Estimation of Structural Reliability
under Combined Loads

by

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Summary

For the overall safety evaluation of seismic category I structures subjected to various load combinations, a quantitative measure of the structural reliability in terms of a limit-state probability can be conveniently used. For this purpose, the reliability-analysis method for dynamic loads, which has recently been developed by the authors, was combined with the existing standard reliability-analysis procedure for static and quasi-static loads. The significant parameters that enter into the analysis are: the rate at which each load (dead load, accidental internal pressure, earthquake, etc.) will occur, its duration and intensity. All these parameters are basically random variables for most of the loads to be considered. For dynamic loads, the overall intensity is usually characterized not only by their dynamic components but also by their static components. The structure considered in the present paper is a reinforced-concrete containment structure subjected to various static and dynamic loads such as dead loads, accidental pressure, earthquake acceleration, etc. Computations are performed to evaluate the limit-state probabilities under each load combination separately and also under all possible combinations of such loads. Indeed, depending on the limit-state condition to be specified, these limit-state probabilities can indicate which particular load combination provides the dominant contribution to the overall limit-state probability. On the other hand, some of the load combinations contribute very little to the overall limit-state probability. These observations provide insight into the complex problem of which load combinations must be considered for design, for which limit states and at what level of limit-state probabilities.

1. Introduction

For the overall safety evaluation of seismic category I structures subjected to various load combinations, a quantitative measure of the structural reliability in terms of a limit state probability can be conveniently used. For this purpose, the reliability method for dynamic loads, which has recently been developed in a companion paper by Kako, et al [1], was combined with the existing standard reliability analysis procedure for static and quasi-static loads. The significant parameters that enter into the analysis are: the rate at which each load (dead load, accidental internal pressure, earthquake, etc.) will occur, its duration and intensity. These parameters involve uncertainties for most of the loads to be considered. For dynamic loads, the overall intensity is usually characterized not only by their dynamic components but also by their static components. The structure considered in the present paper is a reinforced concrete containment structure subjected to various static and dynamic loads such as dead loads, accidental pressure, earthquake acceleration, etc. Computations are performed and the limit state probabilities are evaluated under each load combination separately and also under all possible combinations of such loads. Indeed, it is observed from these limit state probabilities that, depending on the limit state condition to be specified, one of the load combinations provides the dominant contribution to the overall limit state probability. It is further observed that some of the load combinations contribute very little to the overall limit state probability. These observations provide insight into the complex problem of which load combinations must be considered for design, for which limit states and at what level of limit state probabilities. Such insight will be helpful in examining deterministic safety checking formats for the probability-based structural design.

2. Containment Loads

As described in more detail in a second companion paper by Shinozuka, et al [2], four types of loads are taken into consideration in the present analysis. They are dead and live (D/L) loads, the accidental internal pressure (P) load and earthquake ground acceleration (E). Other loads such as the SRV load will be considered in a future study.

2.1 Dead and Live (D/L) Loads

The dead load is the weight of the dome and the cylindrical wall. The weight density of the reinforced concrete is taken to be 150 lb/ft^3. The dead load is obviously static and assumed to be deterministic. Some live loads act on the containment at the locations where the floors are connected to the containment. The locations and design values of these live loads are shown as follows:

Elevation	856'	828¼'	803¼'	778'	755'
Live Load (kip/ft)	0.707	3.00	0.940	1.02	0.930

For the purpose of the present analysis, the live load is also assumed to be deterministic and equal to the design values.

2.2 Internal Pressure (P; P_L or P_H)

The internal pressure is considered a quasi-static load distributed uniformly on the containment wall. Moreover, it is idealized as a rectangular pulse and will occur at a prescribed expected interval with occurrence rate λ_p (per year), mean duration μ_{dP} (in seconds) and intensity P. The intensity P is treated as a Gaussian random variable with mean $\bar{P}$ and standard deviation σ_p. Two different kinds of internal pressure are considered. One is the accidental pressure P_L due to a large LOCA, but not followed by a hydrogen burn, and the oth-

er is the pressure P_H caused by a hydrogen burn (deflagration) following a large LOCA.

