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Average Deployments Versus Missile and Defender Parameters

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AVERAGE DEPLOYMENTS VERSUS MISSILE AND DEFENDER PARAMETERS

by

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ABSTRACT

Leakage estimates of boost-phase kinetic-energy defenses as functions of launch parameters and defensive constellation size agree with integral predictions of near-exact calculations for constellation sizing. The calculations discussed here test more detailed aspects of the interaction. They indicate that SBIs can efficiently remove about 50% of the RVs from a heavy missile attack. The next 30% can be removed with two-fold less effectiveness. The next 10% could double constellation sizes.

I. INTRODUCTION

This report evaluates the average number of reentry vehicles (RVs) that could be deployed successfully as a function of missile burn time, RV deployment times, and the number of space-based interceptors (SBIs) in defensive constellations.

II. MISSILE KILLS

The attack is assumed to consist of M missiles simultaneously launched from an area A of effective radius $W \approx \sqrt{(A/\pi)}$. The missiles are assumed to accelerate longitudinally for a booster burn time T_b. Then their buses, which contain the

RVs, separate and perform sequential, largely transverse accelerations to aim and deploy each of their m_o multiple independent reentry vehicles (MIRVs), which takes the bus until time T. Thus, the missiles and their buses are accessible for a total engagement time T, although the buses are of declining value after T_b, as the number of RVs remaining on them declines.

A. Launch Parameters

For current heavy, $m_o = 10$ RV missiles like SS-18s, $T_b \approx 300$ s. For them, deployment takes a similar time, so $T \approx 600$ s. For singlet missiles like SS-21s, which currently make up about 10% of the force, T_b is slightly shorter, and deployment takes about 30 s, i.e., about the time required to off-load a single RV from other missiles, so that $T \approx 300$ s.

The SBIs are assumed to have a total velocity increment $V \approx 6$ km/s. Thus they can reach the launch area from a distance

$$Y = W + V \cdot T, \tag{1}$$

from its center. The mapping that minimizes the number of SBIs required for uniform coverage uses the SBIs nearest the center on missiles near the center of the launch area and those near Y on missiles near W.[1] The fraction of the K interceptors in the constellation that are within range of the launch is thus

$$f = z\pi(W + VT)^2/4\pi R_e^2, \tag{2}$$

where $R_e \approx 6400$ km is the earth's radius and z is a numerical factor that represents the constellation concentration possible over a land launch area of modest latitudinal extent.[2] For these conditions, $z \approx 2.5/\sqrt{(W+VT)}$, where W and VT are in thousands of kilometers.[3]

B. Missiles Killed

The number of SBIs over the launch area at launch time is $\pi W^2 K''$, where K" is the local areal density of SBIs overhead. The number of SBIs that would later flow into the launch area through a circle of radius r in an interval δt at time t is $2\pi(r+Vt)K''\delta r$, where δr is the width of the ring from which they came. Those arriving at t were at t = 0 on a ring of radius r + Vt and of width $\delta r = V \cdot \delta t$. All of the SBIs would fly in at essentially the same constant velocity V, so $\delta r/\delta t \approx V$, and the rate of influx at t is

$$N'(t) \approx \pi W^2 K'' \delta(t) + 2\pi(r+Vt)K''V, \qquad (3)$$

where the first term represents the SBIs that were over the launch area at the time of launch and the second, those flowing in from the outside later. Setting r = W gives the influx of SBIs into the launch corridor. Its integral gives the total number of SBI, and hence kills up to time t:

$$N = \Sigma_0{}^t \, dt \, N' = \pi W^2 K'' + 2\pi(Wt + Vt^2/2)K''V$$
$$= \pi K''(W + Vt)^2. \qquad (4)$$

For t = T, this gives $N(T) = \pi K''(W + VT)^2$. For the SBIs to engage each missile or bus at least once by T requires N = M, or

$$K = (4\pi R_e{}^2/z)K'' = (4\pi R_e{}^2/z)M/\pi(W + Vt)^2 = M/f, \qquad (5)$$

which is the constellation closure relationship at the level of single intercepts per missile. Note that T_b does not enter closure, which depends only on the total time that the missiles and buses are accessible.

C. Time for First Engagement

A parameter of some interest is the time at which all missiles have been engaged at least once. This time, T_1, marks the end of the first round. It can be determined by inverting Eq. (4) and setting $N(T_1) = M$ to give

$$T_1 = [(M/\pi K'')^{1/2} - W]/V, \qquad (6)$$

which is shown in Fig. 1 for current conditions, i.e., A = 10 $(Mm)^2$, M = 1000 heavy missiles, T = 600 s, and a deployment time of T_b = 300 s. For K < 2000 SBIs, the curve for T_1 is almost vertical. With that number of SBIs, there is not enough time to engage all of the missiles and buses before they deploy all of their RVs. At K ≈ 5000 SBIs, T_1 drops below T, and any RVs that would have been deployed later are instead destroyed. By K ≈ 12000, T_1 drops below T_b, and all missiles are engaged before they release their buses. The only RVs that survive are those on missiles that are engaged unsuccessfully.

Figure 2 shows the T_1 curves for near-, mid-, and long-term engagements, i.e., launches from areas with radii 100%, 50%, and 25% of the current 1800 km of heavy missiles with engagement times of T = 100%, 50%, and 25% of the current 600 s. The curve that is lowest at the right is for the near term; the next up is for the midterm; the top one is for the long term. The top horizontal line is for T = 600 s. The next down is both T_b = 300 s for the near-term T = 600 s, and the total engagement time for the near term. The bottom curve is for T = 150 s.

