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# Hardness Correlation for Uranium and its Alloys

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## HARDNESS CORRELATION FOR URANIUM AND ITS ALLOYS

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### ABSTRACT

The hardness of 16 different Uranium-Titanium (U-Ti) alloys was measured on six (6) different hardness scales ( $R_A$ ,  $R_B$ ,  $R_C$ ,  $R_D$ , Knoop, and Vickers). The alloys contained between 0.75 and 2.0 weight percent Ti. All of the alloys were solutionized ( $850^{\circ}\text{C}$ , 1 hr.) and ice water quenched to produce a supersaturated martensitic phase. A range of hardnesses was obtained by aging the samples for various times and temperatures. The correlation of various hardness scales was shown to be virtually identical to the hardness-scale correlation for steels. For more-accurate conversion from one hardness scale to another, least-squares-curve fits were determined for the various hardness-scale correlations.

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## Table of Contents

- I. Abstract
- II. Introduction
- III. Procedure
- IV. Hardness Correlations
- V. Microstructural Correlations
- VI. Summary
- VII. References

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## Introduction<sup>1</sup>

This report is intended to assist laboratory and shop technicians in converting hardness values among different Rockwell, Knoop, and Vickers hardness scales for uranium and its alloys. Hardness, although not a fundamental material property, is a useful engineering measurement because it allows for a direct comparison between materials. The hardness may rather be a function of many actual material properties and the hardness test itself (including parameters such as load, indenter shape, penetrator velocity, and loading time). Many different tests are used to measure hardness, each with a different indenter geometry and load. Even the definition of hardness may vary from test to test. Some tests, such as the Brinell test, define hardness using the applied load and the area of the indentation. Others, such as the Rockwell test, define hardness using the depth of the indentation. Therefore, there is no reason to expect a theoretical conversion between the various hardness scales. However, empirical correlations have been observed between the various hardness scales for certain materials. These correlations are useful in engineering practice because they allow direct comparison of materials. Care must always be exercised when using these correlations. They are applicable only to the class of material for which they were determined. Even under ideal conditions, it must be understood that these correlations are not exact. Since there is not a standardized scale, different investigators will use different hardness scales, depending on specific circumstances.

Hardness correlation behavior for uranium and its alloys has not been systematically investigated. It was the objective of this study to measure hardness of many uranium alloys on several hardness scales [ $R_A$ ,  $R_B$ ,  $R_C$ ,  $R_D$ , Knoop ( $L = 300g$ ) and Vickers ( $L = 300g$ )] and determine the empirical correlation between the scales, employing standard hardness testing procedures.

### Procedure

A set of 16 different U-Ti alloys was used in this characterization study. The alloy content of the samples ranged from 0.75 to 2.0 weight percent Ti. Figure 1 shows the geometry of samples used. When slicing the samples from the parent rod, extreme care was taken to insure that the two flat surfaces were parallel to one another. This short, cylindrical shape produced enough surface area for reliable hardness determinations, but produced a specimen of sufficiently small thermal mass to allow rapid sample response to the temperature of the heat treating environment.

The alloys were heat treated using standard laboratory procedures. The alloys were solutionized in vacuum at 850°C for 1 hour and then quenched in ice water. At temperatures exceeding 800°C the Ti is fully soluble in the b.c.c.  $\gamma$  phase. Upon quenching, the diffusional decomposition of the  $\gamma$  phase is suppressed, and a Ti supersaturated  $\alpha'$  martensitic phase (orthorhombic) forms.<sup>2</sup> The samples were then aged in a molten salt bath at temperatures ranging from 310 to 480°C for times from 1.0 to 60.0 minutes. Table I lists the alloys used in this study and their aging times and temperatures. Upon aging, the alloys, through a classical age hardening reaction, show a marked increase in strength and hardness.<sup>2</sup> These alloys and aging treatments were chosen because they span the full range of hardnesses typically encountered in engineering uranium alloys.

Standard metallographic procedures were used to examine the samples.<sup>3</sup> The alloys were polished through 0.06  $\mu\text{m}$   $\text{Al}_2\text{O}_3$  and then electrochemically etched in an unstirred 50/50 solution of  $\text{H}_3\text{PO}_4$  and  $\text{H}_2\text{O}$ . A potential of 5 V was used and the electrolyte was maintained at room temperature.

Figure 1

## SAMPLE DIMENSIONS

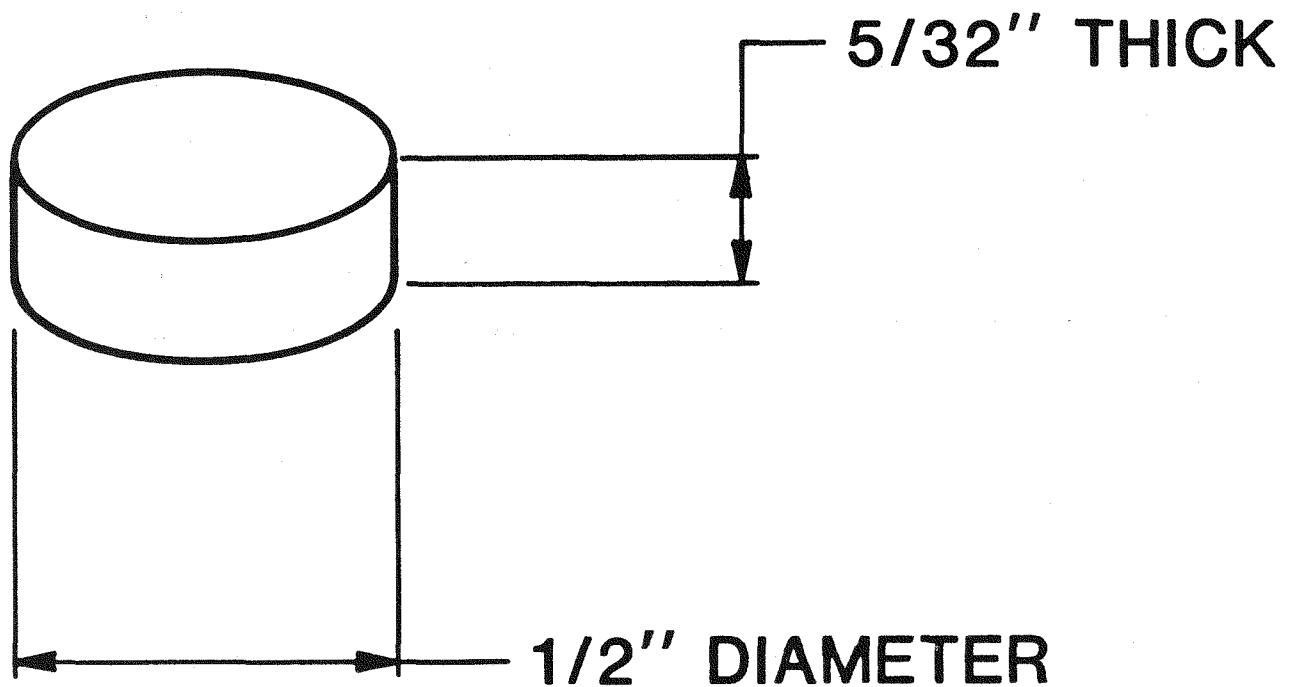


Table I  
**HEAT TREATMENT SCHEDULE**

SPECIMEN NUMBER	ALLOY	TEMPERATURE	TIME
1	U-2.0%Ti	310°C	60min.
2	U-1.5%Ti	370°C	2.5min.
3	U-2.0%Ti	370°C	24min.
4	U-1.0%Ti	385°C	1.0min.
5	U-1.0%Ti	385°C	38min.
6	U-2.0%Ti	385°C	60min.
7	U-0.75%Ti	400°C	1.5min.
8	U-1.5%Ti	400°C	24min.
9	U-1.5%Ti	420°C	10min.
10	U-2.0%Ti	420°C	60min.
11	U-1.5%Ti	420°C	60min.
12	U-1.0%Ti	420°C	60min.
13	U-0.75%Ti	430°C	10min.
14	U-0.75%Ti	445°C	10min.
15	U-1.5%Ti	455°C	38min.
16	U-1.0%Ti	480°C	1.5min.

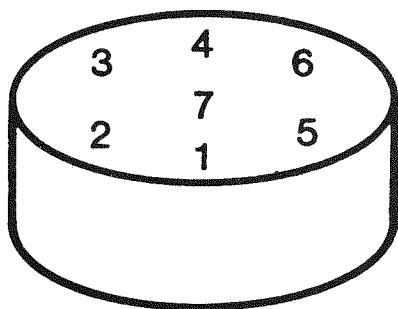
This potential is appropriate for the sample area encountered in most metallurgical samples mounted in standard 1 in. (2.54 cm) metallographic mounts.

Following the heat treatments, both flat sides of the short, cylindrical samples were polished through 600 grit SiC. Care was taken to insure that the two flat surfaces remained visibly parallel to each other. Seven (7) Rockwell hardnesses were measured on each Rockwell scale ( $R_A$ ,  $R_B$ ,  $R_C$ , and  $R_D$ ). The  $R_A$  measurements were taken, using standard hardness testing procedure,<sup>1</sup> on one side of the sample. The sample was then flipped and the seven  $R_B$  measurements were taken on the opposite side. The location of the hardness measurements are shown schematically in Figure 2. The sample flats were then reground, to a depth greater than ten times the prior indentation (several mils) to remove all damage caused by the hardness tests. The surfaces were then repolished through 600 grit SiC. Once again, care was taken to insure that the two flat surfaces remained parallel to each other. Following the same procedure, the seven  $R_C$  and  $R_D$  measurements were made on opposite sides of the samples. Taking hardness measurements on both sides is an often used practice at Sandia.

The samples were then mounted and polished through  $0.03 \mu\text{m}$   $\text{Al}_2\text{O}_3$  for the Knoop and Vickers hardness tests. Once again, seven (7) Vickers and seven (7) Knoop measurements were made. Since these tests leave smaller indentations, all 14 measurements could be made on the same sample surface. A 300 g load was used for all Knoop and Vickers hardness tests. The microhardness testing was done in accordance with the standards set forth by ANSI/ASTM E384-73<sup>4</sup>.

