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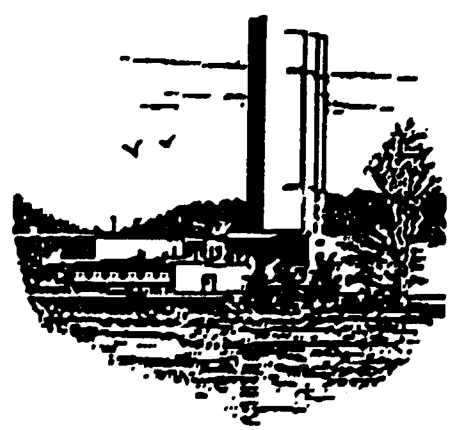

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THE t-J MODEL AT SMALL t/J:

NUMERICAL, PERTURBATIVE AND SUPERSYMMETRIC RESULTS†

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ABSTRACT

We discuss some recent results for one- and two-hole states in the t-J model at small t/J. These include numerical results (bandwidth determinations and accurate t/J values for 4×4 lattice one-hole ground-state level crossings), hopping-parameter perturbation theory (which gives the small-t/J one-hole bandwidth in terms of the static-vacancy ground state), and results at the supersymmetric point $t/J = 1/2$ (exact results for energies and bandwidths). The perturbative results lead us to a new conjecture regarding the staggered magnetization of higher-spin states in the two-dimensional Heisenberg model. We also discuss extrapolation of small-t/J results to high-T_c parameter values; in the two-hole ground states we find $(t/J)^\lambda$ behavior in the rms hole-hole separation, and an extrapolation to $t/J = 3$ gives a bulk-limit rms hole-hole separation of ≈ 7Å

INTRODUCTION: THE t-J MODEL AT SMALL t/J

The "t-J model"[1], which is described by the Hamiltonian

$$H_{tJ} = -t \sum_{<ij>,\sigma} (c^\dagger_{i\sigma} c_{j\sigma} + h.c.) + J \sum_{<ij>} (\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} n_i n_j) \tag{1}$$

with an implicit restriction to unoccupied or singly-occupied sites, has attracted considerable interest as a model of the high temperature superconductors. This is in large part due to suggestions that the closely related two-dimensional Hubbard and Heisenberg spin systems might provide useful models of high temperature superconductors[2], and to the close proximity of the disruption of long-range antiferromagnetic order and the onset of superconductivity as hole doping is increased[3]. Although the t-J model is now believed to be unphysical due to its prediction of hole phase separation[4], it nonetheless continues to be of great interest as a prototype high temperature superconductor model, and may require relatively little modification to correct its unphysical features. The parameters appropriate for the high temperature superconductors are $J \approx 125$ meV (which is relatively well established through comparisons of Heisenberg model predictions with neutron scattering[5]), $t/J \approx 3$ (which is estimated in band

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structure calculations and is much less well established), $a_0 = 3.79$ Å, and fillings close to one electron per site.

The t-J model is unfortunately not amenable to numerical studies using Monte Carlo methods, due to the "minus sign problem" of dynamical many-fermion systems. For this reason numerical studies of energies and other matrix elements have used Lanczos techniques almost exclusively (for a recent review see Dagotto[6]). As Lanczos methods require the storage of several vectors of dimension equal to that of the Hilbert space, which for a given filling fraction increases exponentially with the number of sites N, these studies have been limited to systems of relatively small size, at most $N = 20$ sites[7], and many references consider only the 4×4 lattice.

Although the rather large value of $t/J \approx 3$ is most relevant to the high temperature superconductors, there are several reasons for investigating the small-t/J limit. The first is that at $t/J = 0$ this model is essentially the Heisenberg antiferromagnet with static vacancies, which has been studied using spin-wave theory[8] and Monte Carlo techniques[9]. (These static-hole studies are not very reassuring for Lanczos work, as they find moderately large finite size effects.) The small-t/J limit can also be studied using perturbation theory in the hopping parameter t; this leads to a relation between the structure of the lowest one-hole band and the static hole ground-state wavefunction. Another interesting feature of the small-t/J regime is the occurrence of a supersymmetry at $t/J = 1/2$; this leads to exact relations between the energies of states with different hole number, which are independent of the lattice size. Some of these relations appear in our numerical results as level crossings at $t/J = 1/2$. Finally, some observables such as energies scale quite accurately as powers of t/J; one might hope to establish such behavior at small t/J, where the holes are relatively immobile and finite size effects are less important, and then extrapolate to $t/J \approx 3$ in the bulk limit. We find that this procedure apparently does work for the two-hole ground states of the t-J model, and we estimate the size of these bound states at $t/J = 3$. We shall now discuss these topics (bandwidths, supersymmetry and bulk-limit extrapolations) in more detail.