For the accidental pressure P_L, the occurrence rate λ_{P_L} and the mean duration μ_{dP_L} are taken to be 1.0×10^{-4}/year and 1.0×10^6 seconds, respectively, while the intensity P_L is Gaussian with a mean value of 15 psi and standard deviation of 3 psi. If the probability is assumed to be 0.1 for a LOCA to be followed by a hydrogen burn, the occurrence rate of the hydrogen burn λ_{P_H} is 1.0×10^{-5}. It is further assumed that the mean duration μ_{dP_H} of P_H is 600 seconds and that its intensity is Gaussian with a mean value of 45 psi and standard deviation of 9 psi. For mathematical simplicity, the hydrogen accident is assumed to occur independently of the LOCA without, however, allowing their simultaneous occurrence. Although this scenario is somewhat different from the actual situation, the limit state probability based thereupon is expected to be close to that which would follow from the actual sequence of events.

2.3 Earthquake Ground Acceleration (E)

The earthquake ground acceleration is assumed to act only along the horizontal direction. Moreover, it is idealized as a stationary Gaussian process (of finite duration) with mean zero and Kanai-Tajimi spectrum;

$$S_{ggxx}(\omega) = S_0\{1 + 4\zeta_g^2(\omega/\omega_g)^2\}/\{[1 - (\omega/\omega_g)^2]^2 + 4\zeta_g^2(\omega/\omega_g)^2\} \tag{1}$$

where the parameter S_0 represents the intensity of the earthquake. The values of ω_g and ζ_g depend on the soil conditions of the site. For the present study, $\omega_g = 9\pi$ rad/sec and $\zeta_g = 0.6$ are used. Also, the mean duration μ_{dE} of the earthquake acceleration is assumed to be 10 seconds. The peak ground acceleration A_1, given an earthquake, is assumed to be $A_1 = p_g\sigma_g$ where p_g is the peak factor which is assumed to be 3.0 and σ_g is the standard deviation of the ground acceleration such that

$$\sigma_g = \sqrt{\pi\omega_g\{2\zeta_g + 1/(2\zeta_g)\}}\ \sqrt{S_0} \tag{2}$$

and therefore

$$A_1 = \alpha_g\sqrt{S_0} \qquad \text{with} \qquad \alpha_g = p_g\sqrt{\pi\omega_g\{2\zeta_g + 1/(2\zeta_g)\}} \tag{3}$$

If the earthquake occurs in accordance with the Poisson law at a rate λ_E per year, the probability distribution $F_A(a)$ of the annual peak ground acceleration A is related to the probability distribution $F_{A_1}(a)$ of A_1 in the following fashion.

$$F_A(a) = \exp\{-\lambda_E[1 - F_{A_1}(a)]\} \qquad \text{or} \qquad F_{A_1}(a) = 1 + \frac{1}{\lambda_E}\ \ell n\ F_A(a) \tag{4}$$

Therefore, if a_0 indicates the minimum peak ground acceleration for any ground shaking to be considered an earthquake, $F_A(a_0) = 0$ and hence $\lambda_E = -\ell n\ F_A(a_0)$. Assuming that $F_A(a)$ is of the extreme distribution of Type II, $F_A(a) = \exp[-(a/u)^{-\alpha}]$ with $\alpha = 2.61$ and $u = 0.01$, one finally obtains

$$F_{A_1}(a) = 1 - (a/a_0)^{-\alpha} \qquad\qquad a \geq a_0 \tag{5}$$

Under these conditions, one finds that $\lambda_E = 1.50 \times 10^{-2}$/year provided that $a_0 = 0.05$g. Combining eqs. (3) and (5) and writing Z for $\sqrt{S_0}$, one further obtains the probability distribution and density functions of Z in the forms, respectively,

$$F_Z(z) = 1 - (\alpha_g z/a_0)^{-\alpha}; \qquad f_Z(z) = \alpha(\alpha_g/a_0)(\alpha_g z/a_0)^{-(\alpha+1)} \qquad z \geq a_0/\alpha_g \tag{6}$$