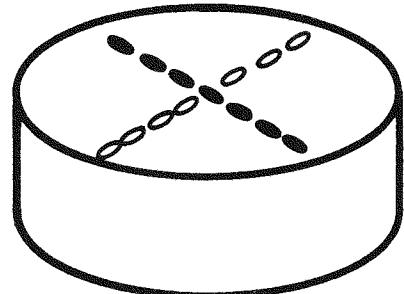
Figure 2

# LOCATION OF HARDNESS DATA



ROCKWELL  
A, B, C, & D

VICKERS  
& KNOOP  
( 300gm LOAD )



VICKERS SHOWN AS ●  
KNOOP SHOWN AS ○

### Hardness Correlations

Table II shows the compiled results of this study. The results for each alloy have been listed in order of increasing hardness. In each case, the mean of seven measurements is listed along with the error (one standard deviation,  $1\sigma$ ). To permit a more ready comparison of the results, each set of hardness data was plotted against each other set of data (i.e.,  $R_A$  vs.  $R_B$ ,  $R_C$ ,  $R_D$ , Vickers, Knoop;  $R_C$  vs.  $R_B$ ,  $R_C$ ,  $R_D$ , Vickers, Knoop;  $R_D$  vs.  $R_B$ , Vickers, Knoop;  $R_B$  vs. Vickers, Knoop; and Vickers vs. Knoop). These correlating plots are shown in Figures 3 through 17. In these plots,  $2\sigma$  error bars are shown for each datum. The hardness data were fitted to a series of least squares lines.<sup>5</sup> (See Appendix) The least squares lines are drawn on the plots given in Figures 3 through 17.

Table III summarizes hardness comparisons for steel<sup>1</sup> and shows that the correlations for steel and uranium are virtually identical. This fact is indeed borne out by the data, although there is no theoretical reason to expect this sort of behavior. Notice that the slopes and intercepts for the corresponding fits for steel and uranium are similar. In addition, all of the correlations between the various scales have small standard errors, indicating the fits are quite good. Considering these correlations, it would seem safe to say that for the hardness range studied, the Kehl "Table of Approximate Hardness Conversion Numbers for Steel"<sup>1</sup> could be used for uranium conversions for general purposes.

Where more accurate measurements are required, then the method of least squares, shown in the Appendix, should be used.

Table II  
HARDNESS VALUES FOR URANIUM-TITANIUM ALLOYS

SPECIMEN NO. & COMPOSITION	ROCKWELL				VICKERS 300g load	KNOOP 300g load
	A	B	C	D		
No. 7 U-0.75%Ti	68.3±0.3	110.2±0.5	34.7±0.8	52.9±0.6	330.7±13.8	358.0±23.2
No. 4 U-1.0%Ti	69.0±0.2	111.0±0.5	37.2±0.4	53.4±0.3	358.9±10.5	375.9±15.0
No. 13 U-0.75%Ti	70.1±0.4	112.8±0.4	39.5±0.5	54.8±0.3	382.4±14.1	406.6±25.4
No. 5 U-1.0%Ti	70.9±0.4	113.1±0.3	41.0±0.7	55.8±0.5	370.9±16.6	387.4±29.4
No. 14 U-0.75%Ti	71.5±0.3	113.5±0.4	41.4±0.6	56.8±0.3	397.3±14.4	411.0±27.5
No. 16 U-1.0%Ti	71.6±0.2	115.0±0.3	42.3±0.5	57.0±0.4	403.8± 9.7	431.9±13.1
No. 2 U-1.5%Ti	73.0±0.5	115.6±0.5	44.9±0.9	59.0±0.5	434.3±11.9	427.6±26.5
No. 1 U-2.0%Ti	74.3±0.4	117.2±0.3	47.2±0.6	61.6±0.4	454.0±16.0	476.0±18.7
No. 12 U-1.0%Ti	75.1±0.3	117.6±0.4	47.5±0.4	61.8±0.3	464.8±17.3	468.3±39.6
No. 9 U-1.5%Ti	75.4±0.3	118.4±0.4	49.9±0.7	63.3±0.7	496.3±17.9	506.9±23.9
No. 8 U-1.5%Ti	75.7±0.5	118.7±0.3	49.3±0.5	62.8±0.6	506.6±26.3	513.3±31.8
No. 3 U-2.0%Ti	77.1±0.3	119.9±0.4	53.0±0.5	64.9±0.4	529.0±10.8	549.7±15.8
No. 6 U-2.0%Ti	78.2±0.1	120.7±0.4	54.3±0.6	66.6±0.3	561.3±11.3	576.6±17.5
No. 11 U-1.5%Ti	78.0±0.2	121.0±0.3	55.0±0.6	66.9±0.3	581.0±23.7	565.7±38.2
No. 15 U-1.5%Ti	79.5±0.2	122.2±0.2	57.0±0.3	68.2±0.2	621.3±18.3	637.4±30.2
No. 10 U-2.0%Ti	79.6±0.2	122.1±0.2	57.4±0.3	68.6±0.4	626.3±21.1	619.6±24.0

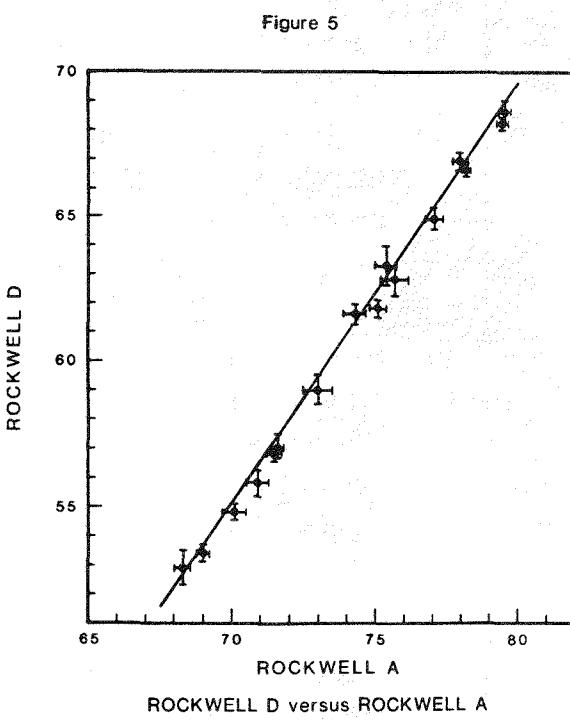
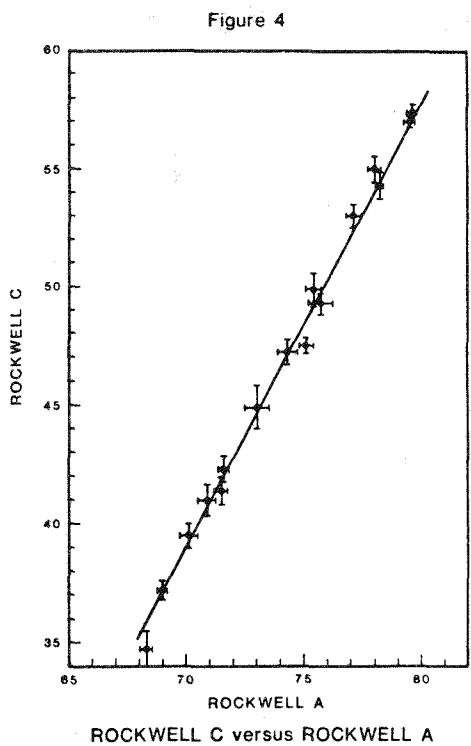
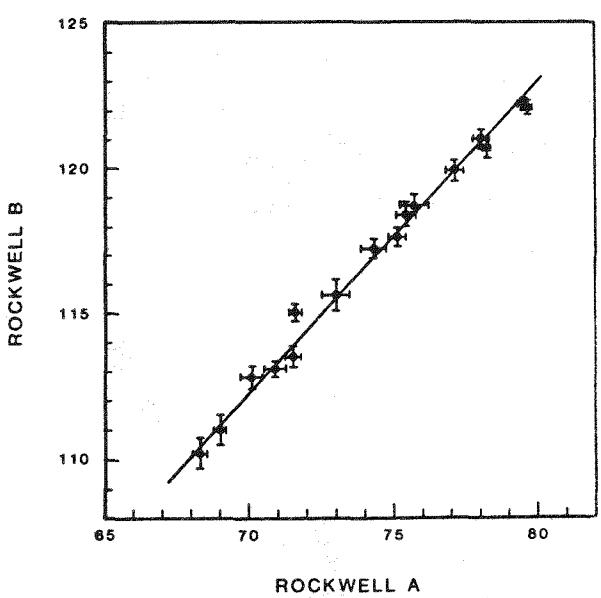


