing over time.

In summary, if marginal vulnerability costs increase more slowly than r (or decrease), then failure to impose taxes equal to the marginal vulnerability costs leads to too much consumption at all times, too little supply in early years, too much savings for later years, and too much imports in early years. Conversely, if marginal vulnerability costs increase more rapidly than r, failure to impose taxes leads to too much consumption at all times, too much supply in early years, too much imports in later years. The impact on imports in early years depends on the relative magnitude of the consumption effect and the supply affect; imports in early years may either be too high or too low.

III. Summary and Conclusions

This paper has compared optimal and market-determined extraction paths for a depletable resource which can be extracted from a set of reserves of varying extraction costs and magnitudes. The total quantity of the resource extracted from each reserve over time is limited by the magnitude of the original reserve. Under the appropriate convexity conditions, the optimal allocation can be supported by a purely competitive market.

While competitive markets may lead to optimal allocations, many market phenomena bias the temporal pattern of extraction from optimality. Percentage depletion allowances lead to an over-extraction of resources at the present time at the expense of future feasible extraction rates. As a corollary, past extraction patterns under a depletion allowance regime have led to over-extraction in the past at the expense of reserves currently remaining. Non-internalized externalities associated with the extraction or use of depletable resources probably bias markets toward current over-extraction at the expense of future extraction alternatives. Price controls may lead to temporal biases, but the direction of bias cannot be predicted without additional data. Depending upon the rate of change of the "excess price" -- the difference between market clearing price and controlled price -- price controls may lead to current over-use, current under-use, or may have no influence on extraction patterns. Monopolies may lead to a more complicated pattern of bias. Generally, monopolies will lengthen the time until resources are ultimately depleted. Depending upon the shape of the demand function, however, monopolies may lead to current under-use to the benefit of future availabilities, or may lead to a current over-use, and under-use at some future time, with a subsequent increased availability in the latest years. Finally, unless the appropriate tariffs are imposed, vulnerability costs of importing vital products from insecure sources may bias extraction patterns from domestic reserves and will lead to over consumption of the product. The direction of intertemporal bias depends upon the rate of growth of the marginal vulnerability costs.

It has been shown that each result cited above can be derived from one general theory which uses the concept of a market imperfection function. Each institution discussed has associated with it a market imperfection function whose properties can be analyzed. We can determine the directions of intertemporal bias by examining simple properties of the associated market imperfection function, without explicitly solving for the equilibria under the various market forms. Thus this paper provides a very general criterion which can be used to examine intertemporal biases stemming from a wide range of institutions.

A few caveats are in order. This paper ignores the exploratory processes for resources, concentrating only upon the intertemporal allocation of known reserves. All analysis is performed under conditions of certainty -- all actors are aware of the resource quantities under their control and the costs of extracting those resources, all actors can predict the price trajectories for the entire future, all actors are aware of future governmental actions. Clearly, however, the certainty assumption is rather untenable when we consider the time span relevant to the depletion of most natural resources. Nor can we even rely upon information processed by futures markets, since such markets are non-existent or cover a too-limited time horizon. When uncertainty is incorporated into the analysis, systematic differences between societal risk aversity and the risk aversity of resource owners may provide yet another bias of markets from optimality.[8]

The predominant pattern of biases in resource markets seems to be one of current over-use at the expense of future availability. Monopoly (and possibly price controls) may counteract these forces, to the extent that monopolistic practices do in fact occur in resource industries. If the biases predicted by this theory do in fact exist, simple reliance on markets as they currently are constituted to optimally allocate depletable resources over time seems unwise. However, the analysis of this paper does suggest specific biases and thereby provides tools for analysis of policy issues. Thus, while it is difficult to justify dependence on existing market forces, reforms such as a repeal of the depletion allowance, a complete internalization of externalities, or tariffs on vulnerable imports, could improve the allocation of depletable resources occurring under the market system.

APPENDIX

Proof of Theorem 1

Proof: We can subtract (10) from (11) to obtain the following inequality valid for all i:

$$[g(Q') + P(Q') - P(Q)]D(t) - \Delta\lambda^i - \gamma^i(\Delta s^i) \begin{cases} \geq 0 & \text{for } s^{i'} > 0, \\ \leq 0 & \text{for } s^i > 0, \end{cases}$$

where $\gamma^i(\Delta s^i) = [MC^i(S^{i'}) - MC^i(s^i)]D(t)$ and $\Delta\lambda^i = \lambda^{i'} - \lambda^i$. It should be noted that

$$\text{sgn}\ \delta^i(\Delta s^i) = \text{sgn}\left\{ s^i - s^i \right\} \text{, if strict convexity}$$

holds. $\delta^i(\Delta s^i) = 0$ if marginal cost is independent of s^i.