There is a significant separation between the first and second curves; less between the second and the third. In Eq. (6) the first term scales as $1/\sqrt{K''} \alpha 1/\sqrt{(zK)}$. Because $z \alpha 1/\sqrt{(W+VT)}$, when W and T are reduced together as $W \approx VT$, as they would be to stress defenses, the first term scales as $W^{1/4}$, which is quite weak. The second term scales as W, which is comparatively strong. The two terms are of comparable magnitude for near-term conditions, but the second term scales away for W small, leaving

$$T_1 \rightarrow [(M/(2W)/2.5K)^{1/2}]2R_e/V \approx 1.5\sqrt{(M/K)}W^{1/4}R_e/V, \qquad (7)$$

as seen in Fig. 2.

The near-term T_1 cuts its T at $K \approx 5000$ and T_b at $K \approx 12000$. The midterm curve crosses its T = 300 s at $K \approx 15000$ and T_b at $K \approx 30000$. The long-term T_1 does not cross T = 150 s until $K > 30000$. Thus, the T_1 curves are similar, but their crossings, which determine the difficulty of engaging all missiles or buses, are quite different.

III. RV KILLS

Although simple closure assures that all of the missiles and buses are killed, some RVs can still be released if the SBIs arrive after T_b. The correction can be evaluated simply.

A. RVs Destroyed

If the bus starts with m_o RVs and releases them at a constant rate from T_b until time T, then for $t < T_b$ an intercept would destroy all m_o RVs, an intercept at $T_b < t < T$ would destroy

$$m = m_o(T - t)/(T - T_b), \quad (8)$$

and intercepts after T would kill none. This relationship can be used with Eq. (3) for N' and multiplied by p, the single-shot kill probability, to give the average number of RVs killed by time $T_b < t < T$, which is

$$R = \Sigma_0^{Tb} dt\ p[\pi W^2 K''\delta(t) + 2\pi(W+Vt)K''V]m_o$$
$$+ \Sigma_{Tb}^{t} dt\ 2\pi p(W+Vt)K''Vm_o(T-t)/(T-T_b)$$
$$= \pi pK''(W+VT_b)^2 Vm_o + p[2\pi K''Vm_o/(T-T_b)][WT(t-T_b)$$
$$+ (VT-W)(t^2-T_b^2)/2 - V(t^3-T_b^3)/3], \quad (9)$$

where the first term represents the number of kills during the boost phase and the second those during deployment. Setting t = T gives the total number of kills during boost and deployment:

$$R(T) = R_b + 2\pi pK''Vm_o[WT+(VT-W)(T+T_b)/2-V(T^2+TT_b+T_b^2)/3], \quad (10)$$

where $R_b \equiv \pi pK''(W + VT_b)^2 m_o$ is the number of boost phase kills. It can be shown directly that $R(T) \rightarrow R_b$ as $T \rightarrow T_b$, i.e., that the contribution from deployment time goes to 0 as the deployment time becomes short.

B. Fast Missiles

There are several useful limits. The first limit is for fast, MIRVed missiles for which $T_b \rightarrow 0$ and R reduces to

$$R_{FM} \approx \pi pK''m_o(W^2 + WVT + V^2T^2/3). \quad (11)$$

For small T, R is just $\pi K''W^2 m_o$, the kills by the SBIs already present. R_{FM} increases as T^2 for long deployments. For point launches, i.e., $W \rightarrow 0$, $R_{FM} \rightarrow (VT)^2/3$, corresponding to the averaging of $m \approx m_o(1 - t/T)$ over $N'(t) \approx 2\pi V^2 tK''$.

C. Heavy Missiles

The second limit is for current heavy missiles, for which $T_b \approx T/2$ and R reduces to

$$R = \pi pK''m_o(W^2 + 3WVT/2 + 7V^2T^2/12), \quad (12)$$

which scales as Eq. (11) but increases ≈ 50% more rapidly for intermediate T and about 7/12 ÷ 1/3 = 7/4 times as rapidly for large T. Note that by Eq. (5), $K'' = M/\pi(W + VT)^2$. Therefore,

$$R/m_oM = p(W^2 + 3WVT/2 + 7V^2T^2/12)/(W + VT)^2. \quad (13)$$

so that the fraction of the RVs killed in single engagements is a relatively slowly varying function of T. For T small, the fraction is ≈ 1; for T large, it drops to ≈ 7/12.

D. Constant Deployment Rate

The third limit, rapid deployment, is difficult to see from Eq. (6) because of its assumption that all m_o RVs are deployed within $T - T_b$, which for serial deployments would fail for very short engagement times. If Eq. (9) is altered to treat the RV deployment at a constant rate β, the number of RVs killed in all of T becomes

$$R = R_b + \Sigma_{Tb}{}^{T}dt\ 2\pi p(W + Vt)K''V(m_o - \beta t)$$
$$= R_b + 2\pi pK''V[Wm_o(T-T_b)+(Vm_o-W\beta)(T^2-T_b{}^2)/2-V\beta(T^3-T_b{}^3)/3] \quad (14)$$

which in the limit $T \to T_b$, or fast deployment, is

$$R \to R_b + 2\pi pK''m_oVW(T-T_b), \quad (15)$$

which is the basis for evaluating the serial deployment of smaller numbers of RVs. Note that in this limit, if W were also reduced proportionally, R_b would be reduced to the number of SBIs overhead at the time of launch, which is proportional to W^2 and hence would tend to 0 as W was compressed.