3. Limit State Probabilities under Combined Loads

As described in detail in a third companion paper by Chang, et al [3] and also outlined in a paper by Shinozuka, et al [2], the state of structural response is considered to have reached the limit state if the rebars begin to yield (in tension or compression) and/or if the crushing strength of the concrete is reached at the extreme fibre of the wall cross-section anywhere in the containment structure; this implies that the structure is in the limit state if the limit state is reached in at least one of the finite elements. The limit state condition introduced above can be analytically expressed as

$$f_s \geq f_y \qquad \text{and/or} \qquad f_c \geq 0.85 f_c' \qquad (7)$$

where f_s is the stress in the rebars and f_c the compressive concrete stress at the extreme fibres. Since the stresses f_s and f_c are functions of the stress vector $\{\tau\}$, the limit state condition in eq. (7) is in general given in the form of $g(\{\tau\}) \leq 0$ where $g(\cdot)$ is an appropriate function. The equality $g(\{\tau\}) = 0$, representing $f_s = f_y$ and $f_c = 0.85 f_c'$, usually indicates a closed (hyper-) surface or a limit state surface in the $\{\tau\}$ space. To be consistent with the SAP V finite element code used, the stress vector $\{\tau\}$ is given by

$$\{\tau\} = [\tau_1 \quad \tau_2 \quad \tau_3 \quad \tau_4 \quad \tau_5 \quad \tau_6]^T \qquad (8)$$

where the first three are the membrane stress components and the last three the bending moment components of the usual definition; $\tau_1 = \tau_{xx}$, $\tau_2 = \tau_{yy}$, $\tau_3 = \tau_{xy}$, $\tau_4 = m_{xx}$, $\tau_5 = m_{yy}$ and $\tau_6 = m_{xy}$. The i-th stress components $\tau_i^{(D/L)}(e)$, $\tau_i^P(e)$ and $\tau_i^E(e)$ in finite element (e) due, respectively, to D/L, P and E are schematically shown in Fig. 1 as functions of time.

The limit state probability P_f for the structure is defined as the probability that the structural response will reach the limit state during its expected service life T and written as

$$P_f = P_f^{(D/L+P_L)} + P_f^{(D/L+P_H)} + P_f^{(D/L+E)} + P_f^{(D/L+P_L+E)} + P_f^{(D/L+P_H+E)} \qquad (9)$$

In eq. (9), the first term of the right-hand side is the limit state probability of the structure under the action of D/L and P_L only, the second under D/L and P_H only, and so forth. Eq. (9) follows from the fact that, at any time instant, the structure is subjected to one of the following mutually exclusive load combinations: D/L, D/L+P_L, D/L+P_H, D/L+E, D/L+P_L+E, and D/L+P_H+E and from the assumption that the limit state probability under D/L alone is zero. The individual terms in eq. (9) can in turn be written as

$$P_f^{(\cdot)} = T \lambda^{(\cdot)} P^{(\cdot)} \qquad (10)$$

in which $\lambda^{(\cdot)}$ is the rate of occurrence of the load combination $(\cdot)$ while $P^{(\cdot)}$ is the conditional limit state probability given the load combination $(\cdot)$. Following Wen [4], if the structure is subjected to independent loads $L_1, L_2, \ldots, L_N$ which can occur simultaneously and if load L_i arrives in accordance with the Poisson law with an expected arrival rate λ_i and each occurrence lasts on the average μ_{di}, then the expected rate $\lambda^{(L_i+L_j)}$ of the load combination L_i+L_j ($i \neq j$) is

$$\lambda^{(L_i+L_j)} \cong \lambda_i \lambda_j (\mu_{di} + \mu_{dj}) \qquad (11)$$

Similarly, the expected arrival rate for the load combinations $L_i+L_j+L_k$ ($i \neq j, j \neq k, k \neq i$) is

$$\lambda^{(L_i+L_j+L_k)} \cong \lambda_i \lambda_j \lambda_k (\mu_{di}\mu_{dj} + \mu_{dj}\mu_{dk} + \mu_{dk}\mu_{di}) \qquad (12)$$

