PROCEEDINGS

CONF-830428--

MASTER

Fourth U.S. Stellarator Workshop

OAK RIDGE NATIONAL LABORATORY
OAK RIDGE, TENNESSEE 37830

APRIL 14-15, 1983

SPONSORED BY

Fusion Energy Division
Oak Ridge National Laboratory

AND BY

Courant Institute of Mathematical Sciences
New York University

PROCEEDINGS

CONF-830428--

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FOURTH U. S. STELLARATOR WORKSHOP

Oak Ridge National Laboratory
Oak Ridge, Tennessee

April 14-15, 1983

R. A. Dory - Workshop Chairman

H. Weitzner - Executive Committee Chairman

Date Published: April 1983

Sponsored By

Fusion Energy Division
Oak Ridge National Laboratory

And By

Courant Institute of Math. Sciences
New York University

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CONTENTS

AVANT PROPOS

The Fourth in the Series of U. S. Stellarator Workshops was held at the *Fusion Engineering Design Center* (FEDC) *in Oak Ridge, Tennessee* on April 14 and 15, 1983. It was sponsored jointly by the Courant Institute of Mathematical Sciences of New York University and the Fusion Energy Division of Oak Ridge National Laboratory.

Viewgraph materials provided by the speakers are reproduced in these proceedings. The clear preponderance of discussion lay in the areas of understanding the transport properties of stellarators, including effects of electric fields and direct particle losses, of determining the gains which might accrue by exploiting the freedom of choosing a nonplanar (i.e., helical) magnetic axis as in the Heliac idea, and of exploiting the substantial gains which have been made in analyzing full 3-D systems.

In view of recent events in the U. S. Fusion Program, the charge by DOE Office of Fusion Energy in 1981 to the sponsors and the Advisory Board for the Workshops has been reconsidered. The original charge was to define the needs of the U. S. Stellarator program and suggest a program to accomplish them. The concensus of the first three workshops was the need for a large stellarator experiment in the U. S. to study (inter alia) beta limits and low collisionality transport, and for stronger support for the U. S. Stellarator program. Since the previous workshop in March 1982, there have been several significant events: the Magnetic Fusion Advisory Committee meetings, approval of the ATF-1 torsatron proposal at Oak Ridge, and redefinition of the role of the stellarator program within the U. S. Fusion program. The present DOE view of the stellarator program role is: (1) in the near term it must contribute to better understanding of general toroidal systems; and (2) in the longer term it must contribute to development of a superior reactor concept.

A panel discussion reflecting these developments and beginning a new phase of the workshops, was held, with the topic: "What information is needed in the U. S. Fusion Program in addition to that from ATF and IMS in the U. S. and the Heliotron/Wendelstein/Liven' devices abroad?" The panel, representing diverse interests within the stellarator community consisted of: H. Weitzner (NYU), H. Berk (U. Texas), J. Callen (U. Wisconsin), R. Dory (ORNL), W. Dove (DOE), J. Johnson (PPPL), and P. Politzer (MIT).

Among the noteworthy comments made by the Panel members, by distinguished visitors from abroad (Prof. A. Schlueter of Germany and Dr. K. V. Roberts of the UK), and by other workshop participants were*:

- ORNL and the ATF program would welcome suggestions, ideas and help from the community, as well as involvement in conduct of the ATF-1 program: for specific studies to be made on ATF-1, for possible new diagnostics, and for ways to increase and to exploit the flexibility built into the coil and auxiliary systems. In turn, ORNL would help where it can to provide

*For the sake of rapid distribution of these proceedings, the comments of panel members and the audience have been abstracted, paraphrased and included without attribution, but with apologies.

technical contributions, as well as programmatic support to other groups in their efforts to advance the stellarator program in the U. S. Fusion effort.

- The stellarator community has not yet made a completely convincing case from its reactor studies for the viability of stellarator reactors.
- The main physics themes and areas needing to be developed were listed:
  > Helical axis systems -- full experimental tests are not presentally provided in the U. S. program.

  > RF heating options. Antennas. Propagation.

  > Modular coil analysis and experiments.

  > Divertor studies. Impurity control. Particle sources and losses.

  > Reactor systems analyses. Low aspect ratio is a key opportunity.

  > Diagnostics for 3-D and long pulse effects.

  > Theories of flow, viscosity and rotation effects; parallel currents.

  > Simple physical pictures to enable understanding of complex systems. For example, understanding t, shear, V'' as key physics properties rather than ℓ number or coil topology.

  > The trade between t, shear, and V'' needs further experimental clarification.

At a meeting of the Executive Board, it was decided to continue the workshops at reasonable intervals, with January 1984 as the next occasion. ORNL has accepted responsibility for the workshops but the location need not be at Oak Ridge. The next meeting of the Executive Board will take place at the APS-DPP fall meeting, at which time a site and program will be determined for the January 1984 Workshop.

On The Occasion of ATF-1 Approval

In nineteen hundred eighty one,
Designing tokamaks was still great fun.
But when folks from Oak Ridge proposed ISX-C,
It failed to amuse the DoE.

On that year's twentieth of May,
Soon to prove a fateful day,
A letter from Anne did then arrive,
"Dear Murray, that same old stuff just won't jive."

Shepherd John called in Tom and Jim.
The pow-wow session was indeed very grim.
"Propose something we must in tokamak's stead;
Our only hope is to forge ahead."

Midnight oil they did burn.
For sleepless nights they did churn.
Argued and debated even after they turned red.
Fell and tumbled as they did steadily tread.

In the end there was great elation.
Announce they did their revelation:
"Journeyed we have through the toroidal map;
Decided we have on a stellarator trap."

The war bugle sounded.
The battle cry began.
Brad, Ben, Vicky, Jim, Jeff,
All took part in the plan.

Magnetic field lines they did trace
And soon conquered the ℓ,m,R,a, space.
Perfect they did the Chodura-Schlüter code;
High and low they pursued every mode.

Stellarator expansions they did do,
Doubly proving each result to be true.
Flux coordinates they did use;
Not one particle could anomalously diffuse.

Equilibrium and stability they did explore;
Many a regime none had visited before.
First and second stability they did cement;
Out popped a beta of eight percent.

In DoE's court they put the ball back;
Erol knew who should review this non-tokamak.
A blue ribbon panel he did convene,
The best tokamak people one's ever seen.

To the panel they did go;
Received a report card all aglow.
Onward and upward they persuaded the DoE:
"A better bargain there never could be.

Eighteen million is a large sum indeed,
But what you'll get is what you'll need.
This device allows currentless research;
This device permits high beta search."

The buck finally stopped on John's home ground,
Who sighed at the outstretched hands all around.
"Which horse must I now give away,
To feed this one that does loudly neigh?"

The eighteenth of February of eighty three
Is a red letter day as we shall see.
With a single stroke of his feathered pen,
He approved the plan there and then.

There was jubilation throughout the land
For the go-ahead of a project so bold and so grand.
The long wait is over here in the U.S.;
Our own fusion stellarator we again possess.

So cheers to you, the hill billy chaps:
You put stellarators back in our fusion maps.
Good Luck, high hopes, and much success.
May your splendid effort progress.

T. K. Chu
April 14, 1983
Princeton

ATF-1 PROGRAM STATUS *

J. F. LYON, ORNL
ATF-1 PHYSICS COORDINATOR

4TH US STELLARATOR WORKSHOP
OAK RIDGE, APRIL 14, 1983

*Research sponsored by the Office of Fusion Energy, OER, U. S. Department of Energy under contract W-7405-eng-26 with the Union Carbide Corporation.

ORNL CONTRIBUTORS TO ATF-1 PROGRAM

CONFIGURATION DESIGN

R. A. Dory, D. Goodman, J. H. Harris, V. E. Lynch, B. F. Masden, J. Sheffield

MHD EQUILIBRIUM AND STABILITY

B. A. Carreras, L. A. Charlton, W. A. Cooper, L. Garcia, T. C. Hender, H. R. Hicks, S. P. Hirshman, J. A. Holmes

TRANSPORT

R. H. Fowler, W. A. Houlberg, J. F. Lyon, J. A. Rome, K. C. Shaing

HEATING

A. C. England, D. J. Hoffman, S. Hokin

ENGINEERING DESIGN

R. L. Brown, W. D. Cain, K. K. Chipley, M. J. Cole, P. H. Edmonds, W. A. Gabbard, T. C. Jernigan, R. L. Johnson, J. F. Monday, G. H. Neilson, B. E. Nelson, P. B. Thompson

TOPICS

- DEVELOPMENTS SINCE PROPOSAL/APS
- ATF-1 PROGRAM
- CONFIGURATION FLEXIBILITY STUDIES
- MHD EQUILIBRIUM AND STABILITY
- TRANSPORT STUDIES
- ATF-1 DESIGN STATUS
  - OVERALL DESIGN FEATURES
  - COILS, VESSEL, STRUCTURE
  - SCHEDULE

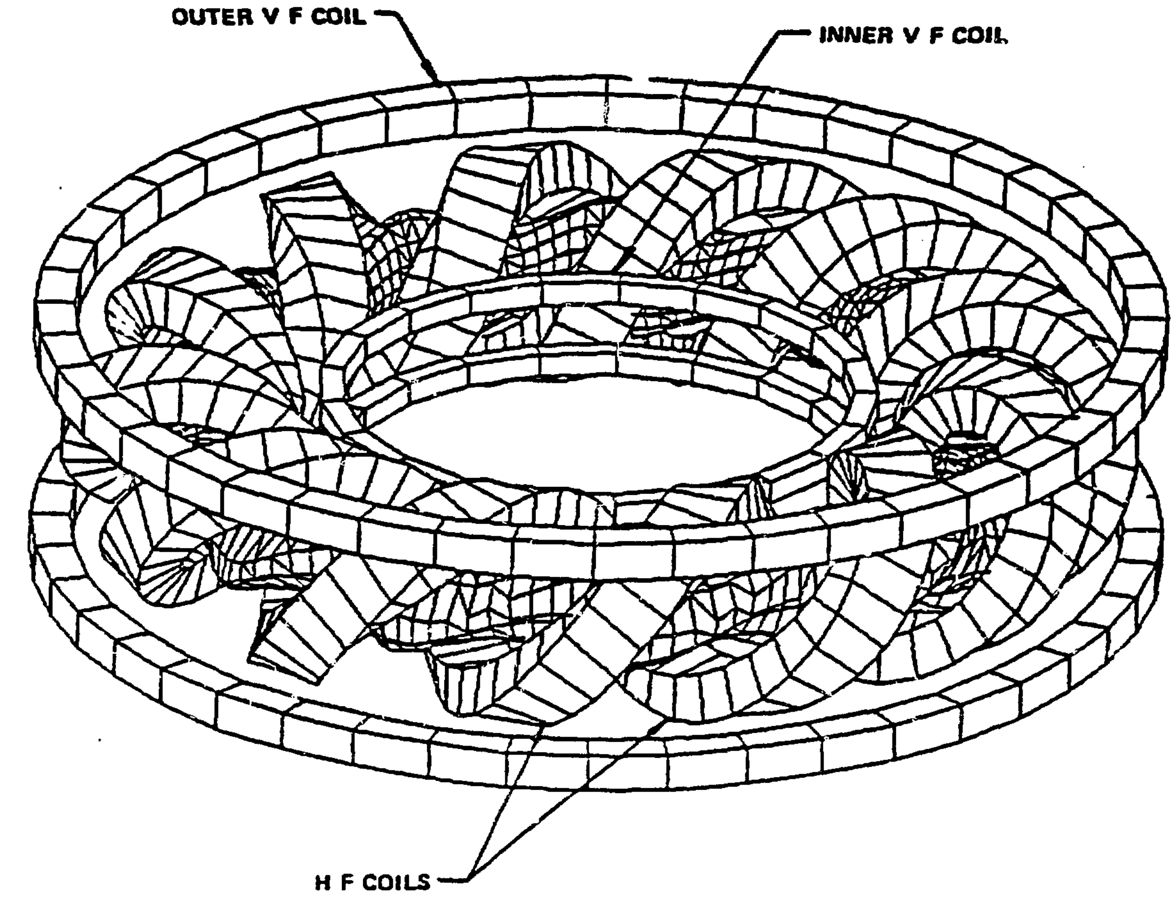
ATF - 1 PROGRAM DEVELOPMENTS SINCE LAST OCTOBER

- FAVORABLE REVIEW BY DOE TOKAMAK/STELLARATOR PANEL
- CONCEPTUAL DESIGN IMPROVEMENTS
  - VACUUM VESSEL, HELICAL COIL DESIGN
  - TRANSPORT IMPROVEMENT WITH ELECTRIC FIELD
- CONSTRUCTION APPROVAL BY DOE-OFE
  - DOE TOROIDAL PROGRAM PLAN
- INITIATE DETAILED DESIGN
  - PROTOTYPE HELICAL COIL PROCUREMENT STARTED

ATF-1 PROGRAM OBJECTIVES

- NEAR-TERM: CONTRIBUTE TO BETTER UNDERSTANDING OF OVERALL TOROIDAL PHYSICS
- LONG-TERM: CONTRIBUTE TO DEVELOPMENT OF A SUPERIOR REACTOR CONCEPT
- ACCOMPLISH THROUGH STUDIES OF:
  - ACCESS TO SECOND STABILITY REGIME AND MHD BETA LIMITS
  - TRANSPORT IN LOW COLLISIONALITY REGIME
  - CURRENT-FREE, LONG PULSE PHYSICS ISSUES
  - IMPURITY BEHAVIOR AND CONTROL
  - ROLE OF ELECTRIC FIELD, BOOTSTRAP CURRENT
  - EFFECT OF HELICAL CONFIGURATION PARAMETERS AND PLASMA CURRENT ON BETA LIMITS AND TRANSPORT
  - TOKAMAK/STELLARATOR HYBRIDIZATION

THE ATF-1 IS A MODERATE ASPECT RATIO TORSATRON WITH HIGH BETA CAPABILITY AND THE FLEXIBILITY TO STUDY A WIDE RANGE OF TOROIDAL CONFIGURATIONS



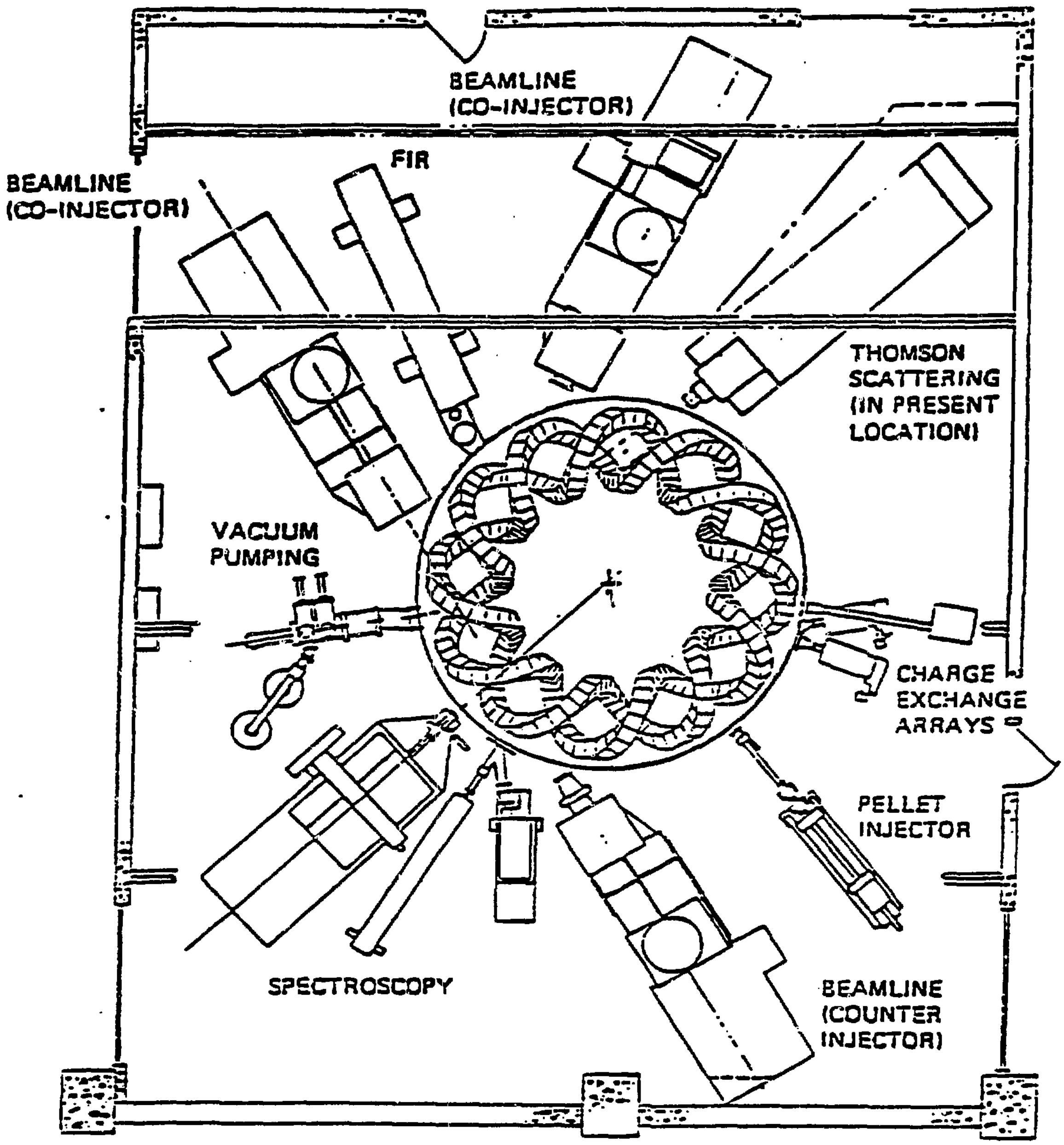
$R = 2.1$ m, $\bar{a} \simeq 0.3$ m, $R/\bar{a} \simeq 7$, $B \leq 2$ T

LARGE SIDE (60 x 90 cm) AND TOP (50 cm DIAM) ACCESS PORTS

P_{IN} = 4.5 MW NEUTRAL INJECTION (INITIALLY)

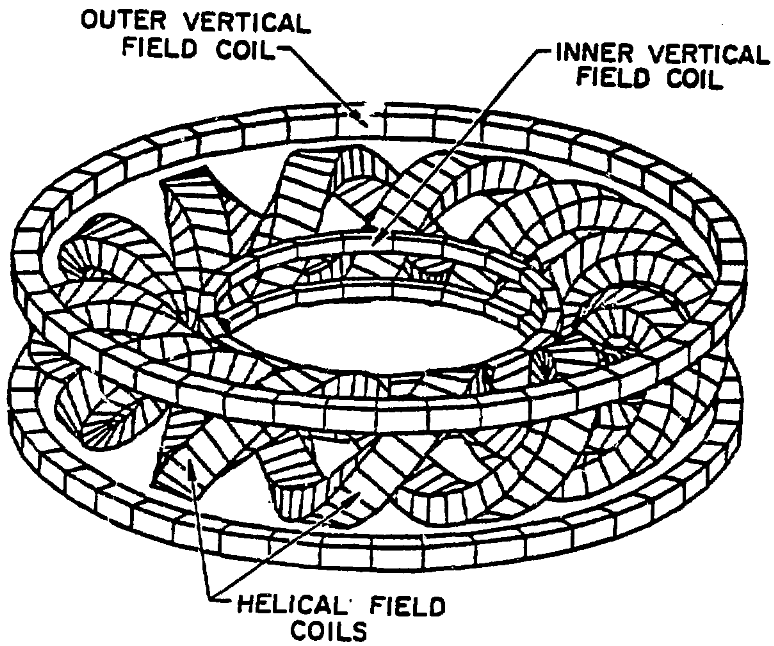
P_{ECH} = 0.3 MW AT 35 GHz

ATF-I WILL REPLACE ISX-B



IT WILL FIT IN THE PRESENT ISX-B EXPERIMENTAL ENCLOSURE AND WILL USE THE ISX-B POWER SUPPLIES, DIAGNOSTICS, CONTROL SYSTEM, AND UTILITIES.

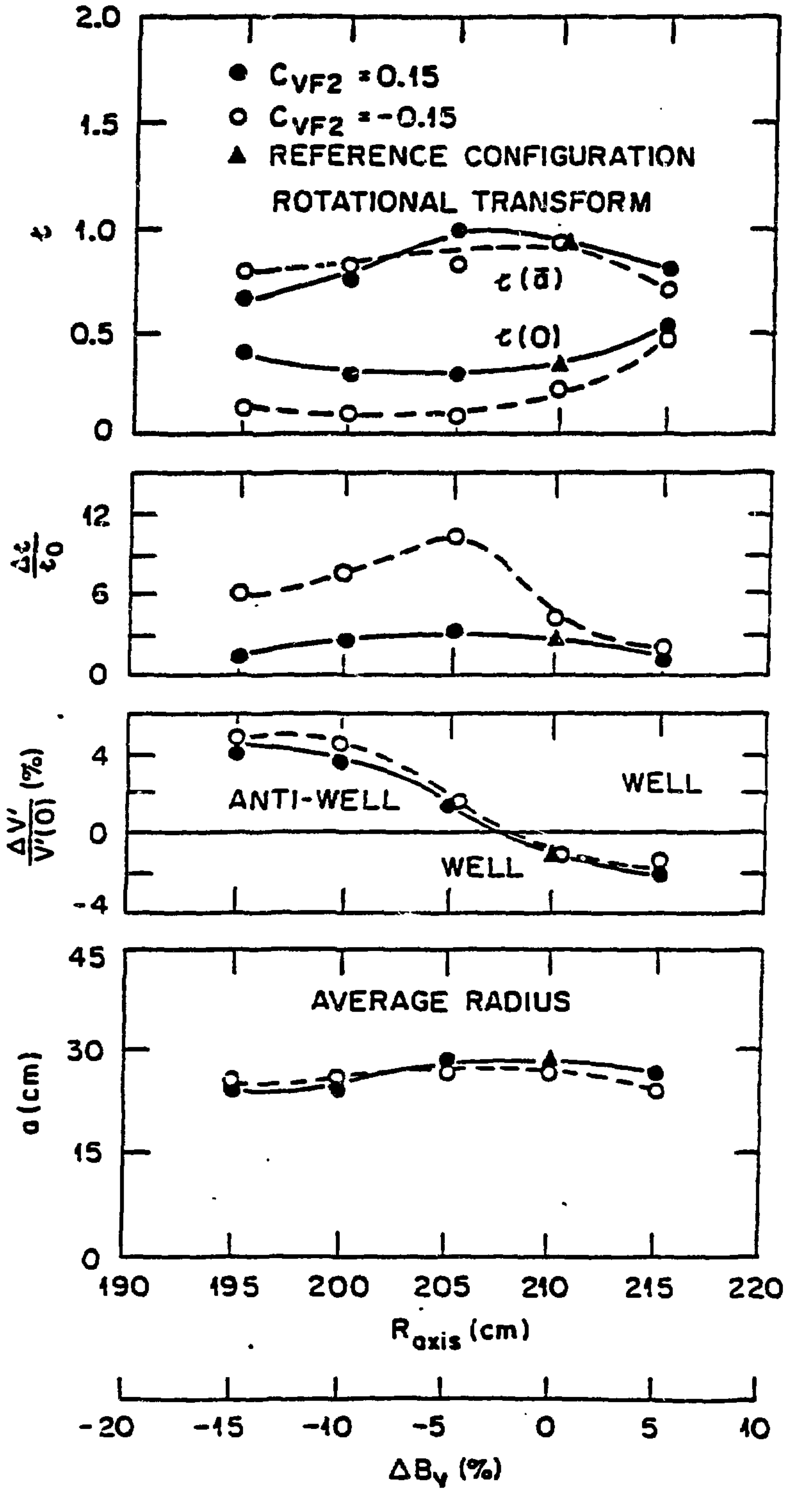
CONFIGURATION FLEXIBILITY STUDIES



- ATF-1 COIL SET ALLOWS FLEXIBILITY IN MAGNETIC CONFIGURATION BY VARYING RELATIVE CURRENTS (A) IN INNER AND OUTER VF COILS AND (B) IN TWO SEPARATE HELICAL WINDINGS

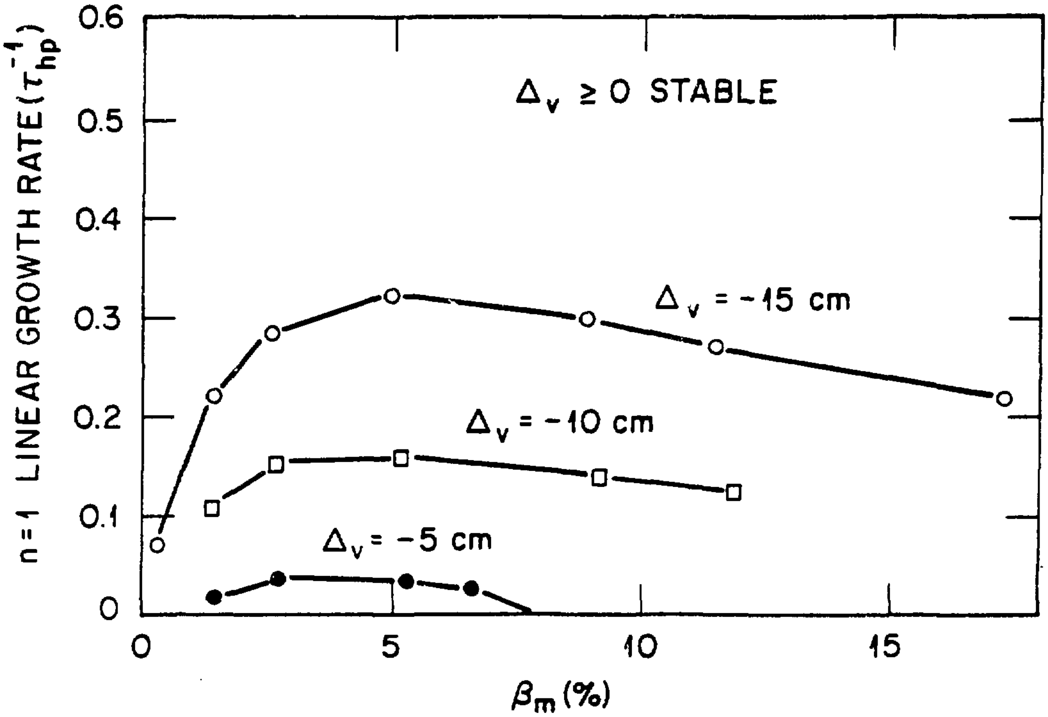
- THIS FLEXIBILITY PERMITS
  - WIDE RANGE IN ROTATIONAL TRANSFORM, SHEAR WELL DEPTH AND PLASMA CROSS SECTION
  - HELICAL OR PLANAR MAGNETIC AXIS
  - ADDITION OF PLASMA CURRENT AND TF COILS FOR TOKAMAK/STELLARATOR HYBRID STUDIES

