132
8/29/79

LA-7955-MS

Informal Report

MASTER

Dr. 3077

Alternate Interpretation of the Subgrid Scale Eddy Viscosity

University of California

LOS ALAMOS SCIENTIFIC LABORATORY
Post Office Box 1663 Los Alamos, New Mexico 87545

This report was not edited by the Technical Information staff.

This report is based on work performed at Applied Theory, Inc., and Aerojet Nuclear Company. It is essentially identical to an Aerojet Nuclear Company internal memorandum (JDR-9-74) dated 19 March 1974.

UNITED STATES
DEPARTMENT OF ENERGY
CONTRACT W-7405-ENG. 36

LA-7955-MS
Informal Report
UC-34
Issued: August 1979

Alternate Interpretation of the Subgrid Scale Eddy Viscosity

John D. Ramshaw

ALTERNATE INTERPRETATION OF THE SUBGRID SCALE EDDY VISCOSITY

by

John D. Ramshaw

ABSTRACT

It is shown that the subgrid scale (SGS) eddy viscosity for numerical calculations of turbulent flow has the effect of artificially enlarging the Kolmogorov microscale until it becomes comparable to the finite-difference mesh spacing. The SGS eddy viscosity is therefore closely analogous to the von Neumann-Richtmyer artificial viscosity for shock waves.

Deardorff[1-3] and others (see references cited by Deardorff) have discussed a subgrid scale (SGS) eddy viscosity method for the numerical simulation of turbulent flows. The SGS eddy viscosity is introduced to represent the Reynolds stresses which arise due to spatial averaging over the cells of a finite-difference mesh. We wish to point out that the SGS eddy viscosity may alternatively be regarded as an artificial viscosity closely analogous to that introduced by von Neumann and Richtmyer[4] for the numerical treatment of shock waves. This interpretation lends useful insight into the SGS eddy viscosity method and the conditions for its validity.

In both turbulent flows and shock waves there exist physical length scales too small to be represented in practice in a finite-difference calculation. For shock waves, von Neumann and Richtmyer dealt with this problem by introducing an artificial viscosity designed to increase the thickness of a shock of arbitrary strength to a length of the order of the mesh spacing Δx. We show below that the same approach is reasonable for turbulence calculations; an artificial viscosity can readily be defined by requiring that the smallest turbulent eddies be enlarged to a size of order Δx. The resulting artificial viscosity has exactly the form of the SGS eddy viscosity.

In the inertial subrange, the velocity gradient characteristic of an eddy of size λ is of order $\varepsilon^{1/3}\lambda^{-2/3}$, where ε is the energy dissipated per unit time per unit mass.[5] The velocity gradient increases with decreasing λ, and ultimately becomes so large that molecular viscosity can no longer be neglected. This happens when $\lambda \sim \eta$, where $\eta = (\nu^3/\varepsilon)^{1/4}$ is the Kolmogorov microscale[6] (ν is the kinematic viscosity). η is the size of the smallest eddies in the flow, in which the viscous dissipation takes place. The velocity gradient $\partial u_i/\partial x_j$ in the smallest eddies must be of the magnitude necessary to dissipate energy at the rate ε: $2\nu S_{ij}S_{ij} \sim \varepsilon = \nu^3/\eta^4$, where $S_{ij} = (1/2)(\partial u_i/\partial x_j + \partial u_j/\partial x_i)$. In order to ensure that η is of order Δx, we artificially set $\eta = C\,\Delta x$ (where C is a dimensionless constant of order unity) and thereby obtain an artificial viscosity ν^* defined by

$$\nu^* = (C\Delta x)^2\ (2S_{ij}S_{ij})^{1/2} \quad . \tag{1}$$

The artificial viscosity ν^* arrived at in this manner is seen to be identical in form to the SGS eddy viscosity of Deardorff. However, the interpretation is different: conventionally ν^* is regarded as representing the effect of turbulent motions occurring on length scales smaller than Δx, while the present viewpoint regards ν^* as enlarging the smallest eddies until they are large enough ($\sim\Delta x$) to be resolved by the finite-difference mesh.