The obvious extension of eqs. (11) and (12) is also valid. On the other hand, according to Shinozuka and Tan [5], the probability $P_0, P_i, P_{i+j}, P_{i+j+k}, \ldots$ that at any time instant the structure is subjected, respectively, to none of the loads, the load L_i alone, the load combination L_i+L_j ($i \neq j$), the combination $L_i+L_j+L_k$ ($i \neq j, j \neq k, k \neq i$), ... are given by

$$P_0 \doteq 1/\prod_{m=1}^{N}(1+\rho_m), \quad P_i \doteq \rho_i/\prod_{m=1}^{N}(1+\rho_m), \quad P_{i+j} \doteq \rho_i\rho_j/\prod_{m=1}^{N}(1+\rho_m), \quad P_{i+j+k} \doteq \rho_i\rho_j\rho_k/\prod_{m=1}^{N}(1+\rho_m), \ldots \quad (13)$$

where $\rho_i = \lambda_i \mu_{di}$. Throughout, ρ_i are assumed to be $\rho_i \ll 1$, while N is not large.

Using eqs. (11), (12) and (13), the expected duration of each occurrence of L_i (and L_i alone) is, as expected, given by

$$\lim_{t \to \infty} (tP_i)/(t\lambda_i) \doteq \mu_{di} \quad (14)$$

where t is the length of time in which the structure is subjected to the loading environment. The expected duration $\mu_d^{(L_i+L_j)}$ of each occurrence of the load combination L_i+L_j is

$$\mu_d^{(L_i+L_j)} \doteq \lim_{t \to \infty} (tP_{i+j})/(t\lambda^{(L_i+L_j)}) = \mu_{di}\mu_{dj}/(\mu_{di} + \mu_{dj}) \quad (15)$$

Similarly,

$$\mu_d^{(L_i+L_j+L_k)} \doteq \mu_{di}\mu_{dj}\mu_{dk}/(\mu_{di}\mu_{dj} + \mu_{dj}\mu_{dk} + \mu_{dk}\mu_{di}) \quad (16)$$

and so forth. For simplicity, let L_i, L_j and L_k denote D/L, P (P_L or P_H) and E, respectively. Since the D/L loads are always acting on the structure, and only P and E occur in accordance with the Poisson law with respective mean durations, $P_i = P_{D/L} = P_0$, $P_{i+j} = P_{D/L+P}$, $P_{i+k} = P_{D/L+E}$ and $P_{i+j+k} = P_{D/L+P+E}$. Referring to Fig. 1, the frequency interpretation of these probabilities are;

$$\left.\begin{array}{l} P_{D/L} = \lim_{t \to \infty} (t_1^{D/L} + t_2^{D/L} + \ldots)/t \\ P_{D/L+P} = \lim_{t \to \infty} (t_1^{D/L+P} + t_2^{D/L+P} + \ldots)/t \\ P_{D/L+E} = \lim_{t \to \infty} (t_1^{D/L+E} + t_2^{D/L+E} + \ldots)/t \\ P_{D/L+P+E} = \lim_{t \to \infty} (t_1^{D/L+P+E} + t_2^{D/L+P+E} + \ldots)/t \end{array}\right\} \quad (17)$$

Also, $\lambda^{(\cdot)}$ in eq. (10) can be written as

$$\left.\begin{array}{l} \lambda^{(D/L+P_L)} \doteq \lambda_{P_L}, \quad \lambda^{(D/L+P_H)} \doteq \lambda_{P_H}, \quad \lambda^{(D/L+E)} \doteq \lambda_E \\ \lambda^{(D/L+P_L+E)} \doteq \lambda_{P_L}\lambda_E(\mu_{dP_L} + \mu_{dE}) \\ \lambda^{(D/L+P_H+E)} \doteq \lambda_{P_H}\lambda_E(\mu_{dP_H} + \mu_{dE}) \end{array}\right\} \quad (18)$$

In the paper by Shinozuka, et al [2], the conditional probabilities $P_f^{(D/L+P_L)}$, $P_f^{(D/L+P_H)}$, $P_f^{(D/L+P_L+E)}$ and $P_f^{(D/L+P_H+E)}$ are obtained for the limit state defined earlier and with the aid of the analytical models and parameter values also indicated earlier and summarized in Table 1. Substituting eq. (18) into eq. (10) and using these conditional limit state probabilities, one obtains the (overall) lifetime limit state probability (7.92×10^{-4}) for the structure as shown in Table 2.