WIDE RANGE OF CONFIGURATION PARAMETERS AVAILABLE ON ATF-1, ALLOWS TESTING OF BETA LIMITS FOR RANGE OF VALUES

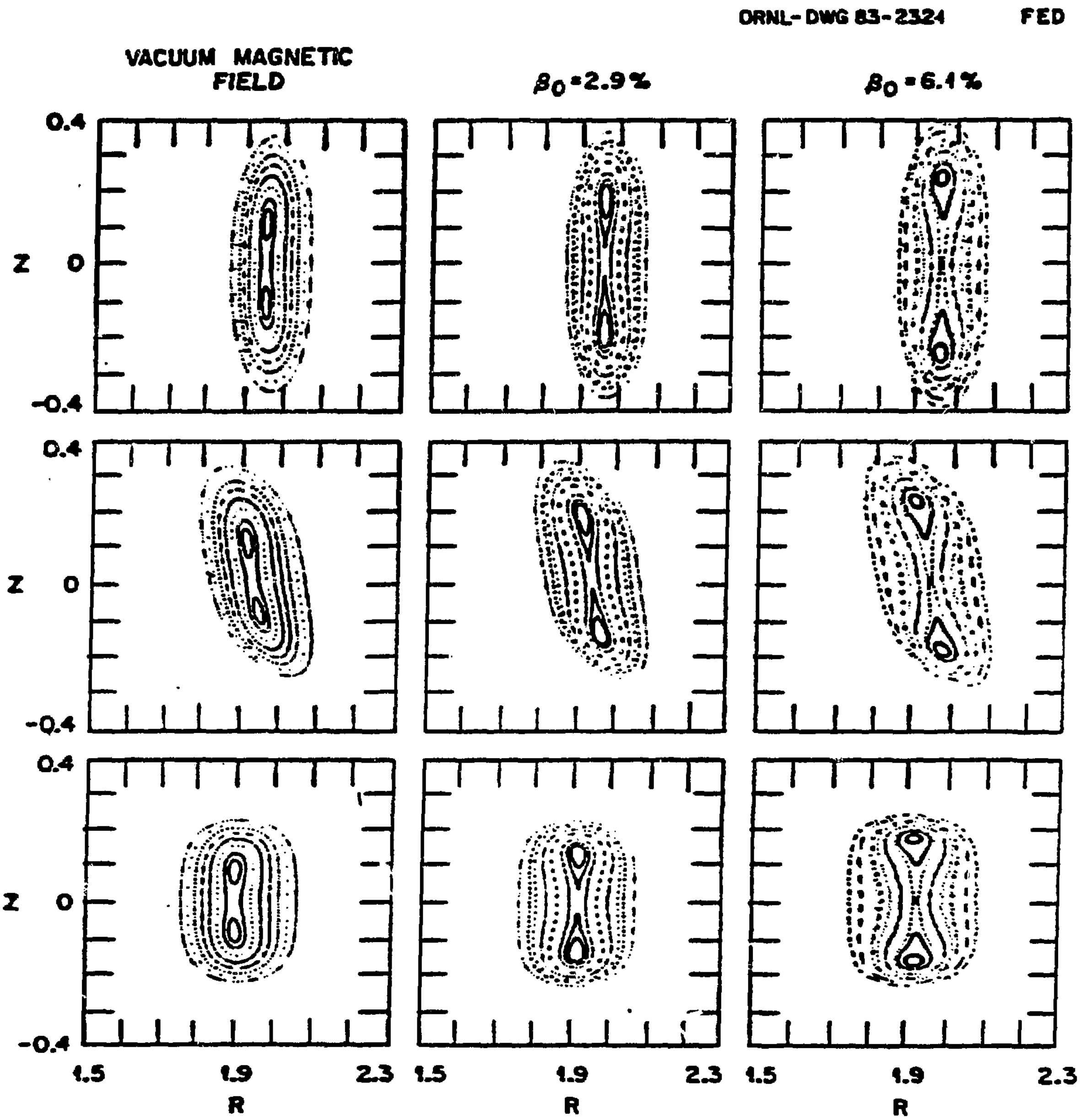


N = 1 LINEAR GROWTH RATE FOR VARIOUS VERTICAL FIELDS

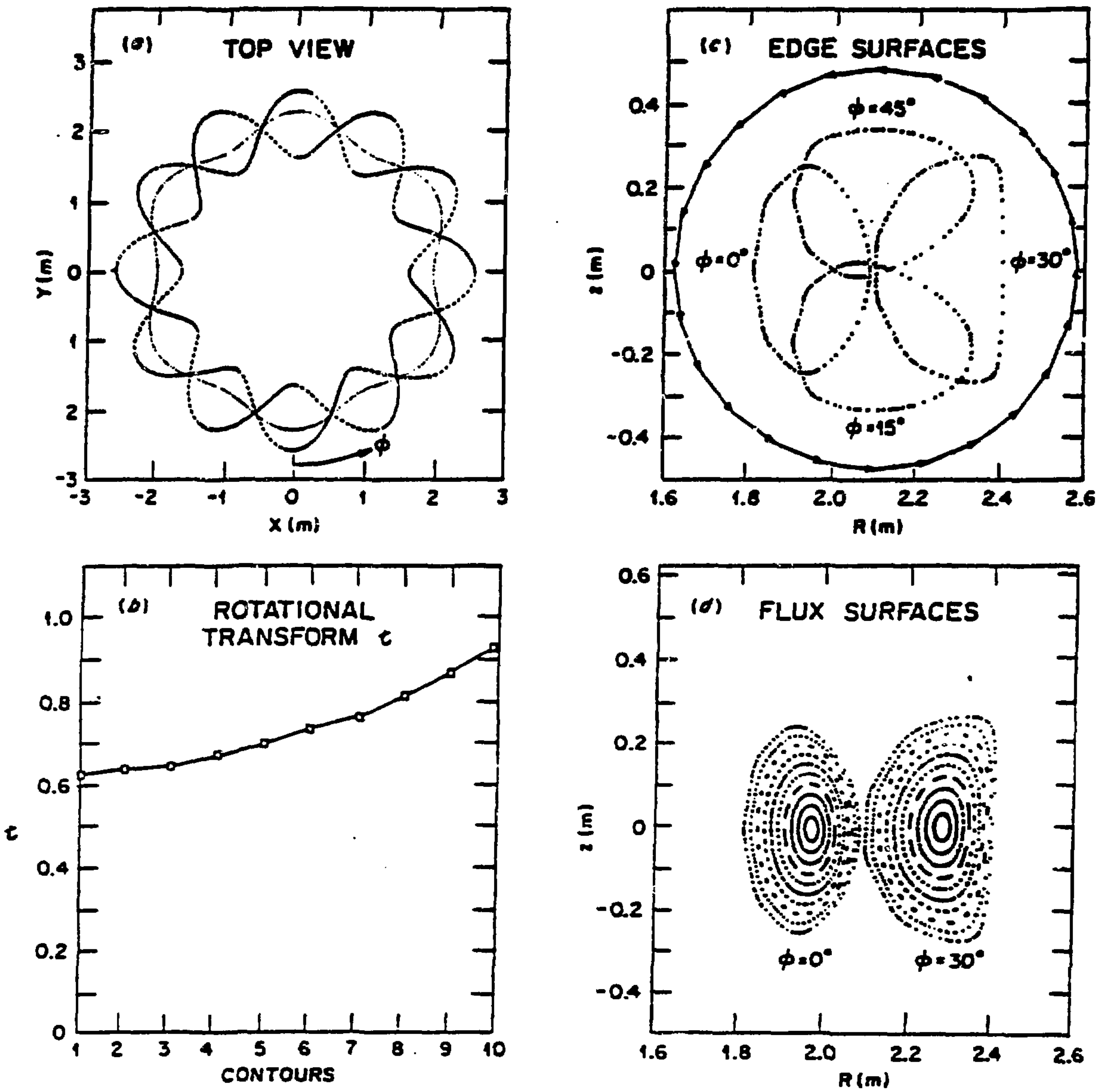
ORNL-DWG 83-2331A FED



"DOUBLET" IN VACUUM FIELD AND AT FINITE BETA

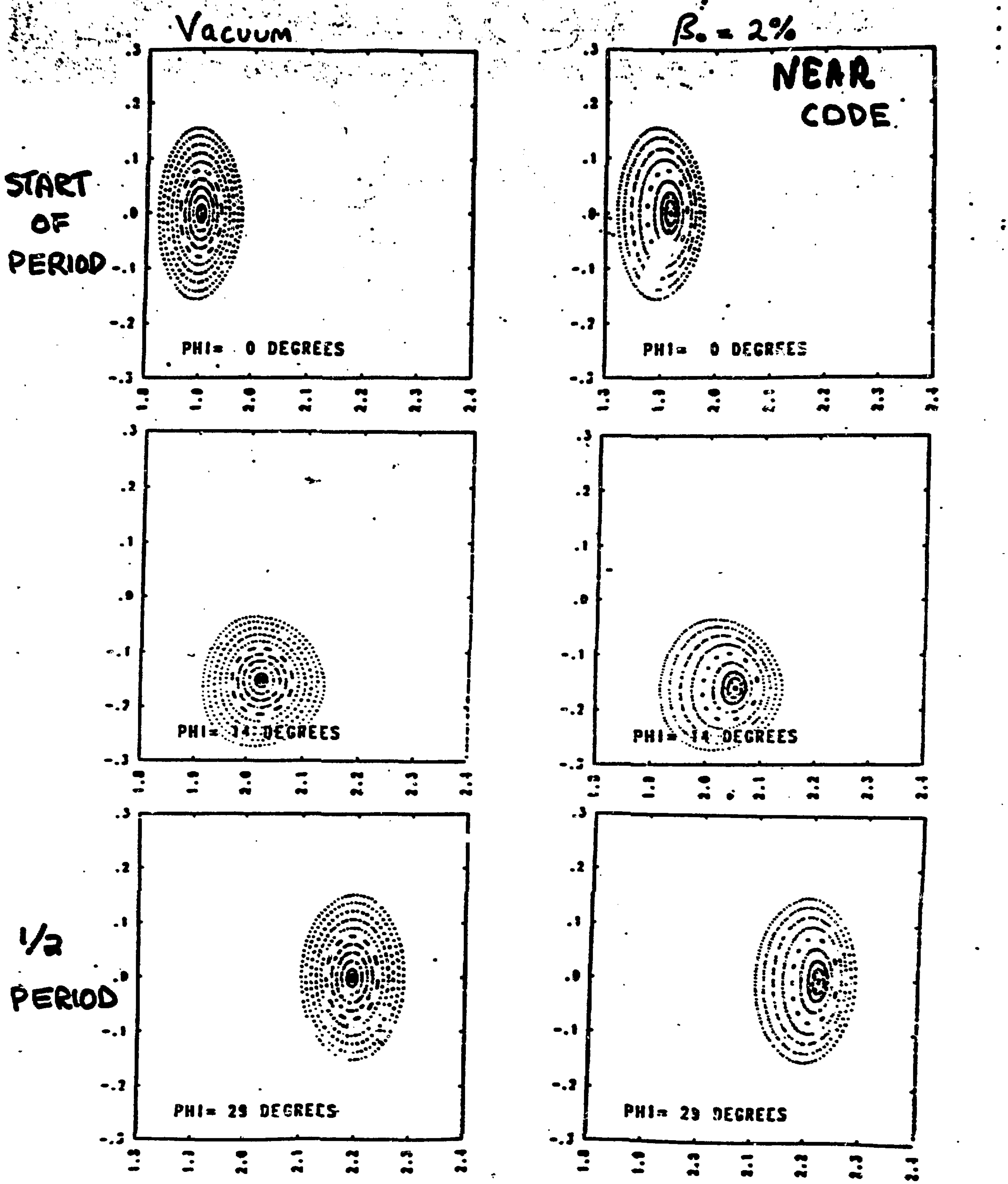


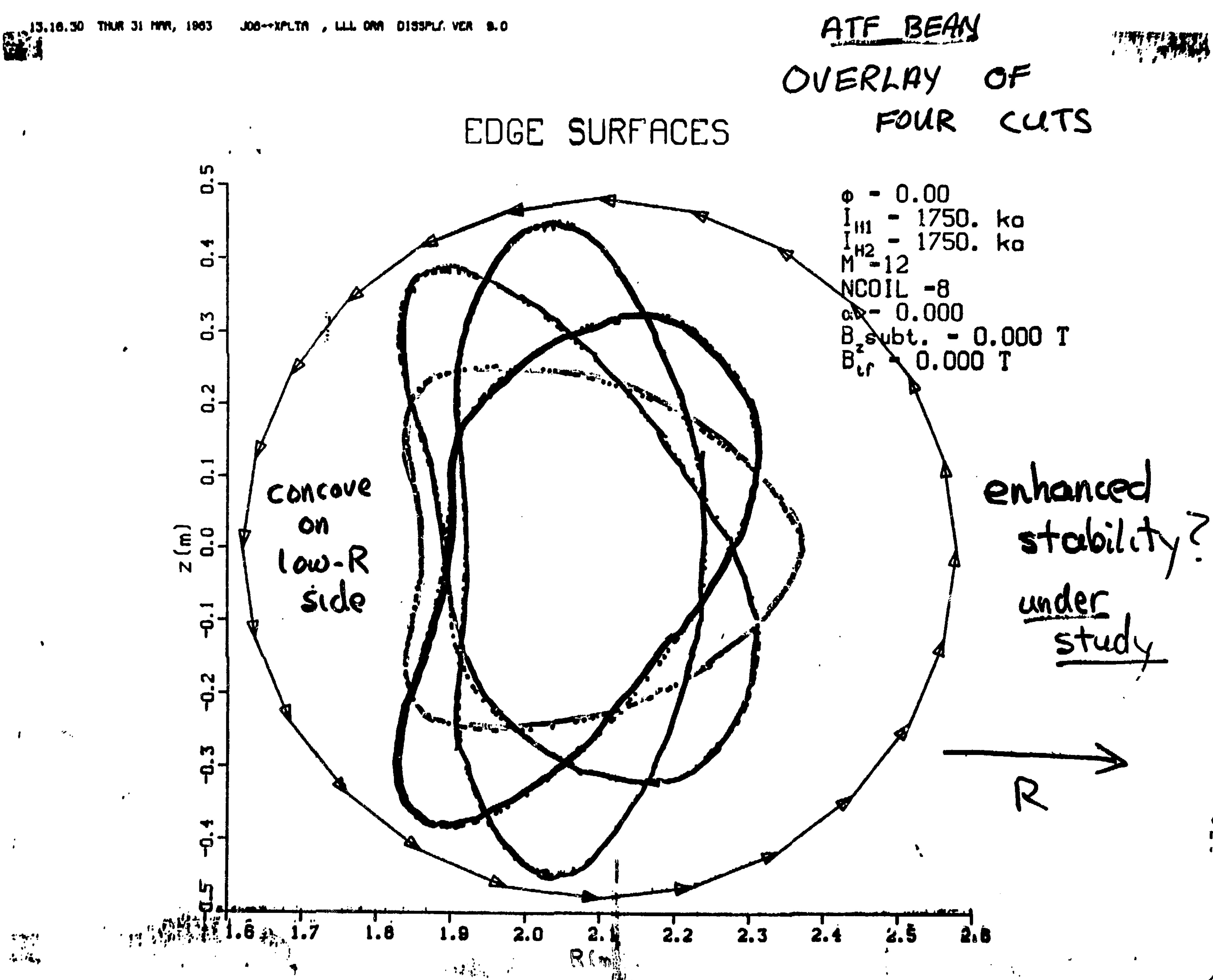
EXAMPLE OF HELICAL AXIS CONFIGURATION PRODUCED WITH CURRENT IN ONE HELIX REDUCED TO 16% OF THAT IN OTHER HELIX



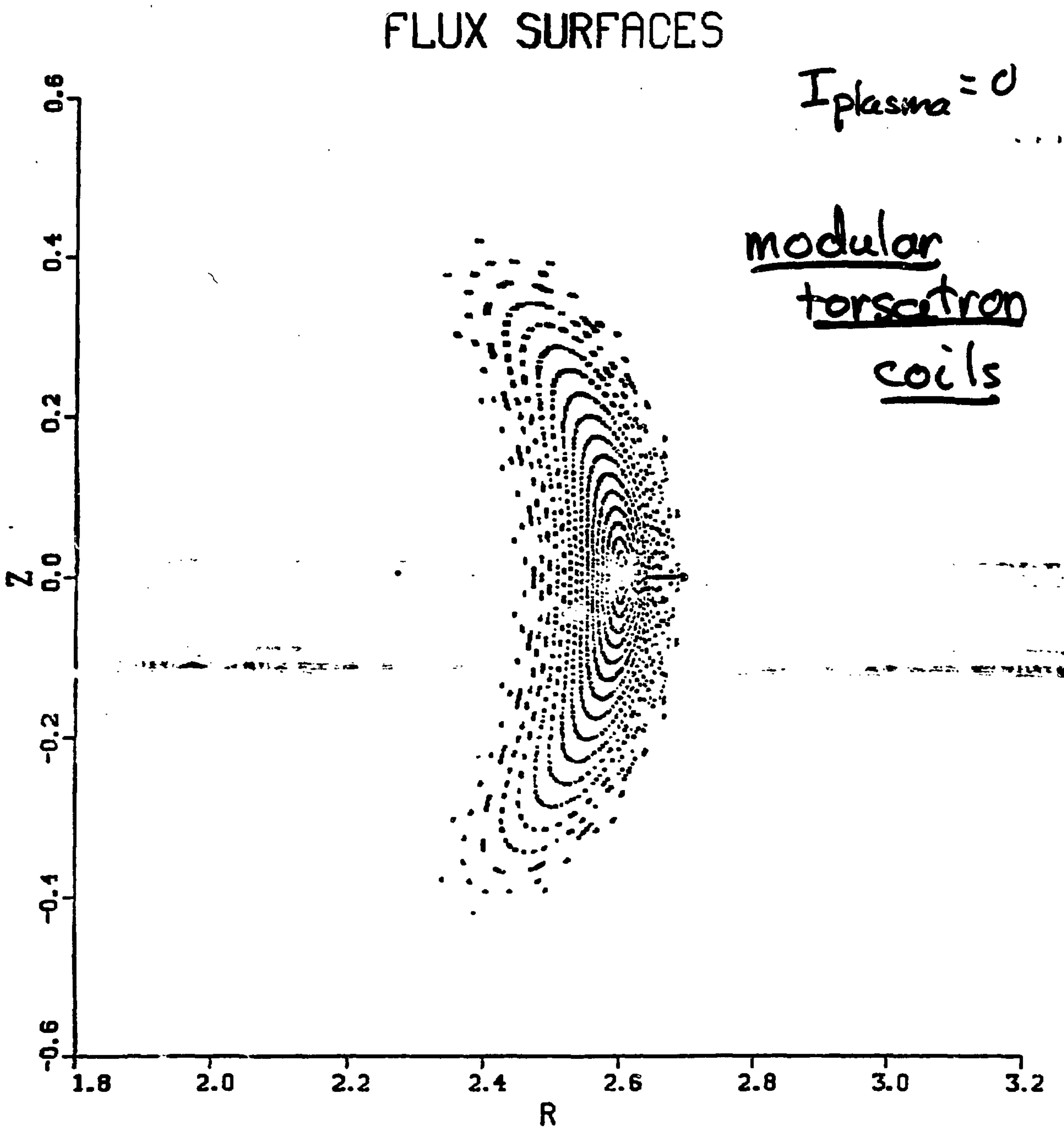
- CAN VARY CONTINUOUSLY FROM PLANAR TO HELICAL AXIS BY VARYING RATIO OF HELICAL COIL CURRENTS

HELICAL MAGNETIC AXIS CONFIGURATION

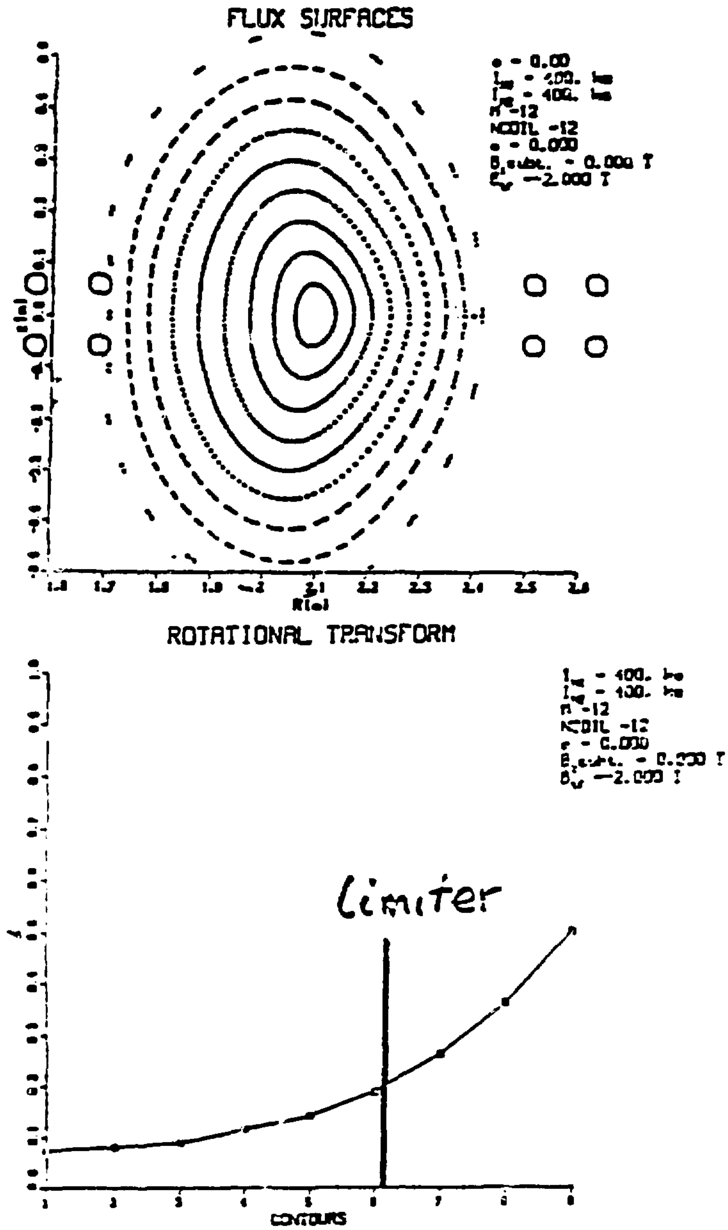



13.16.30 THUR 31 MAR, 1983 JOB→XPLTA , LLL ORA DISSPLA VER 9.0
ATF BEAN
OVERLAY OF FOUR CUTS
EDGE SURFACES
Φ - 0.00
I_{H1} - 1750. ka
I_{H2} - 1750. ka
M - 12
NCOIL - 8
α - 0.000
B_z subt. - 0.000 T
B_{tr} - 0.000 T
concave on low-R side
enhanced stability?
under study
R
z (m)
R (m)
0.5
0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
-0.4
-0.5
1.6
1.7
1.8
1.9
2.0
2.1
2.2
2.3
2.4
2.5
2.6

$\ell=1$ SYMMOTRON BEAN



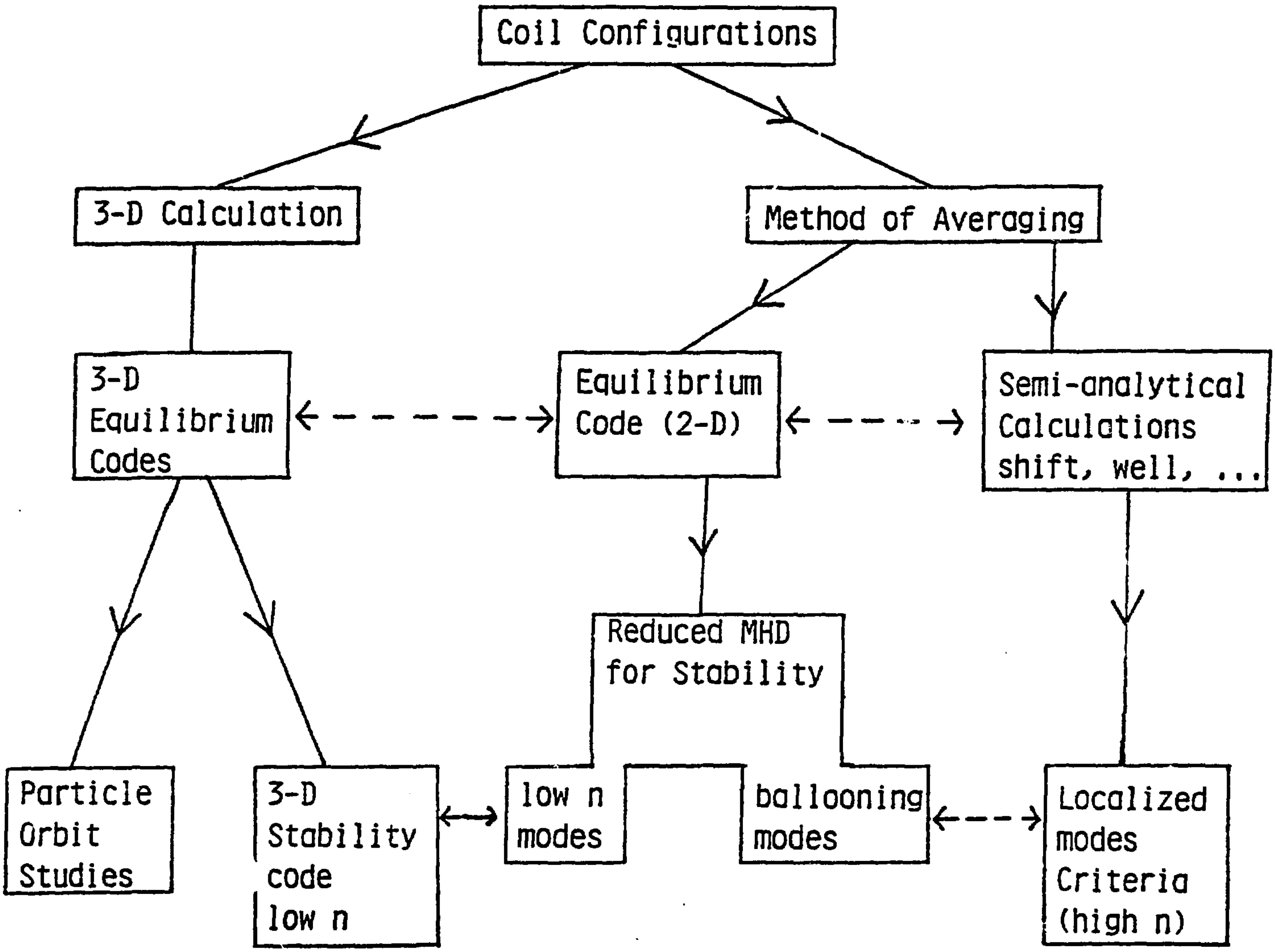
ADDITION OF TF COILS (HERE $B_{TF} = B_H$) FURTHER INCREASES CONFIGURATION FLEXIBILITY



- Base Configuration (no TF)
  - $I_p \sim 40$ kA ($q_a = 1$ limit)
  - Allows current drive/bootstrap current studies

- Add TF coils ($B_{TF} \sim 5$ kG)
  - reduce $\iota\!\!-$
  - $I_p \sim 100$-200 kA for hybrid studies

MHD STUDIES

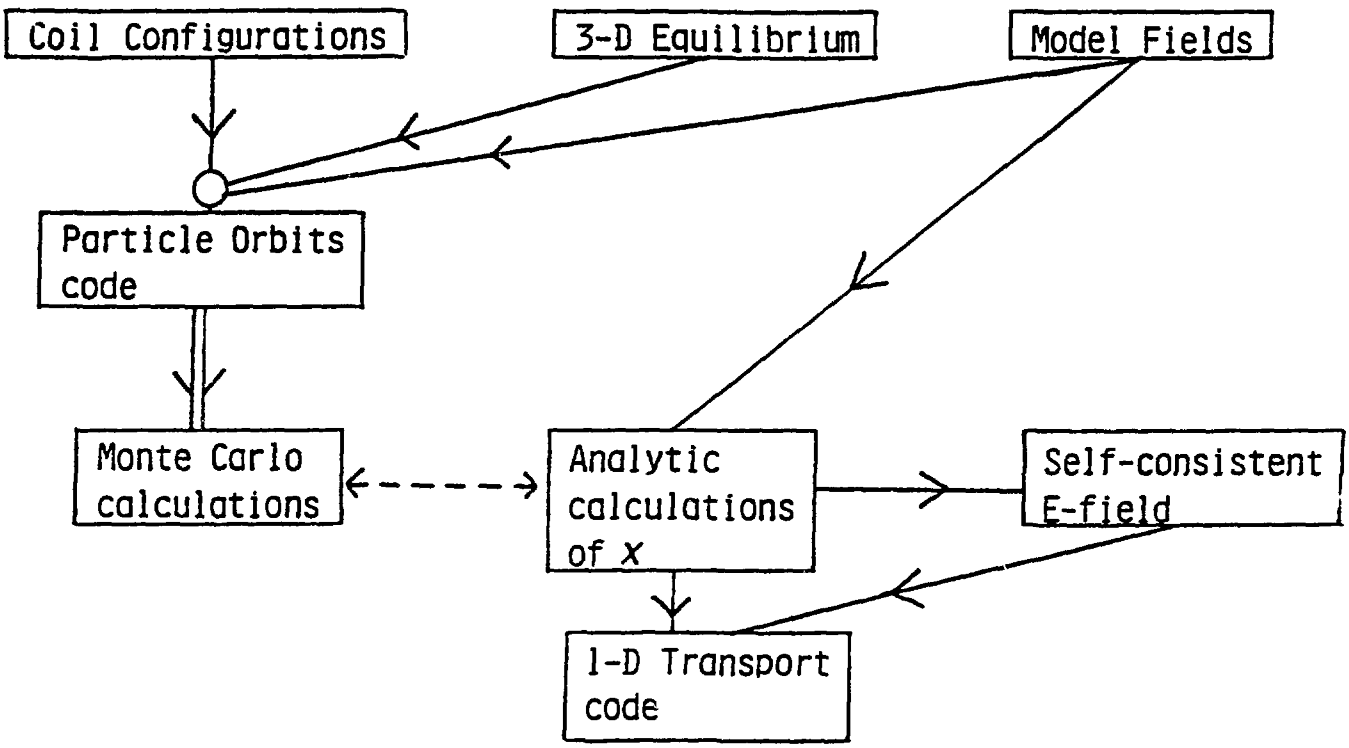

Coil Configurations
3-D Calculation
Method of Averaging
3-D Equilibrium Codes
Equilibrium Code (2-D)
Semi-analytical Calculations shift, well, ...
Reduced MHD for Stability
Particle Orbit Studies
3-D Stability code low n
low n modes
ballooning modes
Localized modes Criteria (high n)

TRANSPORT ⟷ ELECTRIC FIELDS

- RAPIDLY DEVELOPING AREA
- MULTI-LEVEL APPROACH
  - SINGLE ORBIT CONFINEMENT
  - COLLISIONLESS CONFINEMENT OF TEST DISTRIBUTIONS
  - ANALYTICAL TRANSPORT COEFFICIENTS USING MODEL FIELD
  - MONTE-CARLO CALCULATIONS

