NORTHROP GRUMMAN

Technical Progress Report

HIGH BETA AND
SECOND STABILITY REGION
TRANSPORT AND STABILITY ANALYSIS

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ABSTRACT

This report summarizes MHD equilibrium and stability studies carried out at Northrop Grumman's Advanced Technology and Development Center during the 12 month period starting March 1, 1994. Progress is reported in both ideal and resistive MHD modeling of TFTR plasmas. The development of codes to calculate the significant effects of highly anisotropic pressure distributions is discussed along with results from this model.

I INTRODUCTION

_This report summarizes work carried out at the Northrop Grumman Advanced Technology and Development Center (formerly Grumman Corporate Research Center) during the 12-month period from March 1st 1994, and supported by the Department of Energy Grant #DE-FG02-89ER51124. The work described here fulfills the program of our Research Plan for 1994, submitted during December 1993. That Research Plan emphasized theoretical and computational support for the D-T phase of operation in TFTR which started towards the end of 1993 and is still in progress. The intention of our research plan was to offer assistance in interpreting the MHD equilibrium and stability properties of experimental data from these TFTR plasmas. Specifically, the program of work which we proposed was divided among three main tasks as follows:

Task 1: MHD Equilibrium and Stability Analysis of TFTR Experimental Data.

(a)   Ideal and Resistive MHD Stability Analysis of TFTR Experimental Data.

(b)   Resistive Tearing Mode Criteria.

Task 2: Transport Studies and MHD Analysis of Evolving Equilibria.

(a)   Study the effects of anisotropic pressure distributions in TFTR using $p_{\parallel}$ and $p_{\perp}$ calculated by TRANSP.

Task 3: Implement Improvements in Equilibrium and Stability Models.

(a)   Implement Improvements in Equilibrium and Stability Models

Substantial progress is reported in each of the above topics. Specifically, Task 1(a) and (b) are discussed in §II.A and II.B while the topic of Tasks 2 is summarized in §II.C. Task 3 appears in §III. We note that part of the work discussed here has been described in two papers presented at the 1994 International Sherwood Meeting held in Dallas, TX. and three more papers were presented at the 1994 APS, Division of Plasma Physics Meeting held in Minneapolis, MN during November 1994. Two journal articles have appeared in the literature[1,2]. These papers together with other associated presentations during the present period are enumerated in the Appendix to this document.

I.A   Acknowledgments

It is a pleasure to acknowledge the continuing interest and cooperation of numerous colleagues at TFTR and the Theory Division at Princeton, Plasma Physics Laboratory. As in previous years, we continue to maintain and expand our interaction with various other plasma physics groups throughout the country. Their informal input to our work is also acknowledged.

II TFTR MODELING

II.A Task 1(a): Ideal and Resistive MHD Stability Analysis of TFTR Experimental Data