Table 2 indicates that the major contribution to the overall limit state probability comes from the combination D/L+E (7.23×10^{-4}). The second largest contribution comes from

the combination $D/L+P_H$ (6.88×10^{-5}). The combinations $D/L+P_L+E$ and $D/L+P_H+E$ produce limit state probabilities a few orders of magnitude smaller than those resulting from $D/L+E$ and $D/L+P_H$ in spite of the fact that the conditional limit state probabilities under $D/L+P_L+E$ and $D/L+P_H+E$ are as large as 10^{-3} and 10^{-1}, respectively. This is due to the extremely small expected number of simultaneous occurrences of P_L+E (1.90×10^{-6}) and of P_H+E (1.16×10^{-10}) during the expected service life. Also, in Table 2, the critical finite elements 97-120 under $D/L+P_L$ and $D/L+P_H$ are located at the same elevation level ($36\frac{1}{2}'$ above the base) and have the same limit state probability due to the structural and loading symmetry. Critical elements 6, 7, 18 and 19 under $D/L+E$ and $D/L+P_L+E$ are located in the lowest finite element layer and immediately adjacent to the axis along which the earthquake ground acceleration acts. Finally, critical elements 102, 103, 114 and 115 are located at a level $36\frac{1}{2}'$ above the base, and immediately adjacent to the axis of the earthquake ground acceleration (when projected onto the horizontal plane) for the load combination $D/L+P_H+E$.

4. Concluding Remarks

A reliability analysis method for seismic category I structures subjected to various load combinations is developed and numerical examples are worked out under various assumptions and idealizations. The method essentially uses the frequency domain analysis when dealing with the seismic load. In this respect, it is important to confirm more carefully the validity of the assumed analytical form of the spectral density of the earthquake ground acceleration. The adequacy of the assumption that the acceleration can be idealized as a stationary Gaussian process of finite duration is, however, generally accepted. The importance of the task of taking into consideration in the analysis the uncertain and probabilistic nature of the other analytical models and parameter values used is recognized. However, the limited amount of time and resources made available to the authors prevented them from accomplishing the task at this time. In this regard, statistical and sensitivity analyses to reinforce and complement the reliability analysis presented here are currently underway at Brookhaven National Laboratory.

References

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Acknowledgement

Table 1. Load Parameters (Expected Lifetime T = 40 Years)

Load	Load Parameters
Dead & Live Loads (D/L)	* Deterministic and time invariant
Internal Pressure (P_L) Due to a LOCA	* Occurrence rate $\lambda_{P_L} = 1.0 \times 10^{-4}$/year * Mean duration $\mu_{DP_L} = 10^6$ seconds * P_L = Gaussian with $\overline{P}_L$ = 15 psi and σ_{P_L} = 3 psi
Internal Pressure (P_H) due to Hydrogen Burn	* Occurrence rate $\lambda_{P_H} = 1.0 \times 10^{-5}$/year * Mean duration μ_{DP_H} = 600 seconds * P_H = Gaussian with $\overline{P}_H$ = 45 psi and σ_{P_H} = 9 psi
Earthquake Load (E)	* Stationary random process (a segment of 10 seconds) with a Kanai-Tajimi spectrum $$S_{ggxx}(\omega) = S_0 \frac{1 + 4\zeta_g^2(\omega/\omega_g)^2}{[1 - (\omega/\omega_g)^2]^2 + 4\zeta_g^2(\omega/\omega_g)^2}; \quad \omega_g = 9\pi \text{ rad/sec}$$ $\zeta_g = 0.6$ * Distribution function of $Z = \sqrt{S_0}$ $\quad F_Z(z) = 1 - (\alpha_g z/a_0)^{-\alpha}; \quad a_0 = 0.05g$ and $\alpha = 2.61$ $\quad$ where $\alpha_g = p_g\sqrt{\pi\omega_g\{1/(2\zeta_g) + 2\zeta_g\}}$ with $p_g = 3.0$ * Occurrence rate $\lambda_E = 1.50 \times 10^{-2}$/year * Mean duration μ_{DE} = 10 seconds