PERTURBATIVE ONE-HOLE BAND STRUCTURE

The simple $\vec{k}$ dependence of the one-hole band at small t/J can be understood using perturbation theory in the hopping parameter t. The Heisenberg antiferromagnet with static vacancies is taken to be the unperturbed system, and the hopping terms are treated as a perturbation;

$$H_0 = J \sum_{<ij>} (\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} n_i n_j) , \tag{2}$$

$$H_I = -t \sum_{<ij>,\sigma} (c^{\dagger}_{i\sigma} c_{j\sigma} + h.c.) . \tag{3}$$

One then develops a perturbative expansion in the hopping parameter t, using a basis of static-vacancy states in the Heisenberg antiferromagnet[10]. (For related discussions see Dagotto, Joynt, Moreo, Bacci and Gagliano[11] and Elser, Huse, Shraiman and Siggia[12].) All translations of static one-hole states are degenerate under H_0, so one applies degenerate perturbation theory and finds energy shifts which are proportional to t at leading order. Solution of the one-hole secular equation shows that the resulting linear combinations of static-hole states are momentum eigenstates, with a multiplet structure given by

$$\lim_{t/J \to 0} e_h(\vec{k}) - e_h(t=0) = Z_W \cdot 2t\,(\cos k_x + \cos k_y) . \tag{4}$$

The small-t/J "bandwidth renormalization" Z_W (normalized to unity for a free fermion, $W_h = 8t$) is equal to an off-diagonal matrix element of the ground-state wavefunction of a static hole in a Heisenberg background,

$$Z_W = \sum_{S} \Psi_0^*(S', \vec{j}\,') \cdot \Psi_0(S, \vec{j}) . \tag{5}$$

The sum over S is understood to run over all basis states in the appropriate sector of Hilbert space, for example $S^z_{tot} = 1/2$. $\Psi_0(S', \vec{j}')$ is the amplitude to find a spin configuration S' in a static-hole ground state with a hole at $\vec{j}' = \vec{j} + \hat{x}$, where the pair $(S', \vec{j} + \hat{x})$ is constructed from $(S, \vec{j})$ by exchanging the hole at $\vec{j}$ with the spin at $\vec{j} + \hat{x}$. In the basis used to derive (5) the spin-flip terms in $\vec{S}_i \cdot \vec{S}_j$ in the Hamiltonian (2) have positive matrix elements, so that the $\{\Psi_0\}$ are real but do not all have the same sign and hence Z_W in (5) is not positive definite. Negative $W_h = 8Z_W t$ simply implies an inverted multiplet, with $\vec{k} = (0,0)$ rather than (π, π) at the bottom of the band.

A direct Lanczos evaluation of the $S = 1/2$ static-hole matrix element in (5) on the 4×4 lattice gives a bandwidth renormalization of $Z_W = 0.14880571(1)$ and hence a one-hole bandwidth of $\lim_{t/J \to 0} W_h = 1.1904457(1) \cdot t$, which is consistent with the independent $t/J > 0$ Lanczos results shown in Figure 1. The $\vec{k}$-dependence of the theoretical small-t/J dispersion relation (4) is also consistent with the numerical results shown in Figure 1 at small t/J. (The predictions (4) are shown as solid lines.) This band is shown for a somewhat larger range of t/J in Figure 2.

Figure 1. Spin-1/2 one-hole band structure for small t/J.

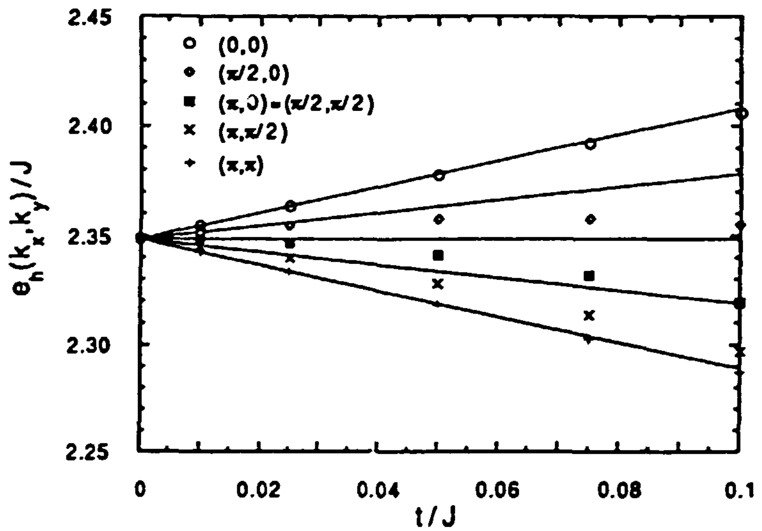
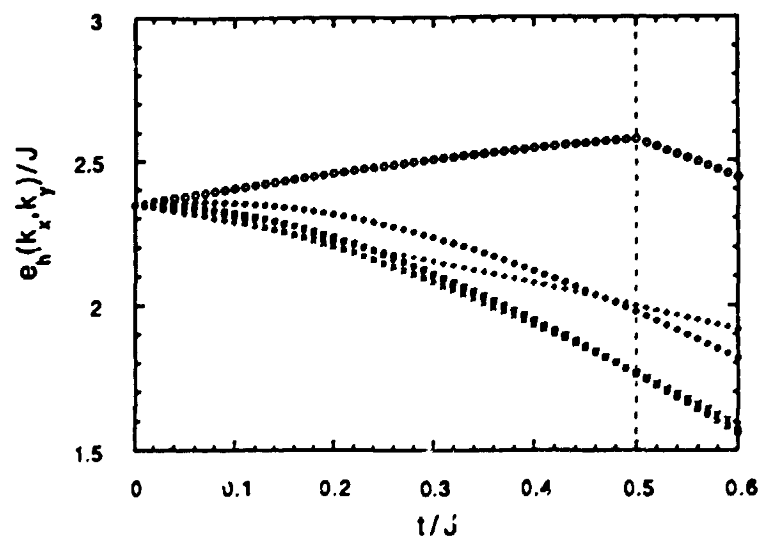