The analysis summarized in this section concentrates attention on early, high current ($q_{edge} \sim 5$), D-T experiments in TFTR where slowly growing, coherent MHD modes were observed. The most frequently observed mode had a 4/3 structure though higher order m/n instabilities were also excited. We note that, from an MHD standpoint, these D-T supershot plasmas have essentially the same characteristics as those in deuterium only. Moreover, transport analysis of these plasmas using the TRANSP code suggest that the axial value of q was smaller than unity over a substantial fraction of the plasma radius. This computational prediction was subsequently supported by MSE diagnostic measurements of the magnetic pitch angle as these became available. The observations of q
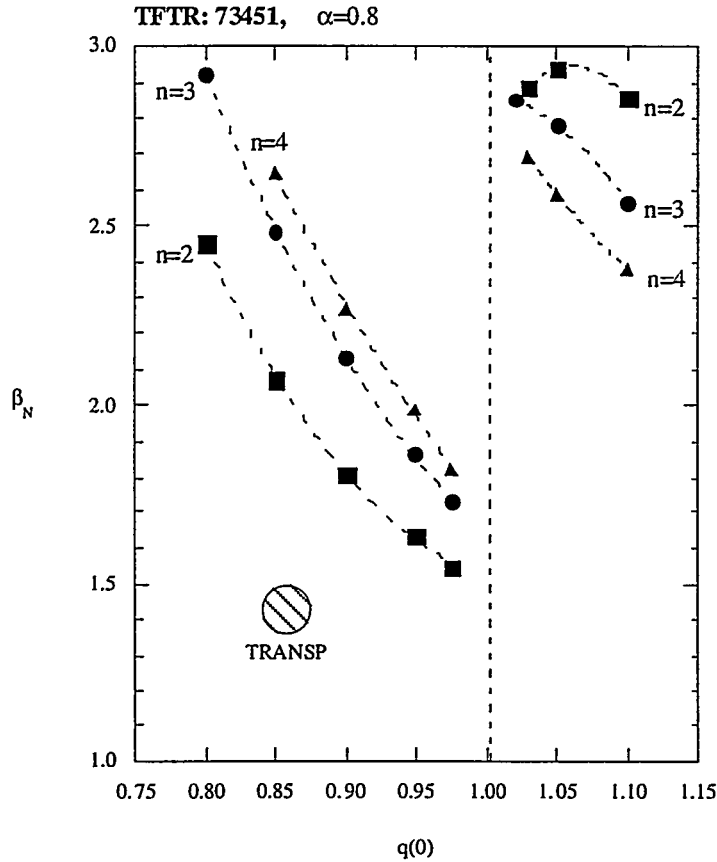
TFTR: 73451, α=0.8

Fig. II.A-1 Beta Limits for low -n modes in TFTR, D-T plasma.

below unity in these and earlier supershot experiments present a significant challenge for ideal MHD theory which predicts that the 1/1 internal mode is unstable when the Bussac beta, $\beta_{p1} > 0.3$. In the experiments of interest here, $\beta_{p1} \sim 0.6$ but the ideal 1/1 mode is apparently stable. Bussac's ideal MHD criterion is invariably violated in supershots when q<1 and where the central pressure gradients are believed to be large as the result of circulating hot ions produced by intense beam heating. This statement, however, is based entirely on TRANSP calculations of the neutral beam pressure and the premise that the total pressure distribution is isotropic. The possible implications of the highly anisotropic pressure distributions in TFTR is discussed in §II.C

In our previous report we noted that the stability properties of the 1/1 internal kink is sensitive to the precise details of the pressure distribution in the immediate vicinity of the q=1 surface. Thus, when a relatively narrow flat spot is inserted in the pressure profile near and inside the q = 1 surface the 1/1 mode is stabilized. Such a flat spot could be produced, for example, by a saturated 1/1 resistive mode located near q=1. While the existence or otherwise of flat spots in the pressure profile in supershot plasmas is a controversial topic at TFTR there is no clear experimental evidence to contradict this hypothesis. Indeed, some modification of the predicted pressure profiles obtained from TRANSP is required to stabilize the Mercier modes which are otherwise excited. When the 1/1 mode is extinguished in this way, ideal modes with n ≥ 2 are computed whose marginal behavior depend on q_0 as indicated in Fig. II.A-1. This figure shows that, when $q_0 > 1$, the larger n-modes have lower beta limits which is characteristic of ballooning mode behavior. When $q_0 < 1$, however, the order is reversed, as is also shown Fig. II.A-1, with the n = 2 mode being the most unstable. It is not clear from these results why the most frequently occurring mode has a 4/3 structure, especially when q<1. When the effect of finite plasma conductivity is included slowly growing resistive versions of these modes appear whose growth rates scale as $S^{-3/5}$. We are currently in the process of studying the non-linear evolution of these resistive MHD instabilities using a version of the ARES code developed recently under Grumman auspices.