This equation can be rewritten. Define $h(t)$ as

$$\text{(A.1)} \qquad h(t) = [g(Q') + P(Q') - P(Q)]D(t).$$

Then the earlier equation becomes:

$$\text{(A.2)} \qquad h(t) - \Delta\lambda^i - \gamma^i(\Delta s^i) \begin{cases} \geq 0 & \text{for } s^{i'} > 0, \\ \leq 0 & \text{for } s^i > 0, \end{cases}$$

It can now be shown that for all i, $\Delta\lambda^i$ is limited:

$$\text{(A.3)} \qquad \Delta\lambda^i \leq \begin{cases} h(t), \text{ for some } t, \text{ if } \int_0^\infty s^i(t)dt = S_0^i \\ 0 \quad \text{otherwise} \end{cases}$$

$$\text{(A.4)} \qquad \Delta\lambda^i \geq \begin{cases} h(t), \text{ for some } t, \text{ if } \int_0^\infty s^i(t)dt = s_0^i \\ 0 \qquad \text{otherwise.} \end{cases}$$

If $\underset{t}{\text{Min}}\, h(t)$ and $\underset{t}{\text{Max}}\, h(t)$ exist, then for inequality (i) binding, Eq. (20) and (21) imply that $\underset{t}{\text{Min}}\, h(t) \leq \Delta\lambda^i \leq \underset{t}{\text{Max}}\, h(t)$.

Assume that $\Delta\lambda^i > h(t)$ for all t. Then, by Equation (A.2), $s^{i'} \leq s^i$ for all t and $s^{i'} < s^i$ whenever $s^i > 0$. Therefore, inequality (1) will not be binding for the alternative regime. Thus, if inequality (1) is binding, it follows that $\Delta\lambda^i \leq h(t)$ for some t. The first part of inequality (20) is established. If constraint (1) is not binding under the alternative regime, then $\lambda^{i'} = 0$ and $\lambda^i \geq 0$. Hence $\Delta\lambda^i \leq 0$, and the second part of inequality (A.3) is established. Inequality (A.4) is established in an identical manner.

<u>Part (a)</u>: Assume now that $g(Q'(0)) > 0$ and that condition 1 holds. It will be shown that this implies that

$$\text{(A.5)} \qquad \Delta\lambda^i < g(Q'(0)), \text{ for all } i.$$

By inequality (A.3), if constraint (1) is not binding, inequality (A.5) holds trivially.

Let $\Delta\lambda^*$ equal the maximum of all $\Delta\lambda^i$ for which $\int_0^\infty s^{i'}(t)dt = s_0^i$, and let T^* be the set of all t for which $h(t) \geq \Delta\lambda^*$. By inequality (A.3), T^* is not the null set. Assume that $\Delta\lambda^* > 0$. If this inequality does not hold, then inequality (A.5) is established trivially. Let J be the set of all i for which $\Delta\lambda^i = \Delta\lambda^*$ and for which $\int_0^\infty s^{i'}(t)dt = s_0^i$.

Now for $i \notin J$ it follows from inequality (A.2) that $\Delta s^{i'}(t) \geq 0$ for all $t \in T^*$. It can thus be shown that $Q'(t^*) \geq Q(t^*)$ for some $t^* \in T^*$.

Assume that contrary, that $Q'(t) < Q(t)$ for all $t \varepsilon T^*$. Then it must follow that $\sum_{i\varepsilon J} \Delta s^i(t) < 0$ for all $t \varepsilon T^*$. Whenever $t \not\varepsilon T^*$, from inequality (A.2) it follows that

$$\sum_{i\varepsilon J} \Delta s^i(t) \leq 0. \text{ Therefore}$$

$\int_0^\infty \sum_{i\varepsilon J} \Delta s^i(t)dt < 0$, inequality (1) must not be binding for some $i\varepsilon J$, and $\Delta\lambda^i \leq 0$ for all $i \varepsilon J$. This contradiction establishes that $Q'(t^*) \geq Q(t^*)$ for some $t^* \varepsilon T^*$.