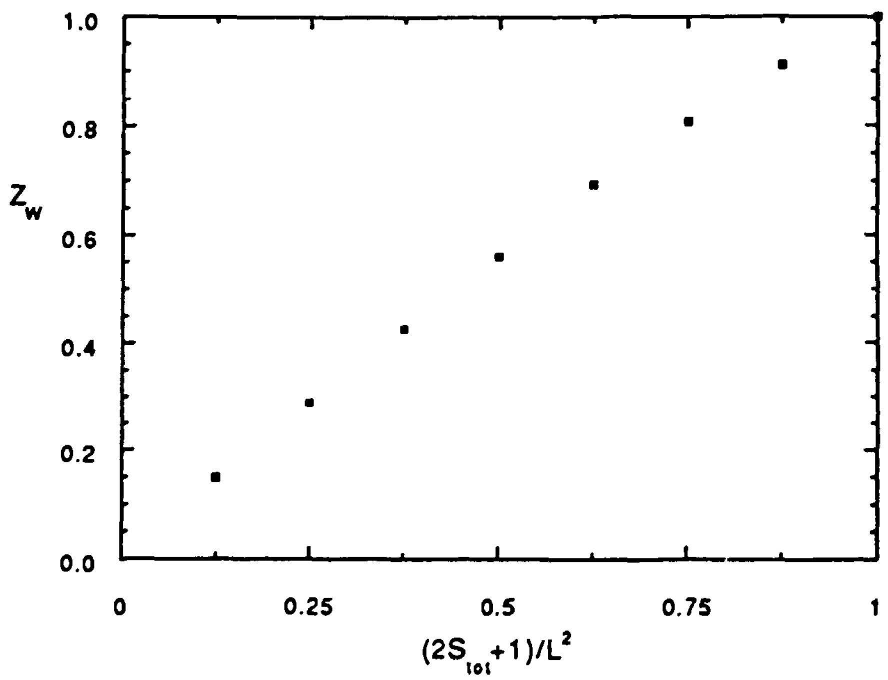
Figure 2. Spin-1/2 one-hole band for 0≤t/J≤0.5
and the supersymmetric point t/J=1/2.



Although Figure 1 and equation (4) appear to confirm that $\lim_{t/J\to 0} W_h = c_1 t$, there is considerable evidence that the coefficient c_1 vanishes in the bulk limit, probably as $c_1 \propto 1/L^2$. Elser, Huse, Shraiman and Siggia[12] argue that c_1 vanishes due to the nonzero bulk-limit staggered magnetization, which acts as a dimerization of the system and reduces the size of the Brillouin zone, resulting in a degeneracy of bulk-limit states which differ by $\Delta\vec{k} = (\pi, \pi)$. In language more appropriate to our bandwidth formula, we would say that the linear-t bandwidth is zero in the bulk limit because the static-hole matrix element in (5) vanishes in this limit. This matrix element is zero because the ground-state staggered magnetizations associated with the initial and final static holes on different sublattices (at $\vec{j}$ and $\vec{j}'$) have opposite signatures. This presumably leaves a bulk-limit one-hole bandwidth of $\lim_{t/J\to 0} W_h = c_2 t^2/J$. Note however that static-hole states with zero staggered magnetization, such as states with sufficiently large total spin, retain a linear-t bandwidth in the bulk limit at small t/J

An interesting question which has not been widely considered is the dependence of the one-hole bandwidth on the total spin. Previous studies have considered $S_{tot} = 1/2$ almost exclusively, as this is the one-hole ground state on a finite lattice. We can use our small-t/J bandwidth formula (5) to determine the linear-t coefficient of $W_h(S_{tot})$ on the 4×4 lattice from the static-hole ground state of the appropriate S_{tot}. Although we have argued that the linear-t term in the small-t/J bandwidth vanishes in the bulk limit for $S_{tot} = 1/2$, this is clearly not the case for all S_{tot}. For example, in the ferromagnetic state with $S_{tot} = S_{max} = (L^2 - 1)/2$, $W_h = 8t$ and $Z_W = 1$ is an exact result. The modulus of Z_W, determined from (5), is shown in Figure 3.

