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# A Doubled Nd-Glass Laser System for Incoherent Thomson Scattering from Low Density Plasmas

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by

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## ABSTRACT

Conventional Thomson scattering using a ruby laser evidences difficulty in scattered signal detection as electron densities drop below $10^{13}$ $cm^{-3}$. Studies of plasmas of interest, such as θ-pinch endloss flow, can require density resolution in that range. Use of a Nd-Glass Laser as a source and frequency doubling the output can result in increases in the incident energy and quantum efficiency of the detectors such that gains in scattered signal of greater than an order of magnitude over a conventional ruby laser system can be achieved. Due to the ~25Å bandwidth of the Nd-Glass Laser line, however, the electron temperature information in the incoherent scattered signal must be extracted from a Voight Profile, not from the usual Gaussian profile as when a ruby laser is used.

## INTRODUCTION

In a pulsed plasma experiment where electron densities on the order of $10^{16}$ to $10^{10}$ $cm^{-3}$ occur, and where a synchronous detector technique cannot be used, there are two possible alternatives for improvement of the standard Thomson scattering techniques[1] for diagnosing the plasma: (1) use of a ruby laser with high quantum efficiency ERMA (extended red multialkali) detectors, or (2) use of an alternative laser source. With ERMA photomultipliers, measurements around $10^{12}$ $cm^{-3}$ are possible with a ruby laser source.[2] However, at such densities the uncertainties in the density determination can reach about 50 percent.[2] The second alternative has been suggested in the literature[3], and one case of the use of such a system for plasma diagnostics has been noted.[4] However, the characteristics of this diagnostic variation have not been developed or reported. The viability of this approach to low electron density diagnostics will be established in this work, as well as the specification of the proper technique for electron temperature evaluation. The method outlined here presumes the use of an optical multichannel analyzer with an SIT having S-20 type photocathode response.

## ADVANTAGES OF THE SYSTEM

Basically, three advantages accrue with the use of frequency doubled Nd-glass. One principle advantage, that of improved detector efficiency, had been noted by G. S. Voronev.[4] In the case considered here, the quantum efficiency for an S-20 photocathode increases from 0.02 at 694.3 nm for a ruby laser to 0.13 at 530 nm for a doubled

Nd-glass or Nd-YAG laser. A second advantage comes from the larger laser pulse energies that can be delivered. Ruby lasers can achieve a maximum of about 15 J for a 30 nsec Q-switched pulse. However, a doubled glass system is capable of output energies around 76 J; this limiting value is arrived at from the doubling crystal characteristics. Using CDA as the doubling media in a one pass configuration,[5] the following parameters[6] are used for the estimation of the doubled output energies available: power conversion efficiency of 30 percent, damage threshold of 300 $W/cm^2$, and crystal face diameter of 6 cm. With the presumption of a 30 nsec pulse width, a conservative output of 50 J will be used here. With this doubled output, the Nd-glass laser itself requires a 30 nsec Q-switched output of 170 J. Consequently, a more realistic comparison would be made with existing or available components. Using an Apollo ruby laser with a 7 J output, it was found that if this laser was converted to use Nd-glass rods of the appropriate size, an output of 20 J would be possible before doubling; this produces a doubled output of 6 J. The third advantage inherent in a doubled glass system is that for the same incident energy there is less heating effect, and hence perturbation, on the plasma. This effect is expressed by[1]

$$\frac{\Delta T_e}{T_e} = 3.24 \frac{n_e \ell n \Lambda}{f_i^2 A T_e^{5/2}} W_i \tag{1}$$

where $T_e$ is electron temperature in eV , $n_e$ is electron density in $cm^{-3}$, $\ell n\lambda$ is the Coublomb logarithm, $f_i$ is the incident frequency in Hertz, A is the focused spot size in $cm^2$, and $W_i$ is the incident energy in Joules. Thus,

$$\frac{(\Delta T_e/T_e)_{ruby}}{(\Delta T_e/T_e)_{doubled\ glass}} = \left(\frac{f_{i_{glass}}}{f_{i_{ruby}}}\right) = 1.7 \tag{2}$$

## COMPARISON OF FREQUENCY DOUBLED SYSTEMS

Nd-glass is not the only laser where doubling the frequency of the output places it in a more efficient regime of detection. The tabulation below presents data for systems on a comparative basis, first where the maximum reasonable output energy is taken for each system and second where available off-the-shelf rods and optics are used. Again the same S-20 detector response is presumed, and the same plasma parameters and laser pulse width for both are assumed. The scattered signals are normalized to the ruby result:

Laser System Comparison--Ultimate Energy Output

| System | Scattering Energy | Scattered Signal |
|---|---|---|
| Ruby | 15 J | 1 |
| Doubled Nd-Glass | 50 J | 21.7 |
| Doubled Ruby | 4.5 J | 2.1 |
| Doubled Nd-Yag (2 Amplifiers) | 1.8 J | 0.78 |

Laser System Comparison--Available Energy (1 Amplifier)

| System | Scattering Energy | Scattered Signal |
|---|---|---|
| Ruby | 7 J | 1 |
| Doubled Nd-Glass | 6 J | 5.6 |
| Doubled Ruby | 2.1 J | 2.1 |
| Doubled Nd-Yag | 0.6 J | 0.56 |

## INTERPRETATION OF SCATTERED SPECTRUM

While a doubled Nd-glass system can provide benefits in scattered signal magnitude, there is also one difficulty with this system; in standard incoherent Thomson scattering diagnostics, the laser line is usually treated as a delta function in wavelength. Nominally, however, an Nd-glass laser has a bandwidth of about 5 nm. Once the beam had undergone frequency doubling, this bandwidth is halved to 2.5 nm. This bandwidth is on the same order as the Doppler broadened bandwidth of a scattered signal from a plasma with a few electron volts in temperature.

Accordingly, the laser bandwidth should be taken into account. To do this, it can be presumed that the electrons have a Maxwellian velocity distribution and that the laser line is Lorentzian in shape.[7] Thus, the photocathode current generated by the scattered signal becomes[1]

$$I_{pc} = \frac{r_o^2 d\Omega n_e L\eta e}{ha2\sin\frac{\theta}{2}\sqrt{\pi}} \left|\hat{s}x(\hat{s}x\hat{E}_{io})\right|^2 P_i d\lambda \int_o^\infty d\lambda' \frac{\exp\left[\dfrac{-c^2(\lambda-\lambda')^2}{4a^2\lambda'^2\sin^2\frac{\theta}{2}}\right]}{(\lambda'-\lambda_i)^2+\Delta\lambda^2}\cdot\frac{\Delta\lambda}{\pi} \qquad (3)$$

Performing a variable change of $x + \lambda_i$ for $\lambda'$, Equation 3 becomes

$$I_{pc} = \frac{r_o^2 d\Omega n_e L\eta e}{ha2\sqrt{\pi}\sin\frac{\theta}{2}} \left|\hat{s}x(\hat{s}x\hat{E}_{io})\right|^2 P_i d\lambda \int_{-\lambda_i}^\infty dx \frac{\exp\left[-\dfrac{c^2\{x-(\lambda-\lambda_i)\}^2}{4a^2\sin^2\frac{\theta}{2}(\lambda_i+x)^2}\right]}{x^2+\Delta\lambda^2}\frac{\Delta\lambda}{\pi} \qquad (4)$$

where $r_o$ is the classical radius of an electron, $d\Omega$ is the solid angle viewed by the detection apparatus, $n_e$ is the electron density, L is the length of the scattering volume, $\eta$ is the detector quantum efficiency, e is the electronic charge, h is Planck's constant, a is the electron thermal speed or $\sqrt{kT_e/m_e}$, $\theta$ is the angle between incident and scattered directions, $\hat{s}$ is the unit vector in the scattering direction, $\hat{E}_{io}$ is the unit vector in the direction of the incident electric field, $P_i$ is the incident laser power, c is the velocity of light, $d\lambda$ is the width of the detection channel in wavelength, $\lambda_i$ is center of the laser line, $\Delta\lambda$ is the half bandwidth at half maximum of the laser line, and $\lambda$ is the center of detection channel.

The integral in equation (4) takes on a familiar form under certain conditions: (1) if x can be ignored compared to $\lambda_i$, and (2) if the lower limit can be extended from $-\lambda_i$ to $-\infty$.