II.B Task 1(b) Resistive Tearing Mode Stability Criteria

In his review of MHD stability theory Wesson[3] quoted a simple formula which purports to predict the stability or instability of resistive tearing modes. Subsequently, this same formula was derived more rigorously by Strauss[4] and, recently, by Hegna and Callen[5]. The latter derive an expression for the resistive MHD index, Δ', treating 1/m as a small number, where m is the poloidal mode number. Using Δ' = 0 as the threshold

condition for instability a simple stability criterion was derived. Specializing to cylindrical geometry, this stability criterion can be cast in terms of the toroidal mode number, n, to obtain

$$n < \overline{nmax} = \frac{r_s}{s} \cdot \left(\frac{1}{B_z/R} \right) \cdot \frac{dJ_{\parallel}}{dr} \qquad (1)$$

as the condition for instability. Here, s is the shear and $dJ_{\parallel}/dr$ is the current density gradient both measured at the rational surface located at r_s. For given equilibrium parameters the prediction of this formula is best visualized graphically.

Fig. II.B-1   Tearing Mode Stability Diagram
This diagram illustrates the prediction of equation (1)
in cylindrical geometry using $q = 1.2*(1+(r/0.4)^8)^{1/4}$.
In this example, only tearing modes with m/n=3/2, 4/3
and 5/4 are unstable.

Thus, for example, Fig. II.B-1 shows a tearing stability diagram corresponding to the 'flat' current density profile of Furth, Rutherford and Selberg (FRS)[6]. This example shows how the criterion (1) predicts, in this particular case, that the 2/1 tearing mode is stable while higher order m/n modes can be unstable. Since this is precisely the behavior that is observed in TFTR, and if the prediction is correct implicitly provides information regarding the current profile, it is clearly of interest to assess the validity of the criterion (1) more closely. For this purpose, we have used the ARES code to calculate the stability properties of FRS profiles under various conditions and have compared these with the predictions of

(1). The conclusion, however, is that the formula (1) does not accurately predict the onset of the low order m/n tearing modes of interest when compared with the solution of the full resistive MHD equations. In particular, the stability of the 2/1 mode, predicted by (1) and indicated in Fig. II.B-1 , is not in accord with computation. In practice we find that, using the FRS profiles, the 2/1 tearing mode is always unstable contradicting the result exhibited in Fig. II.B-1 and, indeed, the experiment.
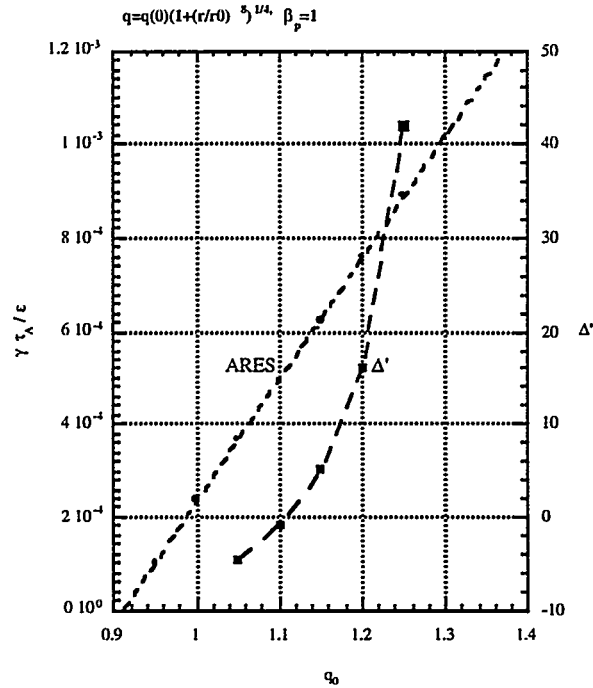

Fig. II B-2 Tearing Mode Growth Rates as Function of q(0)
This diagram shows how the 3/2 tearing mode approaches stability
as q(0) is reduced. The corresponding analytic values of Δ' are also
shown. Computationally, the tearing mode growth rate is still
substantial at the point $q_0=1.12$ where the analytic formula yields $\Delta'=0$.