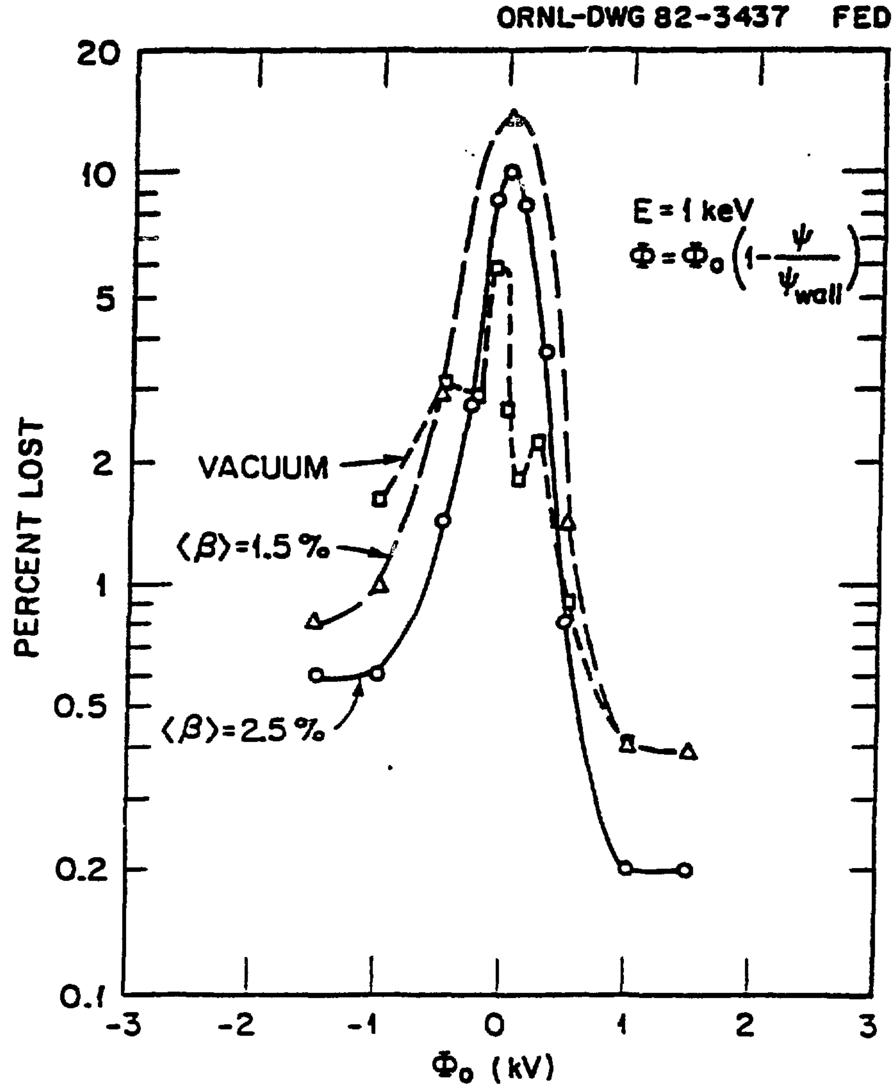
CORRELATION

TRANSPORT STUDIES



• MODEST ELECTRIC FIELDS OF EITHER SIGN

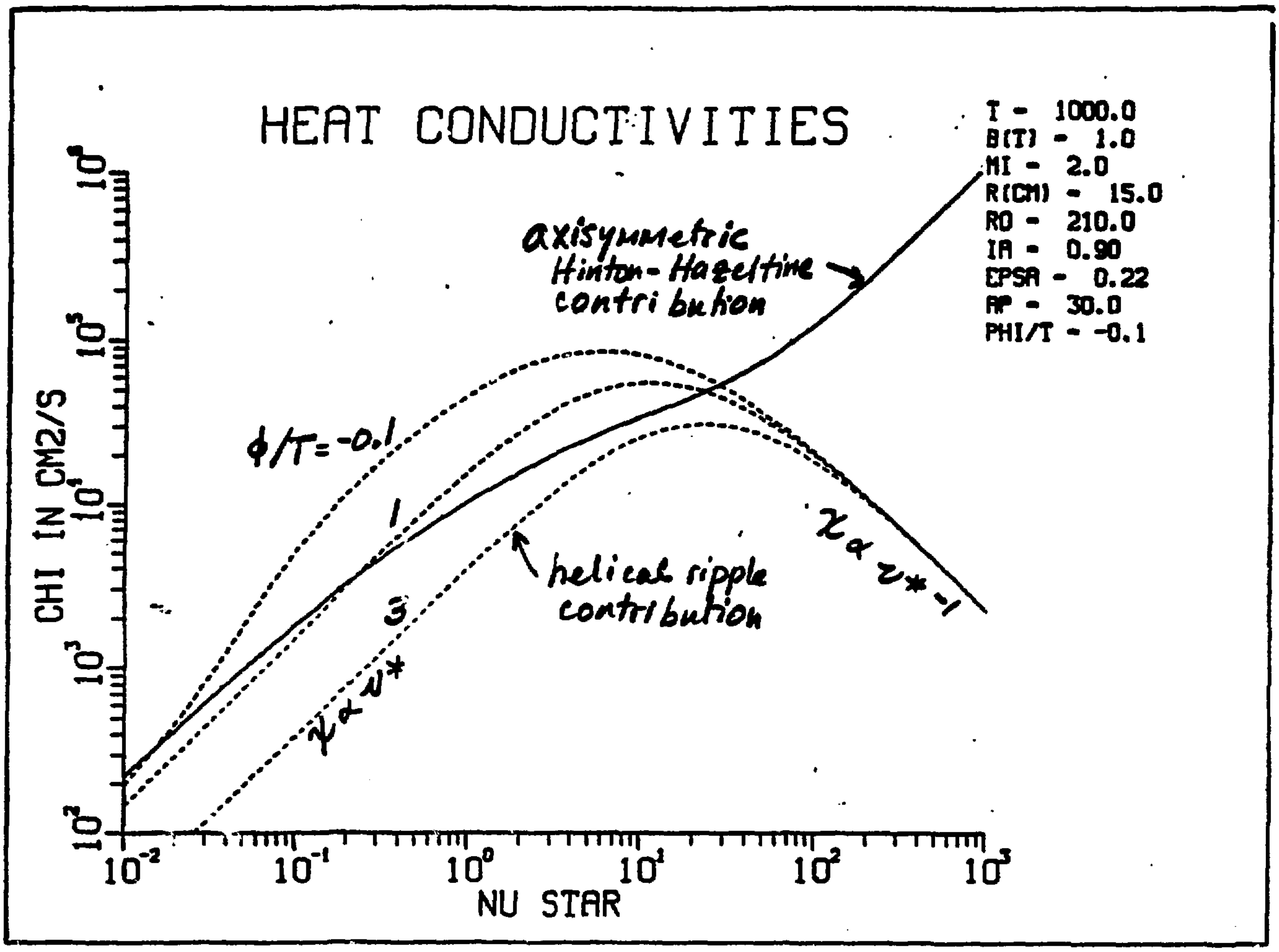
DRAMATICALLY REDUCE DIRECT LOSSES



COLLISIONLESS ORBITS

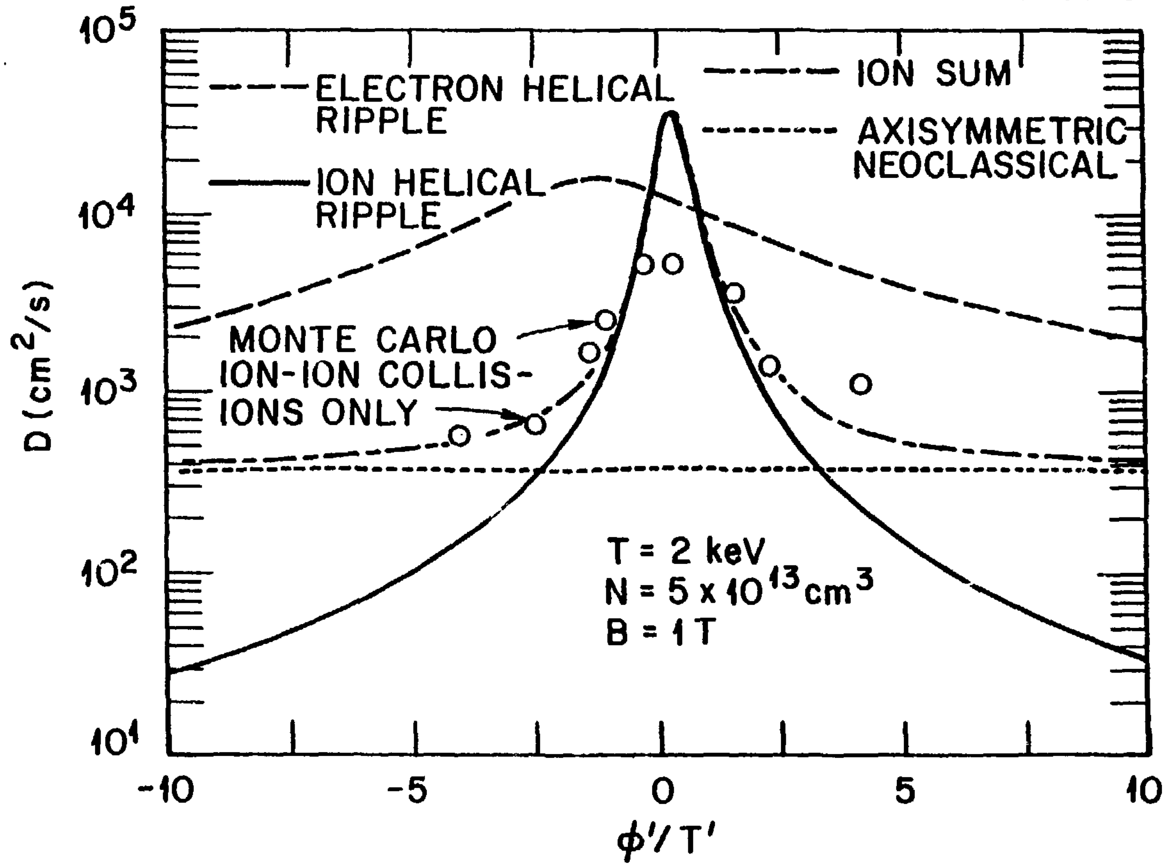
CONFINEMENT IMPROVEMENT WITH ELECTRIC FIELD

- MODERATE ELECTRIC FIELD MOVES TRANSITION TO FAVORABLE SCALING ($\chi \sim \nu^*$ INSTEAD OF $\chi \sim \nu^{*-1}$) TO HIGHER COLLISIONALITY



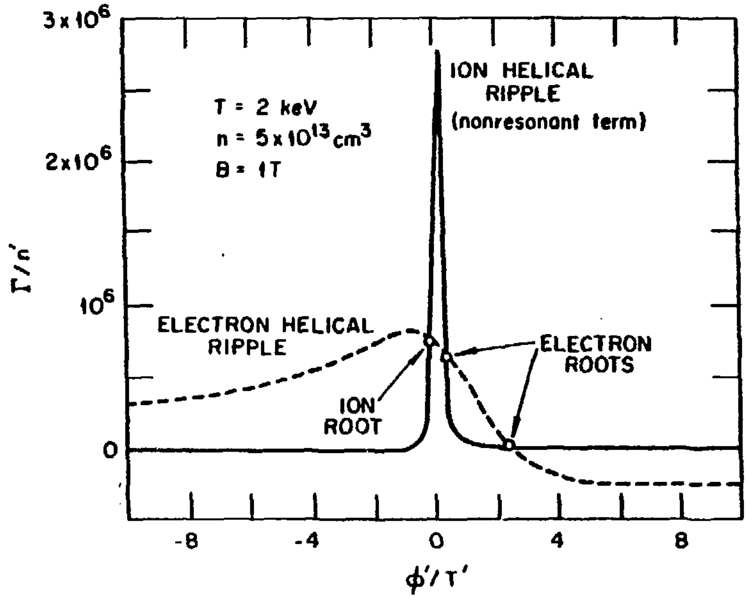
COMPARISON BETWEEN THE ANALYTICALLY CALCULATED ION DIFFUSION COEFFICIENTS AND THE MONTE-CARLO CALCULATIONS

FED/VU 83-67



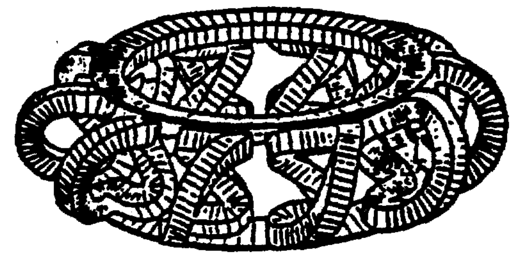
SELF-CONSISTENT ELECTRIC FIELD

- ATF EXAMPLE SHOWING THREE ROOTS FOR THE RADIAL ELECTRIC FIELD WHICH LEAD TO AMBIPOLAR FLUXES. THE ROOT WITH THE LARGEST RADIAL ELECTRIC FIELD LEADS TO MUCH REDUCED LOSSES.



$$\Gamma = -\epsilon_t^2 \epsilon_h^{1/2} v_D^2 n \int_0^\infty \frac{dx\, x^{5/2} e^{-x} \dfrac{\nu(x)}{\epsilon_h} \left[\dfrac{n'}{n} + \dfrac{e\phi'}{T} + \left(x - \dfrac{3}{2}\right)\dfrac{T'}{T}\right]}{\underbrace{\dfrac{\epsilon_t}{\epsilon_h} 1.67 (\omega_E + \omega_{\nabla B})^2}_{\substack{\text{NON-RESONANT} \\ \sim\nu}} + \underbrace{\left(\dfrac{\epsilon_t}{\epsilon_h}\right)^{3/2} \left[\overset{\text{SUPER-BANANA}}{\dfrac{\omega_{\nabla B}^2}{4}} + \overset{\text{PLATEAU}}{0.6 |\omega_{\nabla B}| \dfrac{\nu(x)}{\epsilon_h} \left(\dfrac{\epsilon_t}{\epsilon_h}\right)^{-3/2}}\right]}_{\text{RESONANT TRANSPORT}} + \underbrace{3\left(\dfrac{\nu(x)}{\epsilon_h}\right)^2}_{\sim 1/\nu}}$$

OTHER ATF-RELATED STUDIES



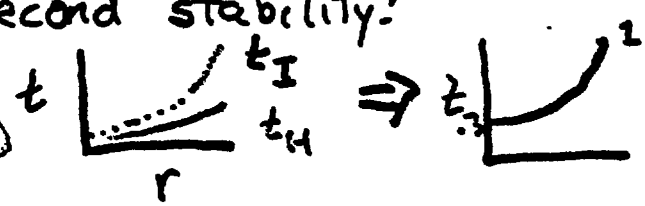
$\ell=2$ SYMMOTRON

(modular torsatron)

- MODULAR COIL SYSTEMS
  - equivalence to continuous helices
  - reactor issues: B_{max}, access, etc.
  - can make variety of configurations: shear, transform, shaping (beans)

- LOW ASPECT-RATIO SYSTEMS
  - goal: "compact" to reduce cost of reactors
  - preserve good features of ATF-1 physics
    → second stability, high-β
  - ATF-1 : $R/a \simeq 7$; promising results at $R/a \simeq 5$
    currently examining $R/a = 3$ candidates
    → helical axis useful to keep transform high

- TOKAMAK / STELLARATOR HYBRIDS
  - reduced current drive needs; disruption elimination
  - high-β via second stability:
    low t stell. + RF skin current drive



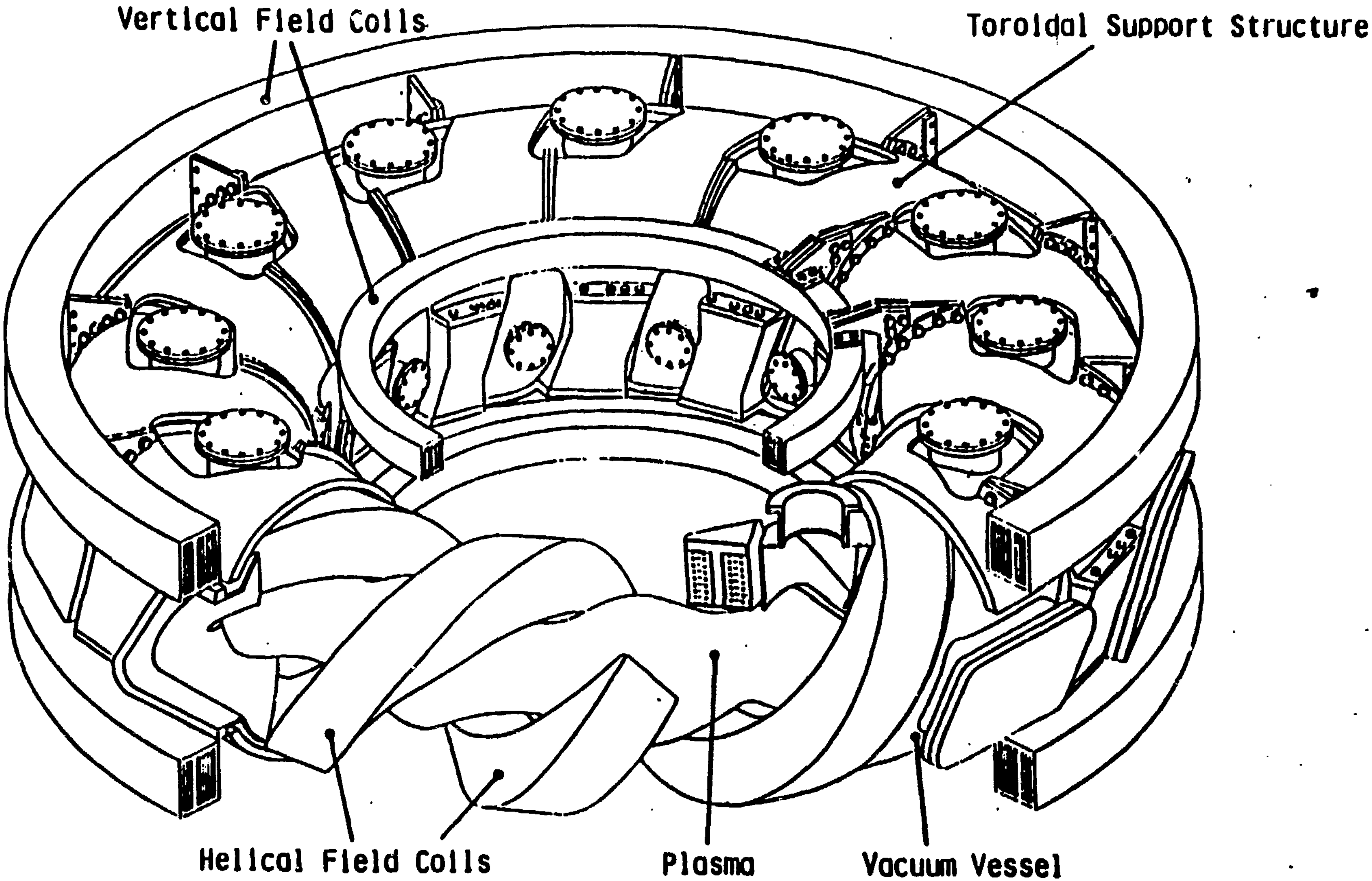
ATF-like stable

  - beans

✳ ATF-1 exp't + extension studies ⇒ IMPROVED TOROIDAL REACTOR

ATF-1 DESIGN STATUS - TOPICS

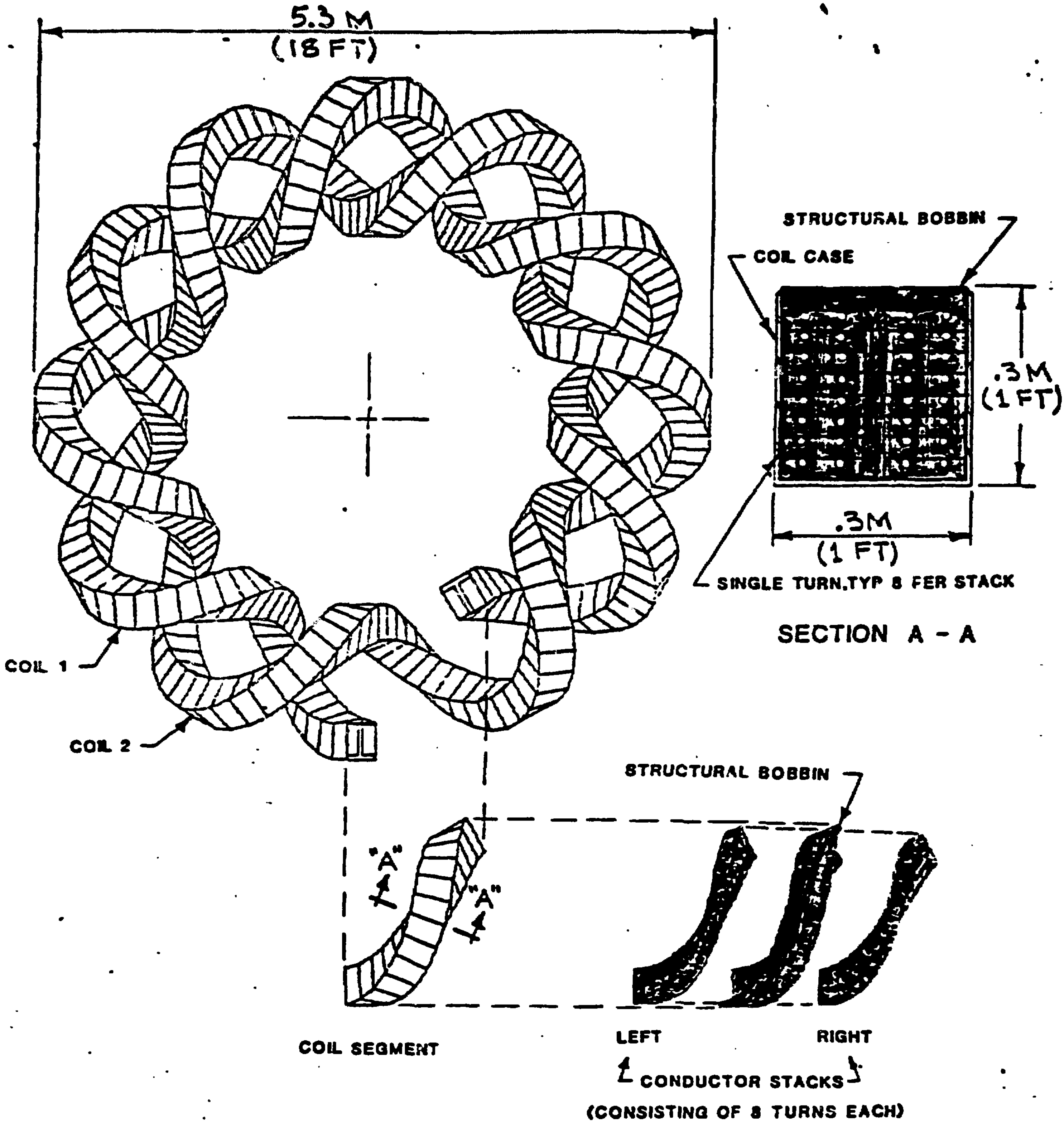
- OVERALL DESIGN FEATURES
- HELICAL FIELD COILS
- VERTICAL FIELD COILS
- VACUUM VESSEL
- STRUCTURAL SUPPORT
- SCHEDULE


Vertical Field Coils
Toroidal Support Structure
Helical Field Coils
Plasma
Vacuum Vessel

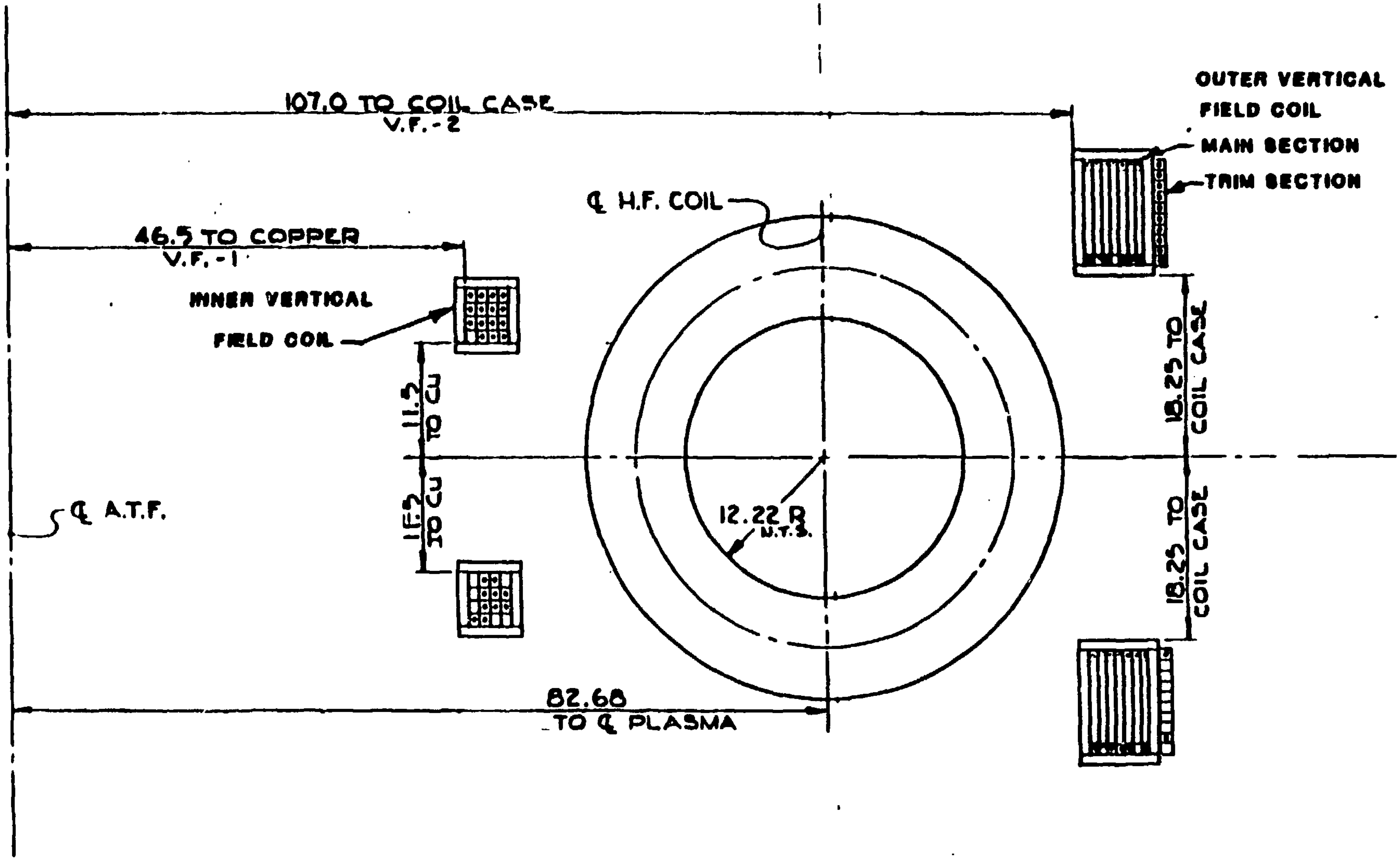
ATF-1

ATF-1 Design Changes Since Proposal (October 1982)

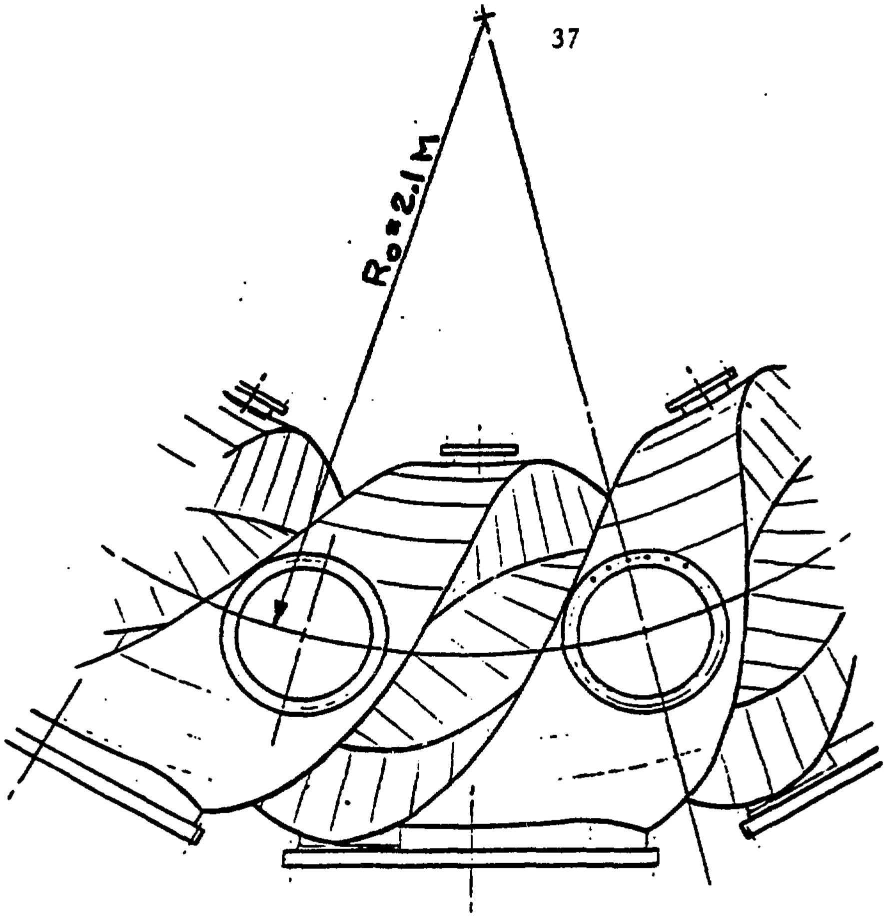
- Vacuum Vessel Expanded
    - Large Ports
    - Room for Divertor Hardware
- Structural Concept Modified
    - Radial Plate Structure Inadequate
    - Toroidal Shell Structure Substituted