The conditions noted above will now be discussed. It can be observed that the valuesof x that contributes to the integral are << $\lambda_i$, thus permitting expansions in $x/\lambda_i$. So, letting $\alpha$ be $\lambda - \lambda_i$ and

$$\Delta\lambda_{1/e} = 2a \sin \frac{\theta}{2} \frac{\lambda_i}{c} \tag{5}$$

the integral becomes

$$\frac{\Delta\lambda}{\pi} \int_{-\lambda_i}^{\infty} \frac{\left[- \frac{(x-\alpha)^2}{\Delta\lambda_{1/e}^2} (1 + \frac{x}{\lambda_i})^{-2}\right]}{x^2 + \Delta\lambda^2} dx \tag{6}$$

Expanding to first order in $x/\lambda_i$ gives

$$\frac{\Delta\lambda}{\pi} \int_{-\lambda_i}^{\infty} \frac{dx}{x^2 + \Delta\lambda^2} \exp\left[- \frac{(x - \alpha)^2}{\Delta\lambda_{1/e}^2}\right] \left(1 + \frac{2x(x - \alpha)^2}{\lambda_i \Delta\lambda_{1/e}^2}\right) \tag{7}$$

This contains the term desired and the estimation of the first order error. Examining only the error term with the substitution of

$$y = \frac{(x - \alpha)}{\Delta\lambda_{1/e}} \tag{8}$$

yields

$$\text{ERROR} \approx \frac{2\Delta\lambda\Delta\lambda_{1/e}}{\pi\lambda_i} \int_{-\lambda/\Delta\lambda_{1/e}}^{\infty} \frac{e^{-y^2} (\Delta\lambda_{1/e}y + \alpha)y^2 dy}{(\Delta\lambda_{1/e}y + \alpha)^2 + \Delta\lambda^2} \tag{9}$$

Since $\lambda/\Delta\lambda_{1/e}$ is >> 1, the lower limit of this integral can be extended to $-\infty$. Also, the y in the denominator will be set to zero and this will only serve to increase the value of the integral. Thus, equation (9) becomes

$$\text{ERROR} \approx \frac{2\Delta\lambda\Delta\lambda_{1/e}}{\pi\lambda_i} \int_{-\infty}^{\infty} \frac{e^{-y^2} (\Delta\lambda_{1/e}y + \alpha)y^2 dy}{\alpha^2 + \Delta\lambda^2} \tag{10}$$

This integral can now be evaluated with the result

$$\text{ERROR} \approx \frac{1}{2\sqrt{\pi}} \frac{\Delta\lambda}{\lambda_i} \frac{\alpha\Delta\lambda_{1/e}}{\alpha^2 + \Delta\lambda^2} \tag{11}$$

This is a small term due to $\Delta\lambda << \lambda_i$. Also, $\alpha$ is on the order of $\Delta\lambda_{1/e}$. This establishes the validity of the neglect of x compared to $\lambda_i$.

Next, the error involved in setting the lower limit of equation (4) equal to $-\infty$ can be shown to be even less than that of equation (11). In this case,

$$\frac{\Delta\lambda}{\pi} \int_{-\lambda_i}^{\infty} \frac{dx}{x^2 + \Delta\lambda^2} \exp\left[-\frac{(x-\alpha)^2}{\Delta\lambda_{1/e}^2}\right] = \frac{\Delta\lambda}{\pi} \int_{-\infty}^{\infty} \frac{dx \exp\left[-\frac{(x-\alpha)^2}{\Delta\lambda_{1/e}^2}\right]}{x^2 + \Delta\lambda^2} - \frac{\Delta\lambda}{\pi} \int_{-\infty}^{-\lambda_i} \frac{dx \exp\left[-\frac{(x-\alpha)^2}{\Delta\lambda_{1/e}^2}\right]}{x^2 + \Delta\lambda^2} \tag{12}$$

Thus, the error term to examine here is

$$\frac{\Delta\lambda}{\pi} \int_{-\infty}^{-\lambda_i} \frac{dx}{x^2 + \Delta\lambda^2} \exp\left[-\frac{(x-\alpha)^2}{\Delta\lambda_{1/e}^2}\right] \tag{13}$$

Letting y be $(x + \lambda_i)/\Delta\lambda_{1/e}$ reduces equation (13) to

$$\text{ERROR} \approx \frac{\Delta\lambda}{\pi\Delta\lambda_{1/e}} \int_{-\infty}^{o} \frac{dx \exp\left[-\left(y - \frac{\lambda}{\Delta\lambda_{1/e}}\right)^2\right]}{\left(y - \frac{\lambda_i}{\Delta\lambda_{1/e}}\right)^2 + \left(\frac{\Delta\lambda}{\Delta\lambda_{1/e}}\right)^2} \tag{14}$$