Figure 3. Bandwidth renormalization $Z_W = \lim_{t/J\to 0} W_h/8t$ versus S_{tot}.



Some striking features of $|Z_W|$ are immediately apparent; it is approximately proportional to $2S_{tot} + 1$, and an obvious conjecture is that Z_W is nonzero if the total spin is a finite fraction of the maximum allowed spin S_{max}. The approximate linearity of Figure 3 can be summarized by

$$\lim_{t/J\to 0} |W_h(S_{tot})| \approx 8t \cdot \left\{ \frac{2S_{tot} + 1}{2S_{max} + 1} \right\} . \qquad (6)$$

In view of the argument of Elser, Huse, Shraiman and Siggia that the linear-t bandwidth vanishes in the bulk limit due to the nonzero staggered magnetization, the nonzero bandwidth in (6) implies a conjecture for the pure Heisenberg antiferromagnet, which is that *the bulk-limit staggered magnetization of the 2D Heisenberg antiferromagnet vanishes in the lowest-lying state in any sector with* $S_{tot}/L^2 > 0$. In other words, there is no ground-state staggered magnetization in the bulk limit of the 2D Heisenberg model if the fraction of up (or down) spins differs from 1/2. (This assumes that a single static hole does not have an important effect on the modulus of the bulk-limit staggered magnetization.) This conclusion does not require that $Z_W(S_{tot})$ has the particular approximate form suggested in (6), only that it is nonzero in the bulk limit for $S_{tot}/S_{max} > 0$. Another remarkable feature of $Z_W(S_{tot})$ is that it alternates in sign with increasing spin,

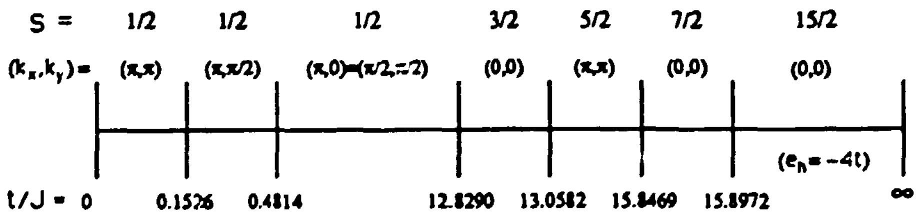
$$\frac{Z_W(S_{tot})}{|Z_W(S_{tot})|} = (-1)^{S_{tot}-1/2} . \tag{7}$$

We shall return to this result in our discussion of the supersymmetric point.

We note in passing that degenerate perturbation theory is not similarly applicable to leading-order bandwidth calculations for two or more holes, since these basis states have H_0 eigenvalues which depend on the relative hole locations. For these systems nondegenerate perturbation theory can be used to determine the two-hole bandwidth, for example, which should therefore satisfy $\lim_{t/J\to 0} W_{hh} = \kappa_2 t^2/J$. This form is consistent with our numerical results on the 4×4 lattice[10] with $\kappa_2 = 2.575(4)$.

As we increase t/J, the simple one-hole dispersion relation (4) begins to show higher-order effects due to multiple hops. This is visible in Figures 1 and 2 as a departure from linearity in the $S_{tot} = 1/2$ one-hole band, which becomes quite pronounced for $t/J \gtrsim 0.1$. As t/J increases there are evidently several changes in the ground-state quantum numbers; the t/J values of these changes and the quantum numbers of each ground-state level are given in Figure 4.

Figure 4. One-hole ground-state level crossings and quantum numbers on the 4 x 4 lattice.



The $(\pi/2, \pi/2)$ level, which becomes the ground state above $t/J \approx 0.4814$ on the 4×4 lattice, is believed to be the true bulk-limit ground state for t/J values of interest here, as the *smaller-t/J crossings recede to $t/J = 0$ in the bulk limit as L^{-2}*. (This is because these level crossings are due to the linear-t bandwidth term, which is expected to vanish for $S_{tot} = 1/2$ in the bulk limit.) At large t/J ($\gtrsim 13$), level crossings to ground states of higher total spin occur, as noted in Figure 4. These indicate the generation of a ferromagnetic Nagaoka polaron, and similar ground-state level crossings presumably first appear at a similar t/J value in the bulk limit.