It follows directly from the avove that $P(Q'(t^*)) \leq P(Q\ (t^*))$, for some $t^* \varepsilon T^*$. Furthermore the above demonstration can be trivally extended to show that if $Q'(t) < Q(t)$ for some $t \varepsilon T^*$, then $Q'(t) > Q(t)$ for some $t^* \varepsilon T^*$. Hence either $P(Q'(t^*)) < P(Q(t^*))$ for some $t^* \varepsilon T^*$ or $P(Q'(t)) \leq P(Q(t))$ for all $t \varepsilon T^*$. Hence $h(t^*) \leq g(Q'(t^*))D(t^*)$, and by condition 1 we obtain for all i,

$$\Delta\lambda^i \leq h(t^*) \leq g(Q'(t^*))D(t^*) \leq g(Q'(0)),$$

with the last inequality becoming a strict inequality if $P(Q'(t^*)) < P(Q(t^*))$ or if $P(Q'(t^*)) \leq P(Q(t^*))$ for some $t^* > 0$. Hence, unless $t^* = 0$ is the time which uniquely maximizes $h(t)$, inequality (A.5) must be satisfied. If $h(t)$ is uniquely maximized at $t = 0$, then it is sufficient to show that $\Delta\lambda^i \neq h(0)$. Assume the converse, that $\Delta\lambda^i = h(0)$ and that $\Delta\lambda^i > h(t)$ for all $t > 0$. Then for $t > 0$, $\Delta s^i(t) \leq 0$ and $\Delta s^i(t) < 0$ if $s^{i'}(t) > 0$. At $t = 0$, if $\Delta s^i(0)$ is infinitely large then $P(Q'(0)) < P(Q(0))$, and inequality (A.5) is established. If $\Delta s^i(0)$ is not infinitely large, then $\int_0^\infty \Delta s^i(t)dt$ is strictly negative and constraint (1) would not be binding under the alternative regime and $\Delta\lambda^i \leq 0$, a contradiction. Thus, inequality (A.5) is established.

Finally, it will be shown that if $Q(0) > 0$, then $Q'(0) > Q(0)$. Assume the converse, that $Q'(0) \leq Q(0)$; then $h(0) \geq g(0) > \Delta\lambda^i$ for all i. By Eq. (A.2), it follows that $\Delta s^i(0) \geq 0$ and $\Delta s^i(0) > 0$ if $s^i(0) > 0$. Thus a contradiction is established: $Q'(0) > Q(0)$ and $P(Q'(0)) < P(Q(0))$. A similar proof holds for $g(Q'(0)) < 0$.

Parts (b) through (d):

All other parts of the theorem are proved in a similar manner. Under Part b(i) assumptions, $\Delta\lambda^i = g(Q'(0))$ for all i. Under Part b(ii), $\Delta\lambda^i < g(Q'(0))$ if g is positive; the reverse inequality holds for g negative. For Part c, $\Delta\lambda^i > g(Q'(0))$ if g is positive, with the reverse inequality holding for g negative. Note that if ultimate depletion did not occur, then these inequalities could not be established [see Eqs. (A.3) and (A.4)]. In Part d, each $\Delta\lambda^i$ is positive, negative, or zero for $g(Q'(t))$ positive, negative, or zero respectively. Thus, the remaining parts of the theorem are proven in a manner similar to Part a.

■

FOOTNOTES

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1. This formulation implies an assumption that the total quantity which can ultimately be recovered is independent of the extraction rates at each time. This assumption can be relaxed very easily without influencing any results in the subsequent sections of this paper. Such a relaxation would help to explain the concept of the maximal efficient rate of extraction of a resource.

2. In a more general formulation cost may also depend upon the resource quantity not yet extracted: $S_0 - \int_0^t s(\tau)d\tau$. Such a formulation will not change any results of sections II and III. However, whether the results of section IV will be changed is an open question, although I suspect that they will not.

3. Note that if $r(\tau) = r$, which is independent of time, then $e^{-\int_0^t r(\tau)d\tau}$ simplifies to the more familiar e^{-rt}.

4. Thus, marginal extraction costs of the resource from two different reserves (evaluated at the optimal extraction rates) need not be equal to one another if both reserves are operated so as to maximize profit. This is true even if both reserves are characterized by the same cost functions (and the same price trajectories), as long as they contain different initial quantities of the resource. This result is in direct

contrast to Gordon's [5, pp. 282-283] assumption that marginal extraction costs must be the same for all firms. That assumption is used heavily in arriving at his conclusion that competitive markets will not optimally deplete resources.

5. A dot above any variable will indicate the time rate of change of that variable, e.g., $\dot{P} = \frac{dP}{dt}$.

6. This allowance is limited to 50% of net income. However, it will be assumed that this limit is not binding.

7. If $\varepsilon < 1$ then no optimal depletion pattern will exist for the monopolist since a reduction of quantity towards zero can always increase revenue and decrease costs.

8. As has been suggested by Vickrey [15] and by Hayne Leland (private communication).

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