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Table 2 Lifetime Limit State Probabilities (T = 40 Years)

Load Combination	Expected Number of Occurrences $_T\lambda(\cdot)$	Conditional Limit State Probabilities $_p(\cdot)$	Limit State Probabilities $p_f(\cdot)$	Critical Finite Elements
D/L	Always Acting	0	0	--
D/L + P_L	4.00×10^{-3}	Numerically Zero		$97,98,...,120$
D/L + P_H	4.00×10^{-4}	1.72×10^{-1}	6.88×10^{-5}	$97,98,...,120$
D/L + E	6.00×10^{-1}	1.21×10^{-3}	7.23×10^{-4}	$6,7,18,19$
D/L + E + P_L	1.90×10^{-6}	1.15×10^{-3}	2.20×10^{-9}	$6,7,18,19$
D/L + E + P_H	1.16×10^{-10}	4.24×10^{-1}	4.92×10^{-11}	$102,103,114,115$
Overall	--	--	7.92×10^{-4}	--

$\tau_i^{D/L}(e)$

DEAD AND LIVE LOADS

$\tau_i^P(e)$

INTERNAL PRESSURE

duration

intensity P

arrival

$\tau_i^E(e)$

EARTHQUAKE

duration

peak acceleration $\alpha_g Z$

$t_1^{D/L}$ $t_2^{D/L}$ $t_1^{D/L+P+E}$ $t_3^{D/L}$ $t_4^{D/L}$

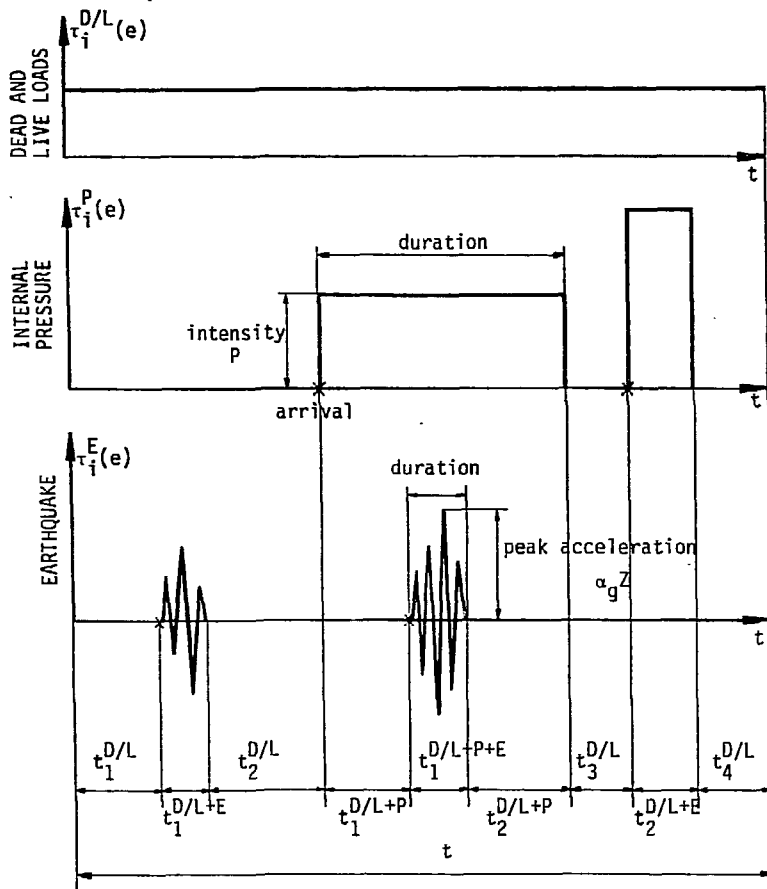$t_1^{D/L+E}$ $t_1^{D/L+P}$ $t_2^{D/L+P}$ $t_2^{D/L+E}$

t

t

t

t

Fig. 1

Figure Captions

1. Load Combinations