E. Exhaustion-Constrained Intercepts

Equation (10) and the limits just derived from it are based on simple closure, i.e. on the assumption that $T_1 = T$, or that all missiles and buses are engaged once. If the SBI constellation is oversized to reduce leakage, T_1 can be less than T. Then the SBIs arriving between T_1 and T have no first-shot targets left. The modification of the analysis is obvious from Fig. 1. For $T_1 > T$, the second integral of Eq. (9) is cut off at T. Equation (10) still obtains, but no longer corresponds to an intercept of all missiles and buses. For $T_b < T_1 < T$, the lower limit of the second integral is unchanged, but the upper limit is replaced by T_1; and $T \to T_1$ in Eq. (9), making it

$$R = R_b + 2\pi pK''Vm_o[WT_1+(VT_1-W)(T_1+T_b)/2-V(T_1{}^2+T_1T_b+T_b{}^2)/3], \quad (16)$$

which is strongly dependent on K through Eq. (6). For $T_1 < T_b$ the second integral is eliminated; and the upper limit of the second is replaced by T_1, so that

$$R = R_b = \pi p K''(W+VT_1)^2 m_o, \qquad (17)$$

which are the relationships used to generate the figures discussed in the next section.

IV. RESULTS

In the equations above, it is the combination $Y = W + VT$ that enters in ways that the attacker could most easily manipulate. Reducing W or T alone would be less stressing to the defense than reducing the two together. Thus, the calculations below study their joint, proportional reduction, as would rational force modernization programs.[4] The launch parameters are varied from the current $W \approx 1800$ km and $T = 600$ s to one-half, one-quarter, and one-eighth of those values, which roughly spans the values likely in the next few decades.

Figure 3 shows the number of RVs engaged for $M = 1000$ heavy missiles for various values of the launch area and engagement time. The abscissa is the number of SBIs deployed; the ordinate is the number of missiles engaged from Eq. (4). The top curve is for the current $W \approx 1800$ km and $T = 600$ s; the second curve is for 900 km and $T = 300$ s; the third curve is for 500 km and 150 s; and the bottom curve is for 250 km and 75 s. The largest values are typical of current heavy missiles. The next three represent progressive generations of technology, culminating in fast missiles without buses that would end their deployment in the sensible atmosphere, presumably without benefit of decoys.

As expected from Eq. (4), all of the missiles are engaged within the engagement time for $K \approx 5000$ SBIs. For $W = 900$ km the number increases to about 15000 SBIs; for $W = 500$ km it increases to $\approx$ 35000 SBIs. For 125 km the number is about 100000, which is about the limit of economic viability. At that point, for \$200M heavy missiles and nominal \$1M SBIs, the SBIs' cost effectiveness would drop to about \$200M/missile·1000 missiles:\$1M/SBI·100,000 SBIs $\approx$ 2:1.

Figure 4 shows the number of RVs destroyed for current launch parameters. The top curve is the total; the middle curve is for the boost-phase kills; and the bottom curve is for the kills during deployment. As the number of SBIs increases, the absolute number and relative importance of the latter decreases because the increased number of SBIs kill the missiles and buses before they deploy many, if any, RVs. The number of RVs killed increases linearly until it reaches about 5000 SBIs. Then it rolls over, showing the diminishing returns from killing buses with fewer RVs.

The effect is slight up to 7000 SBIs and pronounced thereafter. Although it takes only about 4000 SBIs to kill the first half of the 10000 RVs, it takes ≈ 10000-6000 = 4000 SBIs to go from 8000 to 9000 RVs. At low levels the ratio is ≈ 1.25:1 RVs per SBI, but the last increment drops to ≈ 0.25:1 RVs per SBI. The SBIs' marginal utility falls by an order of magnitude in going to low single-shot leakage levels. The asymptote at 9000 RVs reflects the SBIs' single-shot kill probability of 0.9.

Figure 5 shows the comparable scaling for midterm parameters, W = 900 km and T = 300 s. The number of RVs killed again rises almost linearly to ≈ 15,000, the point at which all of the missiles or their buses are engaged during T. The diminishing returns are more pronounced thereafter. It takes about 30000 SBIs to reduce the leakage to 10%— about 5000 SBIs for the last 500 RVs. At that point, the contribution from boost is about 90%; that from deployment is less than 10%. Because such increases in the number of SBIs deployed are neither simple nor cheap and since the kill probabilities assumed are optimistic compared with experience, it is expected that leakage from the boost phase could be substantial in the near-term and midterm.

Figure 6 shows the scaling for essentially a point launch, W = 125 km and T = 75 s. The former is near the limit set by fratricide, which dictates a spacing of ≈ 10 km. For 1000 missiles, that gives a launch diameter of $\approx \sqrt{1{,}000} \cdot 10$ km ≈ 300 km. Deployment would have to be nearly instantaneous, endoatmospheric, and hence without decoys.[5] Deployment could be performed this fast but would probably have to be executed by providing each RV with its own integral precision bus. Such buses are thought to weigh about as much as the RV, so the net effect would be to reduce the number of RVs per missile and their total by about a factor of 2. Figure 4 should be generated with Eq. (14) but is instead calculated with Eq. (17). The message from either is that very large numbers of RVs could leak from affordable boost-phase defenses in the long term.