Figure 6

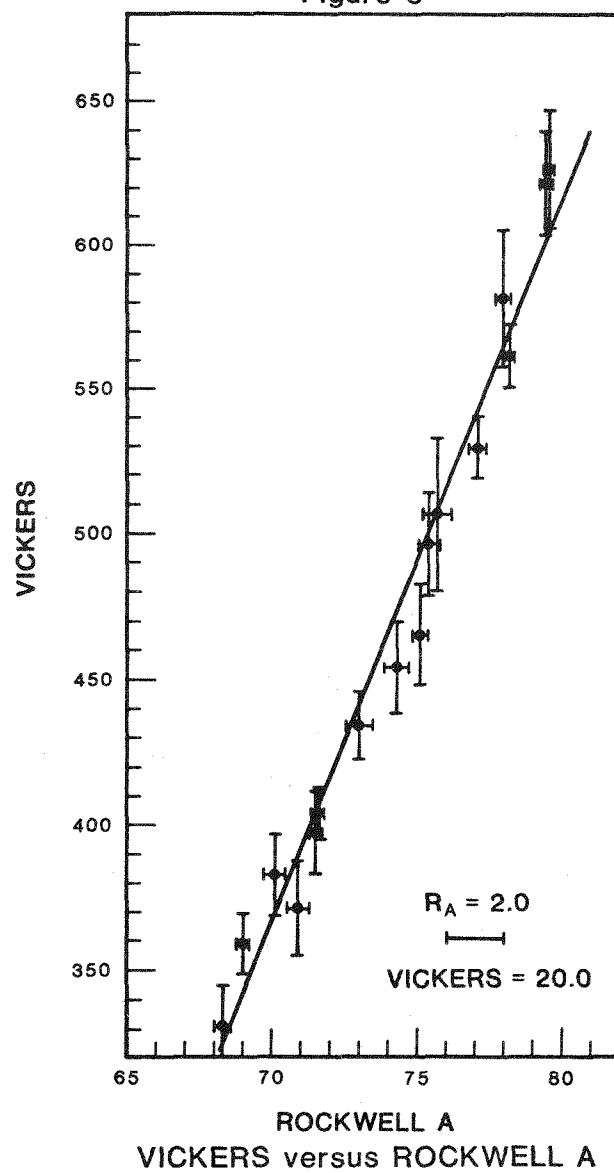


Figure 7

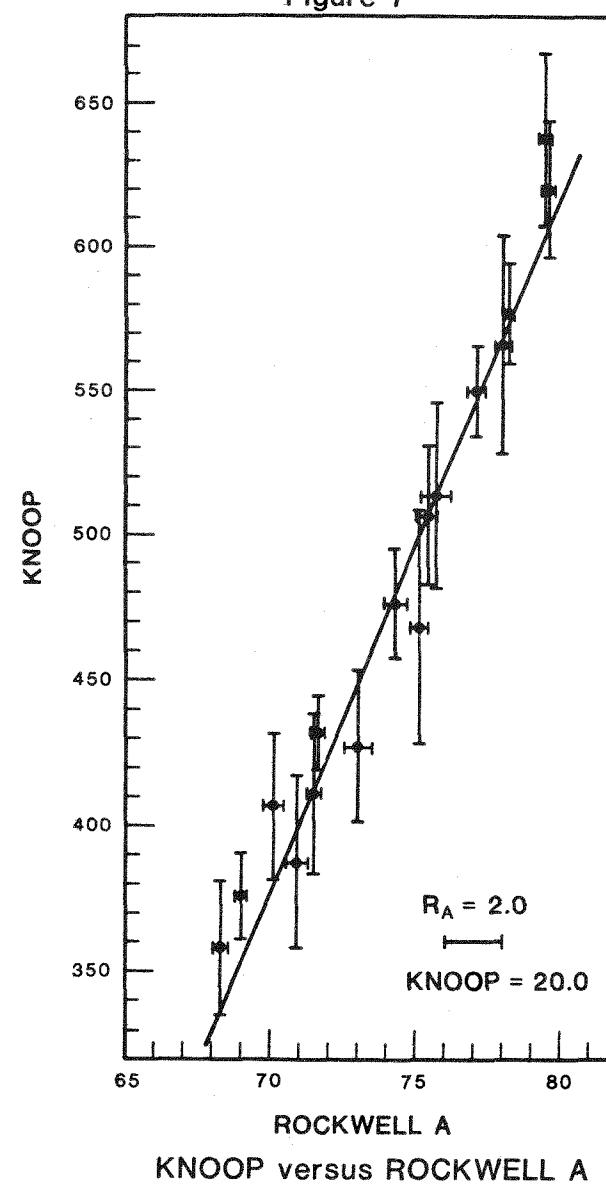
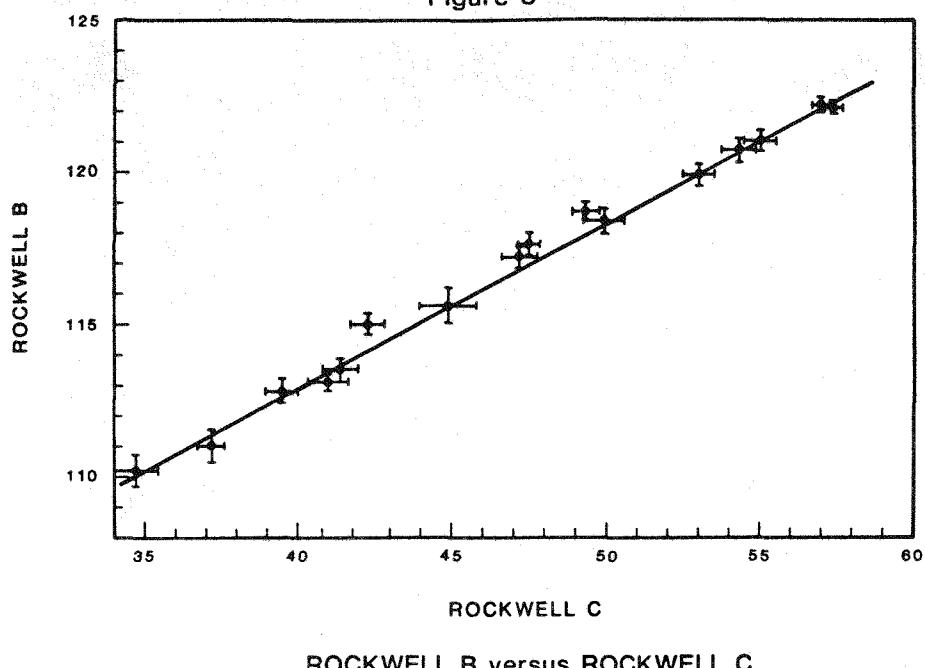
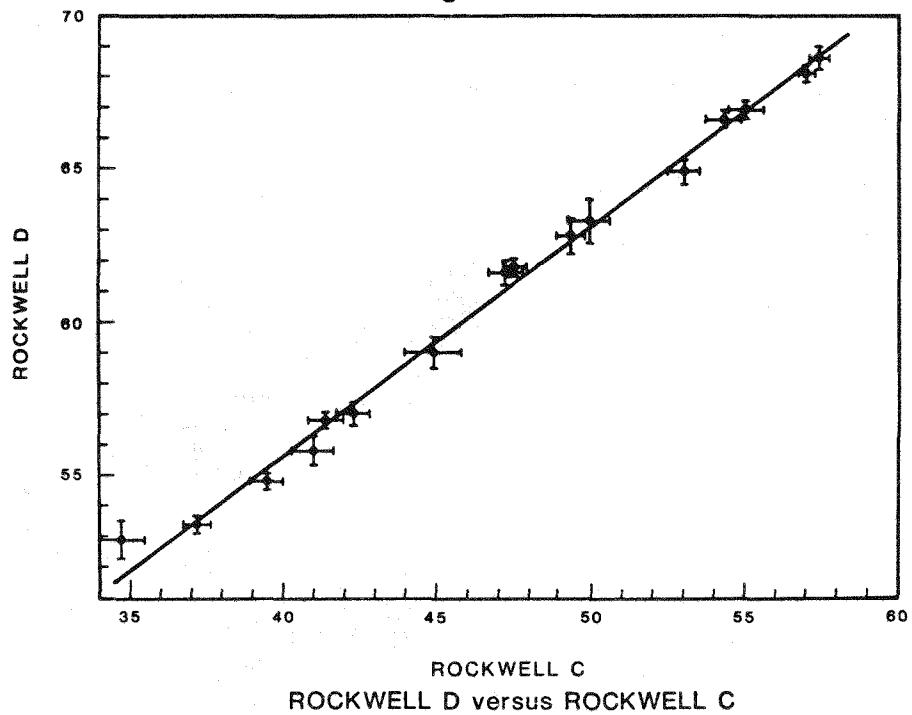