Following Laplace's method[8] the error can be shown to be

$$\text{ERROR} \approx \frac{\Delta\lambda\Delta\lambda_{1/e}}{\pi[\lambda_i^2 + \Delta\lambda^2]} \frac{\Delta\lambda_{1/e}}{2\lambda} e^{-(\lambda/\Delta\lambda_{1/e})^2} \tag{15}$$

Again since

$$\lambda \sim \lambda_i >> \Delta\lambda \sim \Delta\lambda_{1/e} \tag{16}$$

the error term shown by equation (15) can be ignored. When this is done, equation (4) reduces to

$$I_{pc} = \frac{r_o^2 d\Omega n_e L\eta_e}{ha2\sin\frac{\theta}{2}\sqrt{\pi}} \left|\hat{s}x(\hat{s}x\hat{E}_{io})\right|^2 P_i d\lambda \int_{-\infty}^{\infty} dx \frac{\exp\left[\frac{-c^2\{x - (\lambda - \lambda_i)\}^2}{4a^2 \sin^2\frac{\theta}{2}\lambda_i^2}\right]}{x^2 + \Delta\lambda^2} \frac{\Delta\lambda}{\pi} \tag{17}$$

This integral is readily recognizable on the Voigt integral commonly found in spectroscopic studies. Since the laser bandwidth can be measured, and hence is known, the width of the Maxwellian contribution can be determined from the tabulations of this integral.[9] Accordingly, an accurate value of $T_e$ can then be found by accounting for the laser bandwidth.

The above application of the Voigt profile to the temperature determination is unnecessary when the thermal broadening is large enough to ensure that a 2.5 nm width appears relatively narrow. In quantitative terms, this means that

$$\Delta\lambda_{1/e} > 10 \frac{\Delta\lambda}{2\sqrt{\ell n 2}} \qquad (18)$$

Accordingly, this means that the temperature, $T_e$, must be greater than 100 eV in order to ignore the width of the laser line. At the other extreme, the separation of the Maxwellian from the Lorentzian shape becomes questionable when

$$\Delta\lambda_{1/e} < \frac{\Delta\lambda}{20\sqrt{\ell n 2}} \qquad (19)$$

or when $T_e$ is less than .01 eV. Thus, from .01 eV up to 100 eV, the analysis using a Voigt integral holds and should be used in evaluating the electron temperature with a doubled glass system. Above 100 eV the usual analysis with a Gaussian profile in temperature determination can be applied, while below 0.01 eV it is not possible to distinguish temperature information in a doubled glass scattering system.

## CONCLUSIONS

The advantages that a doubled Nd-glass laser system holds for low density determination can be clearly identified. Using the maximum energies for both laser systems and an S-20 photocathode response on the detectors, the ruby laser can resolve densities down to $\sim 10^{12}$ $cm^{-3}$ while the doubled glass can resolve densities down to $\sim 5 \times 10^{10}$ $cm^{-3}$. However, even if the quantum efficiencies of the detector at 694.3 nm and 530 nm could be made equal, the doubled glass laser scattered signal would still be about a factor of 4 greater than the signal from a ruby system. With the improvement in scattered energy, however, there is the slight complication of using a Voigt profile for the electron temperature determination in the range of 0.01 eV < T < 100 eV.

## REFERENCES


[1] J. Sheffield, Plasma Scattering of Electromagnetic Radiation, Academic Press, Inc., New York, 1975.

[2] J. D. Sethian and C. A. Ekdahl, "Cornell Thomson Scattering System," Rev. Sci. Instrum., Vol. 49, pp. 729-735, 1978.

[3] A. W. DeSilva and G. C. Goldenbaum, Methods of Experimental Physics Part 9A, Edited by H. R. Griem and R. H. Louberg, Chap. 3, Academic Press, Inc., New York, 1971.

[4] G. S. Voronev, Proceedings of the International School of Plasma Physics on Acquisition Systems, edited by H. Eubank, Varenna, Italy, pp. 318-320, 1974.

[5] Tu. Lyubavsky and V. Oushinnikev, Solid State Laser Technology, MIR Publishers, Moscow, 1975.

[6] R. S. Adlov and M. Oregag, "Frequency Doubling Crystals: Unscrambling the Acronyms," Electro-Optical Systems Design, pp. 20-24, Dec., 1974.

[7] A. Yariv, Quantum Electronics, Wiley and Sons, Inc., New York, 1975.

[8] J. D. Munay, Asymptotic Analysis, Clarendon Press, Oxford, 1974.

[9] H. G. Kuhn, Atomic Spectra, Academic Press, New York, 1969.