The interpretation of ν^* as an artificial viscosity provides an alternative "theoretical" basis for evaluating the constant C. It is known[6] that viscous dissipation is a maximum at wavenumber $k_m \cong 0.2/\eta$, or wavelength $L_m \cong 31.4\ \eta$. The smallest wavelength which can be represented in the mesh is $2\Delta x$; if this is equated to L_m, the value $C = 0.064$ results. Larger values of C correspond to spreading L_m over a larger number of zones. Indeed, there will always be <u>some</u> viscous dissipation at wavelengths less than L_m; in order to represent these shorter wavelengths in the mesh, it is necessary to use a value of C somewhat greater than 0.064. This consideration may partially explain why values of C between 0.10 and 0.20 have given the best results in practice.[1-3]

The rate of energy dissipation in a turbulent flow is primarily determined by the (essentially inviscid) large-scale motions. The rate of energy dissipation in the smallest eddies is ν^3/η^4, which must continually adjust itself to the rate at which energy is supplied by the larger eddies. In a real turbulent flow, ν is constant and this adjustment occurs through a variation in the eddy size η. The use of ν^* reverses the situation by keeping η constant at the value $C\Delta x$ while ν^* varies as required to maintain the relation $\varepsilon = (\nu^*)^3/\eta^4$.

The use of ν^* is clearly legitimate only if it has a negligible effect on the structure of the larger eddies. Consider a large-scale motion with length scale $\ell >> \Delta x$ and (relative) velocity scale u. Then $\nu^* \sim (C\Delta x)^2 u/\ell$, so that $R^* \equiv u\ell/\nu^* \sim (\ell/C\Delta x)^2 >> 1$. In general, therefore, the large-scale motions remain essentially inviscid in the presence of ν^*. However, the actual value of R^* may still be of interest, since the effective Reynolds number of the calculation is

$$R_e = \left(\frac{1}{R} + \frac{1}{R^*}\right)^{-1} \quad , \tag{2}$$

where R is the true Reynolds number $u\ell/\nu$. (We neglect here the possibility that an additional "numerical viscosity" is inherent in the difference scheme, an effect which must be carefully considered in practice.[2]) If it happens that $R^* << R$ then $R_e \cong R^*$ so that R_e will also be $<< R$. This may or may not affect the calculation, depending on the values involved and the sensitivity of the particular problem to Reynolds number. If R^* is found to be too small, then it must be increased by refining the zoning; the minimum acceptable value of R^* may be regarded as providing a zoning constraint via the relation $R^* \sim (\ell/C\Delta x)^2$. This constraint will not ordinarily be overly restrictive, as indicated by the fact that $R^* \sim 40{,}000$ for the typical values $C = 0.1$ and $\ell/\Delta x = 20$.

REFERENCES

1. J. W. Deardorff, J. Fluid Mech. 41, 453 (1970).
2. J. W. Deardorff, J. Comp. Phys. 7, 120 (1971).
3. J. W. Deardorff, J. Atmos. Sci. 29, 91 (1972).
4. J. von Neumann and R. D. Richtmyer, J. Appl. Phys. 21, 232 (1950).
5. L. D. Landau and E. M. Lifshitz, Fluid Mechanics (Pergamon, London, 1959), p. 121.
6. J. O. Hinze, Turbulence (McGraw-Hill, New York, 1959), p. 184.

Printed in the United States of America. Available from
National Technical Information Service
US Department of Commerce
5285 Port Royal Road
Springfield, VA 22161

Microfiche $3.00

001-025	4.00	126-150	7.25	251-275	10.75	376-400	13.00	501-525	15.25
026-050	4.50	151-175	8.00	276-300	11.00	401-425	13.25	526-550	15.50
051-075	5.25	176-200	9.00	301-325	11.75	426-450	14.00	551-575	16.25
076-100	6.00	201-225	9.25	326-350	12.00	451-475	14.50	576-600	16.50
101-125	6.50	226-250	9.50	351-375	12.50	476-500	15.00	601-up	

Note: Add $2.50 for each additional 100-page increment from 601 pages up.