Figure 8



ROCKWELL B versus ROCKWELL C

Figure 9



ROCKWELL D versus ROCKWELL C

Figure 10

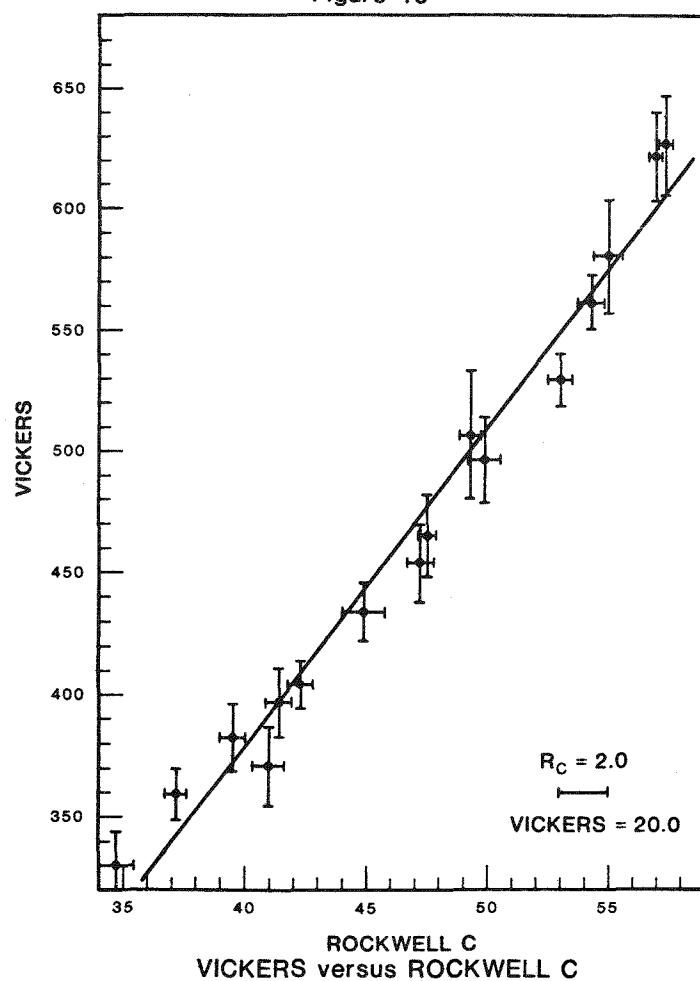


Figure 11

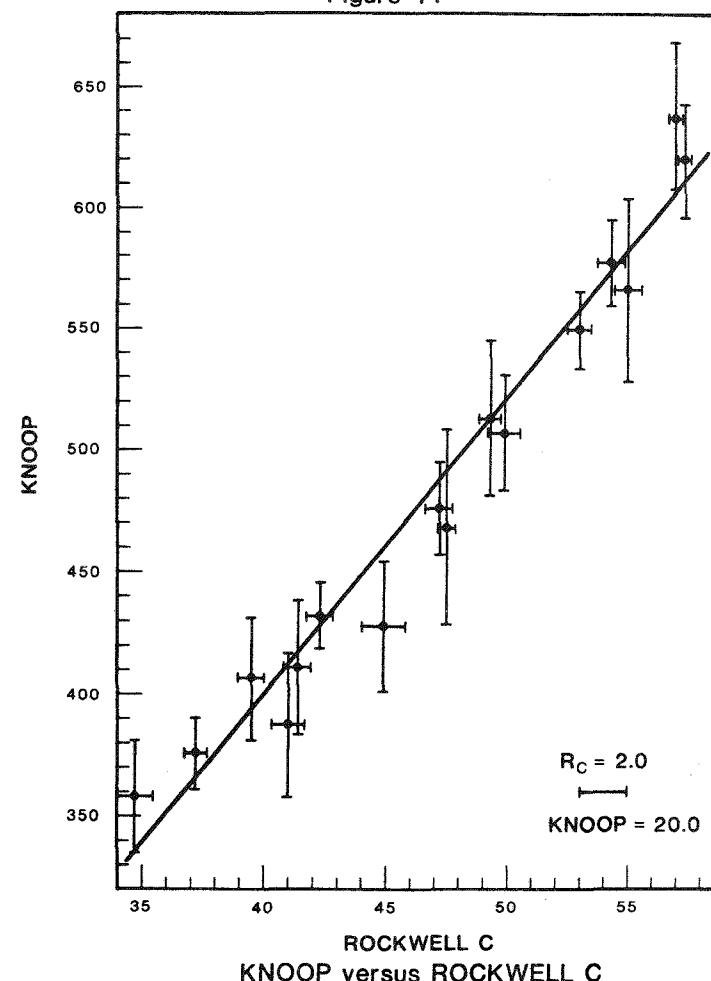


Figure 12

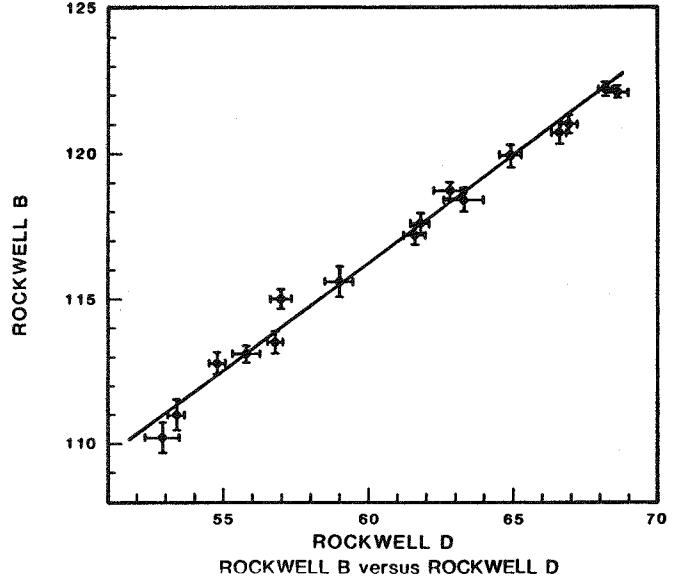


Figure 13

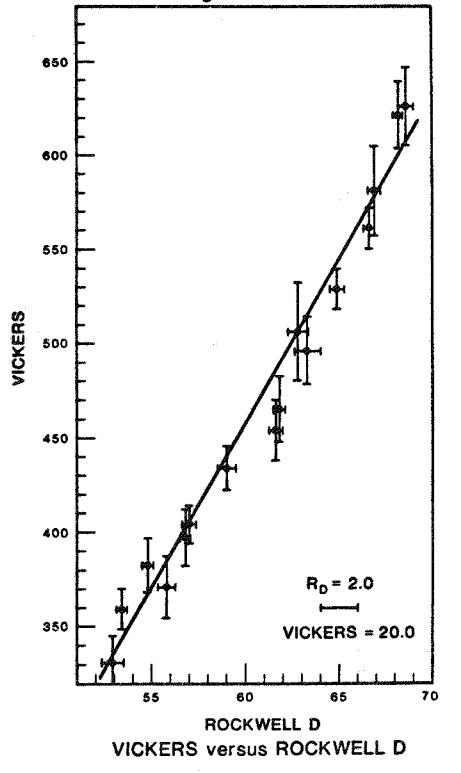


Figure 14

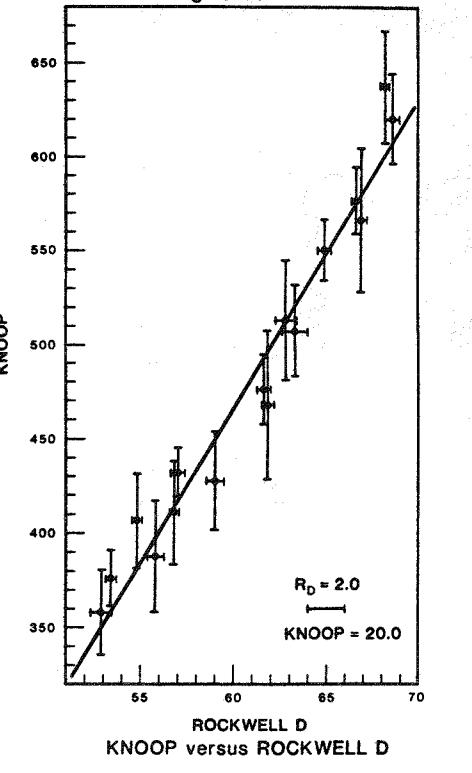


Figure 15

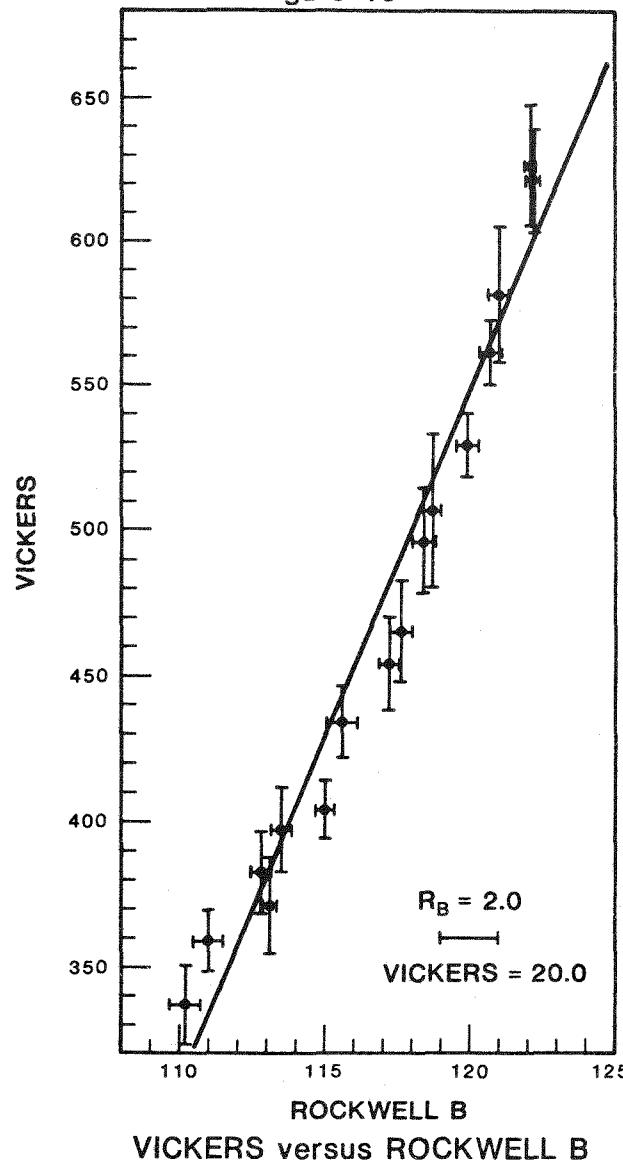
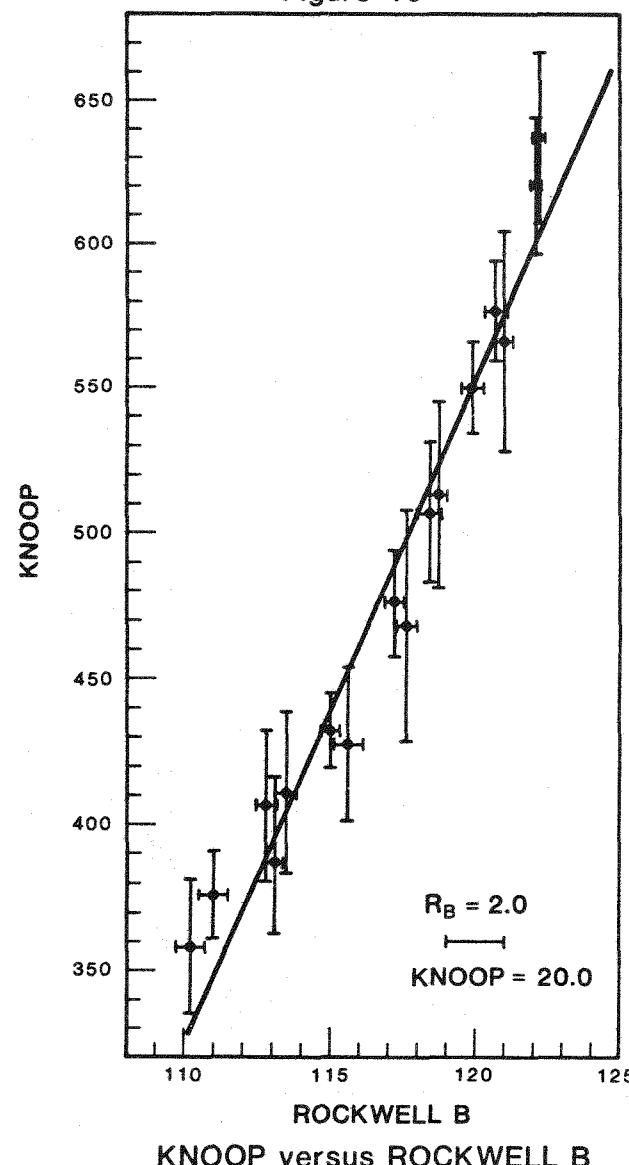