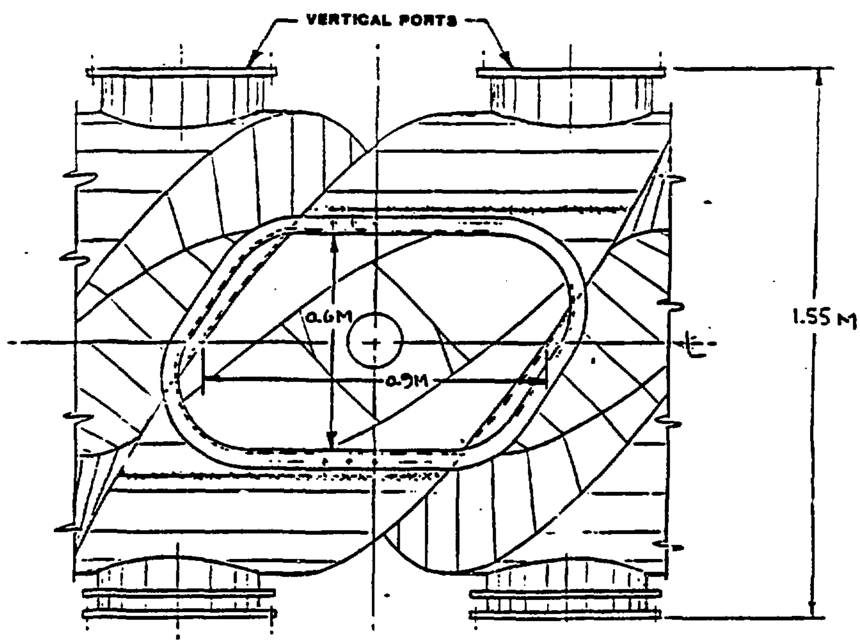
ATF HELICAL COIL SET SHOWING
COIL SEGMENT,BOBBIN AND
WINDING COMPONENTS



SCHEMATIC SECTION THROUGH COIL SET

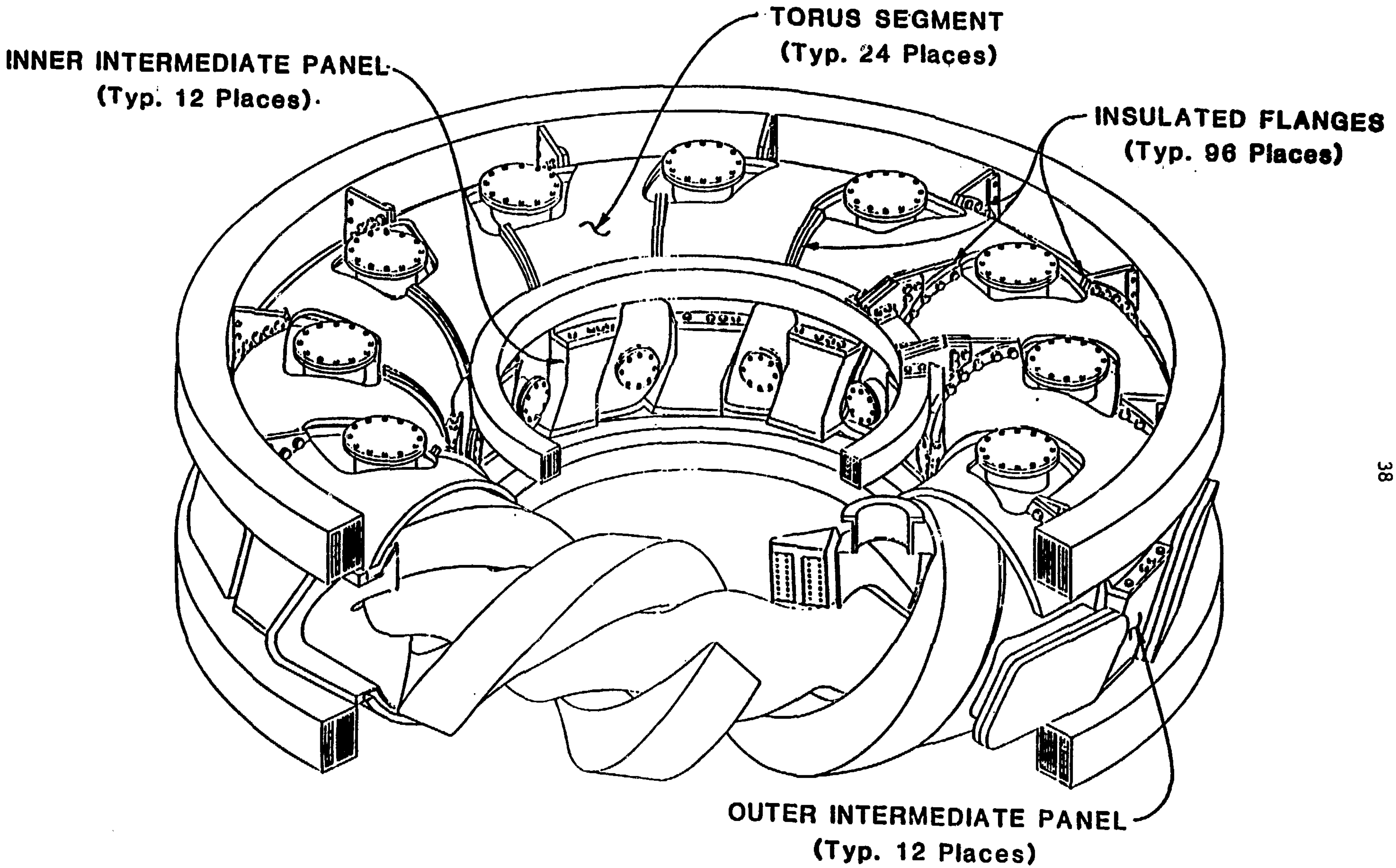


TOP VIEW



SIDE VIEW

VACUUM VESSEL



STRUCTURE SUPPORT SEGMENTED SHELL CONCEPT

ATF-1 SCHEDULE

1983

- FEBRUARY - PROTOTYPE HELICAL COIL PROCUREMENT INITIATED
- MARCH - COMPLETION OF CONCEPTUAL DESIGN AND CONSTRUCTION COST ESTIMATE
- OCTOBER - BEGIN ATF DIAGNOSTIC AND BEAM LINE LAYOUTS

1984

- JANUARY - HF COIL DEVELOPMENT FIXTURE COMPLETE
- MAY - COMPLETE HELICAL COIL TITLE II DESIGN
  - DELIVER PROTOTYPE COIL PARTS
- JUNE - BEGIN ATF DIAGNOSTICS DESIGN
- OCTOBER - TITLE I DESIGN COMPLETE

1985

- JANUARY - ISX-B REMOVAL COMPLETED
- JULY - TITLE II DESIGN COMPLETE
  - HELICAL COIL SEGMENTS DELIVERED
- SEPTEMBER - MACHINE DEVELOPMENT ACTIVITIES COMPLETE

1986

- APRIL - ALL COMPONENTS RECEIVED
- JULY - ASSEMBLY COMPLETE
- SEPTEMBER - PRE-OPERATIONAL TEST COMPLETE

Neoclassical Transport in a Multiple Helicity Torsatron -- Low Collisionality Regime*

K. C. Shaing

ORNL

*Research sponsored by the Office of Fusion Energy, OER, U. S. Department of Energy under contract W-7405-eng-26 with the Union Carbide Corporation.

Motivation

- Construct a <u>simple</u> magnetic field model for a multiple helicity torsatron
- Evaluate the diffusion coefficients
- Transport Optimazation
- Evaluate finite β effect on the diffusion coefficients

Outline of the Talk

- Summary of the fluxes in a single helicity model.
- Magnetic Field Model for a multiple helicity torsatron
- Neoclassical fluxes in $1/\nu$, collisionless detrapping and superbanana plateau regimes.

- Magnetic Field Model:

$$\frac{B}{B_0} = 1 - \epsilon_t \cos\theta - \epsilon_h \cos(\ell\theta - m\phi) \;,\; \underline{\epsilon_h \gg \epsilon_t}$$

- Action & Drifts:

$$J = \frac{16 R_0}{m}\left(\frac{\mu B_0 \epsilon_h}{M}\right)^{1/2}\left[E(k) - (1-k^2)K(k)\right]$$

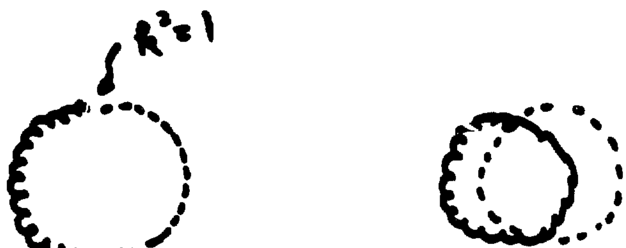
$$\langle \dot{r} \rangle = -\frac{\mu B_0 \epsilon_t}{M\Omega r}\sin\theta$$

$$\langle \dot{\theta} \rangle = -\frac{\mu B_0}{M\Omega r}\left[\epsilon_h'\left(\frac{2E}{K}-1\right) - \epsilon_t'\right] + \omega_E$$

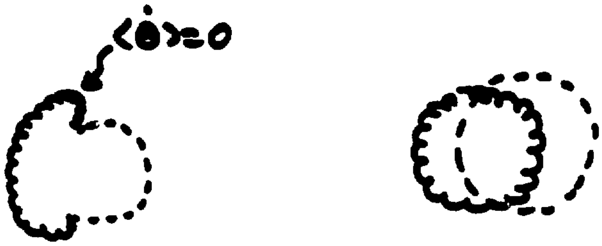
$$k^2 = \left[W - \mu B_0(1 - \epsilon_t \cos\theta - \epsilon_h)\right] / 2\mu B_0 \epsilon_h$$

- Collisionless Orbits (thermal particles):

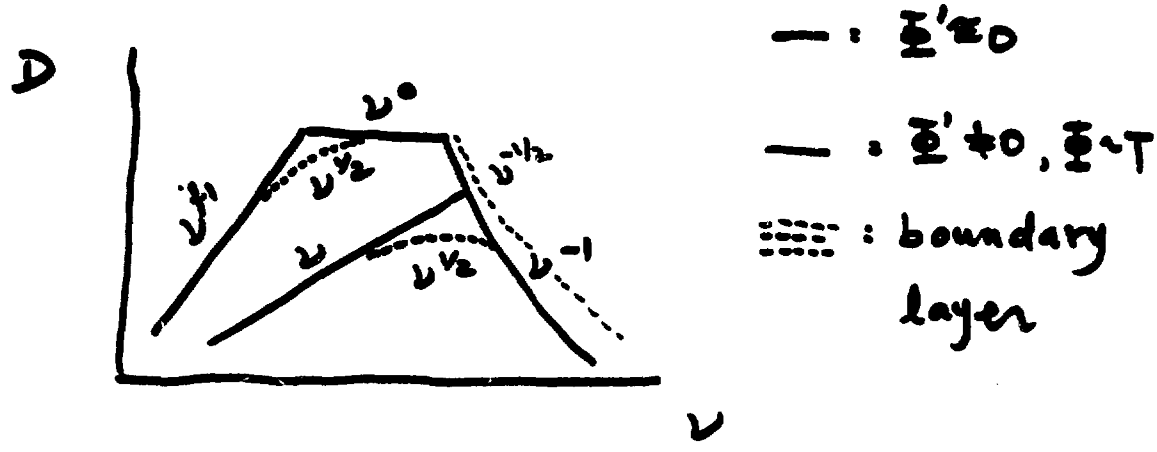
$\Phi' \neq 0$:
$\Phi \sim T$



$\Phi' = 0$



- Overall Transport:



- ν^{-1} regime: $\omega_0 < \nu_{eff} < \omega_{bh}$

$$\Delta r \sim v_d \, \nu_{eff}^{-1}$$

$$D \sim \underbrace{\sqrt{\epsilon_h}}_{f} \underbrace{\left(v_d^2 \, \nu_{eff}^{-2}\right)}_{(\Delta r)^2} \underbrace{\nu_{eff}}_{\Delta t^{-1}} \sim \underline{\sqrt{\epsilon_h} \frac{v_d^2}{\nu_{eff}}}$$

$\Delta t \sim \nu_{eff}^{-1}$

- ν^0 regime (Superbanana Plateau): $\omega_{Ds} < \nu_{eff} < \omega_{Dc}$

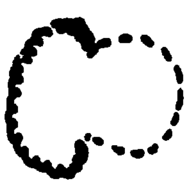
$$\langle\dot{\theta}\rangle' \Delta k^2 \simeq \frac{\nu}{\epsilon_h}\frac{1}{(\Delta k^2)^2}$$

$$\Rightarrow \quad \Delta k^2 \simeq \left(\frac{\nu/\epsilon_h}{\langle\dot{\theta}\rangle'}\right)^{1/3}$$

$$\Rightarrow \quad D \sim \underbrace{\sqrt{\epsilon_h}\left(\frac{\nu/\epsilon_h}{\langle\dot{\theta}\rangle'}\right)^{1/3}}_{f} v_d^2 \underbrace{\left(\frac{\nu}{\epsilon_h}\right)^{-1}\left(\frac{\nu/\epsilon_h}{\langle\dot{\theta}\rangle'}\right)^{2/3}}_{\Delta t}$$

$$\propto \nu^0$$

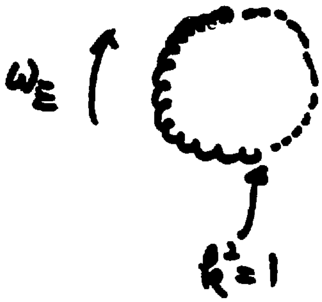
- Superbanana Regime : $\nu_{eff} < \omega_{Ds}$



$$\Delta r \sim \left(\frac{\epsilon_t}{\epsilon_h}\right)^{1/2} r \quad , \quad \epsilon_h \gg \epsilon_t$$

$$D \sim \underbrace{\sqrt{\epsilon_t}}_{f}\ \underbrace{\left(\frac{\epsilon_t}{\epsilon_h}\right) r^2}_{\Delta r^2}\ \underbrace{\frac{\nu}{\epsilon_t}}_{\Delta t^{-1}}$$

- Collisionless Detrapping Regime: $\nu_{eff} < \omega_E$



$$k^2 = \frac{W - \mu B_0 (1 - \epsilon_t \cos\theta - \epsilon_h)}{2\mu B_0 \epsilon_h}$$

$$\Delta k^2 = \epsilon_t/\epsilon_h$$

$$\Delta r \sim v_D \, \omega_E^{-1}$$

$$D \sim \underbrace{\sqrt{\epsilon_h}\,\frac{\epsilon_t}{\epsilon_h}}_{f} \; \underbrace{\left(v_D (\omega_E)^{-1}\right)^2}_{\Delta r^2} \; \underbrace{\frac{\nu}{\epsilon_h}\,\frac{1}{\epsilon_t^2/\epsilon_h^2}}_{\Delta t^{-1}}$$

- Smooth Connection of various regimes:

$$\underline{\Gamma} = -\epsilon_t^2 \epsilon_h^{1/2} v_D^2 n \int_0^\infty \frac{dx\, x^{5/2} e^{-x} \frac{\nu(x)}{\epsilon_h}\left[\frac{n'}{n} + \frac{e\Phi'}{T} + \left(x - \frac{3}{2}\right)\frac{T'}{T}\right]}{\underbrace{\left(\frac{\epsilon_t}{\epsilon_h}\right) 1.67 (\omega_E + \omega_{bB})^2}_{\substack{\text{Collisionless} \\ \text{detrapping}}} + \underbrace{\left(\frac{\epsilon_t}{\epsilon_h}\right)^{3/2}\left[\frac{\omega_{bB}^2}{4} + 0.6|\omega_{bB}| \frac{\nu(x)}{\epsilon_h}\left(\frac{\epsilon_t}{\epsilon_h}\right)^{-3/2}\right]}_{\substack{\text{Superbanana +} \\ \text{Superbanana plateau}}} + \underbrace{3\left[\frac{\nu(x)}{\epsilon_h}\right]^2}_{D^{-1}}}$$

empirical formula

Collisionless detrapping: nonresonant transport

Superbanana + Superbanana plateau: resonant transport

--- similar to the energy smoothing technique

$$B = \sum A(N,L) \cos[L\theta - 12N\phi]$$

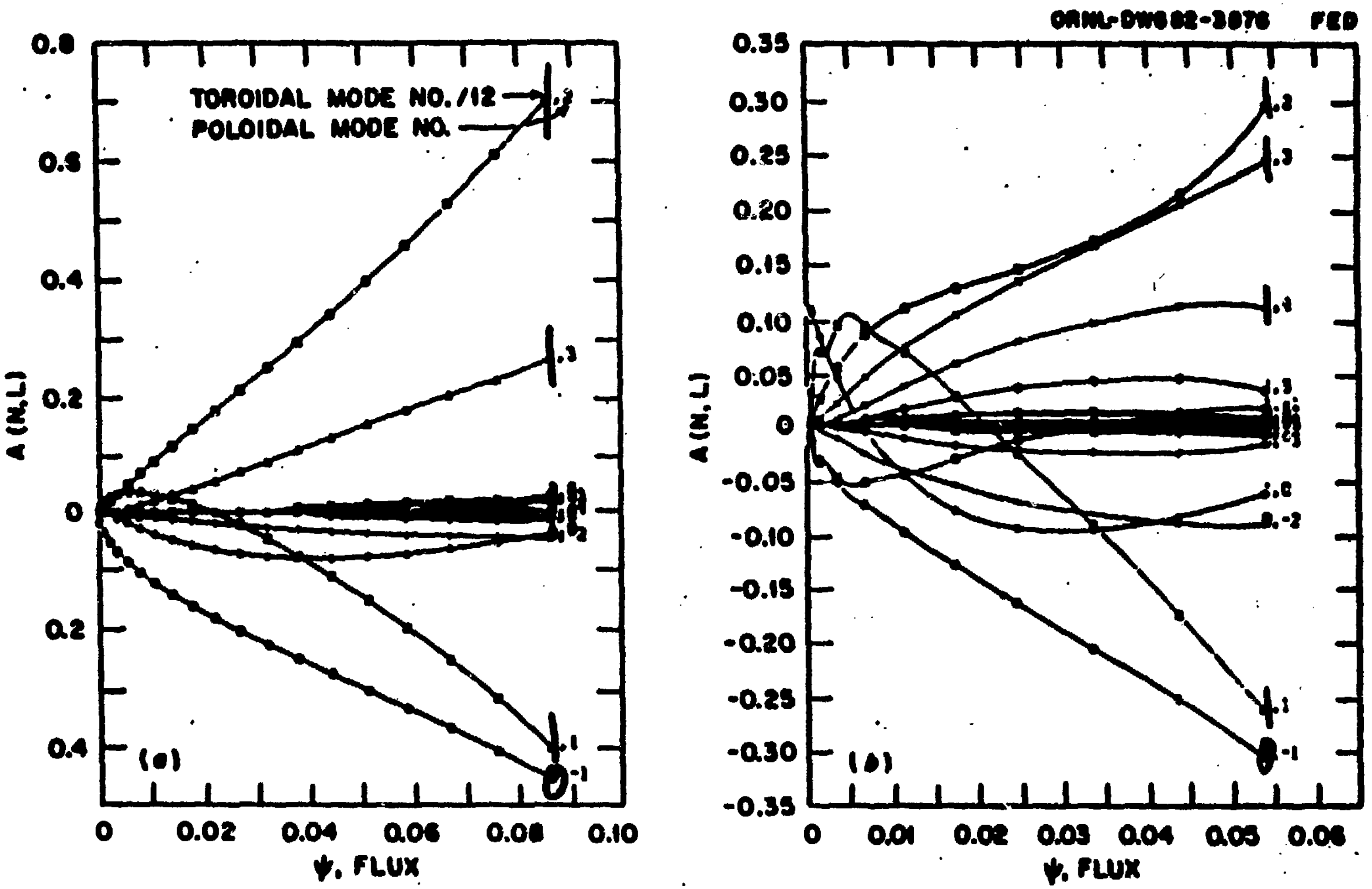


Fig. 1. The amplitude A(N,L) of the Fourier harmonic cos[ℓθ − (N × 12)φ] (a) for a vacuum magnetic field and (b) for a finite beta (<β> = 1.7%) magnetic field in a ℓ = 2, m = 12 torsatron.

Action & Drift Velocities

- $$J = \frac{16 R_0}{m}\sqrt{\frac{\mu B_0 \epsilon_H}{M}}\left[E(k) - (1-k^2)\,K(k)\right],$$

$$k^2 = \frac{W - \mu B_0\left(1 + \epsilon_T - \epsilon_H\right)}{2\mu B_0 \epsilon_H}.$$

bounce averaged drifts

$$\dot{\theta} = \underbrace{-\frac{\mu B_0}{M\Omega r}\left[\frac{\partial \epsilon_H}{\partial r}\left(\frac{2E}{K}-1\right) - \frac{\partial \epsilon_T}{\partial r}\right]}_{\omega_{\nabla B}} + \omega_E,$$

$$\dot{r} = \frac{\mu B_0}{M\Omega r}\left[\frac{\partial \epsilon_H}{\partial \theta}\left(\frac{2E}{K}-1\right) - \frac{\partial \epsilon_T}{\partial \theta}\right],$$

ω_E = $\vec{E}\times\vec{B}$ drift frequency

Magnetic Field Model

$$\frac{B}{B_0} = 1 + \epsilon_t \cos\theta + \epsilon_d \cos l\theta + \sum_{n=-\infty}^{+\infty} \epsilon^{(n)} \cos(n\theta + \eta)$$

$$= 1 + \epsilon_T + \epsilon_H \cos(\eta + \chi)$$

where $\eta = l\theta - m\phi$

$$\epsilon_H = \sqrt{C^2 + D^2}$$

$$\epsilon_T = \epsilon_t \cos\theta + \epsilon_d \cos l\theta$$

$$C = \epsilon^{(0)} + \left[\epsilon^{(+1)} + \epsilon^{(-1)}\right] \cos\theta + \cdots$$

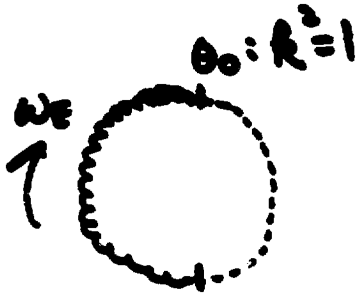
$$D = \left[\epsilon^{(+1)} - \epsilon^{(-1)}\right] \sin\theta + \cdots$$

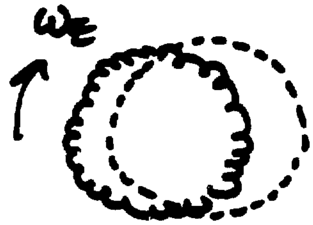
$$\cos\chi = C / \epsilon_H ,$$

$$\sin\chi = D / \epsilon_H .$$

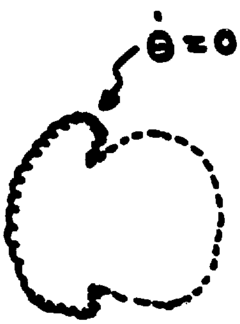
Collisionless Orbits

- $\omega_E \gg \omega_{\nabla B}$





- $\omega_E \lesssim \omega_{\nabla B}$



• ν^{-1} regime : $\omega_D < \nu_{eff} < \omega_{bh}$

$$\Gamma = -\frac{4}{\pi}\frac{1}{M^{7/2}\Omega^2 r^2}\left(\int_0^\infty dE\,\frac{E^{5/2}}{\nu}\frac{\partial f_M}{\partial r}\right)\times$$

$$\int_0^{2\pi} d\theta\,\epsilon_H^{3/2}\left[G_1\left(\frac{\partial\epsilon_T}{\partial\theta}\right)^2 - 2G_2\frac{\partial\epsilon_T}{\partial\theta}\frac{\partial\epsilon_H}{\partial\theta} + G_3\left(\frac{\partial\epsilon_H}{\partial\theta}\right)^2\right]$$

$$G_1 = \frac{16}{9},$$

$$G_2 = \frac{16}{15},$$

$$G_3 = .684$$

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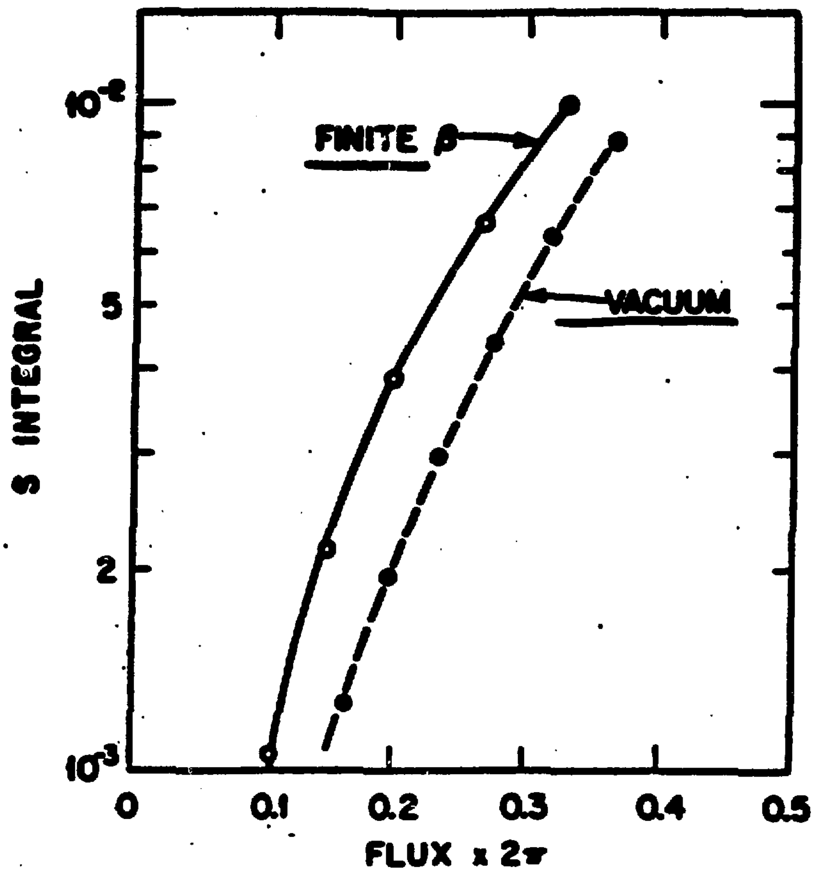


Fig. 3. The transport integral S vs radial flux for the A(N,L) shown in Fig. 1.

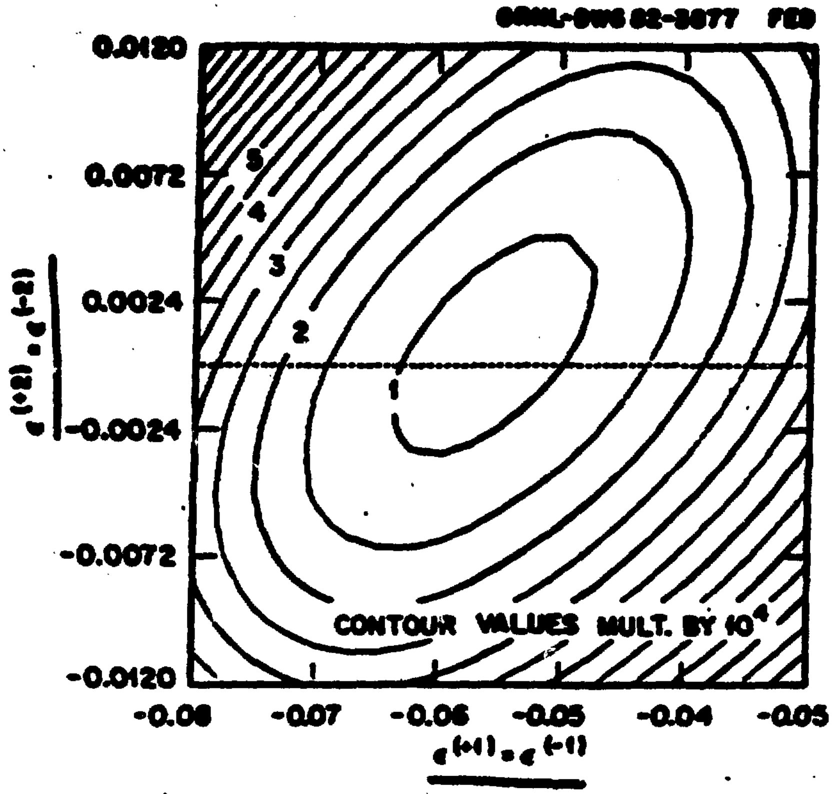
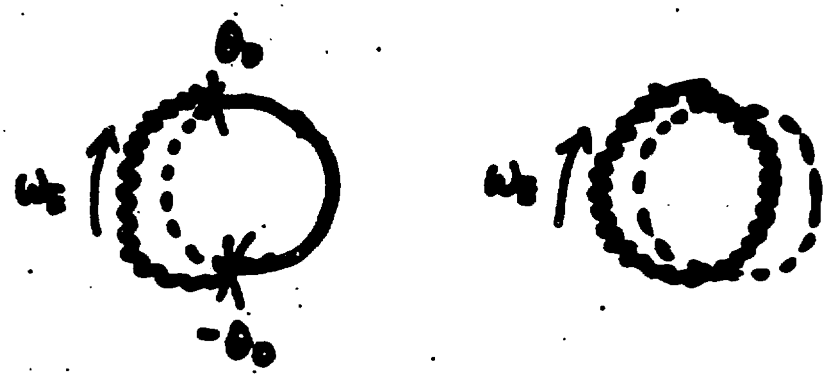


Fig. 2. The contour plot of the transport integral S for the constraints $\epsilon^{(+1)} = \epsilon^{(-1)}$, $\epsilon^{(+2)} = \epsilon^{(-2)}$, $\epsilon^{(0)}/\epsilon_t = 1.56$, $\epsilon_d = 0$, and $\epsilon_t = -7.35 \times 10^{-2}$.

Strong E_r limit

- $\omega_E \gg \omega_{vB}$

- typical collisionless orbits :



$k^2(\theta=\theta_0)=1$ transition banana

$k^2 < k^2_{min}$ circulating banana

- $\omega_E \ll \omega_{bb}$, toroidal banana bounce frequence

- In the limit $\omega_{bb} \rightarrow \infty$, no collisional event will occur at the toroidally trapped portion and thus no transport

- In the case $\omega_E \ll \omega_{bh}$, the transport is induced from the <u>helically trapped portion</u>.