The lack of agreement between theory and computation is illustrated in fig. Fig. II B-2 where Δ' for the m=3 mode, calculated analytically, is plotted as q(0) is varied, which, for the FRS profiles is simply a mechanism for moving the rational surface relative to the maximum in $dJ_{\parallel}/dr$. In this example, the analytic condition $\Delta'=0$ occurs when q(0)=1.12 while the ARES result, on the other hand, shows that the 3/2 tearing mode is still quite virulent at this point. This comparison is not altered significantly even when 1/m corrections are included in the analytic calculation of Δ'. From a number of similar calculations for higher poloidal mode numbers we find that the formula approaches the computational results only when the poloidal mode number, m > 5, which restricts its

usefulness in modeling and interpreting TFTR plasmas. The value of the criterion (1), however, is that it correctly identifies trends as the current profiles are altered. In using the criterion some care is required to ensure that the quantity $\lambda = n_{max} / 2n$ is less than unity since the ordering involved in Hegna's theory breaks down at that point. This inevitably occurs, for example, when the surface averaged parallel current density possesses an off-axis maximum. These 'reverse shear' profiles are presently a topic of interest.

II.C Stability of Anisotropic Plasmas

During 1994 we have achieved our goal of completing the development of a comprehensive set of codes that include the effects of anisotropic pressure distributions in MHD equilibrium and stability calculations. The code development work was supported primarily by Grumman Corporation. Application of these codes, including benchmarking and some modification tasks, was supported by this Department of Energy Grant. The codes, which now contain an option for anisotropic pressure distributions, include an equilibrium code, a ballooning code which also evaluates the Mercier criterion and a low-n MHD stability code.

Auxiliary heated plasmas invariably have anisotropic pressure distributions. Depending on the details of the heating scheme either parallel pressure is dominant or, in other situations the perpendicular pressure is largest. High performance supershots in TFTR are such that the parallel pressure is usually dominant, particularly near the center of the plasma. This is due to the neutral beam injection angle together with the difference in slowing down times in the perpendicular and parallel directions. Fig. II.C-1 shows the fraction of parallel pressure compared to the total for a high powered DT supershot. The fraction of parallel pressure is initially large but decreases as the particles thermalize, eventually leveling off in the 20% - 30% range. The degree of pressure anisotropy is configuration dependent. For example, ICRF heated plasmas in TFTR are such that the perpendicular component of the pressure is largest.
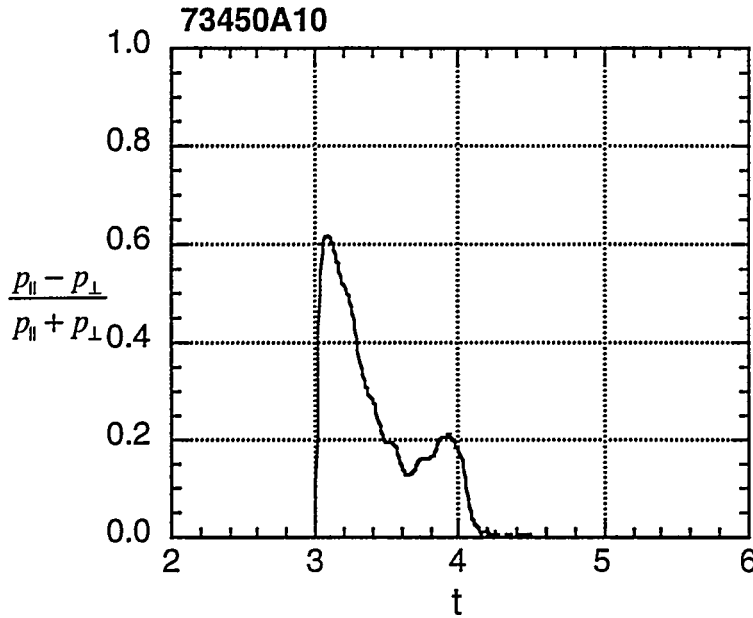
Fig. II.C-1 Fraction of $\delta p_{\parallel}$ vs. time for TFTR DT supershot.