Figure 16



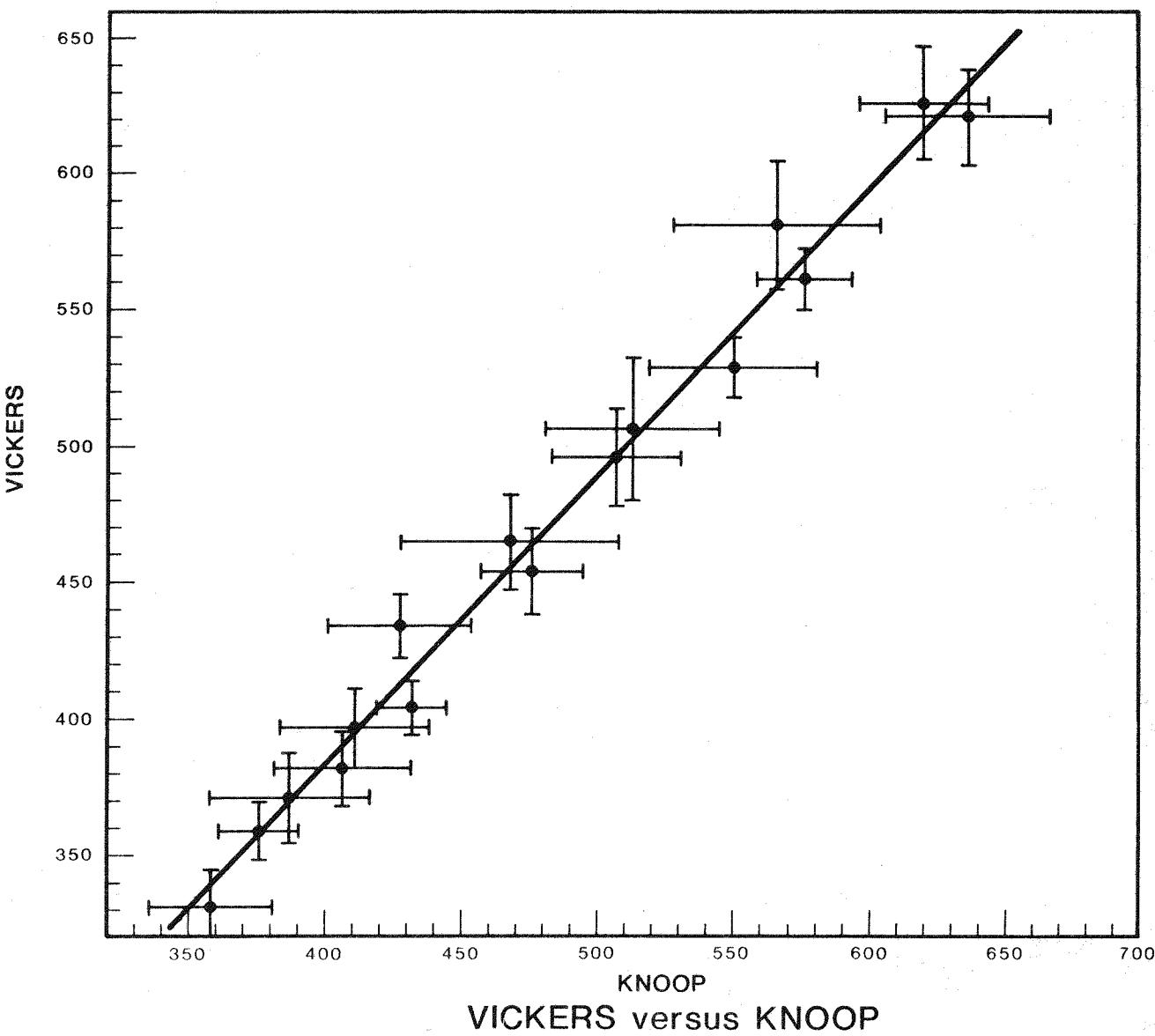


Figure 17

Table III  
**TABLE OF APPROXIMATE HARDNESS CONVERSION  
 NUMBERS FOR STEEL, BASED ON DPH ( VICKERS )<sup>1</sup>**

VICKERS*	ROCKWELL HARDNESS				VICKERS*	ROCKWELL HARDNESS			
	A	B	C	D		A	B	C	D
650	80.0		57.8	69.0	490	74.9		48.4	61.6
640	79.8		57.3	68.7	480	74.5		47.7	61.3
630	79.5		56.8	68.3	470	74.1		46.9	60.7
620	79.2		56.3	67.9	460	73.6		46.1	60.1
610	78.9		55.7	67.5	450	73.3		45.3	59.4
600	78.6		55.2	67.0	440	72.8		44.5	58.8
590	78.4		54.7	66.7	430	72.3		43.6	58.2
580	78.0		54.1	66.2	420	71.8		42.7	57.5
570	77.8		53.6	65.8	410	71.4		41.8	56.8
560	77.4		53.0	65.4	400	70.8		40.8	56.0
550	77.0		52.3	64.8	390	70.3		39.8	55.2
540	76.7		51.7	64.4	380	69.8	110.0	38.8	54.4
530	76.4		51.1	63.9	370	69.2		37.7	53.6
520	76.1		50.5	63.5	360	68.7	109.0	36.6	52.8
510	75.7		49.8	62.9	350	68.1		35.5	51.9
500	75.3		49.1	62.2	340	67.6	108.0	34.4	51.1

\* DIAMOND PYRAMID HARDNESS NUMBER, 50Kg LOAD

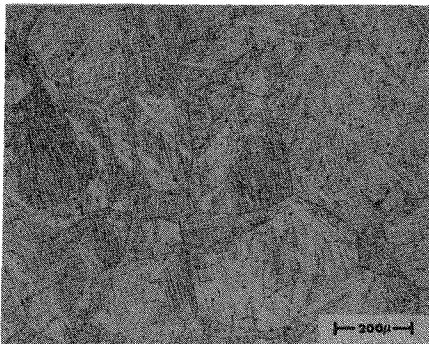
REFERENCE: " PRINCIPLES OF METALLOGRAPHIC LABORATORY PRACTICE "  
 GEORGE L. KEHL, THIRD EDITION, 1949, McGRAW-HILL, PORTIONS TAKEN  
 FROM TABLE 37, PAGES 466 & 467.

### Microstructural Correlations

The hardening behavior observed in this study is consistent with the hardness observations made in earlier uranium studies.<sup>2</sup> The hardness of uranium alloys can be correlated to microstructure. At high temperature (>1050 K) the Ti dissolves fully in the b.c.c.  $\gamma$  phase. If cooled slowly the  $\gamma$  will decompose to  $\alpha$ -U (orthorhombic) and U<sub>2</sub>Ti (complex). This two phase structure is hard and strong, although it possesses little ductility. If cooled quickly from high temperature, the  $\gamma$  transforms diffusionlessly to form a Ti supersaturated martensitic phase, which is very soft and ductile. If the material is reheated to a moderate temperature (<800 K) the alloy will age harden. Small precipitates, related to U<sub>2</sub>Ti, form and harden the material. Also, as hardening continues the martensite begins to decompose to form a lamellar mixture of  $\alpha$ -U + U<sub>2</sub>Ti. As these reactions proceed, the alloy increases in hardness.

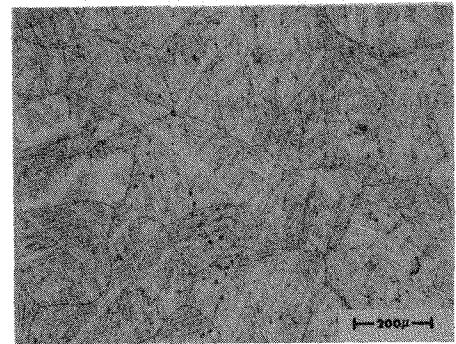
The softest alloys are those that have been aged for short times at low temperatures. The microstructure of the alloys used in this study can be correlated to their hardnesses. Figures 18 through 33 show the microstructures for each alloy, arranged in order of increasing hardness. The softest alloys are all martensitic (Figure 18 through 25). Much of the hardness increase observed through this series of samples is due to solid solution hardening effect. The high Ti martensitic alloys tend to be harder than the low Ti martensitic alloys. Up to this point, any microstructural change due to precipitation which could increase hardness is at too fine a scale to observe in the optical microscope. The harder samples (the microstructures are shown in Figures 26 through 33) all show evidence

Figure 18



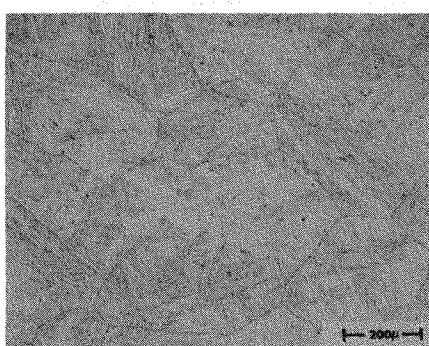
Alloy : U-0.75%Ti ( No. 7 )  
Aged : 400°C/1.5min.  
Microstructure :  
 $\alpha'_a$  ( Acicular Martensite )