A useful parameter in evaluating SBI constellations is their exchange ratio, i.e., the ratio of missiles and RVs destroyed per SBI deployed. The former follows directly from Eq. (4):

$$N = \pi(zK/4\pi R_e^2)(W + VT)^2 = (2.5K/4R_e^2)(W + VT)^{3/2}, \quad (18)$$

up to a maximum of N = M. Thus, $dN/dK = (2.5/4R_e^2)(W + VT)^{3/2}$, which falls in proportion to $W^{3/2}$, as seen in Fig. 1. The exchange ratio for RVs can be obtained by substituting Eq. (6),

$$T_1 = [2R_e(M/zK)^{1/2} - W]/V, \quad (19)$$

into Eq. (10), (16), or (17), as appropriate, and differentiating, which produces the exchange ratios shown in Fig. 7. The top curve is for the near term; the middle curve is for the midterm; and the bottom curve is for the long term. The near-term curve has the value $dR/dK \approx 1.2$ up to about 5000 SBIs. It then drops sharply to ≈ 0 at $K \approx 15000$. The midterm curve starts out about a factor of 3 lower, just the $W^{3/2}$, or $(1/2)^{3/2} \approx 0.35$ reduction resulting from absenteeism in Eq. (18). It starts falling off at $K \approx 15000$. The long-term curve is down about another factor of 3; it does not fall off in the interval shown.

Ideally, one should deploy SBIs until their marginal cost exceeds that of the RVs or of other defenses. For a ≈ \$200M heavy missile with 10 RVs, the value of each RV is ≈ \$20M. Thus, \$1M SBIs with an exchange ratio of 1.2:1 would initially have a cost effectiveness of ≈ 1.2·\$20M:\$1M ≈ 24:1. Beyond 5000 SBIs, as their effectiveness fell, their cost-effectiveness would fall, too. By 15000 SBIs, where $dR/dK \approx 0.02$, their cost effectiveness would be ≈ 0.02·\$20M:\$1M ≈ 0.4:1, adverse, and the SBIs should be replaced by other defenses or discontinued, lest they induce proliferation.

V. SUMMARY AND CONCLUSIONS

Models that are modest extensions of previously tested results can be used to describe the performance of and leakage from boost-phase kinetic-energy defenses as functions of launch parameters and defensive-constellation size. The geometric model used is based on a scaling approximation to detailed kinematic calculations. The model has shown good agreement with the integral predictions of near-exact calculations for constellation sizing, but the calculations discussed here test more detailed aspects of the interaction. Launch geometry alone suggests that the calculations might not be much better than factor-of-2 estimates in absolute terms. For relative variations they should be better.

The calculations indicate that SBI constellations can efficiently remove about 50% of the RVs from a heavy-missile attack, that they can remove the next 30% with roughly factor-of-2 less effectiveness, and that the next 10% could double the constellations' sizes. Lower leakage would require more than one shot. It is treated elsewhere. These proportions hold roughly over a wide range of launch parameters. It is likely that the differential-kill calculations presented above are sufficiently accurate to evaluate the relative cost-effectiveness of further boost-phase attrition and downstream intercepts.

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Fig. 1 Single engagement times vs SBIs.