- Solubility Constraints:

$$\int_0^{2\pi} \frac{d\theta}{\langle \dot\theta \rangle} \langle C(f) \rangle = 0 \quad , \quad 0 \le k_0^2 \le k_{0\min}^2$$

... circulating banana

$$\int_{\theta_0}^{2\pi-\theta_0} \frac{d\theta}{\langle \dot\theta \rangle} \langle C(f) \rangle = 0 + O\left(\frac{\langle \dot\theta \rangle}{\omega_{bb}}\right) ,$$

$k_{0\min}^2 \le k_0^2 \le 1$ ... transition banana

- Boundary Conditions:

$$f = \frac{W \epsilon_t}{M \Omega r \omega_E} \cos\theta \, \frac{\partial f_m}{\partial r} + g$$

at $0 \le k_0^2 \le k_{0\min}^2$: $\underline{g = 0}$

$k_0^2 = 1$: $\underline{f = 0.}$

Results

- Single Helicity $(\partial \epsilon_k/\partial\theta = 0,\ \epsilon_T = -\epsilon_t \cos\theta)$

$$\Gamma = -0.0635\,\frac{N \nu_t \epsilon_t^3 T^2}{M^2 \Omega^2 r^2 \omega_E^2 \sqrt{\epsilon_k}}\,\frac{I}{\left[\ln\left(\frac{1+\epsilon_t/2\epsilon_k}{k_{min}^2 + \frac{\epsilon_t}{2\epsilon_k}}\right)\right]^2}$$

$$\int dx\, x e^{-x}\left[\frac{N'}{N} + \frac{e\Phi'}{T} + \left(x - \frac{3}{2}\right)\frac{T'}{T}\right],$$

$$I = 2\pi \ln\left[\frac{1+\sqrt{1-\epsilon_t/2\epsilon_k}}{2\left(k_{min}^2 + \frac{\epsilon_t}{2\epsilon_k}\right)}\right] + \frac{2\pi}{\sqrt{1-\left(\frac{\epsilon_t}{2\epsilon_k}\right)^2}}\left[1 - \sqrt{1-\left(\frac{\epsilon_t}{2\epsilon_k}\right)^2}\right].$$

- $\epsilon_k \gg \epsilon_t$:

$$\Gamma = -0.2\,\frac{N \epsilon_t \sqrt{\epsilon_k}\,\nu_t T^2}{M^2 \Omega^2 r^2 \omega_E^2}\left(\frac{N'}{N} + \frac{e\Phi'}{T} + \frac{1}{2}\frac{T'}{T}\right)$$

- Multiple Helicity

$$\Gamma = \Gamma_g + \Gamma_h + 2\,\Gamma_{gh}$$

$$\Gamma_g = -0.0635\,\frac{N \nu_e \epsilon_g^2 T^2}{M^2 \Omega^2 r^2 \omega_E^2 \sqrt{G_H^2}}\;\frac{I_g\, I_E}{\left\{\ln\left[\dfrac{2G_H^2 + \sum_{n=1}^{\infty}\left(A_{nn} - A_{nn-1}\right)}{G_{H0} + G_{T0}}\right]\right\}^2}$$

$$\epsilon_g = \frac{G_T^2 + G_H^2}{2}\left(\ln G_H - \langle \ln G_H\rangle\right)^2 - \frac{1}{2}\left(G_H - \langle G_H\rangle\right)^2 + G_T^2$$
$$+ \sum_{n=1}^{\infty}(-1)^n \frac{G_n}{n}\,.$$

$$I_g = \int_0^{2\pi} d\theta \sqrt{\frac{G_H^2}{G_H}}\left[\left(\frac{A_0 + G_H - G_T}{G_{H0} + G_{T0} - A_0} - \frac{A_0 + G_H - G_T}{G_T + G_H - A_0}\right) + \ln\left(\frac{G_T + G_H - A_0}{G_{H0} + G_{T0} - A_0}\right)\right].$$

$$G_{H0} = G_H(\theta = 0)\,,$$

$$I_E = \frac{N'}{N} + \frac{\Omega'}{\Omega} + \frac{1}{2}\frac{T'}{T}\,,$$

- Multiple Helicity (cont.)

$$\Gamma_h = -0.0635 \frac{N v_t \epsilon_H^{0\,2} T^2}{M^2 \Omega^2 \gamma^2 \omega_t^2 \sqrt{\epsilon_H}} I_h I_E ,$$

$$I_h = \int_0^{2\pi} d\theta \sqrt{\frac{\epsilon_H}{\epsilon_T^0}} \left(\ln \epsilon_H - \langle \ln \epsilon_H \rangle\right)^2 \left[\frac{\epsilon_T^0 - \epsilon_H^0 - (\epsilon_T - \epsilon_H)}{2\epsilon_H} k_{rt}^2 + \frac{\epsilon_H^0 k_{rt}^4}{2\epsilon_h} \right] ,$$

$$k_{rt}^2 = (\epsilon_T + \epsilon_H - \epsilon_T^0 + \epsilon_H^0)/2\epsilon_H^0 ,$$

Γ_{jh} : cross term

- If no collisionless Detrapping ($\epsilon_h \gg \epsilon_t$)

  - Boundary Layer + <u>Deeply trapped particle contribution</u> ($\partial \epsilon_H / \partial \theta \neq 0$)

  - $\Gamma = \Gamma_d + \Gamma_b$

$$\Gamma_d = -0.0317 \frac{N \nu_t T^2}{M^2 \Omega^2 r^2 \omega_E^2} I_d I_E$$

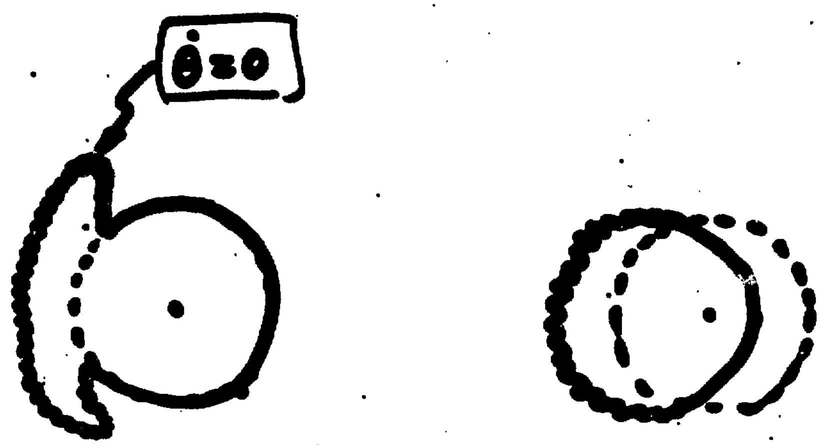
$$I_d = \int_0^{2\pi} d\theta \frac{(\epsilon_H - \langle \epsilon_H \rangle)^2}{\sqrt{\epsilon_H}}$$

$$\Gamma_b = -0.057 \frac{\sqrt{K} \sqrt{\nu_t} T^2}{M^2 \Omega^2 r^3 \omega_E^{3/2}} \sum_n (n a_n^2) \times$$

$$\int dx\, x^{7/4} e^{-x} \left[\frac{N'}{N} + \frac{e\Phi'}{T} + \left(x - \frac{3}{2}\right) \frac{T'}{T} \right].$$

- **Superbanana Plateau Regime: $\omega_E \lesssim \omega_{\nabla B}$**

- typical collisionless orbits:



Superbanana

circulating banana.

- Plateau Regime:

$$\omega_{Db} < \nu_{eff} < \omega_{Dc}$$

$$\left.\begin{matrix} \omega_{Dc} \\ \omega_{Db} \end{matrix}\right\} \text{drift frequency} \left\{\begin{matrix} \text{circulating banana} \\ \text{superbanana} \end{matrix}\right.$$

- Special Case: $\left(\frac{\partial \epsilon_H}{\partial \theta} = 0 \;,\; \epsilon_T = -\epsilon_t \cos\theta \;,\; \epsilon_h \sim \epsilon_t \right)$

- $\omega_E = 0 \;,\; \epsilon_H' < \epsilon_t'$

$$\Gamma = -\frac{4\pi \sqrt{\epsilon_H}\, \epsilon_t^2}{M^{5/2} |\Omega| r \underline{\epsilon_H'}} \int dW\, W^{3/2} \frac{\partial f_M}{\partial r} \int_{\cos(\epsilon_H'/\epsilon_t')}^{\pi - \cos^{-1}(\epsilon_H'/\epsilon_t')} d\theta\, \sin^2\theta$$

$\epsilon_H' \to 0$:

$$\Gamma = -\frac{8\pi \sqrt{\epsilon_H}\, \epsilon_t^2}{M^{5/2} |\Omega| r \underline{\epsilon_t'}} \int dW\, W^{3/2} \frac{\partial f_M}{\partial r}$$

- $\omega_E \Omega \begin{matrix} < 0 \\ > 0 \end{matrix}$

$$\Gamma = -\frac{4\pi \epsilon_t^2}{M^{5/2} |\Omega| r} \frac{\sqrt{\epsilon_H}}{\epsilon_H'} \int_{W_{min}}^{\infty} dW\, W^{3/2} \frac{\partial f_M}{\partial r} \int_{Max(\theta_m, 0)}^{Min(\theta_M, \pi)} d\theta\, \sin^2\theta$$

$$\theta_M = \cos^{-1}\left(-\frac{\epsilon_H'}{\epsilon_t'} \mp \frac{M r |\Omega \omega_E|}{W \epsilon_t'} \right),$$

$$\theta_m = \cos^{-1}\left(\frac{\epsilon_H'}{\epsilon_t'} \mp \frac{M r |\Omega \omega_E|}{W \epsilon_t'} \right),$$

$$W_{min} = \frac{M r |\Omega \omega_E|}{\epsilon_t' + \epsilon_H'}.$$

Results

$$\Gamma = -\frac{2\pi}{M^{m}|a|^2}\int dW \int d\theta \; W^{m} \frac{\partial f_0}{\partial \varepsilon} \frac{\sqrt{G_n}}{G_n'} \times$$

$$\sum_{n,m=1}^{\infty}\left(\frac{M\Omega\gamma\omega_B}{W}X_m + Y_m + Z_m\right)\left(\frac{M\Omega\gamma\omega_B}{W}X_n + Y_n + Z_n\right) \times$$

$$\frac{\sin n\theta \, \sin m\theta}{n} .$$

$$\frac{\partial G_n/\partial\theta}{G_n'} = \sum_{n=1}^{\infty} X_n \sin n\theta ,$$

$$\frac{(\partial G_n/\partial\theta)\, G_T'}{G_n'} = \sum_{n=1}^{\infty} Y_n \sin n\theta ,$$

$$-\frac{\partial \psi_T}{\partial\theta} = \sum_{n=1}^{\infty} Z_n \sin n\theta .$$

Conclusions

- A realistic magnetic field model for a multiple helicity torsatron is obtained.

- A set of neoclassical flux expressions is obtained in the low collisionality regime. ($\nu_{eff} < \omega_{bb}$)

- Finite β can have a significant effect on the diffusion coefficients.

MONTE CARLO TRANSPORT IN ATF*

James A Rome
Ronald H. Fowler
James F. Lyon
Wayne A. Houlberg
Ker Chung Shaing

IV U.S. Stellarator Workshop
April 14, 1983

*Research sponsored by the Office of Fusion Energy, OER, U. S. Department of Energy under contract W-7405-eng-26 with the Union Carbide Corporation.

Purpose: To validate analytic transport coefficients in collisionless regimes.

E-FIELD EFFECTS:

- LOWERS "MT. RIPPLE"
- ALLOWS ENTERING COLLISIONLESS ν REGIME AT LOWER ν
- SEALS OFF LOSS CONES

MODEL FOR PARTICLE AND ENERGY FLUXES

à la Shaing-Houlberg

The electron and ion particle and heat fluxes due to helical trapping are given by:

$$\Gamma_j^{ht} = - \boxed{\varepsilon_t^2 \sqrt{\varepsilon_h}\, v_d^2\, n_j \int_0^\infty x^{5/2} e^{-x} dx \; \tilde{\nu}_j(x) \frac{A_j(x)}{\omega_j^2(x)}}$$

"D" FOR MONTE CARLO CODE CHECK

$$Q_j^{ht} = - \varepsilon_t^2 \sqrt{\varepsilon_h}\, v_{dj}^2 n_j T_j \int_0^\infty x^{7/2} e^{-x} dx \; \tilde{\nu}_j(x) \frac{A_j(x)}{\omega_j^2(x)}$$

$$A_j(x) = \frac{n_j'}{n_j} + \frac{e_j \varphi'}{T_j} + (x - 1.5)\frac{T_j'}{T_j}$$

$$\tilde{\nu}_j(x) = \frac{\nu_j(x)}{\varepsilon_h} = \frac{\nu_{jT}}{\varepsilon_h x^{3/2}}$$

$$x = \frac{E}{T_j}$$

$$\omega_j^2(x) = \frac{\varepsilon_t}{\varepsilon_h}\Big\{\underbrace{1.67(\omega_E + \omega_{\nabla Bj})^2}_{\substack{\text{non resonant} \\ \sim \nu}} + \underbrace{\left(\frac{\varepsilon_t}{\varepsilon_h}\right)^{1/2}\Big[\underbrace{\frac{\omega_{\nabla Bj}^2}{4}}_{\text{superbanana}} + \underbrace{.6|\omega_{\nabla Bj}|\,|\tilde{\nu}_j|\Big]\left(\frac{\varepsilon_h}{\varepsilon_t}\right)^{3/2}}_{\text{plateau}}}_{\substack{\text{resonant transport} \\ \dot{\theta} \simeq 0}}\Big]\Big\} + \underbrace{3\tilde{\nu}_j^2(x)}_{\substack{\text{collisional} \\ \sim 1/\nu}}$$

$$\omega_E = \frac{\varphi'}{eBr}$$

$$\omega_{\nabla Bj} = - v_{dj} \varepsilon_h' x = - \frac{T_j}{e_j Br} \varepsilon_h' x$$

TRANSPORT STUDIES

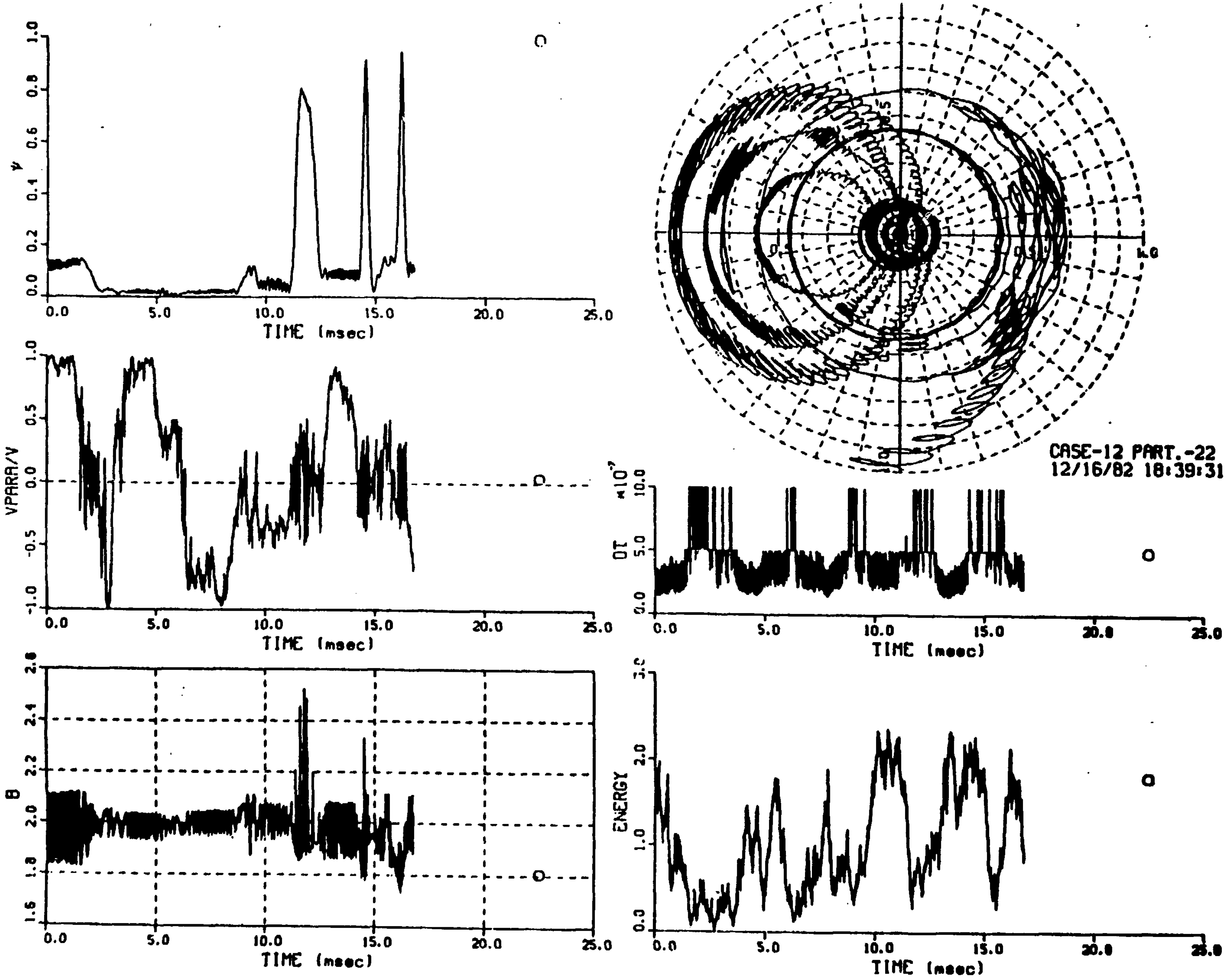
collaboration with K.C. Shaing and W.A. Houlberg

Ions in ATF are in the collisionless ν regime for all reasonable plasma parameters. Electrons are usually in the $1/\nu$ regime and may dominate the loss process.

Immediate goal is to test the analytic theories of Shaing and Kovriznykh in the ν regime to determine which is correct.

Calculational difficulties:

- $t > \tau_{orbit}$ and $t < \tau_{loss}, \tau_{non-localization}$.
- Direct losses contaminate D.
- Quiet start essential.
- Statistics in tail of distribution function poor.



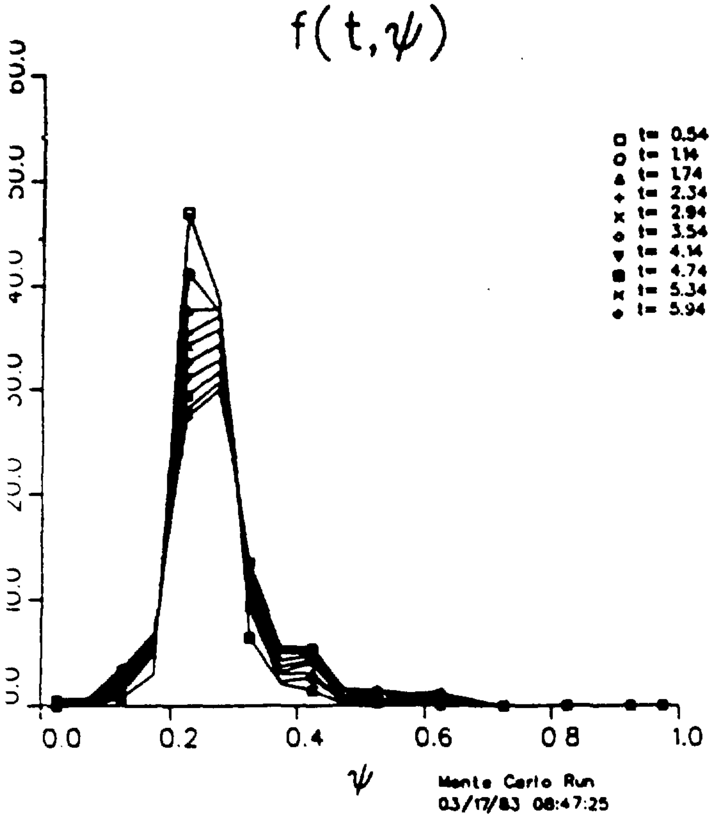
CASE-12 PART.-22
12/16/82 18:39:31
ψ
VPARA/V
B
DT ×10⁷
ENERGY
TIME (msec)

METHOD

- Determine loss region and % loss by following collisionless particles for τ_{90} or τ_{orbit}, whichever is smaller.
- Use end locations of contained particles for quiet start.
- For energy distributions, either:
  - — Use fixed v, no v scattering.
  - — Use 3T/2 and v scattering.
  - — Sample from f_M with v scattering.
- Plot $\langle \psi^2 \rangle - \langle \psi \rangle^2$ versus t and fit to $C_1 + C_2 t$.

$$D(T,\psi) = C_2/2 \text{ with v scattering}$$

$$D(T,\psi) = (1/n)\int D(v,\psi)\, f_M(v)\, d^3v.$$


f(t,ψ)
t= 0.54
t= 1.14
t= 1.74
t= 2.34
t= 2.94
t= 3.54
t= 4.14
t= 4.74
t= 5.34
t= 5.94
0.0
0.2
0.4
0.6
0.8
1.0
ψ
Monte Carlo Run
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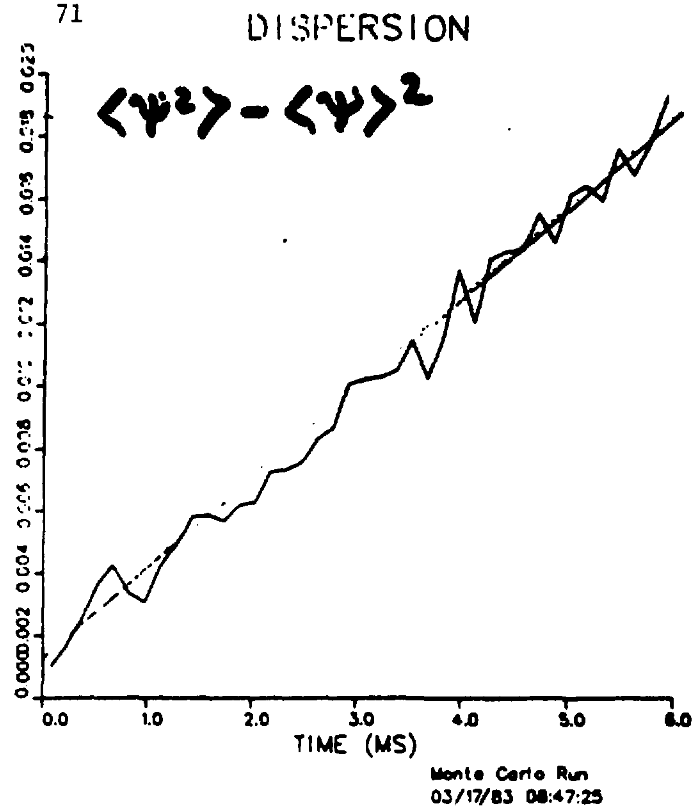
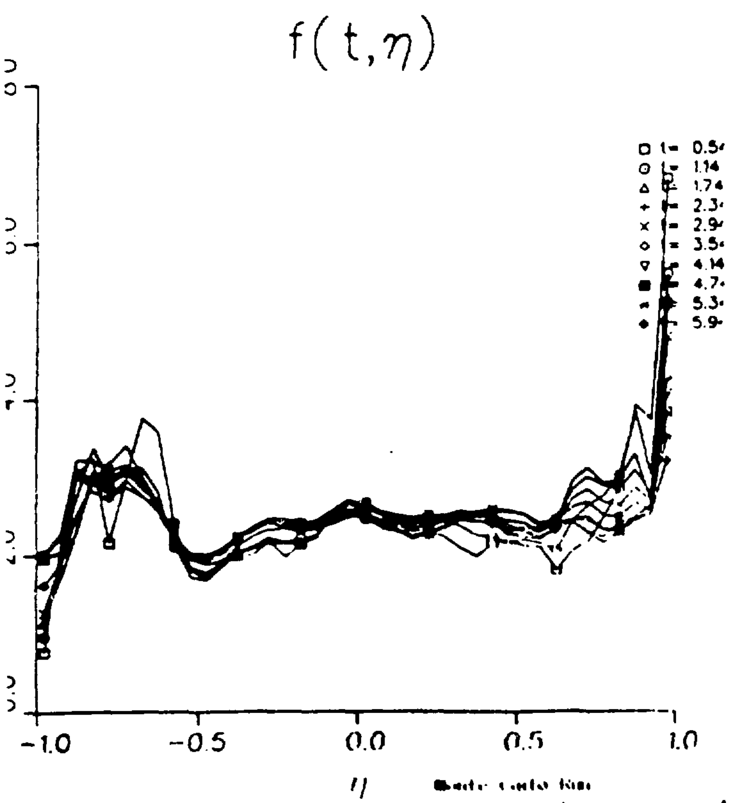
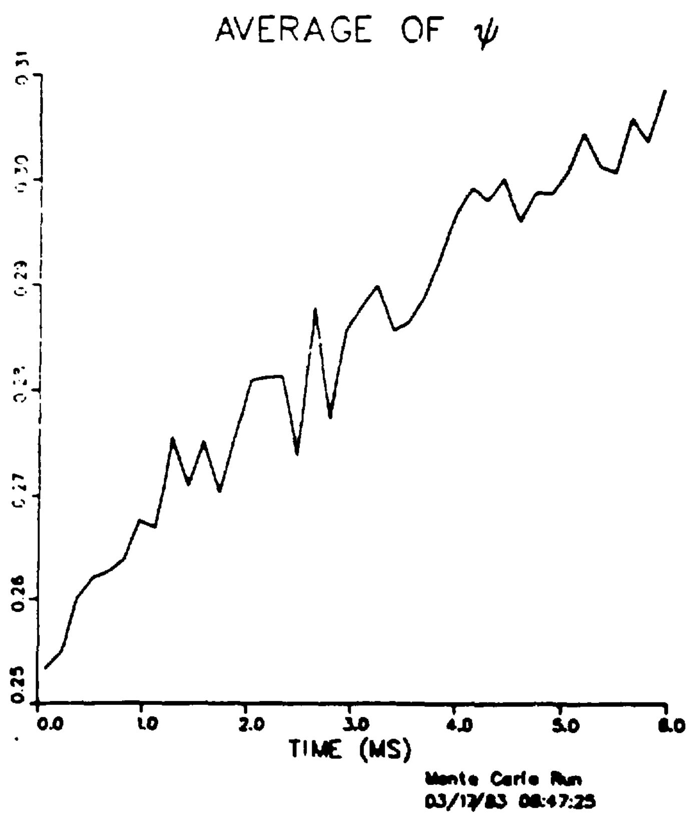
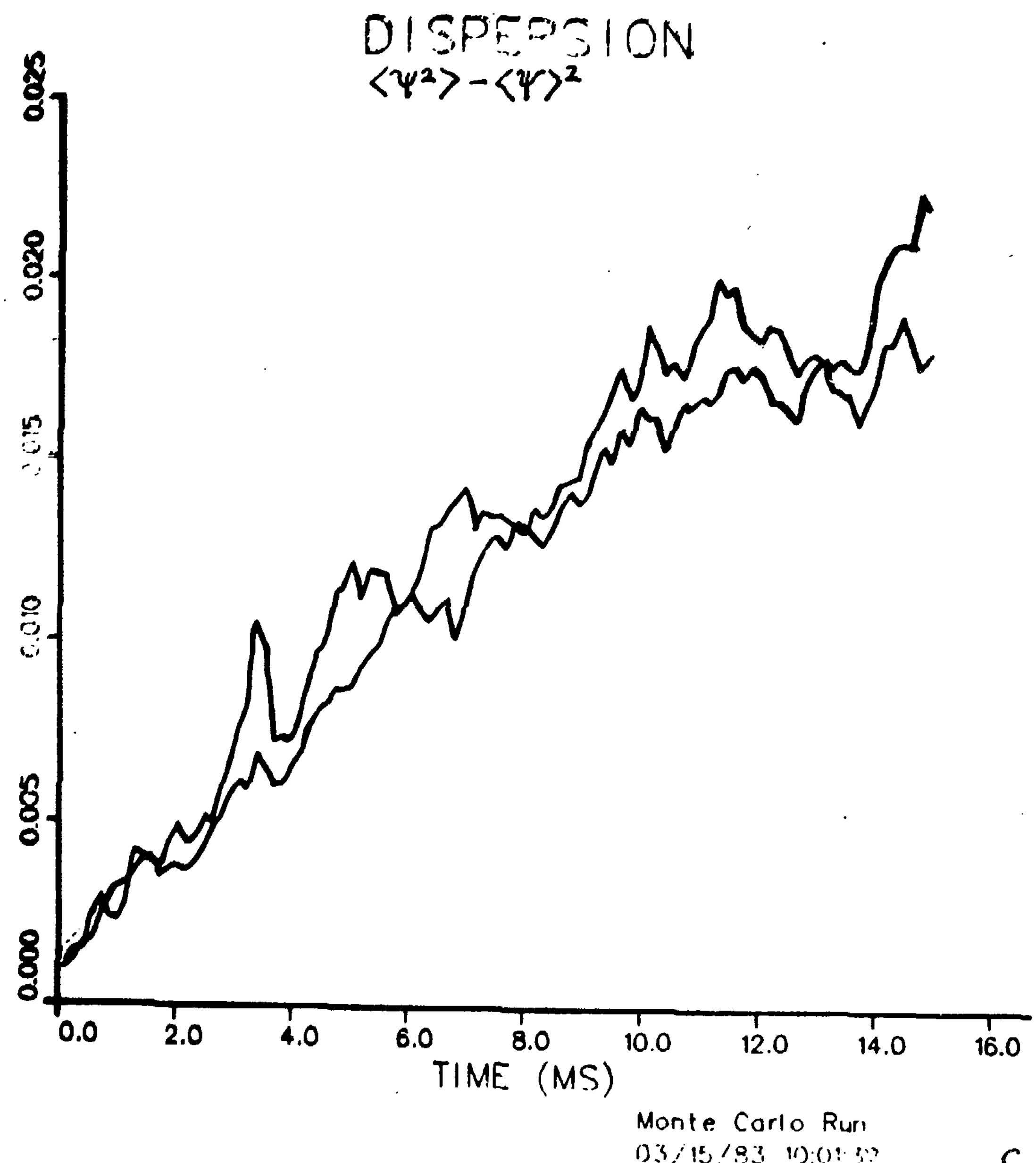