Figure 19



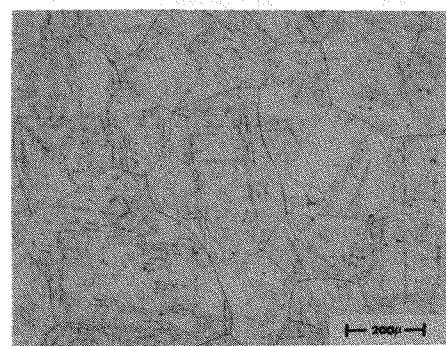
Alloy : U-1.0%Ti ( No. 4 )  
Aged : 385°C/1min.  
Microstructure :  
 $\alpha'_a$  ( Acicular Martensite )

Figure 20



Alloy : U-0.75%Ti ( No. 13 )  
Aged : 430°C/10min.  
Microstructure :  
 $\alpha'_a$  ( Acicular Martensite )

Figure 21

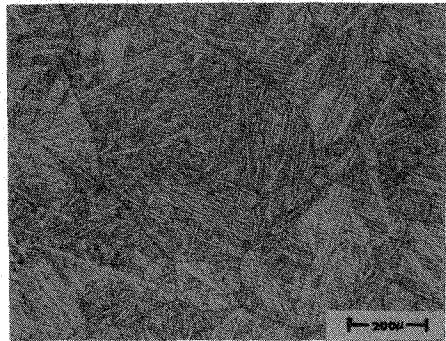


Alloy : U-1.0%Ti ( No. 5 )  
Aged : 385°C/38min.  
Microstructure :  
 $\alpha'_a$  ( Acicular Martensite )

Solution Treatment For All Samples : 850°C/1hr., Ice Water Quenched

Scale Bar : 200μm

Figure 22



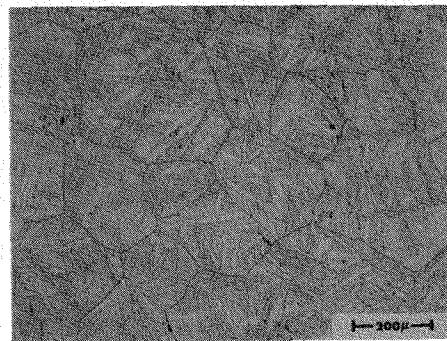
Alloy : U-0.75%Ti ( No. 14 )

Aged : 445°C/10min.

Microstructure :

$\alpha'_a$  ( Acicular Martensite )

Figure 23



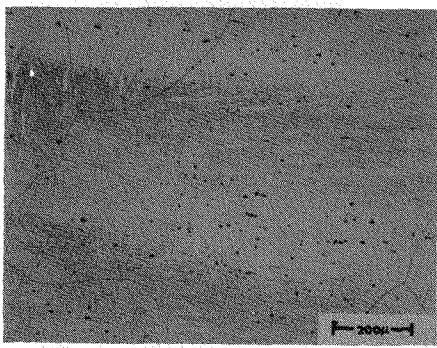
Alloy : U-1.0%Ti ( No. 16 )

Aged : 480°C/1.5min.

Microstructure :

$\alpha'_a$  ( Acicular Martensite )

Figure 24



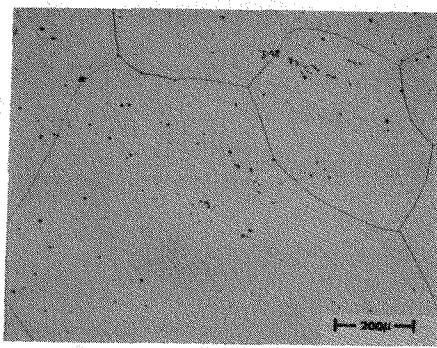
Alloy : U-1.5%Ti ( No. 2 )

Aged : 370°C/2.5min.

Microstructure :

$\alpha'_a$  ( Acicular Martensite )

Figure 25



Alloy : U-2.0%Ti ( No. 1 )

Aged : 310°C/60min.

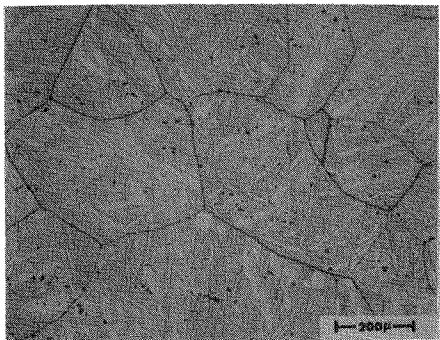
Microstructure :

$\alpha'_b$  ( Banded Martensite )

Solution Treatment For All Samples : 850°C/1hr., Ice Water Quenched

Scale Bar : 200μm

Figure 26



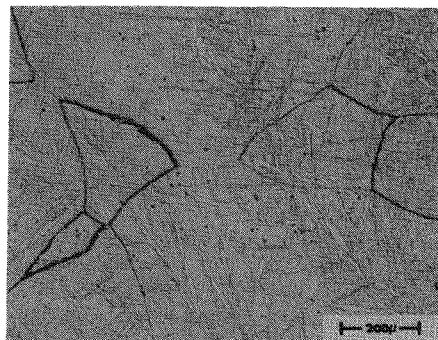
Alloy : U-1.0%Ti ( No. 12 )

Aged : 420°C/60min.

Microstructure :

$\alpha'_a$  ( Acicular Martensite  
plus Grain Boundary  
Decomposition Product )

Figure 27



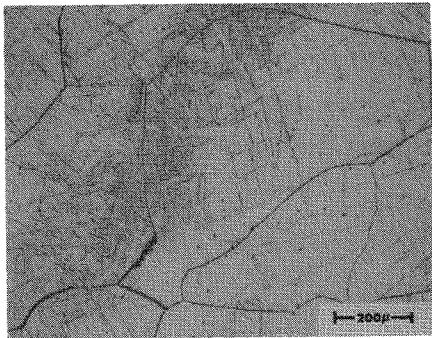
Alloy : U-1.5%Ti ( No. 9 )

Aged : 420°C/10min.

Microstructure :

$\alpha'_a$  ( Acicular Martensite  
plus Grain Boundary  
Decomposition Product )

Figure 28



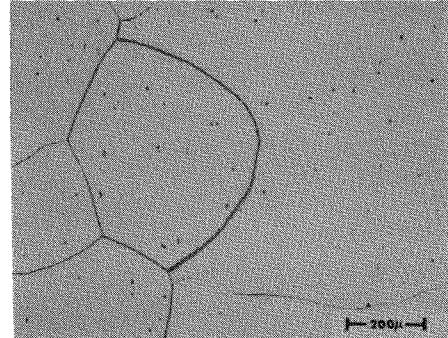
Alloy : U-1.5%Ti ( No. 8 )

Aged : 400°C/24min.

Microstructure :

$\alpha'_a$  ( Acicular Martensite  
plus Grain Boundary  
Decomposition Product )

Figure 29



Alloy : U-2.0%Ti ( No. 3 )

Aged : 370°C/24min.

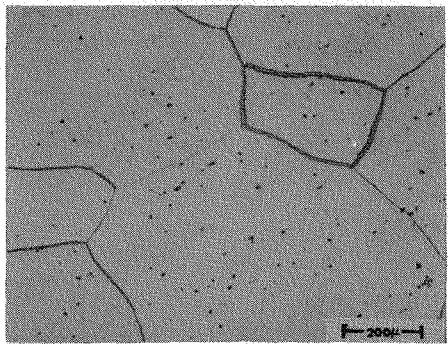
Microstructure :

$\alpha'_b$  ( Banded Martensite  
plus Grain Boundary  
Decomposition Product )

Solution Treatment For All Samples : 850°C/1hr., Ice Water Quenched

Scale Bar : 200μm

Figure 30



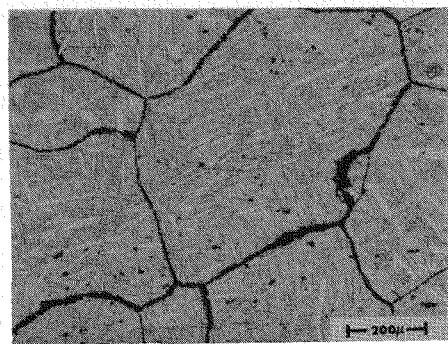
Alloy : U-2.0%Ti ( No. 6 )

Aged : 385°C/60min.

Microstructure :

$\alpha'_b$  ( Banded Martensite  
plus Grain Boundary  
Decomposition Product )

Figure 31



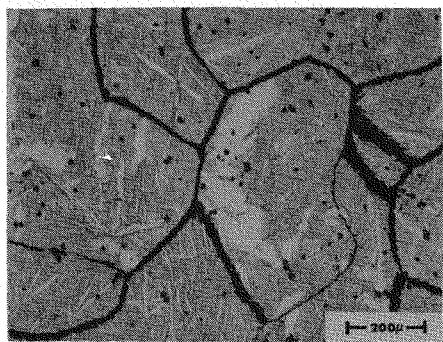
Alloy : U-1.5%Ti ( No. 11 )

Aged : 420°C/60min.

Microstructure :

$\alpha'_a$  ( Acicular Martensite plus  
Decomposition Product )

Figure 32



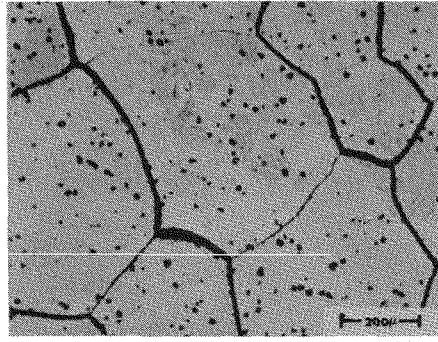
Alloy : U-1.5%Ti ( No. 15 )

Aged : 455°C/38min.

Microstructure :

$\alpha'_a$  ( Acicular Martensite plus  
Decomposition Product )

Figure 33



Alloy : U-2.0%Ti ( No. 10 )

Aged : 420°C/60min.

Microstructure :

$\alpha'_b$  ( Banded Martensite plus  
Decomposition Product )

Solution Treatment For All Samples : 850°C/1hr., Ice Water Quenched

Scale Bar : 200μm

of decomposition of the martensite at the grain boundaries and sometimes also within the grains. This decomposition occurs more extensively as aging times and temperatures are increased. However, it should be recalled, that most of the hardening observed in the hardest samples is still attributable to microstructural changes too fine to be observed with the optical microscope.