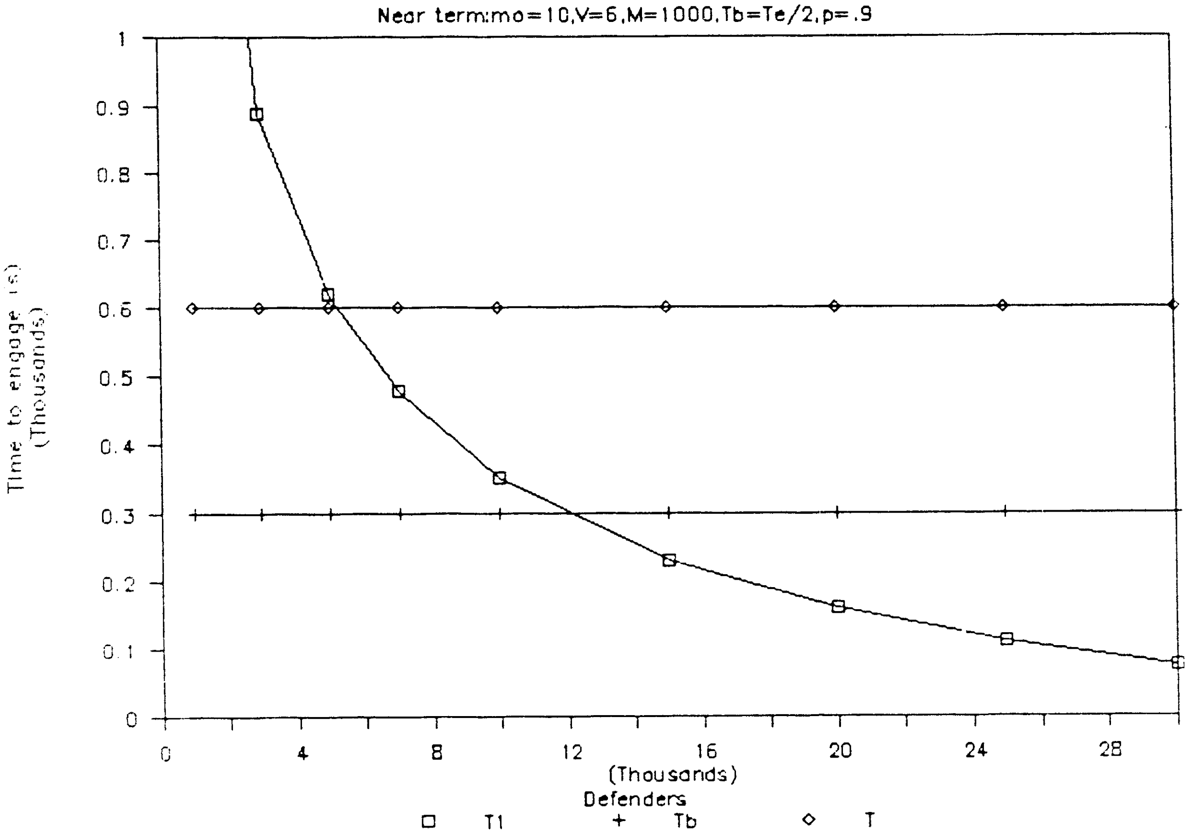


Fig. 2 Missile engagement times vs SBIs.

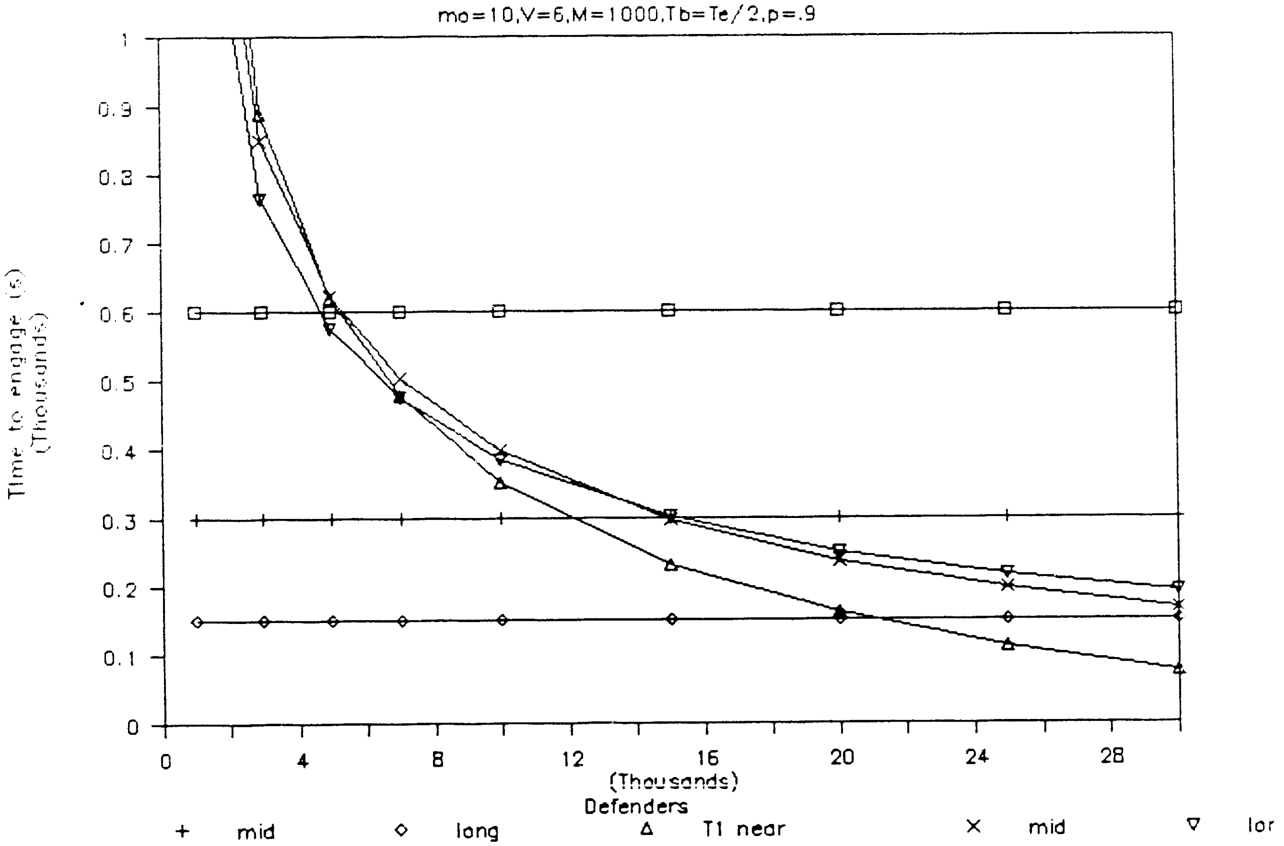


Fig. 3 Missiles engaged vs SBIs.