DISPERSION
$\langle\psi^2\rangle - \langle\psi\rangle^2$
0.0
1.0
2.0
3.0
4.0
5.0
6.0
TIME (MS)
Monte Carlo Run
03/17/83 08:47:25


f(t,η)
-1.0
-0.5
0.0
0.5
1.0
η

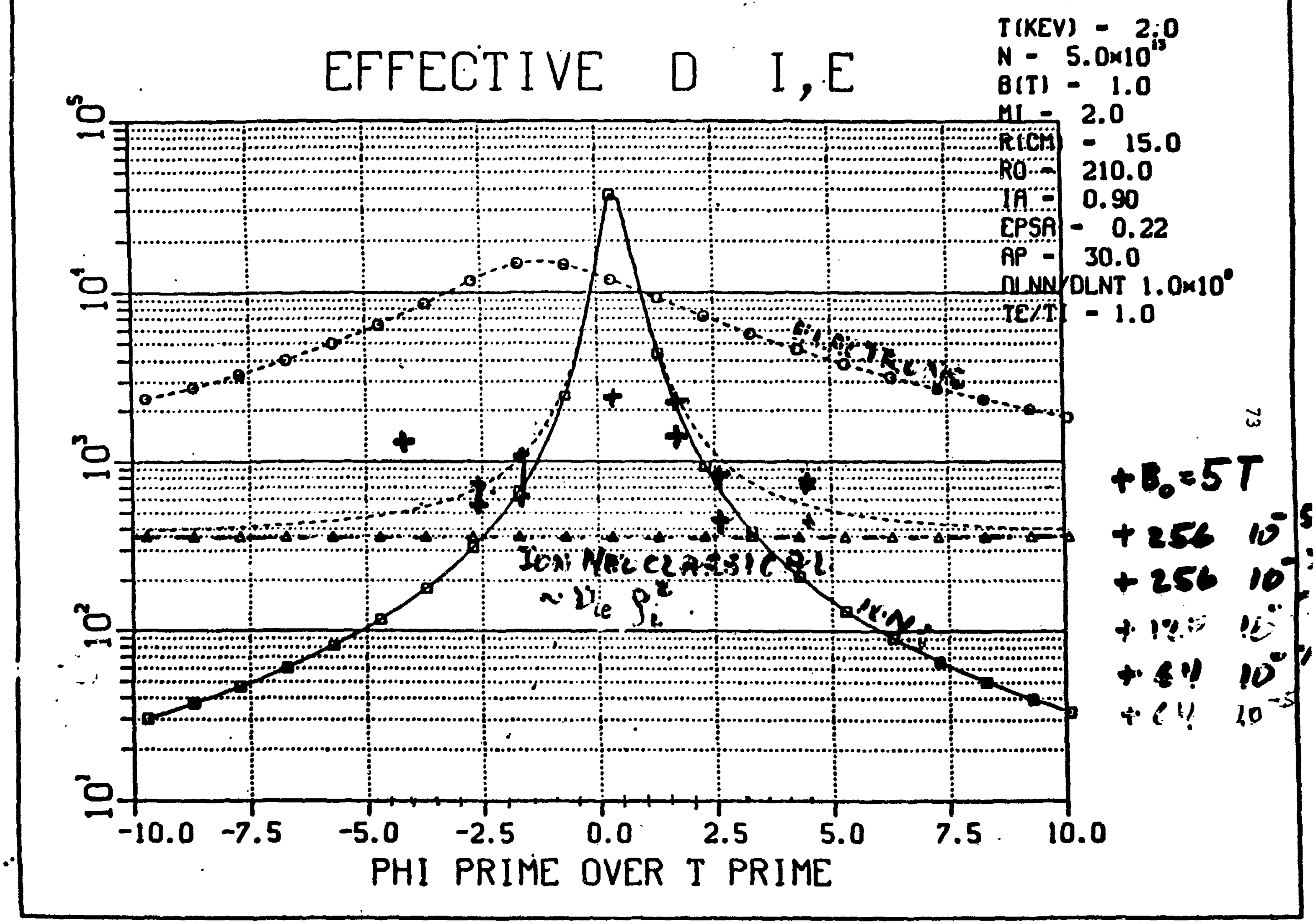

AVERAGE OF ψ
0.25
0.26
0.27
0.0
1.0
2.0
3.0
4.0
5.0
6.0
TIME (MS)
Monte Carlo Run
03/17/83 08:47:25

A COMPARISON OF DIFFERENT STARTING SCHEMES



δf_N START @ $\frac{3}{4}T$
MAXWELLIAN. ORBITS NEAR

COMPARISON OF MONTE CARLO CODE TO SHAING'S THEORY



MODEL B: $B = B_0\left[1 - \frac{1}{A}\frac{r}{a}\cos\theta - .22\left(\frac{r}{a}\right)^2\cos(l\theta - m\phi)\right]$

TODAY'S EXPLANATION

Why don't we see the neoclassical (non momentum-conserving) diffusion due the bananas? It is much bigger than the diffusion calculated for the helically trapped ions.

- For $\varphi'/T' \rightarrow 0$, all bananas are in the loss region. Since we eliminate loss orbits from the Monte Carlo population, we can't see the banana-driven terms.
- As φ'/T' increases, the loss region disappears. But, only a small fraction of trapped particles are banana trapped and not helically trapped:

$$f_T \simeq (\varepsilon_T + \varepsilon_H)^{1/2} - (\varepsilon_H)^{1/2} = (.07 + .055)^{1/2} - (.055)^{1/2} = .12$$

versus $.26 = \sqrt{\varepsilon_T}$

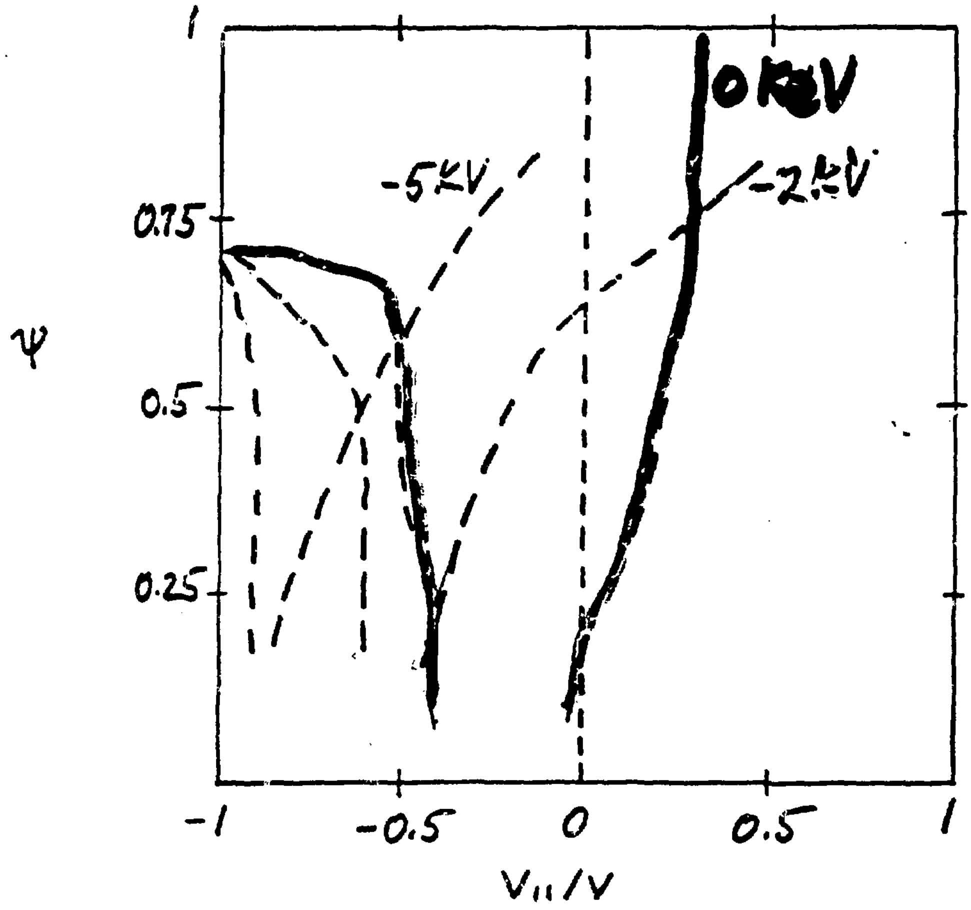
INCREASING B_0 LOWERS ASYMPTOTES

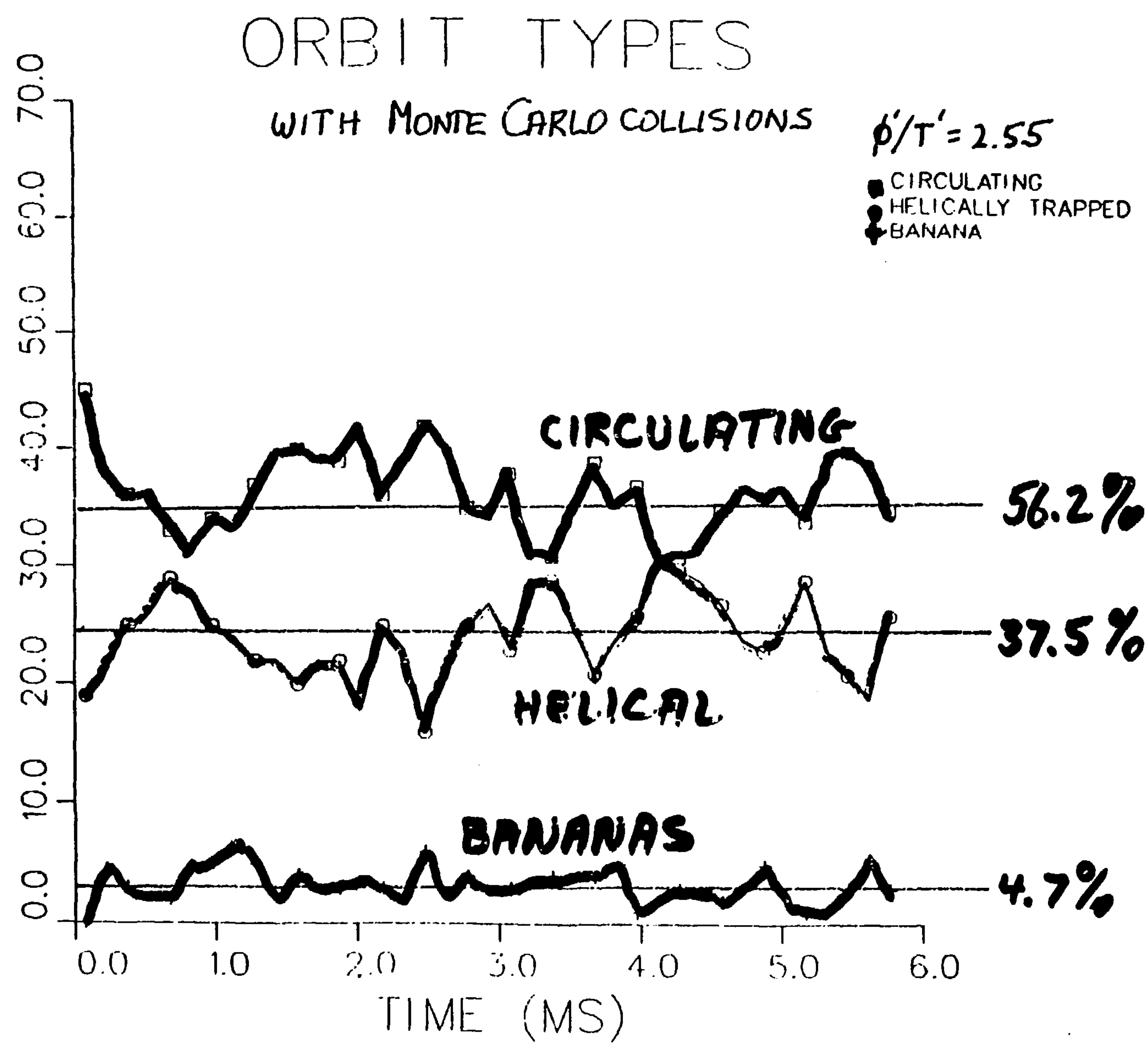
This reduced the effective ν_{ii} diffusion.

- We are studying the dependence of the loss regions and the fraction of banana-trapped ions on φ'/T'.

LOSS REGIONS FOR 5 keV D^+

$B_0 = 1T$, $\Phi = \Phi_0(1-\psi)$




ORBIT TYPES
WITH MONTE CARLO COLLISIONS
ϕ'/T' = 2.55
CIRCULATING
HELICALLY TRAPPED
BANANA
CIRCULATING
HELICAL
BANANAS
56.2%
37.5%
4.7%
70.0
60.0
50.0
40.0
30.0
20.0
10.0
0.0
0.0
1.0
2.0
3.0
4.0
5.0
6.0
TIME (MS)

TRANSPORT MODELING – TOWARD SELF-CONSISTENT DETERMINATION OF THE RADIAL ELECTRIC FIELD*

W. A. Houlberg
ORNL

with support from
K. C. Shaing
J. F. Lyon

Fourth U.S. Stellarator Workshop

Oak Ridge, TN

April 14–15, 1983

*Research sponsored by the Office of Fusion Energy, OER, U. S. Department of Energy under contract W-7405-eng-26 with the Union Carbide Corporation.

STELLARATOR TRANSPORT CODE DEVELOPMENT

Incorporate self–consistent models for particle and energy confinement in a transport code:

- o assess the limitations and important features of the models – both physical and computational.
- c develop the computational tools for inclusion of the radial electric field in direct and diffusive losses.

Coordinate work with the development of kinetic models and benchmarking against Monte–Carlo calculations.

Longer range development to include 3–D MHD equilibria.

MODEL FOR PARTICLE AND ENERGY FLUXES

The electron and ion particle and heat fluxes due to helical trapping are given by:

$$\Gamma_j^{ht} = -\varepsilon_t^2\sqrt{\varepsilon_h}v_{dj}^2 n_j \int_0^\infty x^{5/2}e^{-x}dx\ \tilde{\nu}_j(x)\,\frac{A_j(x)}{\omega_j^2(x)}$$

$$Q_j^{ht} = -\varepsilon_t^2\sqrt{\varepsilon_h}v_{dj}^2 n_j T_j \int_0^\infty x^{7/2}e^{-x}dx\ \tilde{\nu}_j(x)\,\frac{A_j(x)}{\omega_j^2(x)}$$

$$A_j(x) = \frac{n_j'}{n_j} + \frac{e_j\varphi'}{T_j} + (x-1.5)\frac{T_j'}{T_j}$$

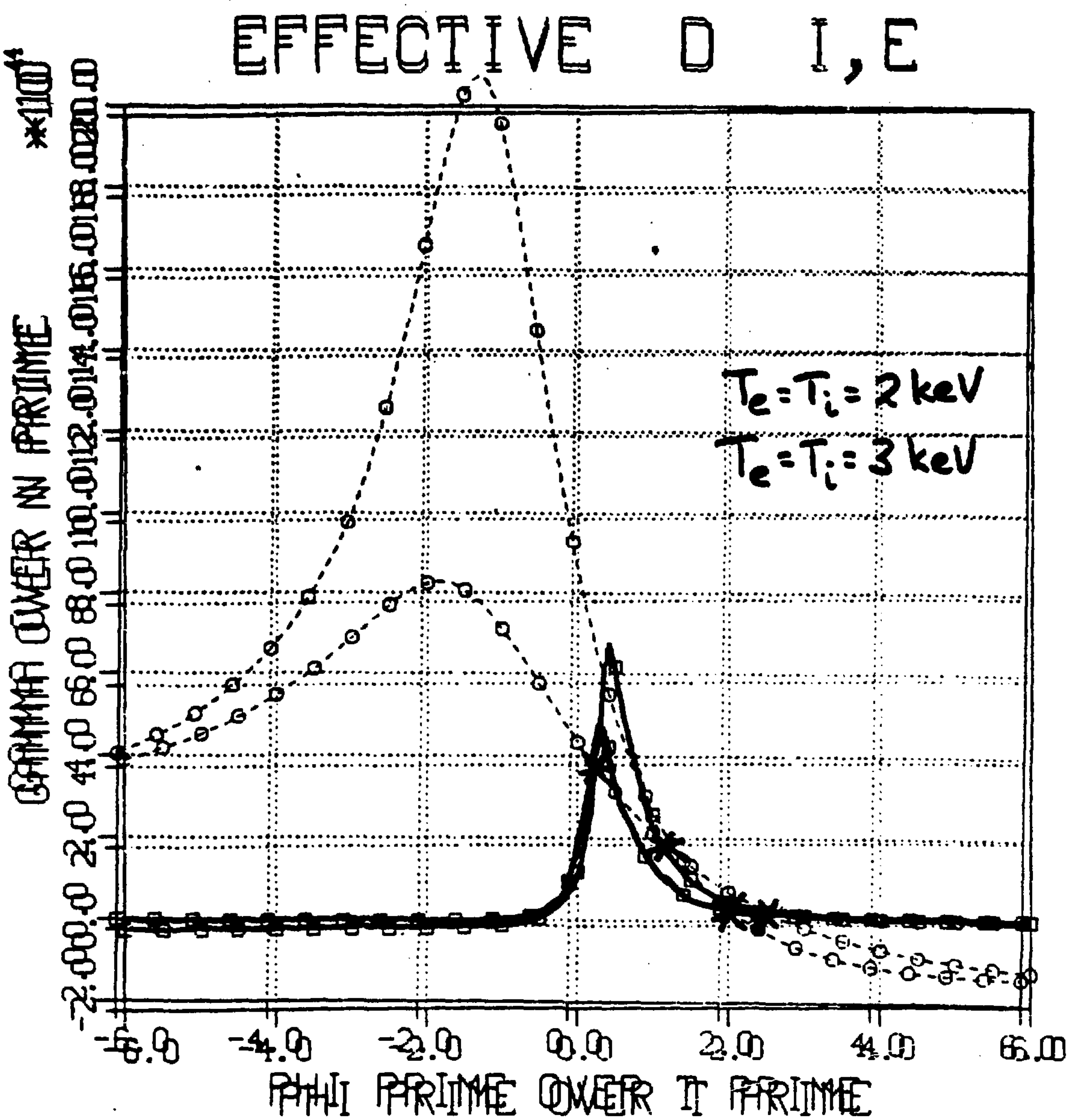
$$\tilde{\nu}_j(x) = \frac{\nu_j(x)}{\varepsilon_h} = \frac{\nu_{jT}}{\varepsilon_h x^{3/2}}$$

$$x = \frac{E}{T_j}$$

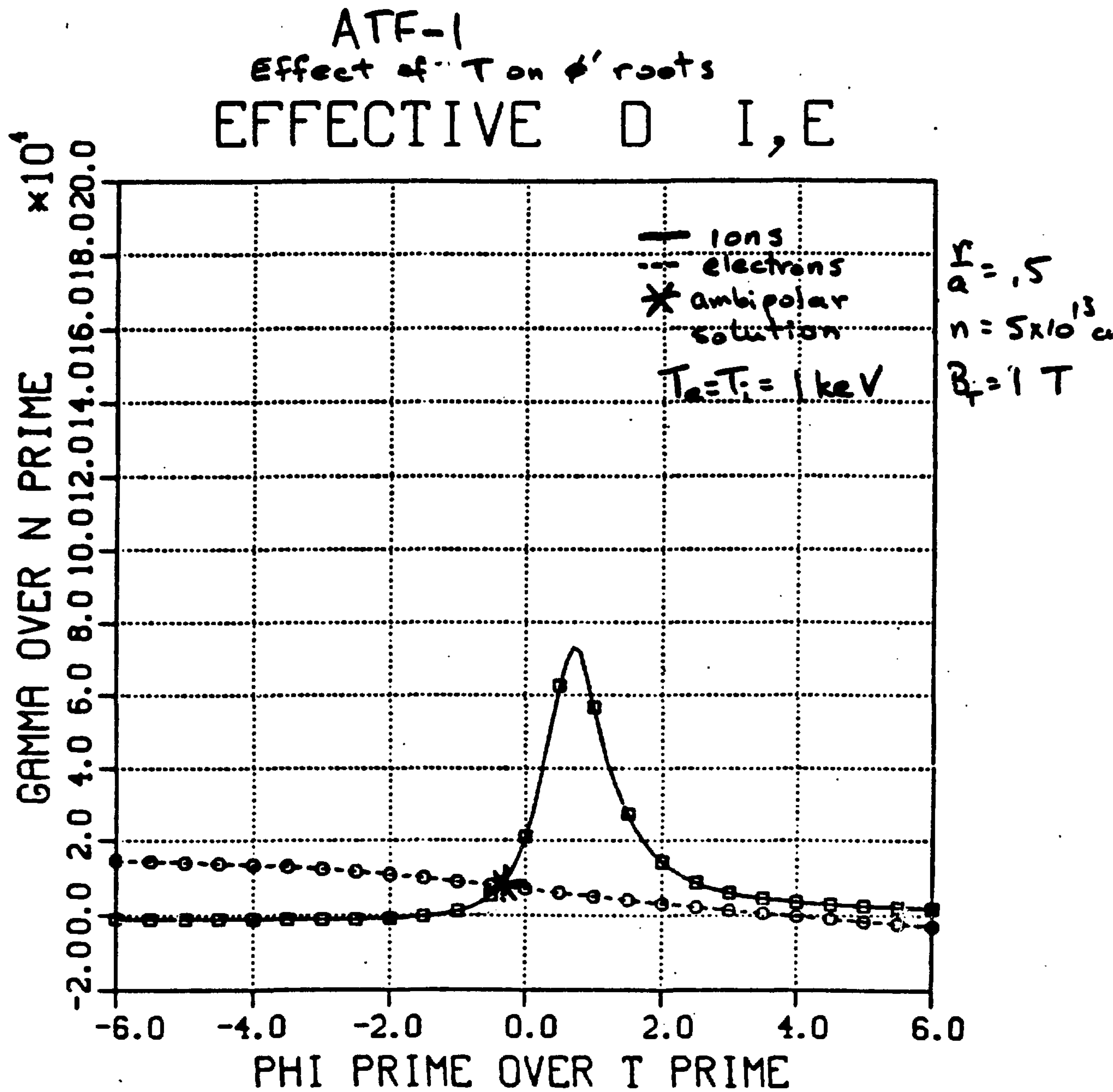
$$\omega_j^2(x) = \frac{\varepsilon_t}{\varepsilon_h}\Big\{1.67(\omega_E+\omega_{\nabla Bj})^2+\underbrace{\Big(\frac{\varepsilon_t}{\varepsilon_h}\Big)^{1/2}\Big[\frac{\omega_{\nabla Bj}^2}{4}+.6|\omega_{\nabla Bj}|\tilde{\nu}_j\Big]\Big(\frac{\varepsilon_h}{\varepsilon_t}\Big)^{3/2}\Big]}_{\substack{\omega_{res}^2 \\ \text{Super banana and plateau}}}\Big\}+3\tilde{\nu}_j^2(x)$$

$$\omega_E = \frac{\varphi'}{eBr}$$

$$\omega_{\nabla Bj} = -v_{dj}\varepsilon_h' x = -\frac{T_j}{e_jBr}\varepsilon_h' x$$

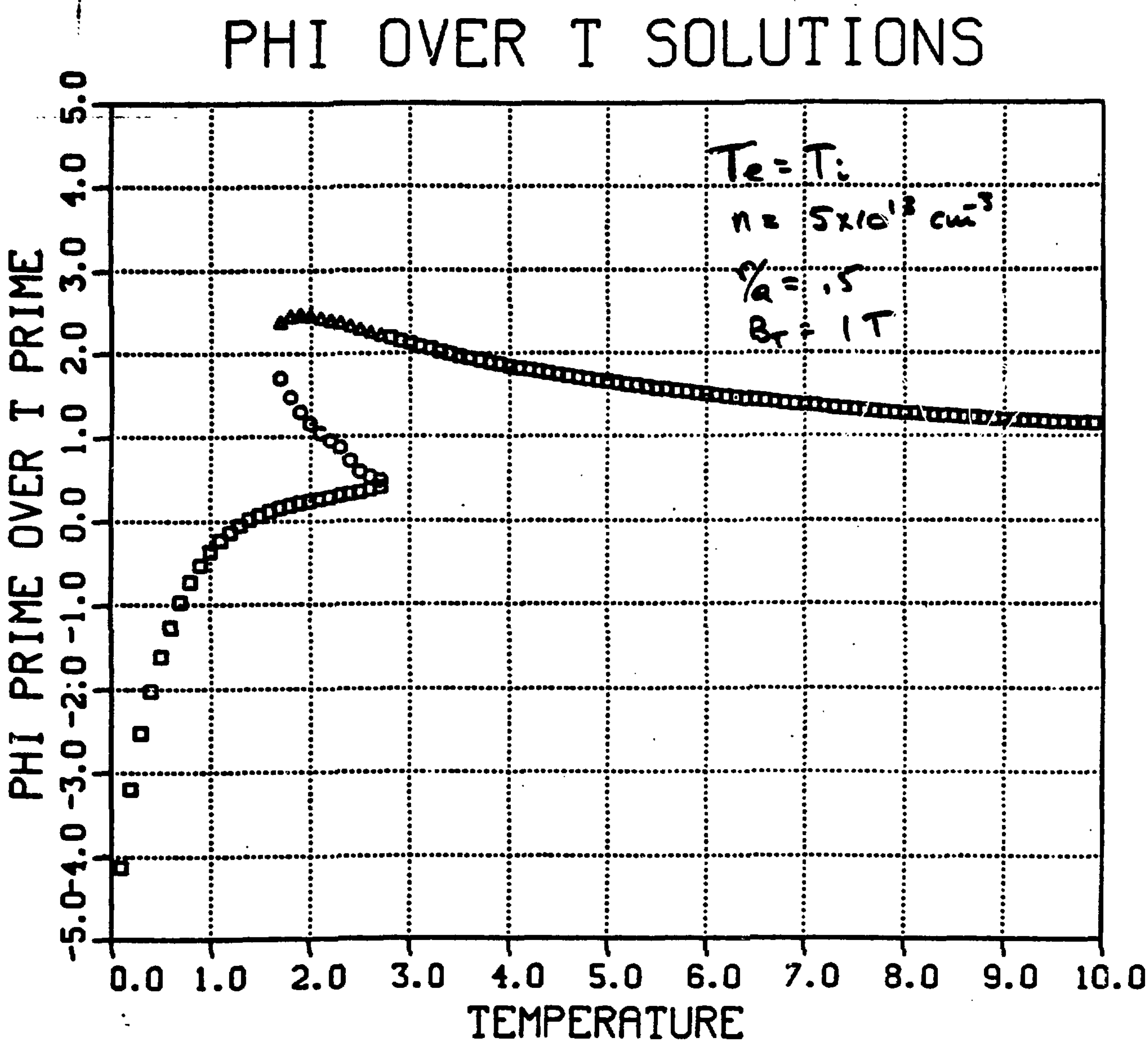


Three roots for $T = 2$ keV $\langle\beta\rangle \approx 4.8\%$
Single root, $\phi > 0$, for $T = 3$ keV $\langle\beta\rangle \approx 7.2\%$



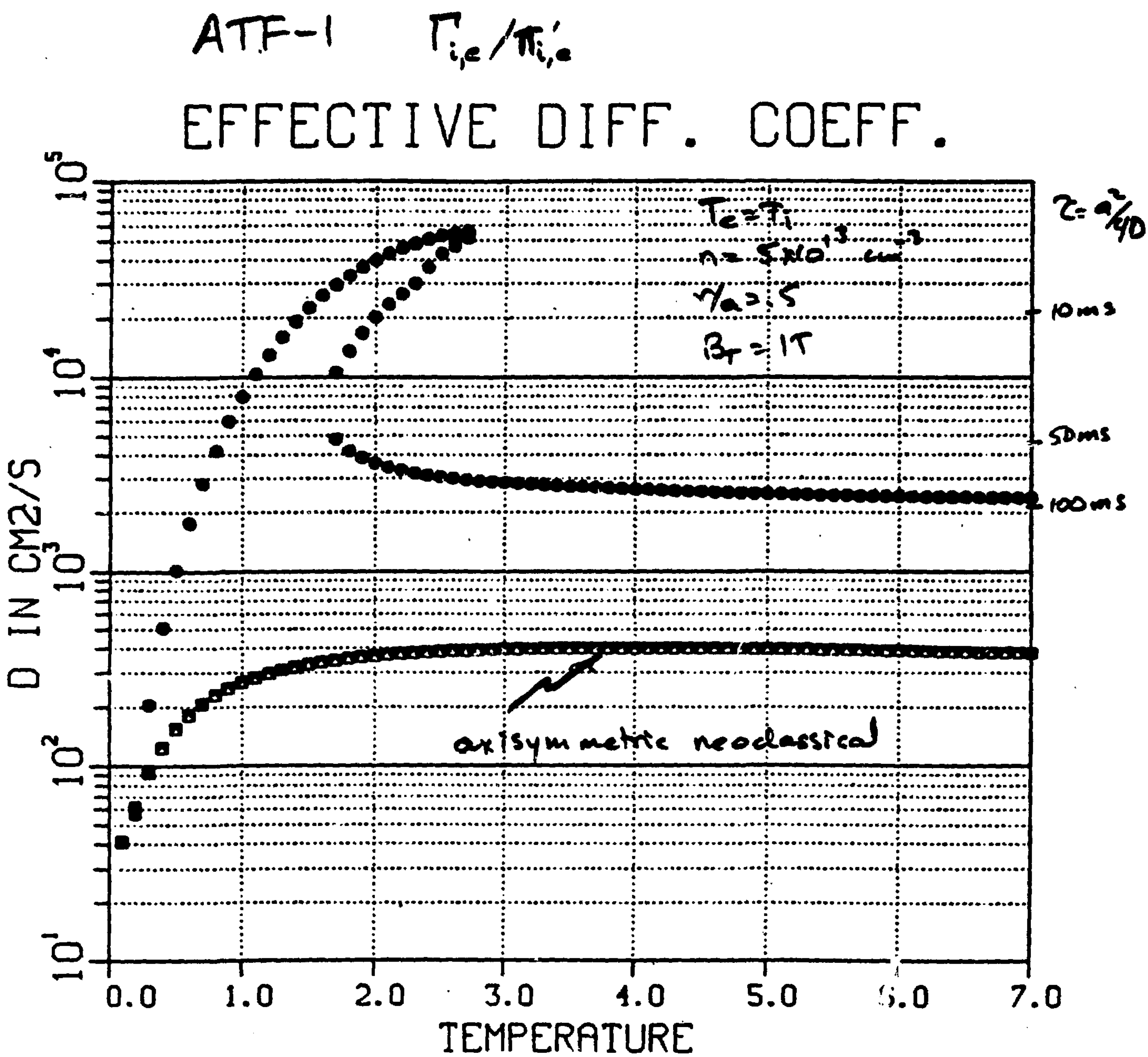
Single root, $\phi < 0$, for T = 1 keV $\langle\beta\rangle = 2.4\%$