THE SUPERSYMMETRIC POINT t/J = 1/2

Returning to Figure 2, a level crossing is apparent in the $\vec{k} = (0,0)$ one-hole states at exactly $t/J = 1/2$, where the $S_{tot} = 1/2$ level crosses a descending $S_{tot} = 3/2$ level. A close investigation of $t/J = 1/2$ reveals more such "accidents"; the energy of the (π,π) one-hole level at $t/J = 1/2$ is exactly $2J$, and the separation between the $(0,0)$ and (π,π) levels is $\approx 0.578529J$, which equals the singlet-triplet gap of the Heisenberg antiferromagnet (without vacancies) on a 4×4 lattice. These simple relations result from an spl(2,1) supersymmetry at $t/J = 1/2$, which has been discussed elsewhere by Förster[13] and Cappon[14].

This supersymmetry can be understood by considering the effect of the off-diagonal terms in the t-J Hamiltonian on two nearest-neighbor sites,

$$H_{tJ}(1,2) = -t\Big(c^{\dagger}_{1\uparrow}c_{2\uparrow} + c^{\dagger}_{1\downarrow}c_{2\downarrow} + h.c.\Big) + \frac{J}{2}\Big((c^{\dagger}_{1\uparrow}c_{1\downarrow})(c^{\dagger}_{2\downarrow}c_{2\uparrow}) + h.c.\Big) + H_{diagonal}\,. \quad (8)$$

Note that the spin-flip and hopping matrix elements have equal magnitudes at $t/J = 1/2$, which suggests that an additional symmetry might be present. This symmetry can be made more obvious by introducing slave-boson hole operators, which casts the first hopping term in the form

$$-t(c^{\dagger}_{1\uparrow}h_1)(h^{\dagger}_2c_{2\uparrow})\,. \quad (9)$$

This is now a quartic in electron and hole operators with the same magnitude matrix element as the quartic spin-flip electron operator in (8). A sum of such quartic operators which is invariant under an infinitesimal rotation between electron (Fermi) and hole (Bose) operators (a supersymmetry transformation) is almost identical to the t-J Hamiltonian (1). There is a minor complication due to the $-\frac{1}{4}n_in_j$ term in (1), which prevents the full t-J Hamiltonian from being supersymmetric. The operator $\tilde{H} = H_{tJ} - 4tN_h$ however is exactly supersymmetric at $t/J = 1/2$, so we find degeneracies between states in an irreducible supermultiplet which have the same hole number, and energy differences of $4t\Delta N_h$ between states in one supermultiplet which have different hole number.

The representation theory of this supersymmetry has been discussed by Cappon[14], who finds three types of irreducible representations. In terms of their total spin and hole content these representations decompose as

$$(S_{tot}, N_h) = (0, n_0) \oplus (1/2, n_0+1) \oplus (0, n_0+2)\,, \quad (10)$$

$$(S_{tot}, N_h) = (s_0, n_0) \oplus (s_0 - 1/2, n_0+1) \oplus (s_0 + 1/2, n_0+1) \oplus (s_0, n+2) \quad (11)$$

and

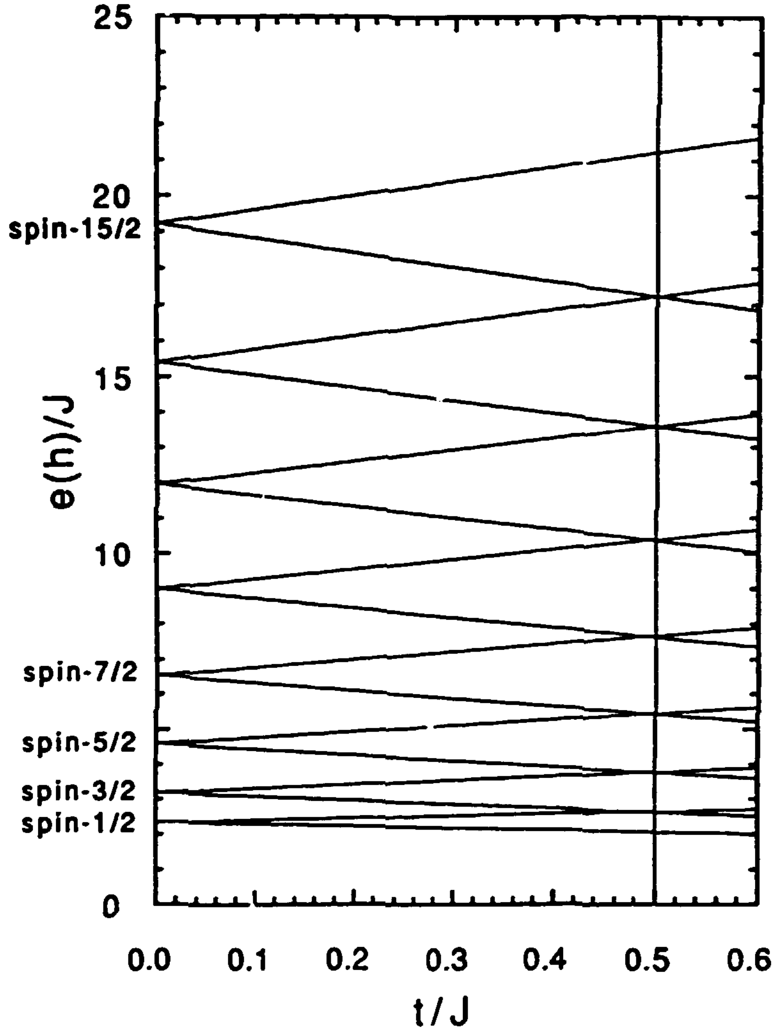
$$(S_{tot}, N_h) = (L^2/2, 0) \oplus (L^2/2 - 1/2, 1)\,. \quad (12)$$