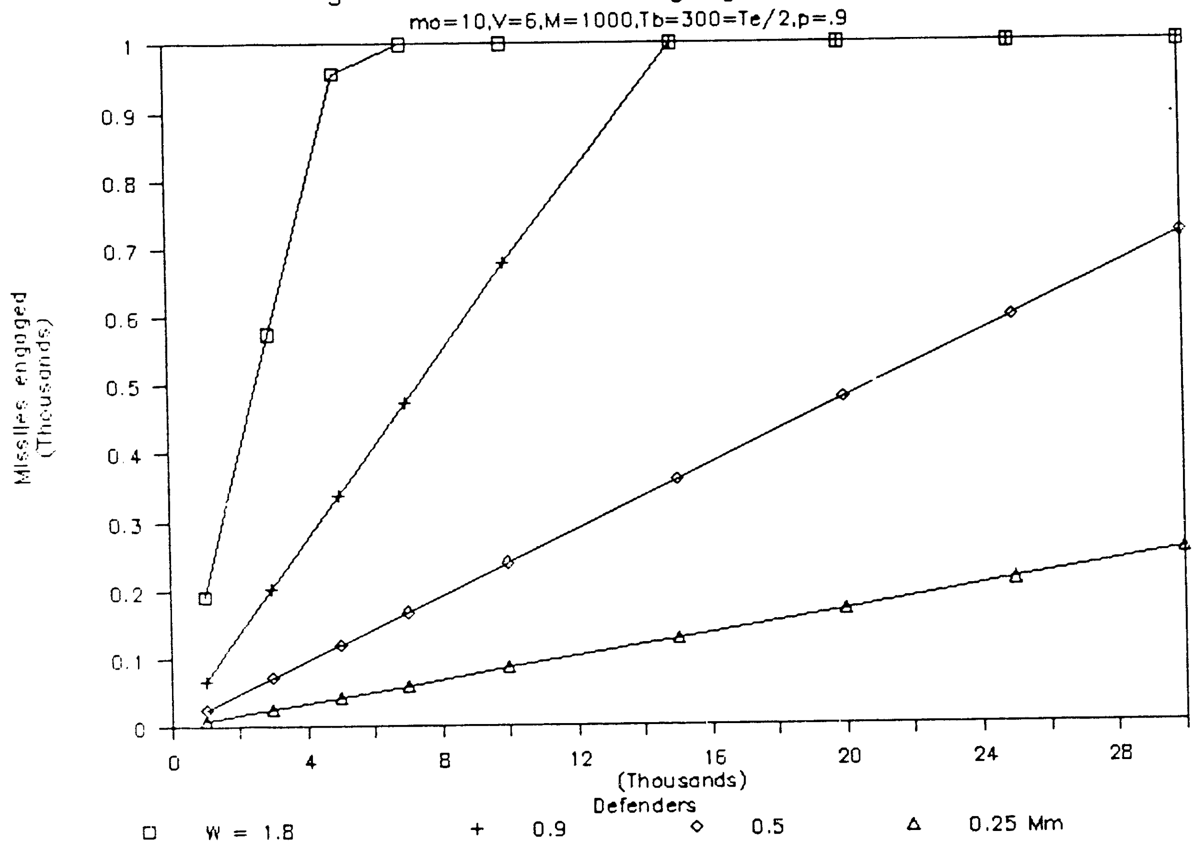


Fig. 4 Reentry vehicles killed vs SBIs.

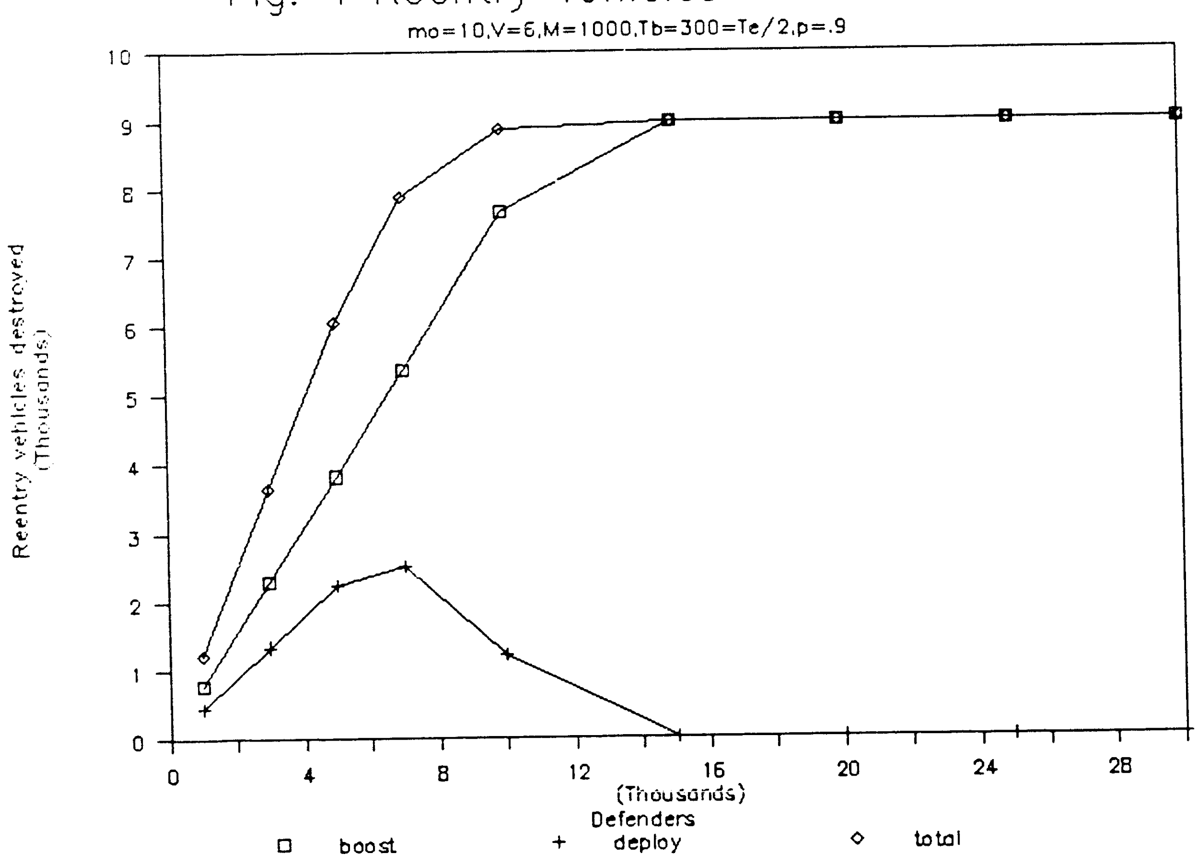


Fig. 5 Reentry vehicles killed vs SBIs.