A general survey was completed to determine the effect, if any, of anisotropic pressure distributions on the MHD stability of such plasmas. The analysis was performed using experimental data from recent DT supershots. A relatively simple model was used for the pressure anisotropy in calculating the equilibrium. The degree of anisotropy was varied over a wide range that included the experimental values. The survey included an analysis of high-n ballooning stability, the Mercier criterion, low-n ballooning stability and kink instability.

For high-n ballooning modes perpendicular pressure tends to be destabilizing while parallel pressure is stabilizing[7]. However, for low beta, circular plasmas such as those in TFTR the effect on the first region stability limits was found to be small . This was confirmed in the analysis of DT supershots. On the other hand, there is a significant effect on the transition region and second region boundaries and, hence, on the radial extent of the unstable region. There is no significant consequence for the TFTR experiments since supershots always lie entirely in the first region of MHD stability. The impact of anisotropic pressure on low-n ballooning was also found to be small. In this case, and in contrast to the high-n calculations, parallel pressure was found to be mildly destabilizing.

Anisotropic pressure distributions have an interesting effect on the Mercier criteria. Previous theoretical analysis by Taylor and Hastie[8] indicated that in plasmas where the

parallel pressure is dominant the critical safety factor for Mercier stability could be as low as $q_0 = 0.8$. This effect was confirmed numerically for supershot equilibria where the plasma remained Mercier stable down to $q_0 \approx 0.92$ in agreement with Taylor and Hastie's prediction for this level of anisotropy. In the last few years, better diagnostic measurements and plasma equilibrium reconstruction of the experiments have shown that in the high current cases q_0 is usually less than unity; estimates of q_0 range from 0.7 to 0.9. MHD theory with scalar pressure typically predicts both Mercier and internal kink instabilities when $q_0 < 1$. Experimentally, however, these plasmas are quiescent with MHD activity, when it occurs, attributed to beta dependent instabilities. This observation is inconsistent with isotropic MHD theory. Previous MHD analyses of experimental plasmas therefore artificially restricted the safety factor $q_0 > 1.0$. Hence, in this particular case, including pressure anisotropy effects brings MHD predictions more in line with the experimental observations.
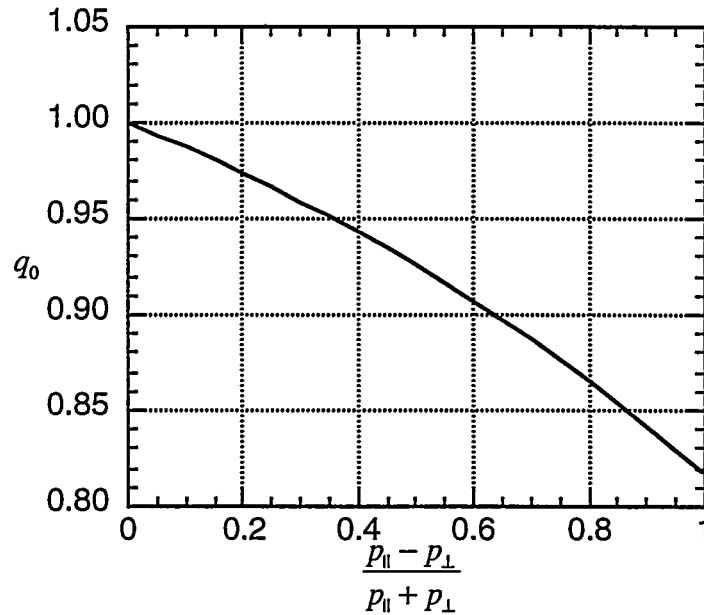

Fig. II.C-2 Critical q_0 for Mercier stability for varying pressure anisotropy.