All states within a supermultiplet have the same momentum $\vec{k}$.

The previous "accidents" in the $S_{tot} = 1/2$ one-hole energies on the 4×4 lattice can now be understood as relations between states within a supermultiplet. In particular, $e_h(\pi,\pi)/J = 2$ indicates that this one-hole state belongs to an $(S_{tot}, N_h) = (0,0) \oplus (1/2,1) \oplus (0,2)$ supermultiplet, with the pure Heisenberg ground state as its spin-zero no-hole partner. Similarly, the degeneracy of $\vec{k} = (0,0)$ spin-1/2 and spin-3/2 levels shows that they belong to a single supermultiplet, necessarily $(1,0) \oplus (1/2,1) \oplus (3/2,1) \oplus (1,2)$, which they share with the lowest Heisenberg triplet level (down in energy by $4t$) and a spin-1 two-hole level (up by $4t$). This also explains why these one-hole levels are separated by the same gap ($\approx 0.578529J$) as the Heisenberg singlet and triplet levels; each one-hole level is $4t$ above its no-hole Heisenberg partner. As the singlet-triplet gap of the Heisenberg model is believed to vanish as L^{-2}, the $e_h(0,0) - e_h(\pi,\pi)$ splitting and hence the linear-t bandwidth coefficient c_1 should also vanish as L^{-2}.

As our attention was originally drawn to this supersymmetry by a degeneracy of $S_{tot} = 1/2$ and $S_{tot} = 3/2$ levels at $t/J = 1/2$, we are naturally led to consider the lowest-lying one-hole bands of higher spin, and to inquire whether or not similar degeneracies occur in these sectors. Rather than tracking each level in detail, as a first indication we simply use the small-t/J bandwidth formula (5), Figure 3, which together with the corresponding static-hole energy gives a linear-t approximate band pattern. The resulting approximate bands on the 4×4 lattice are shown in Figure 5.

Figure 5. 4x4 higher-spin one-hole bands in the small-t/J approximation.



Evidently we have found a sequence of level crossings at $t/J = 1/2$, which in order of increasing energy are
$S_{tot} = 1/2 \rightarrow 3/2$; $\vec{k} = (0,0)$ (discussed above)
$S_{tot} = 3/2 \rightarrow 5/2$; $\vec{k} = (\pi,\pi)$
$S_{tot} = 5/2 \rightarrow 7/2$; $\vec{k} = (0,0)$
$S_{tot} = 7/2 \rightarrow 9/2$; $\vec{k} = (\pi,\pi)$
and *so on until*
$S_{tot} = 13/2 \rightarrow 15/2$; $\vec{k} = (0,0)$,
which is the bottom of the $S_{tot} = 15/2$ band. This remarkable pattern of level crossings requires the inversion of alternate-spin bands, which was noted in (7). These degeneracies allow us to assign the lowest-lying $\vec{k} = (0,0)$ and $\vec{k} = (\pi,\pi)$ one-hole states to specific supermultiplets; evidently they all belong to multiplets of the type

$$(S_{tot}, N_h) = (s_0, 0) \oplus (s_0 - 1/2, 1) \oplus (s_0 + 1/2, 1) \oplus (s_0, 2) \ , \qquad (13)$$

which implies that each one-hole state is $4t$ higher in energy than the lowest-lying spin-s_0 pure Heisenberg state (in t-J model conventions). Comparison of our results with tabulated energies[15] for the higher-spin Heisenberg states on the 4×4 lattice confirms this identification.

EXTRAPOLATION TO BULK-LIMIT PROPERTIES AT $t/J=3$

Although we have found several very interesting results for the t-J model at small t/J, one can object that these have little to do with the original motivation for studying this model, which was to learn about a possible magnetic mechanism for high temperature superconductivity. This goal requires that we attempt to learn something about the relevant parameter regime of $t/J \approx 3$.