mo=10,V=6,M=1000,Tb=300=Te/2,p=.9

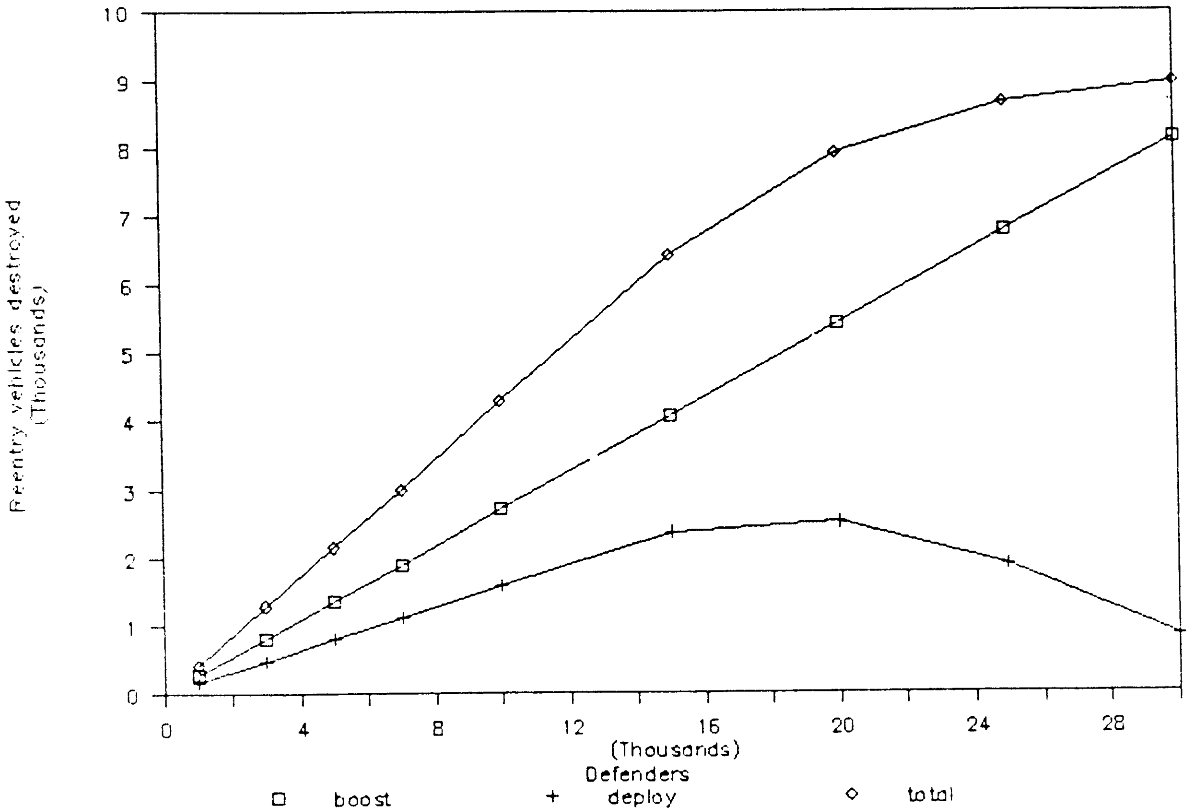


Fig. 6 Reentry vehicles killed vs SBIs.