### Summary

Hardness correlations between the  $R_A$ ,  $R_B$ ,  $R_C$ ,  $R_D$ , Vickers (300 g load), and Knoop (300 g load) hardness scales were determined for uranium alloys.

These data were fit using a least squares technique to enable simple and quick conversion from one hardness scale to another. For rapid approximate conversions, the data has been plotted in a series of figures. The hardness observed have been correlated to microstructure. This property/ microstructure relationship showed that high hardnesses occur with more advanced decomposition of the martensite.

### References

1. G. L. Kehl, Principles of Metallographic Laboratory Practice, McGraw-Hill, New York, New York (1949).
2. K. H. Eckelmeyer, "Diffusional Transformations, Strengthening Mechanisms, and Mechanical Properties of Uranium Alloys," SAND82-0524 (1982).
3. A. D. Romig, Jr. "Metallographic Techniques and Microstructure: Uranium Alloys," SAND81-1014 (1982).
4. Microhardness Testing Standard ANSI/ASTM E384-73, 1982 Annual Book of ASTM Standards, Part 44 "Magnetic Properties; Metallic Materials for Thermostats; Electrical Resistance, Heating, Contacts; Temperature Measurement." Page 776 Standard Test Method for Microhardness of Materials. Published by ASTM.
5. M. R. Spiegel, Statistics, Schaum's Outline Series, McGraw-Hill, New York (1961) p. 217-220, 243.

## **Appendix A**

**DISCUSSION OF LEAST SQUARES AND  
FORTRAN PROGRAM TO CALCULATE  
LEAST SQUARES LINE FOR HARDNESS DATA**

## Appendix A

The method of least squares is one of the most commonly used statistical techniques to fit data. This is one way to obtain the "best fitting line".

Consider a set of data points  $(X_1, Y_1), (X_2, Y_2), \dots, (X_N, Y_N)$  which are to fit to a straight line. For each value of  $X$ , say  $X_1$ , there will be a corresponding value of  $Y$ , say  $Y_1$ . There will be a difference between  $Y_1$  and  $Y$  as calculated from the line. The difference is called the deviation, error, or residual. For every value of  $X_2$  through  $X_N$  there will then be the deviations  $D_1$  through  $D_N$ . This is illustrated in Figure 34.

A measure of the goodness of fit of the line to the data is given by

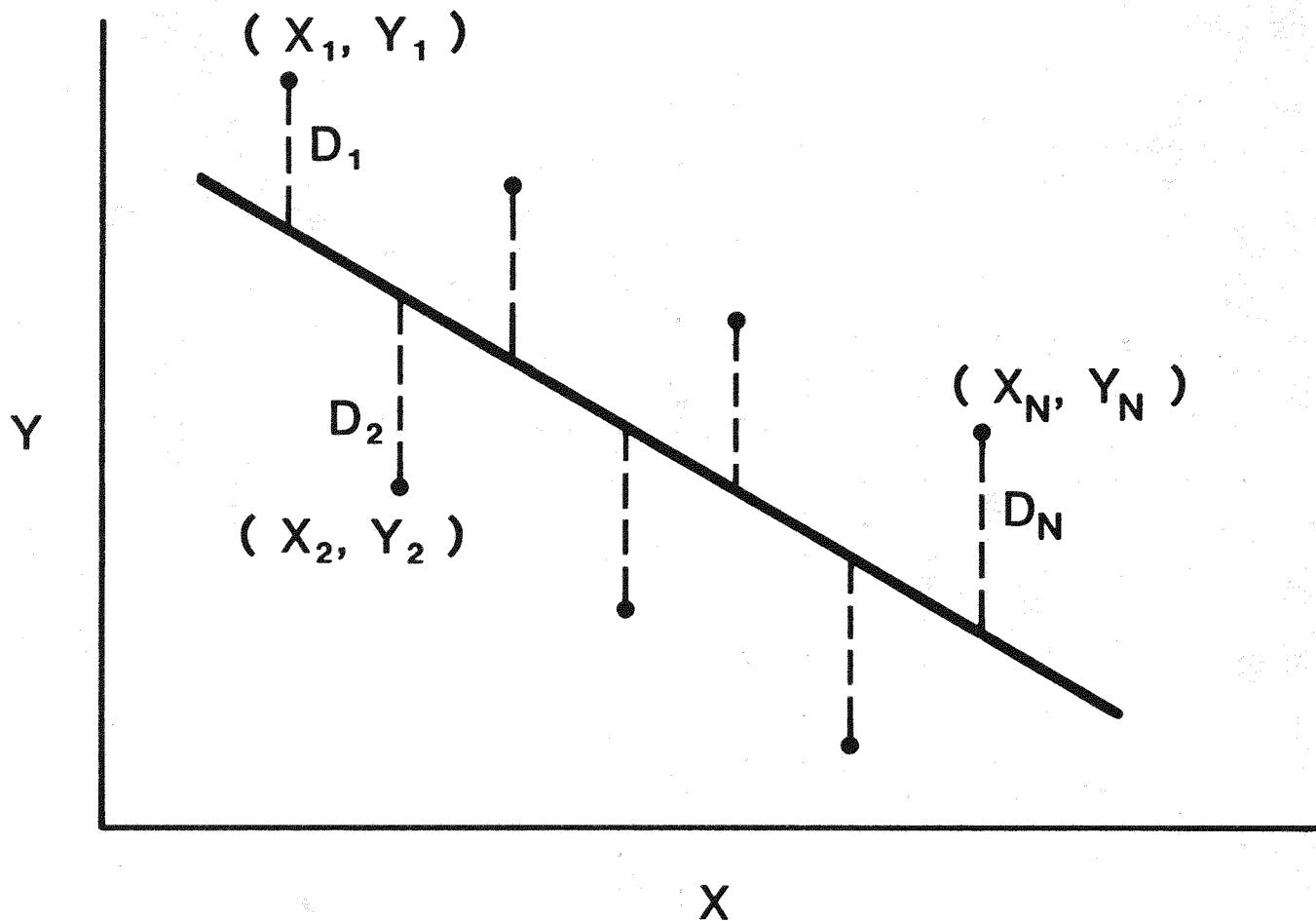
$D_1^2 + D_2^2 + \dots + D_N^2$ . If this sum of squares is small the fit is good, if it is large the fit is bad. The best fitting curve occurs when  $D_1^2 + D_2^2 + \dots + D_N^2$  is a minimum. Such a curve is a least squares curve. When the curve is a straight line it is a least squares line. It is important to notice that the squares of deviations are used, rather than the deviations themselves. Squaring the deviations forces positive and negative deviations to be treated in an identical fashion.

A straight line has the form

$$Y = A_0 + A_1 X \quad (1)$$

where  $X$  and  $Y$  are the independent and dependent variables respectively, and  $A_0$  and  $A_1$  are the intercept and slope, respectively. The constants  $A_0$  and  $A_1$  can be determined for the least squares line. They are:

$$A_0 = \frac{(\Sigma Y)(\Sigma X^2) - (\Sigma X)(\Sigma XY)}{N\Sigma X^2 - (\Sigma X)^2} \quad (2)$$



LEAST SQUARES LINE FOR WHICH

$$D_1^2 + D_2^2 + \dots + D_N^2 = \text{MINIMUM}$$

Figure 34

and

$$A_1 = \frac{N\Sigma XY - (\Sigma X)(\Sigma Y)}{N\Sigma X^2 - (\Sigma X)^2} \quad (3)$$

where  $N$  is the number of data points,  $\Sigma X$  is the sum of all values of  $X$ ,  $\Sigma Y$  is the sum of all values of  $Y$ , and  $\Sigma XY$  is the sum of all the products of  $X$  and  $Y$  (i.e.,  $X_1 \times Y_1$ ,  $X_2 \times Y_2$ , ...,  $X_N \times Y_N$ ).

Since finding the least squares line is a tedious process, a computer program was written to calculate the least squares line for the uranium hardness data determined in this study and the steel hardness data from the literature.<sup>1</sup> The computer program is given in Appendix A. Tables IV and V give the results of these least squares curve fits. The fits have been done for each possible combination of hardness scales. The slope and the intercept are given, as is the error which is a measure of the goodness of the fit.

The use of these tables is simple, as illustrated by the following example. A uranium alloy has been determined to have a hardness of  $R_A = 73$ . What is its hardness on the  $R_C$  scale? From Table III, to convert from  $R_A$  to  $R_C$  one uses

$$R_C = -96.50 + 1.93 R_A$$

or

$$R_C = -96.50 + 1.93(73)$$

$$R_C = 44.4$$

The associated standard error is  $\pm 0.5$ .

Therefore,  $R_A 73 = R_C 44.4 \pm 0.05$ .