One might hope that a small lattice such as the 4×4 system discussed here is sufficiently large to learn about the bulk-limit properties of the lightly hole-doped sectors of the t-J model, and the many studies of the t-J model on small lattices tacitly assume this. Of course we cannot conclude with certainty that a 4×4 lattice is too small for this purpose without obtaining accurate results on larger lattices, but there is evidence nonetheless that the 4×4 system does experience large finite size effects for $t/J \gtrsim 1$, which we shall discuss.

Our small-t/J results can be used to estimate bulk-limit physics if we can convincingly demonstrate simple scaling behavior as a function of t/J for physically interesting quantities, which can then be used to extrapolate to the bulk limit at larger t/J. For sufficiently small t/J the holes are relatively immobile, so one expects properties such as the rms hole-hole separation in the two-hole ground state to be insensitive to the finite lattice extent in this limit. As a first test of possible scaling behavior we consider precisely this hole-hole separation, which is defined by

$$r_{rms} = \left\{ \langle \Psi_{0,hh}(\vec{k}) | \, r^2 \, | \Psi_{0,hh}(\vec{k}) \rangle \right\}^{1/2} . \tag{14}$$

On the 4×4 lattice there are three degenerate two-hole ground states, with $\vec{k} = (0,\pi), (\pi,0)$ and $(0,0)$. The first two have equivalent wavefunctions under rotations, so we consider only $(\pi,0)$ and $(0,0)$.

We anticipate that the hole-hole separation may scale as a power of t/J. This behavior has previously been reported[6,16] for the ground-state energy in various sectors of the t-J model, and can be motivated by the familiar argument that the hole moves in an approximately linear potential due to the trail of energetically unfavorable bonds it creates in hopping through a Néel background. This argument would lead us to expect that the energy of a hole should scale as $e_h/t \propto (t/J)^{-2/3}$, and the rms hole-hole separation as $r_{rms}/a_0 \propto (t/J)^{1/6}$. The approximate t/J dependence actually found for the energy of these d-wave two-hole states[16] is $e_{hh}/t \propto (t/J)^{-0.78}$, which deviates from the naive exponent of $-2/3$ in the direction expected if the linear potential is weakened at large distance by spin flips. A very naive picture of this effect may be obtained by replacing the linear potential by a weaker-than-linear power law[10], $V(r_{hh}) \propto r_{hh}^p$; given the energy exponent of -0.78 cited above we then estimate $p \approx 0.56$ from scaling arguments using the Schrödinger equation, and that the rms hole-hole separation should scale as

$$r_{rms}/a_0 \propto (t/J)^{1/(2p+4)} \approx (t/J)^{0.195} . \tag{15}$$

To test for such simple power-law behavior we show $\ln(r_{rms}/a_0)$ versus $\ln(t/J)$ in Figure 6. At $t/J = 0$ the holes are static and adjacent ($r_{rms} = a_0$), and as $t/J \to \infty$ they approach a limiting rms separation of $r_{rms} \approx 1.987 a_0$ which is fixed by the 4×4 lattice. For an intermediate range of t/J, approximately $0.3 \lesssim t/J \lesssim 1.0$, there is indeed evidence of power-law behavior, with a power remarkably close to the $(t/J)^{0.195}$ predicted by the simple estimate (15). A fit to these two states in this scaling region gives[10]

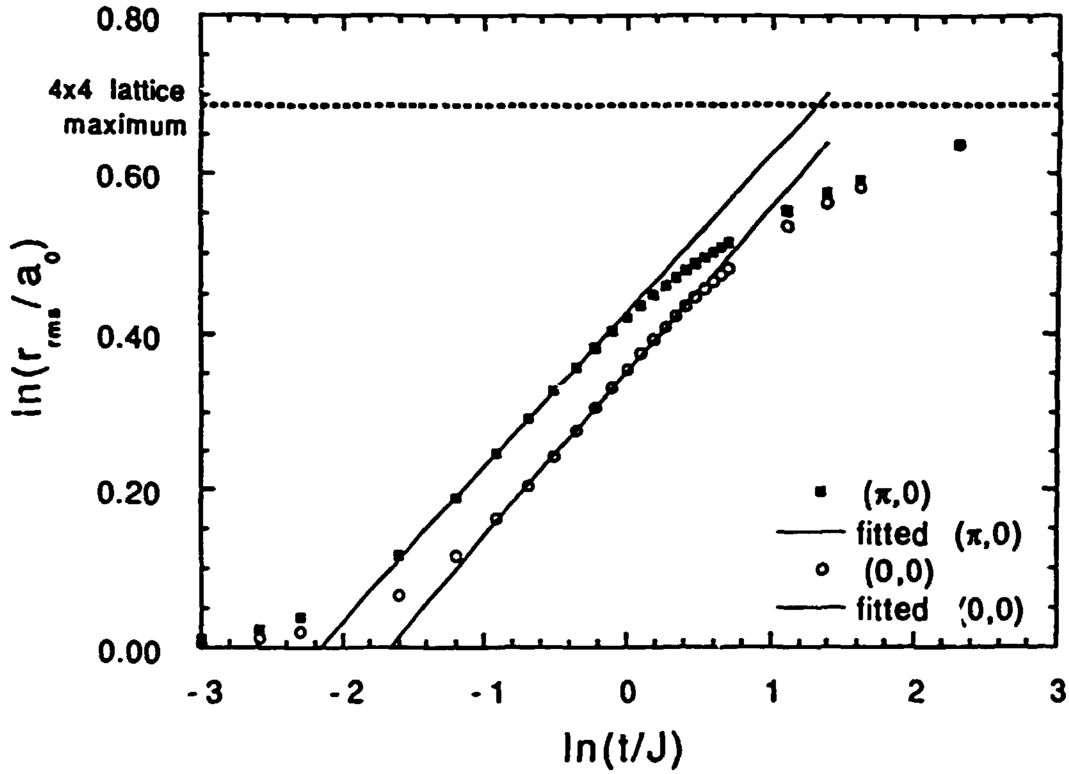
$$r_{rms}(\pi,0)/a_0 = 1.533(2) \cdot (t/J)^{0.198(4)} \tag{16}$$