In contrast to ballooning stability, pressure anisotropy is expected to have a larger effect on kink stability. This is because the kink instability is driven by the parallel current, a substantial part of which is directly proportional to the parallel pressure gradient. Therefore, anisotropic pressure effects should be apparent even in low beta plasmas. To study these low-n instabilities a new code was developed based on the Kruskal-Oberman energy principle[9] which includes the possibility of pressure anisotropy. By using the

Bineau form of the force balance equation[10] considerable analytic simplification is possible in the anisotropic pressure contribution to the energy principle, thereby aiding numerical implementation. Initial studies were carried out for the 1/1 internal kink mode. A typical result for a TFTR, D-T supershot is shown in Fig. II.C-3 . This figure shows how the parallel pressure tends to be destabilizing for the internal kink mode. This differs from our initial results reported in a previous progress report. An error was discovered in the code which invalidated the previous results. According to this present result the experiment should be more unstable to the 1/1 mode when pressure anisotropy is included, although no sawteeth are observed experimentally. Hence the mystery of why tokamak experiments with $q_0 < 1.0$ are apparently stable is still unresolved by this theory.

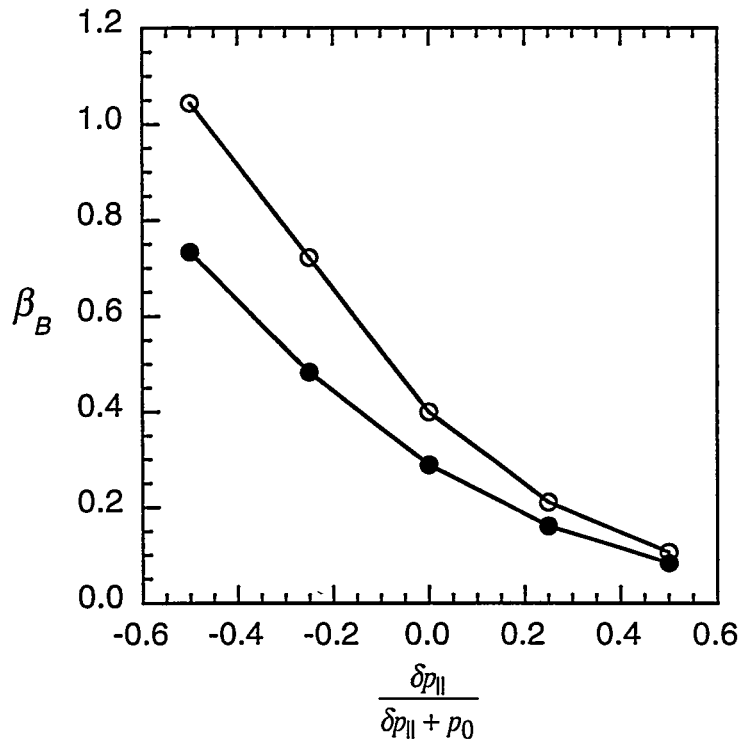Fig. II.C-3 1/1 Internal Kink Beta Limits for TFTR

The effect of anisotropic pressure on external kink mode has also been studied. Again we concentrated attention on recent high beta DT supershot plasmas. For this study the distribution of the parallel current together with the total current was held fixed. The q profile, though, tended to vary somewhat as beta and degree of anisotropy was changed.

As in the isotropic case the external kink stability was found to be mainly influenced by the shape of $j_{\parallel}$. Enhanced parallel pressure was found to be mildly destabilizing. This may have a somewhat stabilizing influence on the experiment since, although the parallel pressure tends to be larger than the perpendicular component near the plasma center, at the edge the perpendicular pressure is often larger. Overall, the effects of the anisotropic pressure were found to be less pronounced than in the case of the internal kink mode probably because the external kink is influenced mainly by conditions near the plasma edge where the pressure is small.