ATF-1 $\phi'(T)$

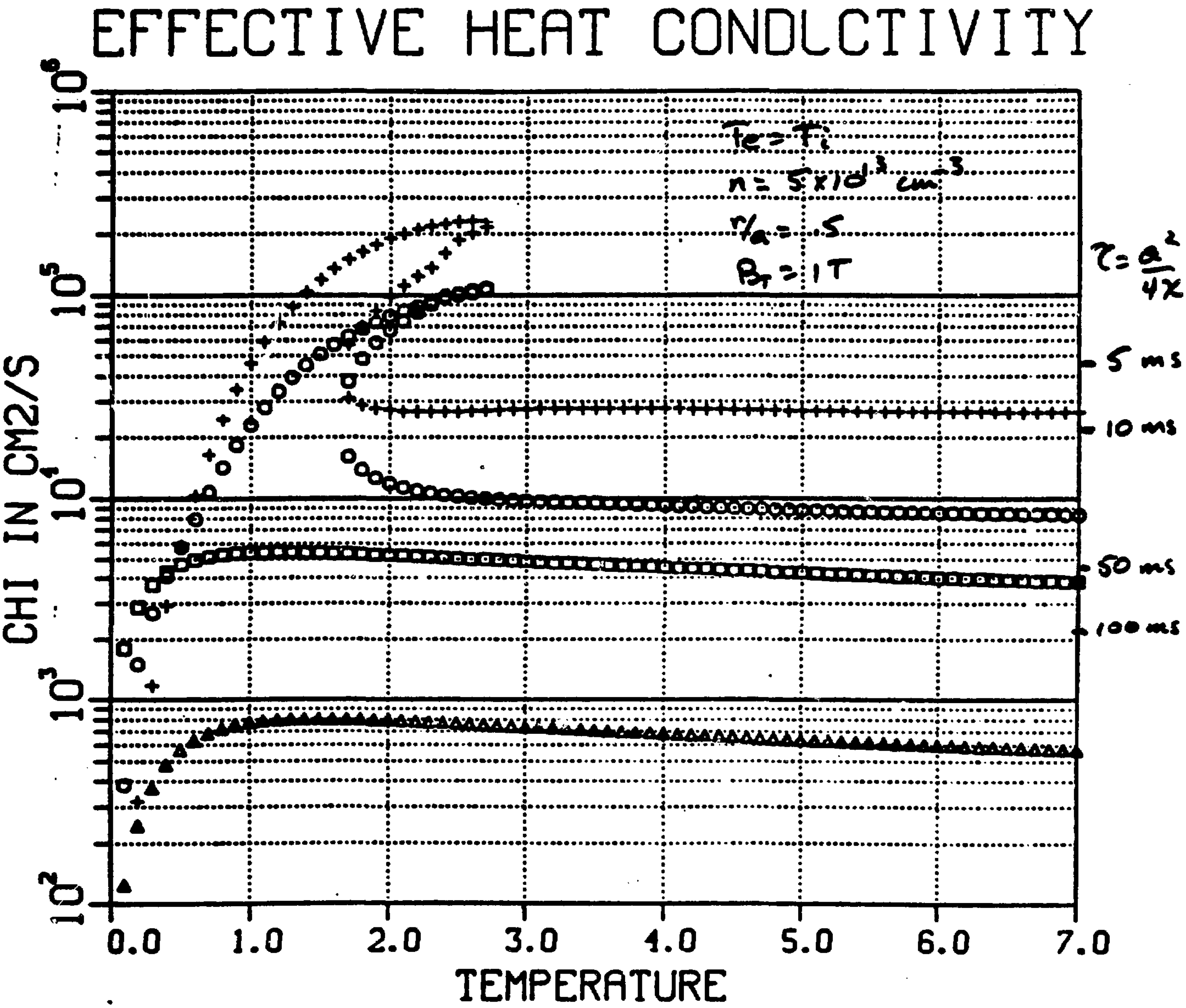


Under these example conditions three roots for ϕ' exist over the temperature range

$1.7 < T < 2.8$ keV


ATF-1 $\Gamma_{i,e}/\pi'_{i,e}$
EFFECTIVE DIFF. COEFF.
$T_e = T_i$
$n = 5 \times 10^{13}\ cm^{-3}$
$r/a = .5$
$B_T = 1T$
$\tau = a^2/4D$
10ms
50ms
100ms
axisymmetric neoclassical
D IN CM2/S
TEMPERATURE
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0

ATF-1 $Q_{i,e}/T'_{i,e}$



+ χ_e^{rip}

o χ_i^{rip}

□ χ_i } axisymmetric neoclassical for reference

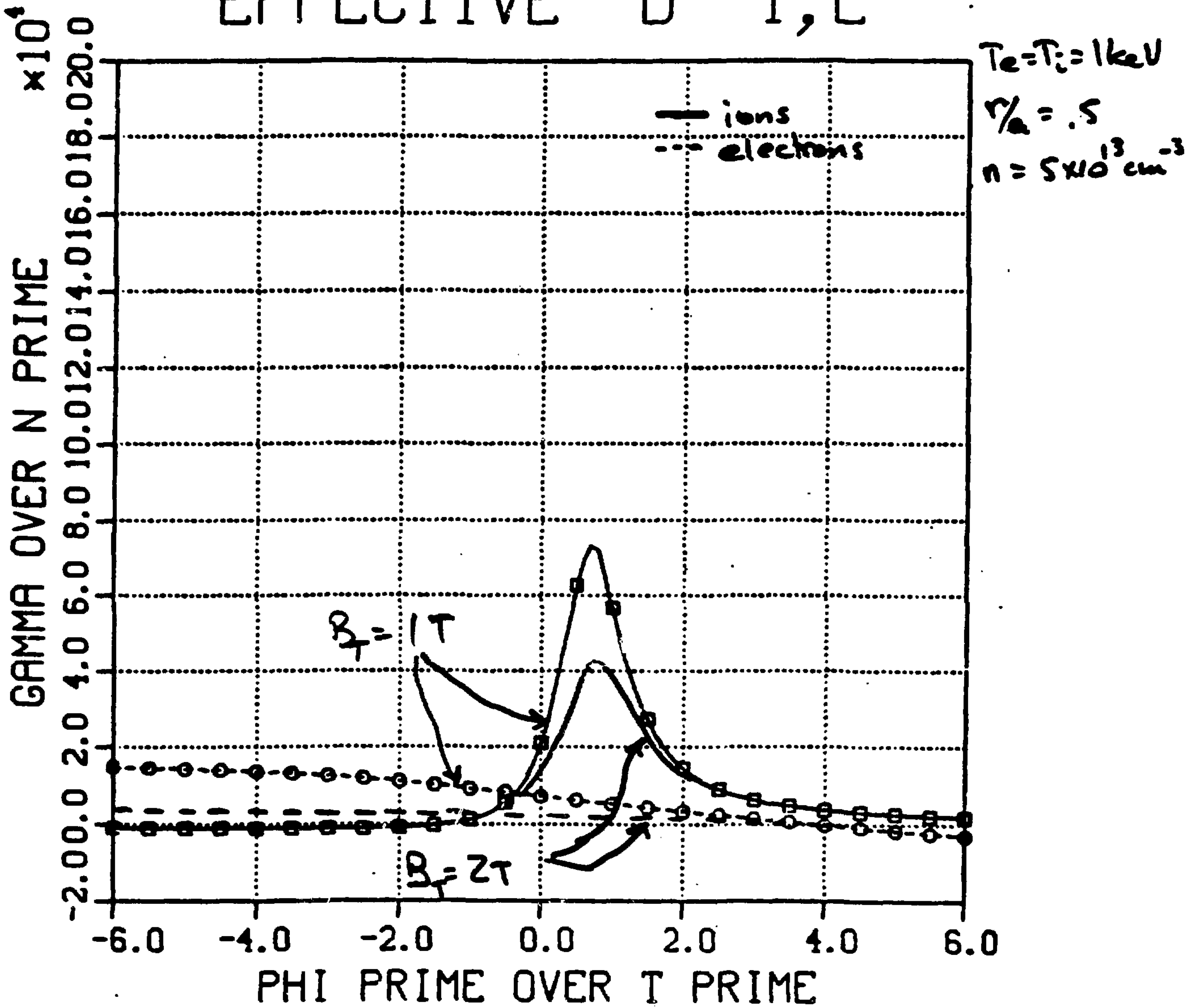
△ χ_e }

Above ~ 500 eV, $\chi_e^{rip} > \chi_i^{rip}$.

ATF-1

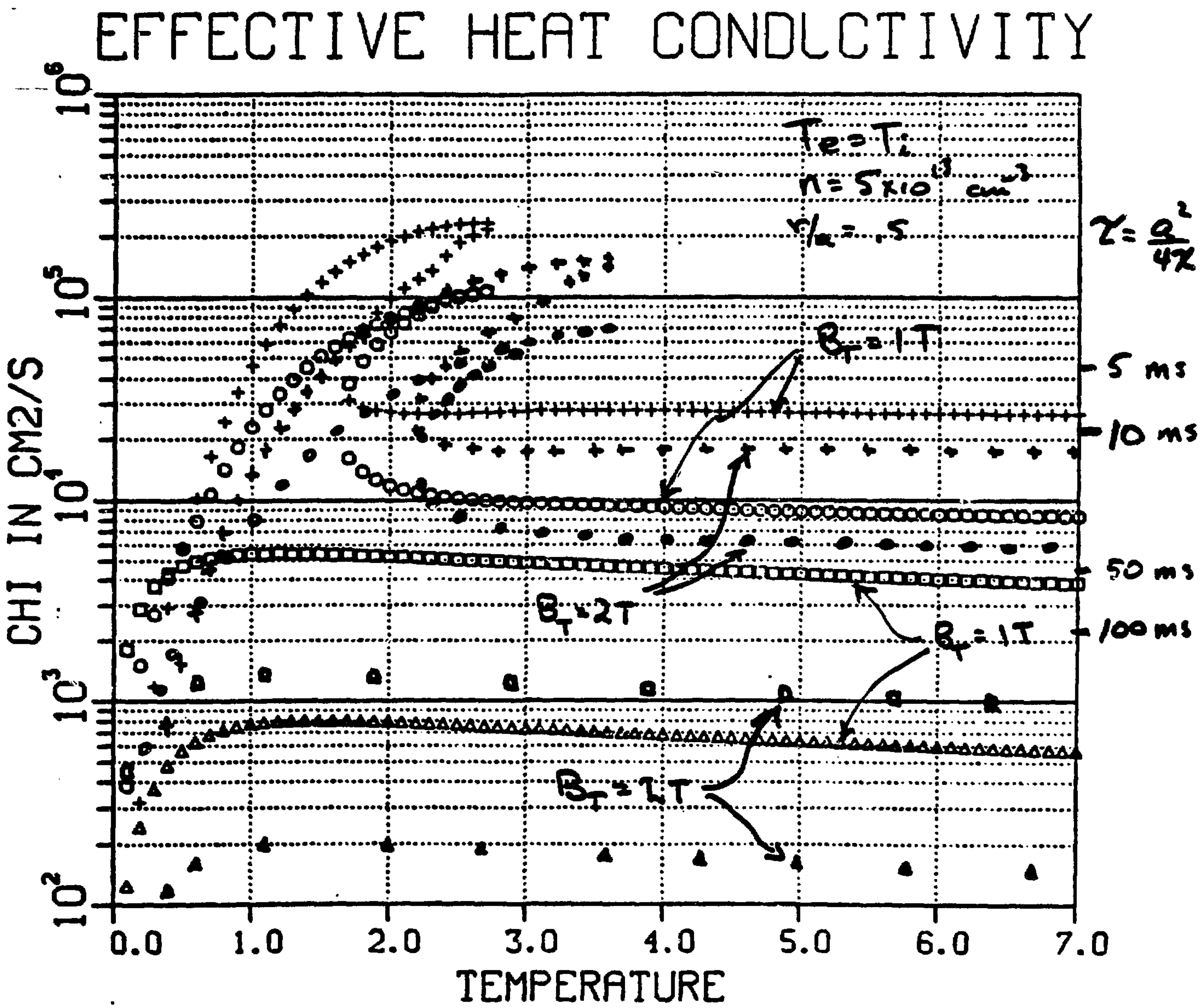
Effect of B_T on ϕ' roots

EFFECTIVE D I,E



- Increased B_T drives the solution toward the ion root.
- The transition from ion to electron roots is most easily obtained at low B_T.

ATF-1 $Q_{i,e}/T'_{i,e}$



+ χ_e^{rip}

• χ_i^{rip}

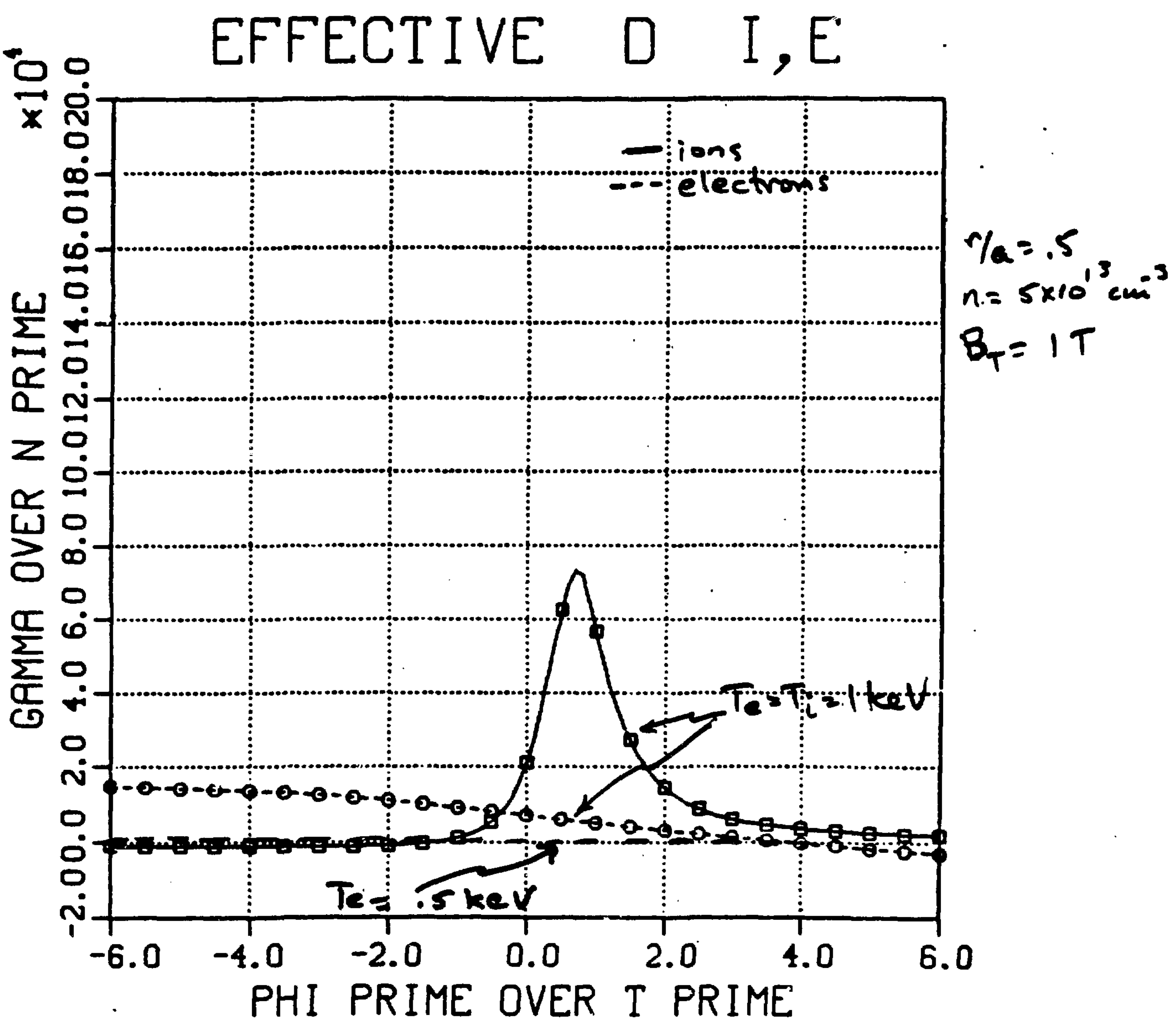
□ χ_i } axisymmetric neoclassical for reference

Δ χ_e }

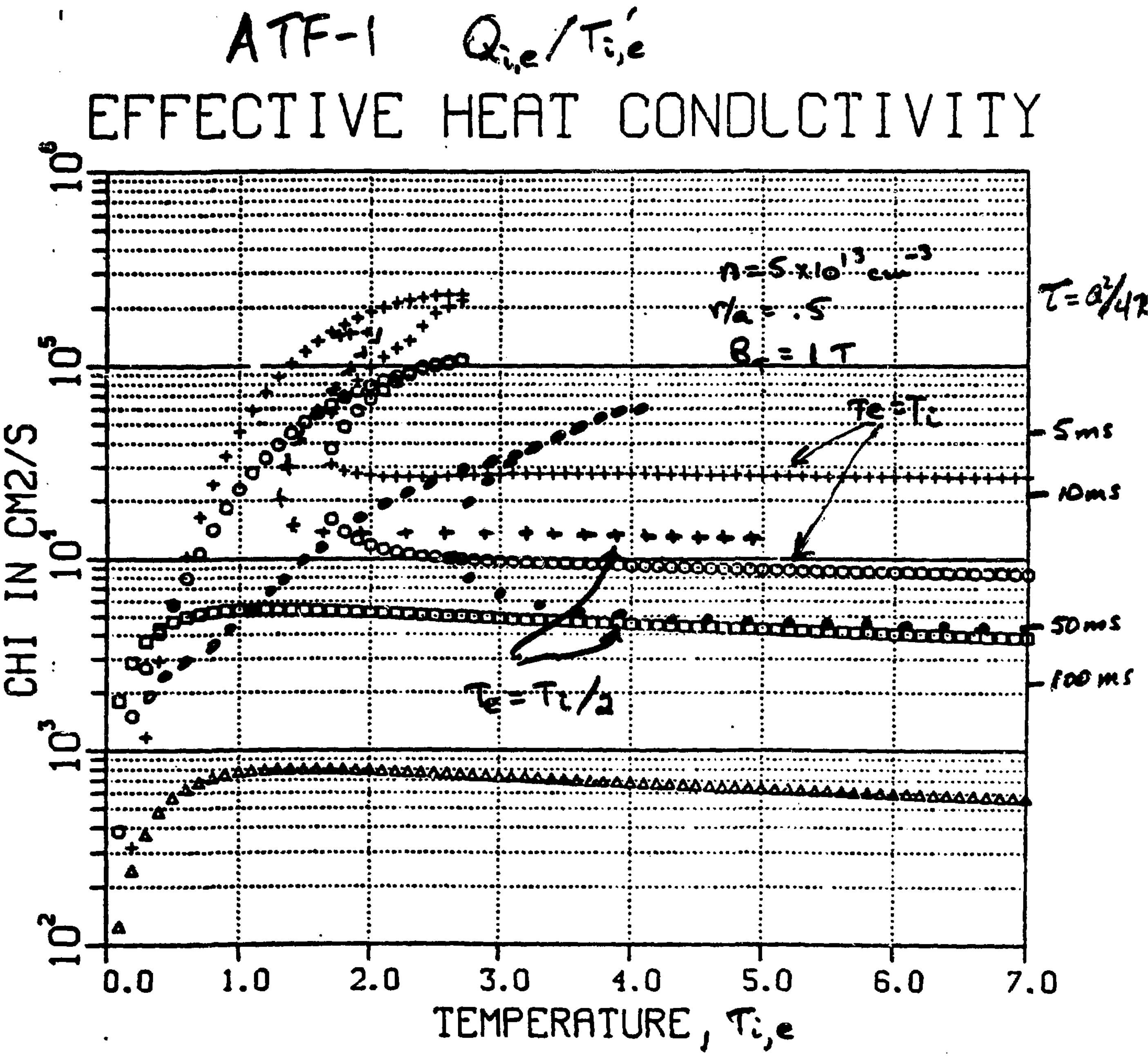
Ripple confinement improves $\sim B_T$

ATF-1

$T_e/T_i = .5$

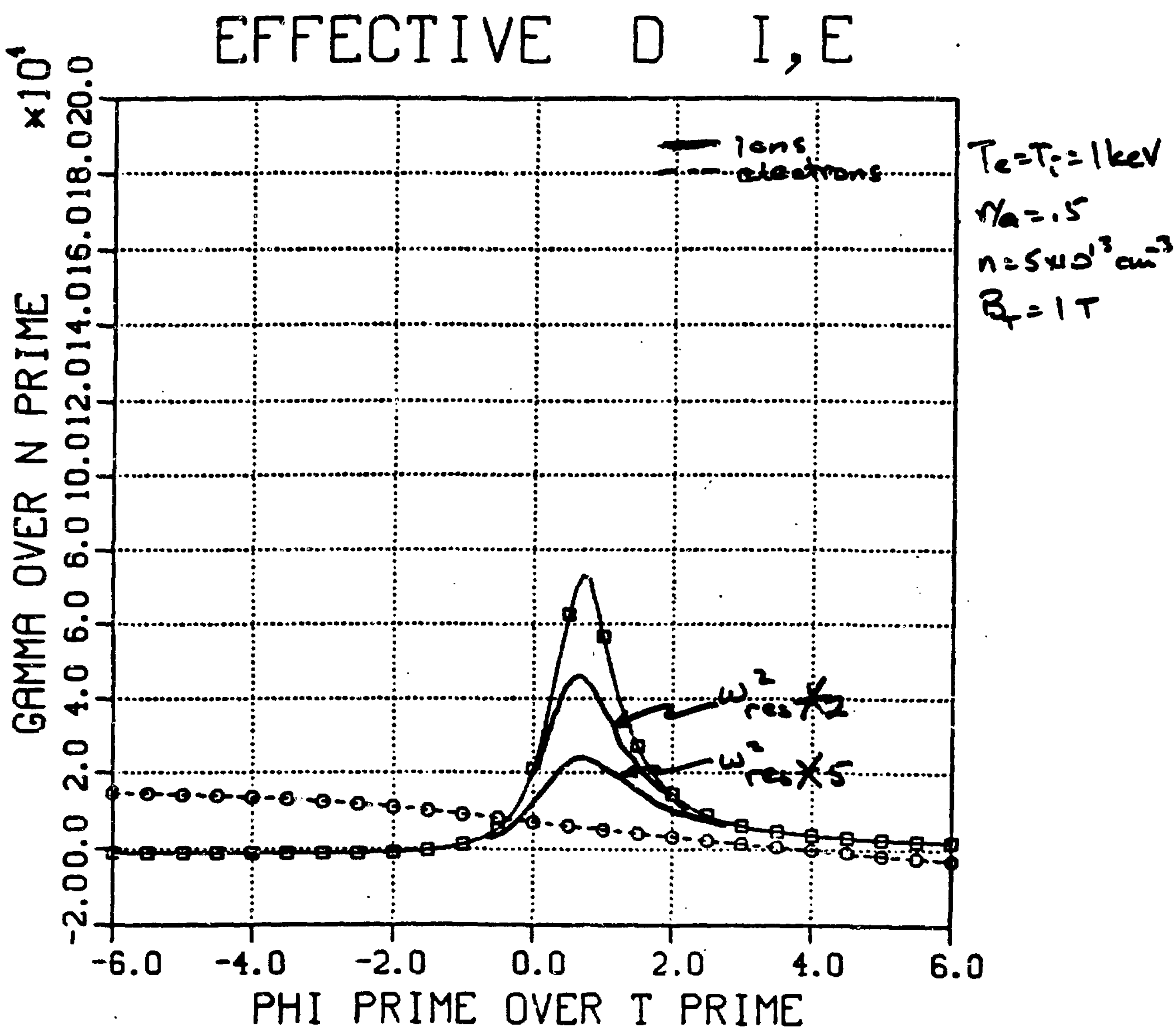


- Reduced electron temperature drives the solution toward the ion root and delays the transition.



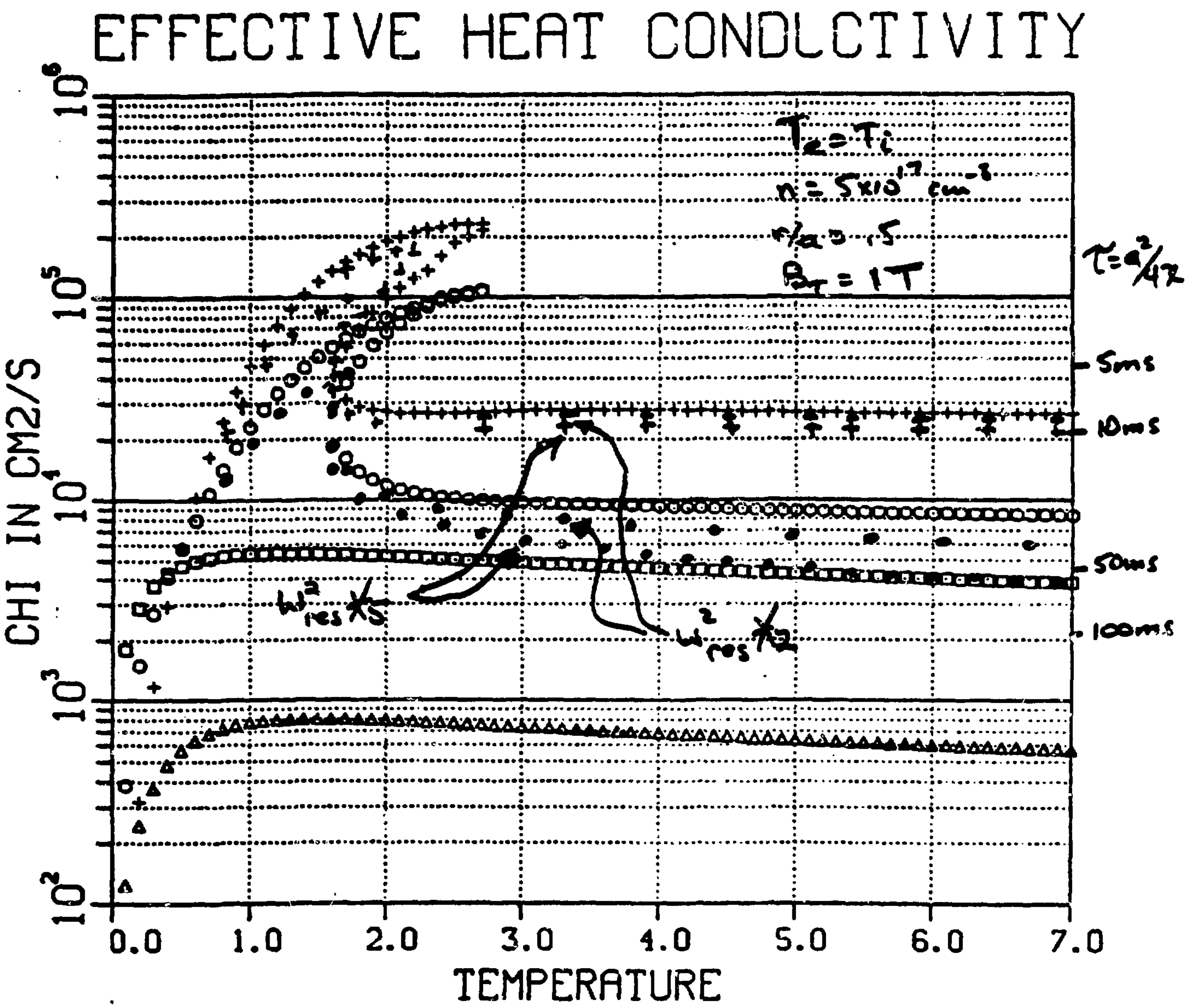
- The electron temperature determines the transition.
- Reduced T_e/T_i leads to a significant improvement in confinement.