and

$$r_{rms}(0,0)/a_0 = 1.423(2) \cdot (t/J)^{0.210(4)} , \tag{17}$$

which are shown as solid lines in Figure 6.

Figure 6. Ground-state rms hole-hole separation versus t/J.



One can see evidence here for finite size effects on the 4×4 lattice; there is a clear departure from the power law (16) for $t/J\gtrsim1.0$ in the $(\pi,0)$ state, and a similar departure at somewhat larger t/J in the $(0,0)$ state. This is the origin of our claim that there are important finite size effects in the two-hole ground states on a 4×4 lattice for $t/J\gtrsim1$. Note however that the characteristic length scales of the two-hole bound states increase very slowly with t/J, so that studies at $t/J = 3$ on only moderately larger lattices would incorporate much smaller finite size effects.

Having found a simple scaling law for the rms hole-hole separation in the small-t/J regime, we can now extrapolate to $t/J = 3$ and estimate properties of the two-hole bound states in the high-T_c regime. In physical units our power-law parametrizations (16) and (17) imply

$$r_{rms}(\pi,0) = 7.2\text{Å} \tag{18}$$

and

$$r_{rms}(0,0) = 6.8\text{Å} \tag{19}$$

in the bulk limit. As these numbers are comparable to the in-plane coherence length of $\xi_{ab}\approx$ 14Å reported for the high temperature superconductors[17], our results appear to support the identification of hole pairs with the Cooper pairs of high temperature superconductivity. (As the coherence length is only a qualitative length scale of the Cooper pair, it is not clear whether or not the factor-of-two difference between ξ_{ab} and r_{rms} is significant. The fact that we are studying an isolated hole pair in the insulating phase rather than in the metal may also affect the estimated pair size.) Of course the t-J model is incomplete in that it does not incorporate hole-hole Coulomb repulsion and hence suffers hole phase separation; we have studied the effect of such an interaction in the t-J model[8,18], and have concluded that this stops phase separation, results in a hole-hole binding energy of $\sim$ 30 meV, and only slightly increases the rms hole-hole separation to about 8Å.

CONCLUSIONS

We have discussed numerical and analytic results for one- and two-hole energies and rms hole-hole separations in the t/J model at small t/J. An application of perturbation theory in the hopping parameter led to a relation between the linear-t term in the one-hole bandwidth and a static-hole ground-state matrix element. We also discussed evidence that this linear-t bandwidth term actually vanishes in the bulk limit, which leads us to a conjecture for the two-dimensional Heisenberg antiferromagnet: The ground states of the $S_{tot}/L^2 > 0$ sectors all have zero staggered magnetization in the bulk limit. We also gave t/J values and quantum numbers associated with the six one-hole ground-state level crossings. A crossing of $S_{tot} = 1/2$ to $S_{tot} = 3/2$ one-hole levels at exactly $t/J = 1/2$ is due to a supersymmetry at that point, which also leads to exact relations between energies of states having different numbers of holes. We find evidence for a sequence of related degeneracies between levels of higher spin. Finally, we discuss the extrapolation of small-t/J results to the high-T_c regime of $t/J \approx 3$. We find that the rms hole-hole separations in the two-hole ground states scale approximately as $(t/J)^{0.29}$, and an extrapolation to high-T_c parameters gives an rms hole-hole separation of ≈ 7Å in the bulk limit. This and other numerical evidence supports the identification of two-hole bound states with the Cooper pairs of high temperature superconductivity.

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