In high beta and more highly shaped plasmas the level of pressure anisotropy that exists in TFTR would be expected to have a more significant influence on MHD stability. The next generation tokamak experiments such as ITER and TPX will be predominately auxiliary heated and have a significantly higher beta than TFTR. Hence, in studies of these tokamaks anisotropic pressure effects should be even more important. The principal result pertaining to ITER may be the ability to operate free of Mercier instabilities with $q_0 < 1$. Moreover, the effects of pressure anisotropy on the ballooning mode and the external kink mode should prove interesting in formulating advanced operating modes for TPX.

III CODE ENHANCEMENTS

Task 3: Implement Improvements in Equilibrium and Stability Models.

The analysis and identification of instabilities observed in tokamak experiments continues to be an evolving and interesting area of research. This requires an on-going effort to maintain and improve our analysis tools. A substantial amount of code development was performed during this year to add new capabilities to our equilibrium and stability codes. This will enable us to address specific stability issues for TFTR and future fusion experiments. The code development was carried out entirely under Grumman Corporation auspices. We mention this here since it directly impacts our accomplishments under this contract. Specifically, our low-n MHD stability code underwent a major upgrade to include anisotropic pressure effects and to permit a vacuum region between the plasma and an external conducting wall. Further, a version of the ARES resistive MHD code was updated to include the nonlinear elements of the MHD equations and is now in the process of being validated. Under the present contract we have benchmarked these codes and implemented improvements to the equilibrium and mapping codes. To facilitate collaborative studies with colleagues at PPPL a mapping code was developed for interfacing our equilibrium code to the PEST code. In addition, a new version of our ballooning mode subroutine, free from the constraint of up-down symmetry, was developed and installed by ourselves in the TRANSP analysis code at PPPL.

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APPENDIX Associated Publications and Presentations

1. M.H. Hughes, M.W. Phillips and E.D. Fredrickson, *Magnetohydrodynamic Stability Studies of Supershot Plasmas,* Phys. Fluids B, **5**(9), (1993), p. 3267.

2. R. Iacono, A. Bhattacharjee, C. Ronchi, J.M. Greene and M.H. Hughes, *Stability of Tearing Modes in Finite-Beta Plasmas,* Physics of Plasmas, **8**, (1994), p. 2645.

3. R. Iacono, A. Bhattacharjee, C. Ronchi, J.M. Greene and M.H. Hughes, *Stability of Tearing Modes in Finite-Beta Plasmas,* 1994 International Sherwood Fusion Theory Conference, Dallas, TX, Paper 1C7

4. M.W. Phillips and M.H. Hughes, *Anisotropic Pressure Effects on Advanced Operating Regimes in Tokamaks,* 1994 International Sherwood Fusion Theory Conference, Dallas, TX, Paper 1D53

5. M.H. Hughes and M.W. Phillips, *MHD Stability Analysis of D-T Plasmas in TFTR,* 1994 International Sherwood Fusion Theory Conference, Dallas, TX, Paper 1D54

6. M.H. Hughes and M.W. Phillips, *Noninear MHD Studies of TFTR Plasmas,* 36th Annual Meeting, APS, Division of plasma Physics, Minneapolis, MN, November 1994

7. M.W. Phillips and M.H. Hughes, *Stability of Anisotropic Plasmas in High Beta Tokamaks,* 36th Annual Meeting, APS, Division of plasma Physics, Minneapolis, MN, November 1994

8. W. Stodiek, M.H. Hughes and M.W. Phillips, *The Nonlinear Stability of the 1/1 Mode in Tokamak Plasma Columns with $q<1$,* 36th Annual Meeting, APS, Division of plasma Physics, Minneapolis, MN, November 1994