**Table IV**  
**LEAST SQUARES CURVE FIT FOR URANIUM ALLOYS**

GIVEN	DETERMINE	INTERCEPT $a_0$	SLOPE $a_1$	STANDARD ERROR
<b>ROCKWELL A</b>	ROCKWELL B	38.80	1.05	0.37
	ROCKWELL C	-96.50	1.93	0.53
	ROCKWELL D	-45.80	1.44	0.35
	KNOOP	-1260.00	23.40	16.70
	VICKERS	-1400.00	25.20	14.60
<b>ROCKWELL B</b>	ROCKWELL A	-35.90	0.94	0.35
	ROCKWELL C	-167.00	1.83	0.65
	ROCKWELL D	-97.60	1.36	0.56
	KNOOP	-2100.00	22.10	18.50
	VICKERS	-2300.00	23.70	17.80
<b>ROCKWELL C</b>	ROCKWELL A	50.00	0.51	0.28
	ROCKWELL B	91.30	0.54	0.36
	ROCKWELL D	26.10	0.74	0.45
	KNOOP	-85.30	12.10	16.80
	VICKERS	-141.00	13.00	14.40
<b>ROCKWELL D</b>	ROCKWELL A	32.00	0.69	0.24
	ROCKWELL B	72.50	0.73	0.41
	ROCKWELL C	-34.70	1.34	0.61
	KNOOP	-507.00	16.20	17.20
	VICKERS	-594.00	17.50	15.00
<b>KNOOP</b>	ROCKWELL A	54.40	0.04	0.70
	ROCKWELL B	96.00	0.04	0.82
	ROCKWELL C	8.60	0.08	1.36
	ROCKWELL D	32.40	0.06	1.04
	VICKERS	-41.50	1.06	11.20
<b>VICKERS</b>	ROCKWELL A	56.00	0.04	0.57
	ROCKWELL B	97.70	0.04	0.74
	ROCKWELL C	11.70	0.08	1.09
	ROCKWELL D	34.70	0.06	0.85
	KNOOP	45.70	0.93	10.50

Table V  
LEAST SQUARES CURVE FIT FOR STEEL

GIVEN	DETERMINE	INTERCEPT $a_0$	SLOPE $a_1$	STANDARD ERROR
ROCKWELL A	ROCKWELL C	-91.20	1.86	0.11
	ROCKWELL D	-45.40	1.43	0.10
	VICKERS	-1360.00	24.80	10.50
ROCKWELL C	ROCKWELL A	49.00	0.54	0.06
	ROCKWELL D	24.70	0.77	0.06
	VICKERS	-140.00	13.30	11.50
ROCKWELL D	ROCKWELL A	31.80	0.70	0.07
	ROCKWELL C	-32.10	1.30	0.08
	VICKERS	-567.00	17.30	11.40
VICKERS	ROCKWELL A	54.90	0.04	0.42
	ROCKWELL C	11.10	0.07	0.86
	ROCKWELL D	33.20	0.06	0.65

As was demonstrated in the Hardness Correlation section, all of the correlations between the various scales used in this study have small standard errors, indicating the least squares fits are quite good. In a few instances a second order curve of the form

$$Y = A_0 + A_1X + A_2X^2 \quad (4)$$

would better fit the data. However, for simplicity and consistancy the decision was made to use only first order fits.

```

0001      PROGRAM LSQFIT
0002      DIMENSION X(100),Y(100),YC(100),XSD(100),YSD(100),XY(100)
0003      DIMENSION BX(20),BY(20),BUF(50)
0004      DIMENSION R(100),RSQ(100)
0005      IMPLICIT BYTE(B)
C
C
C      A PROGRAM TO DO A STRAIGHT LINE LEAST SQUARES CURVE FIT
C      TO HARDNESS DATA CORRELATIONS.
C
C      D.L. HUMPHREYS & A.D. ROMIG, JR.
C      DIVISION 1832
C      SANDIA NATIONAL LABORATORIES
C      ALBUQUERQUE, NEW MEXICO 87185
C
C
0006      WRITE(5,20)
0007 20      FORMAT(5X,'A PROGRAM TO FIT A LEAST SQUARES LINE TO: /,
$ 10X,'HARDNESS CORRELATION DATA',/,15X,
$ 'BY DAVE, THE HUMP, HUMPHREYS',/)
C
0008      WRITE(5,21)
0009 21      FORMAT(//5X,'POSSIBLE VARIABLES ARE: RA, RB, RC, RD, KNOOP, VICKERS')
0010      WRITE(5,22)
0011 22      FORMAT('$',5X,'INDEPENDENT VARIABLE:')
0012      READ(5,23) INX,(BX(I),I=1,INX)
0013 23      FORMAT(0,20A1)
0014      WRITE(5,25)
0015 25      FORMAT(//)
0016      WRITE(5,24)
0017 24      FORMAT('$',5X,'DEPENDENT VARIABLE:')
0018      READ(5,23) INY,(BY(I),I=1,INY)
0019      WRITE(5,25)
0020      WRITE(5,26)
0021 26      FORMAT('$',5X,'NUMBER OF DATA PAIRS:')
0022      READ(5,77) N
0023 77      FORMAT(I3)
0024      WRITE(5,25)
C
0025 32      CONTINUE
C
0026      DO 50 J=1,N
0027      WRITE(5,28) (BX(I),I=1,INX),',',',(BY(I),I=1,INY)
0028 28      FORMAT('$',10X,'ENTER DATA:',20A1,20A1)
0029      WRITE(5,27) J
0030 27      FORMAT('$',5X,'DATA PAIR',I2,':')
0031      ACCEPT*, X(J),Y(J)
0032 50      CONTINUE
C
C
0033      WRITE(5,25)
0034      WRITE(5,30)
0035 30      FORMAT('$',5X,'ALL DATA O.K. (Y/N) ')
0036      READ(5,31) BANS

```

FORTRAN IV V02.5-4

PAGE 002

```
0037 31  FORMAT(A1)
0038  IF(BANS.EQ.'N') WRITE(5,34)
0040 34  FORMAT(10X,'REENTER DATA')
0041  IF(BANS.EQ.'N') GO TO 32
C
0043  DO 52 I=1,N
0044  XSO(I)= X(I)**2
0045  YSO(I)= Y(I)**2
0046  XY(I)= X(I)*Y(I)
0047 52  CONTINUE
C
0048  XSUM=0.
0049  YSUM=0.
0050  XYSUM=0.
0051  XSOSUM=0.
0052  YSOSUM=0.
C
0053  DO 53 I=1,N
0054  XSUM= XSUM + X(I)
0055  YSUM= YSUM + Y(I)
0056  XYSUM= XYSUM + XY(I)
0057  XSOSUM= XSOSUM + XSO(I)
0058  YSOSUM= YSOSUM + YSO(I)
0059 53  CONTINUE
C
C
C
C      CALCULATE COEFFICIENTS
C
0060  A0= ((YSUM*XSOSUM)-(XSUM*XYSUM)) / ((FLOAT(N)*XSOSUM)-(XSUM**2))
0061  A1= ((FLOAT(N)*XYSUM)-(XSUM*YSUM)) / ((FLOAT(N)*XSOSUM)-(XSUM**2))
C
C
C
C      CALCULATE RESIDUALS
C
0062  DO 60 I=1,N
0063  YC(I)= A0 + (A1*X(I))
0064  R(I)= ABS(Y(I)-YC(I))
0065  RSQ(I)= R(I)**2
0066 60  CONTINUE
C
0067  TERR=0.
0068  DO 61 I=1,N
0069  TERR= TERR + RSQ(I)
0070 61  CONTINUE
C
C
C      CALCULATE STANDARD ERROR
C
0071  STDERR= SQRT(TERR/FLOAT(N))
C
C
C      WRITE FINAL RESULTS
C
0072  WRITE(6,100) (BY(I),I=1,INY), ' ', 'V', 'S', ' ', (BX(I),I=1,INX)
0073 100  FORMAT(/,15X,'CORRELATION OF ',40A1,//)
```

```
C
0074      ENCODE (INY+1,98,BUF) (BY(I),I=1,INY),*=?
0075      ENCODE (10,99,BUF(INY+2)) A0
0076      ENCODE (3,98,BUF(INY+12)) ' ','+','
0077      ENCODE (10,99,BUF(INY+15)) A1
0078      ENCODE (INX+1,98,BUF(INY+24)) '*,(BX(I),I=1,INX)
0079      NC = INX + INY + 24
0080      WRITE (6,25)
0081      WRITE (6,101) (BUF(I),I=1,NC)
0082      WRITE (6,25)
0083 101  FORMAT(25X,50A1)
0084 99   FORMAT(E10.3)
0085 98   FORMAT (50A1)
C
0086      WRITE(6,25)
0087      WRITE(6,94) A0,A1
0088 94   FORMAT(/,10X,'A0 = ',E10.3,5X,'A1 = ',E10.3)
C
0089      WRITE(6,25)
0090      WRITE(6,102)
0091 102  FORMAT(20X,'DATA SUMMARY')
0092      WRITE(6,103)
0093 103  FORMAT(5X,'EXPTL X',5X,'EXPTL Y',5X,'CALC Y',5X,'RESIDUAL',/)
C
C
0094      DO 55 I=1,N
0095      WRITE(6,104) X(I),Y(I),YC(I),R(I)
0096 104  FORMAT(5X,4(F6.2,6X))
0097 55   CONTINUE
C
0098      WRITE(6,105) STDERR
0099 105  FORMAT(/,20X,'STANDARD ERROR = ',E10.3)
0100      CALL EXIT
0101      END
```

**FORTRAN IV Storage Map for Program Unit LSQFIT**

**Local Variables, .PSECT \$DATA, Size = 006536 ( 1711. words)**

Name	Type	Offset	Name	Type	Offset	Name	Type	Offset
AO	R*4	006470	A1	R*4	006474	BANS	L*1	006442
I	I*2	006432	INX	I*2	006430	INY	I*2	006434
J	I*2	006440	N	I*2	006436	NC	I*2	006510
STDERR	R*4	006504	TERR	R*4	006500	XSDSUM	R*4	006460
XSUM	R*4	006444	XYSUM	R*4	006454	YSOSUM	R*4	006464
YSUM	R*4	006450						

**Local and COMMON Arrays:**

Name	Type	Section	Offset	-----Size-----	Dimensions
BUF	L*1	\$DATA	004610	000062 ( 25. )	(50)
BX	L*1	\$DATA	004540	000024 ( 10. )	(20)
BY	L*1	\$DATA	004564	000024 ( 10. )	(20)
R	R*4	\$DATA	004672	000620 ( 200. )	(100)
RSQ	R*4	\$DATA	005512	000620 ( 200. )	(100)
X	R*4	\$DATA	000000	000620 ( 200. )	(100)
XSQ	R*4	\$DATA	002260	000620 ( 200. )	(100)
XY	R*4	\$DATA	003720	000620 ( 200. )	(100)
Y	R*4	\$DATA	000620	000620 ( 200. )	(100)
YC	R*4	\$DATA	001440	000620 ( 200. )	(100)
YSQ	R*4	\$DATA	003100	000620 ( 200. )	(100)

**Subroutines, Functions, Statement and Processor-Defined Functions:**

Name	Type	Name	Type	Name	Type	Name	Type	Name	Type
ABS	R*4	EXIT	R*4	FLOAT	R*4	SORT	R*4		

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