mo=10,V=6,M=1000,Tb=300=Te/2,p=.9

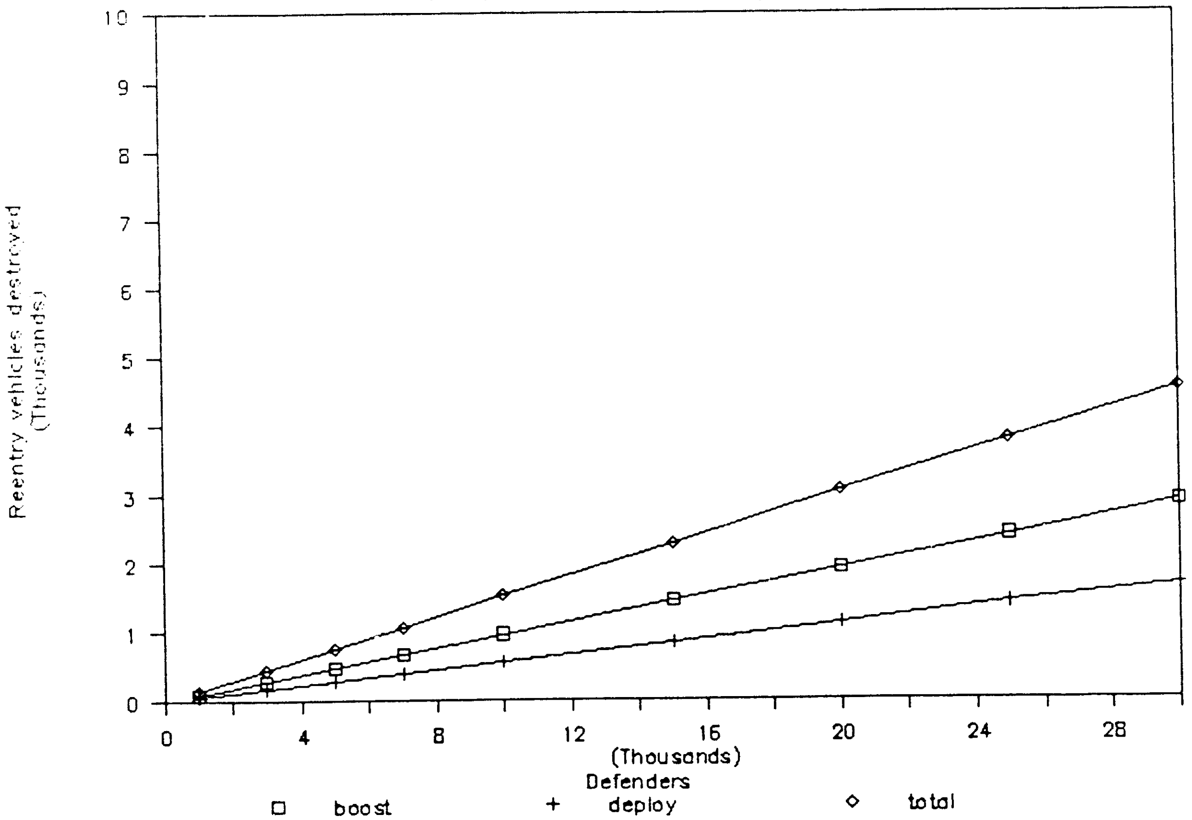
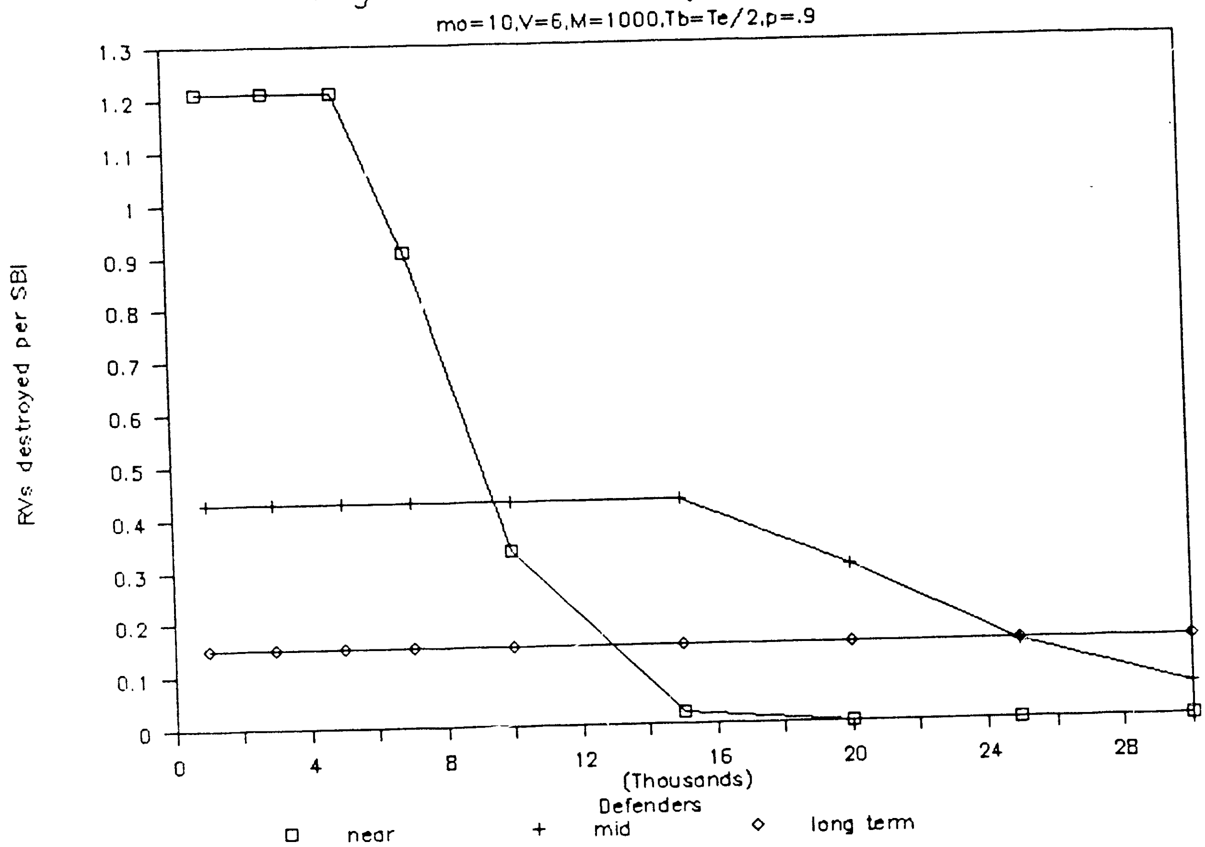


Fig. 7 RVs destroyed vs SBIs.



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