ATF-1 Decreased Resonant Losses



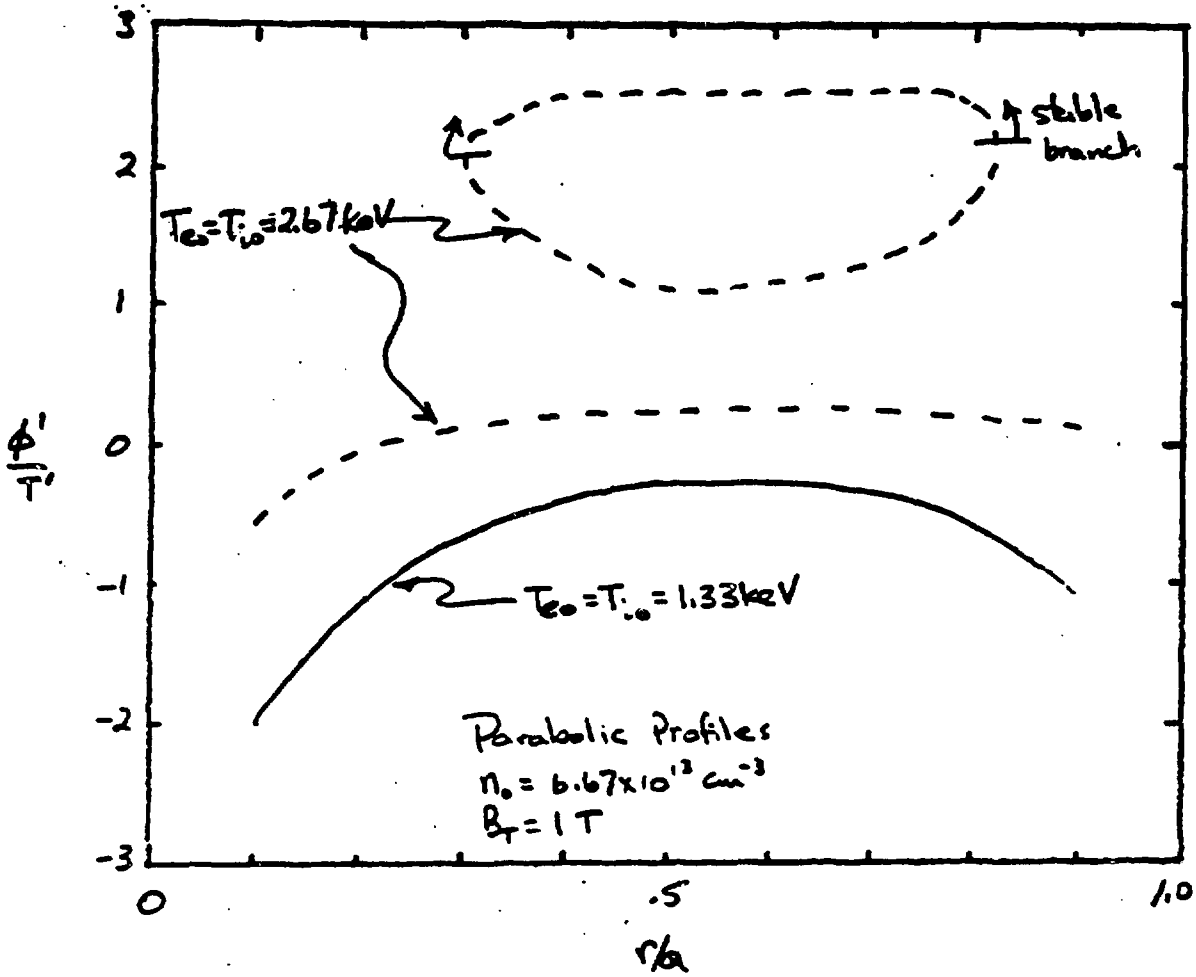
- The resonant loss term is the critical term for determining the maximum particle loss rate in the collisionless regime.

ATF-1 — Resonant Loss Term



- Decreased resonant losses smooth the transition from the ion to the electron regime.
- A more modest improvement in the very low collisionality electron regime is also noted.

ATF-1
Radial ϕ' Profiles



- The ion root exists at all radii for both temperature profiles in this example.
- The electron roots are introduced at a point and the closed contour grows with increasing temperature until it joins the ion curve.
- The algebraic solution of $\Gamma_e(\phi') = \Gamma_i(\phi')$ cannot guarantee a smooth transition from the ion to the stable electron root at all radii — discontinuities in ϕ' will generally occur.

SUMMARY OF STELLARATOR TRANSPORT ANALYSIS

Efforts thus far have concentrated on examining features of the kinetic model:

- o sensitivity to parameters.
- o methods for calculating the self-consistent radial electric field.

The most critical issue appears the transition from the high collisionality, ion-loss regime to low collisionality, electron-loss regime.

Further development and benchmarking of kinetic and Monte-Carlo analyses is required.

ATF-1 and Heliotron-E can address the issue experimentally at low magnetic fields.

WHERE ARE WE IN STELLARATOR TRANSPORT ANALYSIS?

J. D. Callen
University of Wisconsin
Madison, Wisconsin

FOURTH U. S. STELLARATOR WORKSHOP
OAK RIDGE, TENNESSEE
April 14-15, 1983

WHERE ARE WE IN STELLARATOR TRANSPORT ANALYSIS

- *Implicit Assumptions:*

  Drift orbits have small radial steps
  "Small" loss regions on high energy tail of the distrib
  function -- due to bounce as well as drift orbi
  Concentrate on dominant transport processes

  (Similar to tokamak theory in early 70's)

- *Present Issues:*

  Effects of radial electric field -- $\Phi' \to 0$ within plasma
  transport stability of the various Φ' roots
  Transition from high to low collisionality -- energ
  smoothing, $\nu^{1/2}$ region, boundary layer effects, plateau, etc
  Boundary conditions in collisionless detrapping calculation
  More detailed magnetic field models
  Role of Monte Carlo transport calculations

  (Important processes being calculated and investigated:

EFFECTS DETERMINING RADIAL ELECTRIC FIELD

- *Nonaxisymmetric Particle Fluxes:*

  "Superbanana" drift orbits — effects presently included

  Banana tip diffusion
  Ripple-plateau transport effects — effects presently omitted

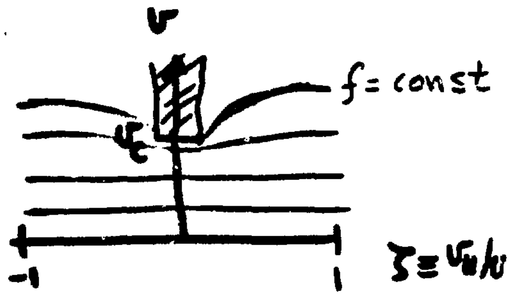
- *Other Effects:*

  Direct particle losses
  Impurity transport
  Stochastic field line regions ($D, \chi_e \sim v_{T_e} D_m$)
  Fluctuation-induced transport

(Ultimately radial electric field and its effects on transport will probably have to be determined experimentally)

ESTIMATE OF DIRECT PARTICLE LOSSES

- *Model of Drift Loss Region:*



$v < v_c$, $E \times B$ drift bigger than ∇B drift and confines particles

$|\xi| \equiv |v_\parallel/v| < \sqrt{\epsilon_h}$ helically-trapped particles

- *Model for Solving for f with $v > v_c$:*

Particle source is energy upscattering

Particle loss is pitch-angle scattering into the loss region

- *Particle and Energy Loss Rates:*

$$\left.\frac{\partial n}{\partial t}\right|_{DL} = \frac{n}{\tau_{DL}} = \int_{v > v_c} d^3v \, C(f) \sim n\nu \frac{v_T}{v_c} e^{-v_c^2/v_T^2}$$

$$\frac{\partial}{\partial t}\left(\frac{3}{2}nT\right) \equiv \frac{nT}{\tau_{EDL}} = \int_{v > v_c} d^3v \, \frac{mv^2}{2} C(f) \sim nT\nu \frac{v_c}{v_T} e^{-v_c^2/v_T^2}$$

TRANSPORT EQUATIONS INCLUDING DIRECT LOSSES

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial V}\left(\Gamma_{V_{rel}} + n\langle \underline{u}_\psi \cdot \underline{\nabla} V \rangle \right) = -\frac{n}{\tau_{DL}}$$

$$\Gamma_{V_{rel}} \equiv \left\langle \int_{\underline{v<v_c}} d^3v \; \underline{v}_D \cdot \underline{\nabla} V f \right\rangle$$

$$= \Gamma_{classical} + \Gamma_{P-S} + \Gamma_{\substack{banana-\\plateau}} + \Gamma_{\substack{non-\\axisymmetr.}}$$

$\langle \underline{u}_\psi \cdot \underline{\nabla} V \rangle$ = motion of flux surfaces

$n/\tau_{DL} = \int_{\underline{v>v_c}} d^3v \, C(f)$ = direct loss rate

REVIEW OF HELICAL AXIS RESEARCH

A. Reiman
Princeton Plasma Physics Laboratory
Princeton University
Princeton, NJ

FOURTH U. S. STELLARATOR WORKSHOP
OAK RIDGE, TENNESSEE
April 14-15, 1983

I. Why helical axis stellarators?
Some of their properties.

II. Proposed designs for helical axis stellarators.

What's so interesting about helical axis stellarators?

I. magnetic well

$\Rightarrow$ large stability β

II. large rotational transform

$\Rightarrow$ large equilibrium β

III. low shear: avoid low order rational surfaces

IV. helical symmetry in large aspect ratio limit

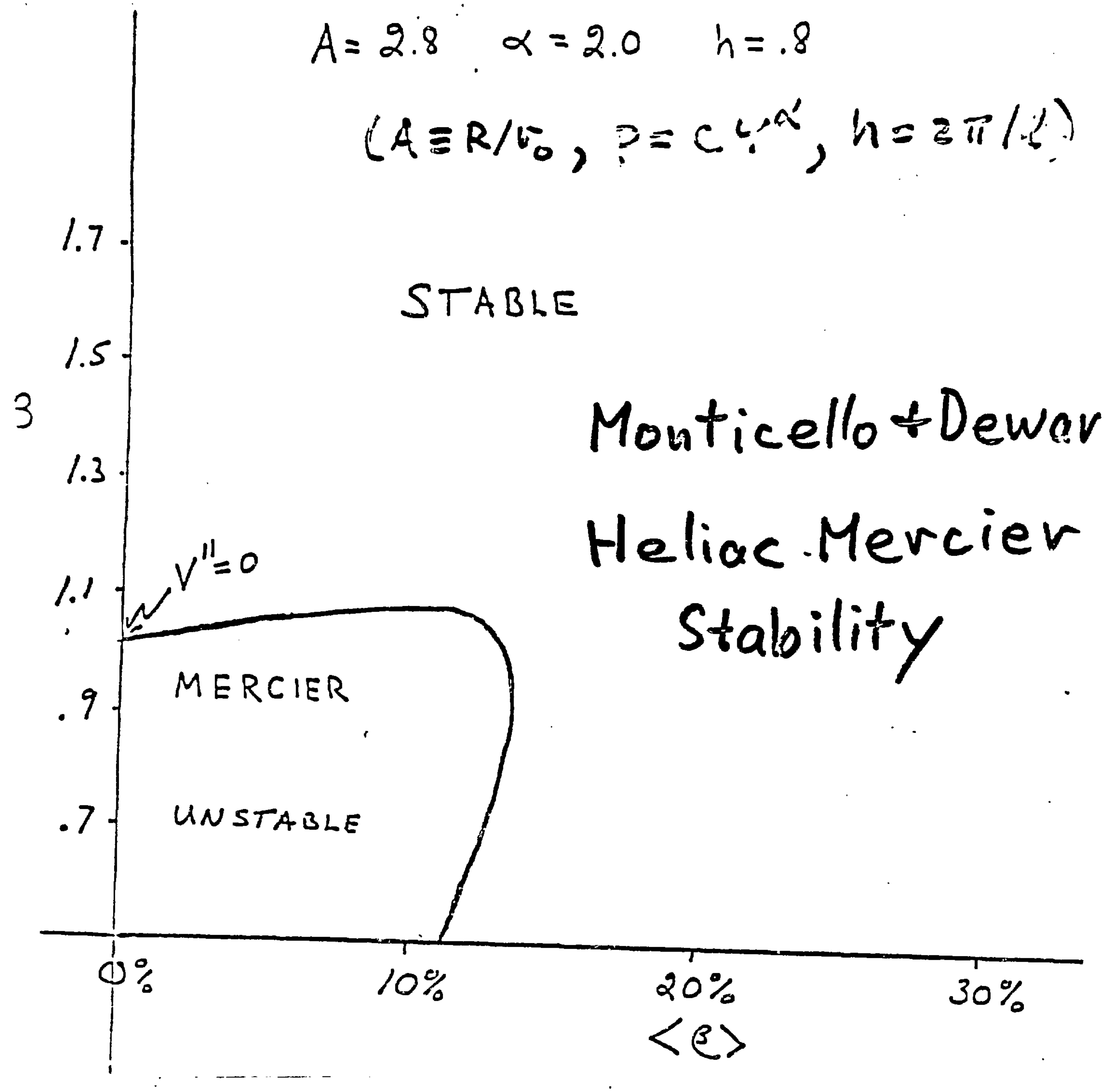
$\Rightarrow$ good transport in limit

Toroidal corrections to well & ι small.

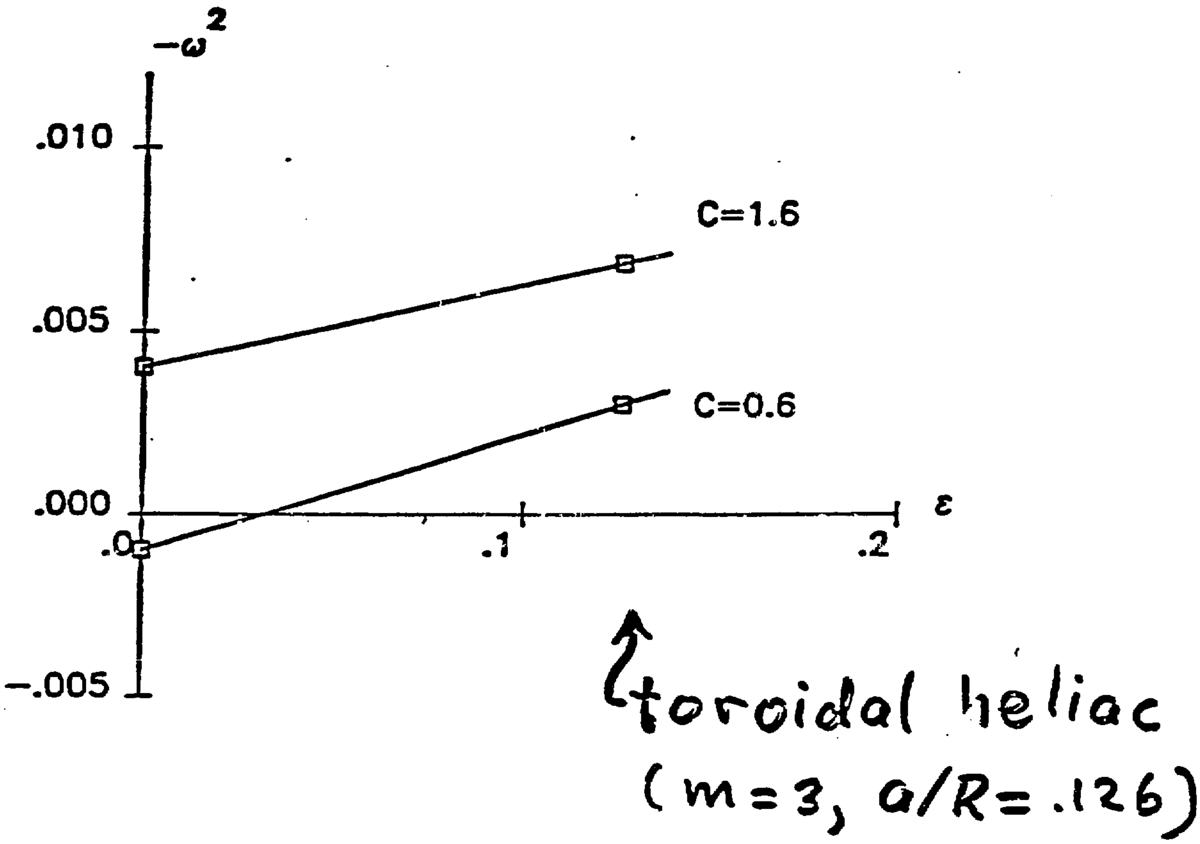
$A = 2.8 \quad \alpha = 2.0 \quad h = .8$

$(A \equiv R/r_0,\ p = c\psi^\alpha,\ h = 2\pi/\ell)$

Monticello + Dewar
Heliac Mercier
Stability



Also: HASE + HERA studies at Garching
Stable equilibria at $\langle\beta\rangle = .3$.



M = 1 MODE OF HELIAC WITH β = .04 IN DEPENDENCE ON INVERSE ASPECT RATIO $\varepsilon=1/A$

FIGURE 1. STABILIZING EFFECT OF TOROIDICITY.

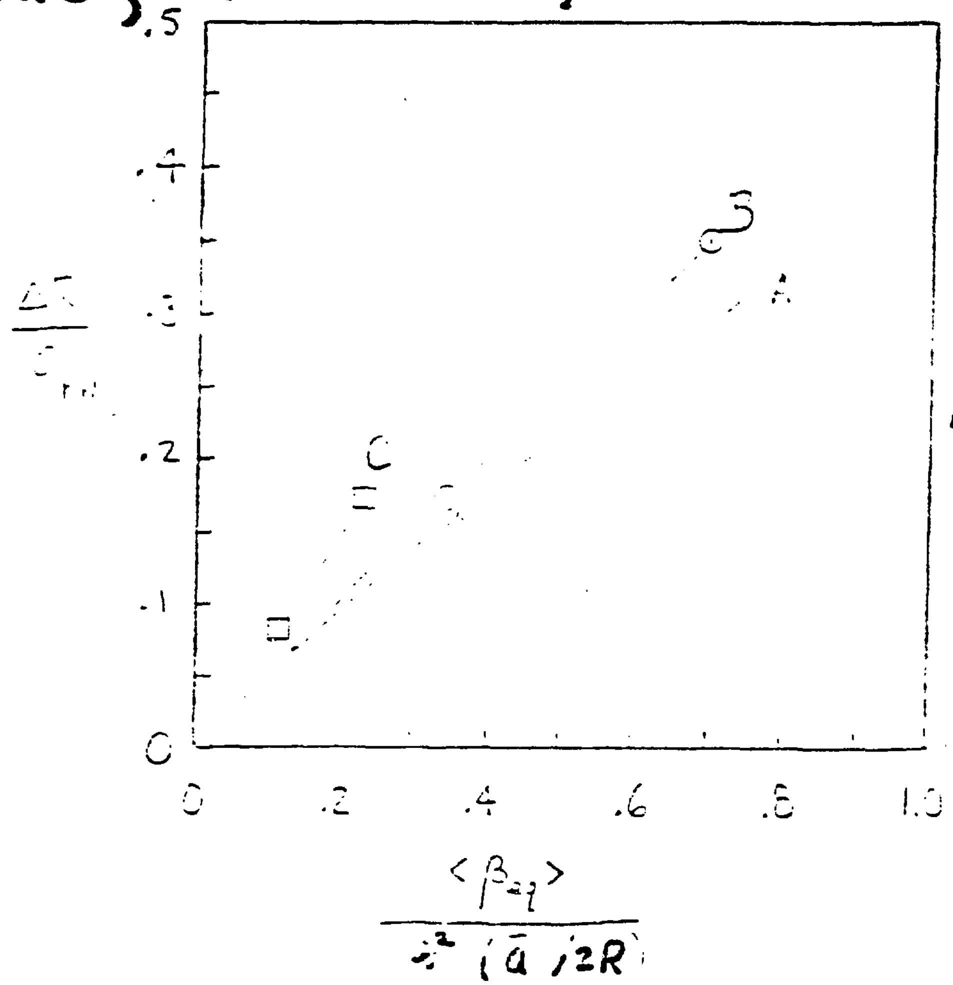
Equilibrium β Limit due to axis shift

broken symmetry $\Rightarrow$ surfaces destroyed

convention: β_c at $\Delta R/a_{hw} = 1/2$

$\Delta R/a_{hw}$ scales like $\langle\beta\rangle A/\iota^2$.

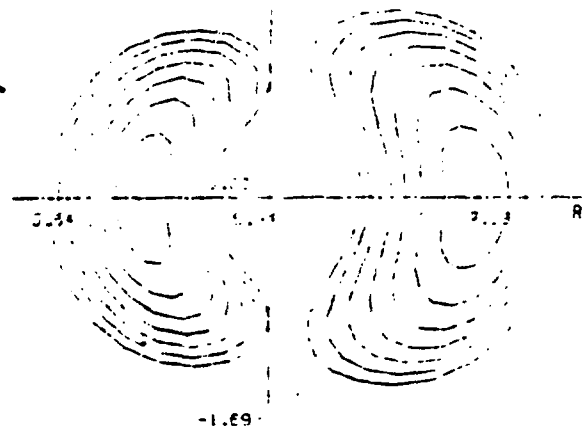
Bauer, Betancourt, Garabedian



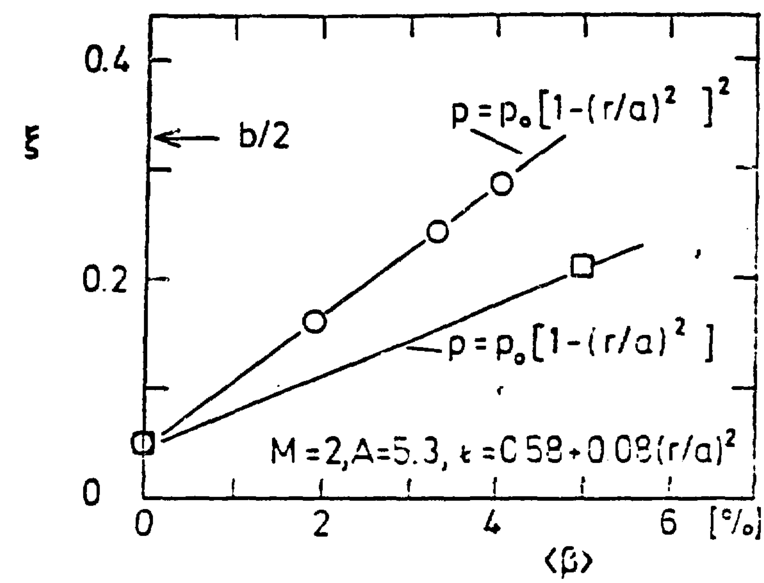
heliac

	m	R/a_p
A	2	4
B	4	8
C	6	8

Hernegger:



M = 2, A = 5.3, ι = 0.58 + 0.08$(r/a)^2$
$p = p_o[1-(r/a)^2]^2$



Scaling:

$$\Delta R/a_{hw} \approx .5 \langle\beta\rangle \frac{A}{\iota^2} \quad \text{for } p = p_o[1-(\tfrac{r}{a})^2]^2 .$$

$$\Delta R/a_{hw} \approx .3 \langle\beta\rangle \frac{A}{\iota^2} \quad \text{for } p = p_o[1-(\tfrac{r}{a})^2] .$$

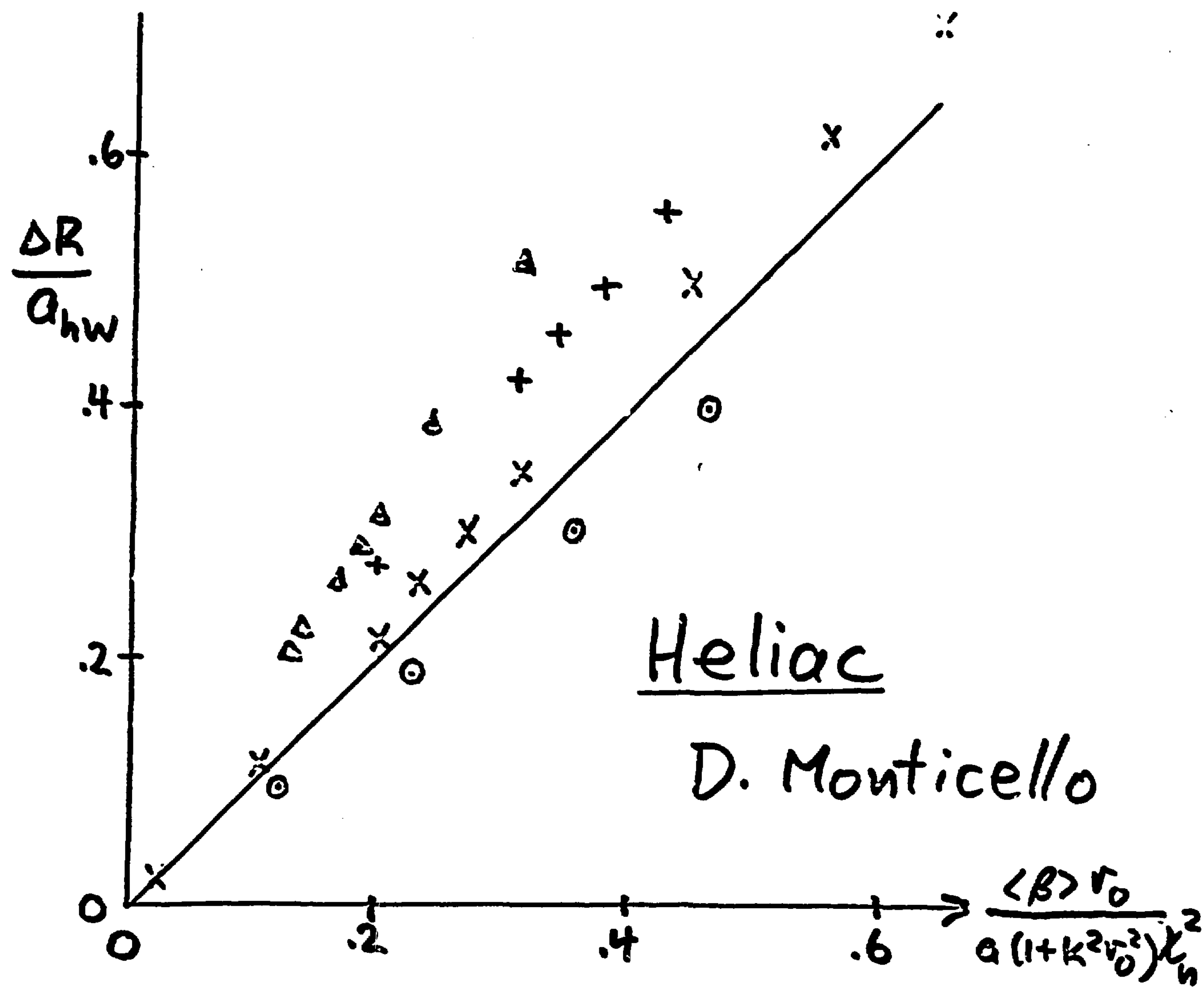
W VII AS : $\approx .3$ for $p = p_o[1-(r/a)^2]^2$.

Reduced by ½ from unoptimized.

Helical Shift

$$\frac{\Delta R}{a_{hw}} \sim \langle\beta\rangle \frac{r_0}{a(1+k^2 r_0^2)\iota_h^2} \quad \text{(Boozer)}$$

$$\iota_h \equiv 1 - \iota/m$$



× $\alpha=2$, $k=.8$, $A=2.8$, $B=1$

⊙ $B=1.2$

+ $A=3.6$

Δ $\alpha=3$

Expansion about Magnetic Axis

Scale lengths for vacuum field:

helical rod. of curvature $r_c = \dfrac{1+k^2r_0^2}{k^2 r_0}$

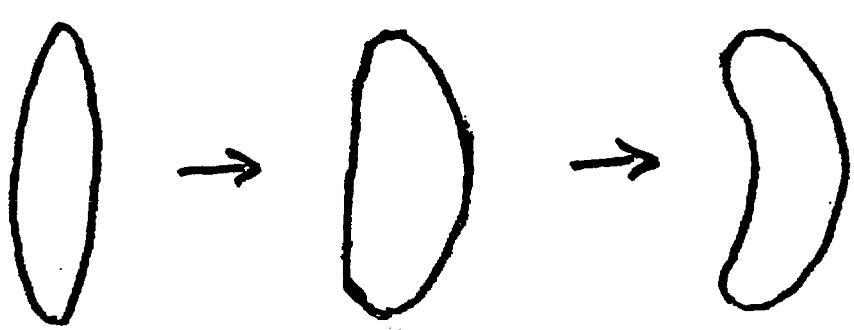
major radius R

Helical effects dominate if $r_c \ll R$.

Magnetic Well:

$$V'' \propto k^2 r_0^2 (1-\varepsilon) - \left[(1+k^2 r_0^2) \frac{r_0 \dddot{\psi}'}{\dot{\psi}} - 1 + \varepsilon \right] \varepsilon$$

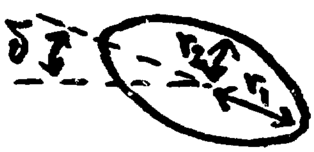
(Zueva & Solov'ev)



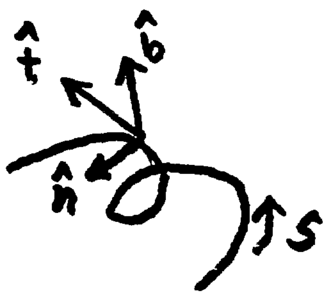
increasing $r_0 \dddot{\psi}/\dot{\psi}$

The Transform

$$\iota = \frac{1}{2\pi} \oint \left\{ \delta' \left[\sqrt{1-\varepsilon^2} - 1 \right] - \chi \sqrt{1-\varepsilon^2} \right\} ds + N_W$$



$$\varepsilon = \frac{r_1^2 - r_2^2}{r_1^2 + r_2^2}$$



$$\chi = \hat{b} \cdot \frac{d\hat{n}}{ds}$$

N_W = no. of rotations of normal vector

<u>Planar axis</u>: $\chi = 0$

$$\frac{\iota}{m} \simeq 1 - \sqrt{1-\varepsilon^2} \simeq \frac{1}{2}\varepsilon^2$$

<u>helical axis</u>: $\delta' = 0$, $\chi = \dfrac{k}{1+k^2 r_0^2}$ $\left(k = \dfrac{2\pi}{\ell}\right)$

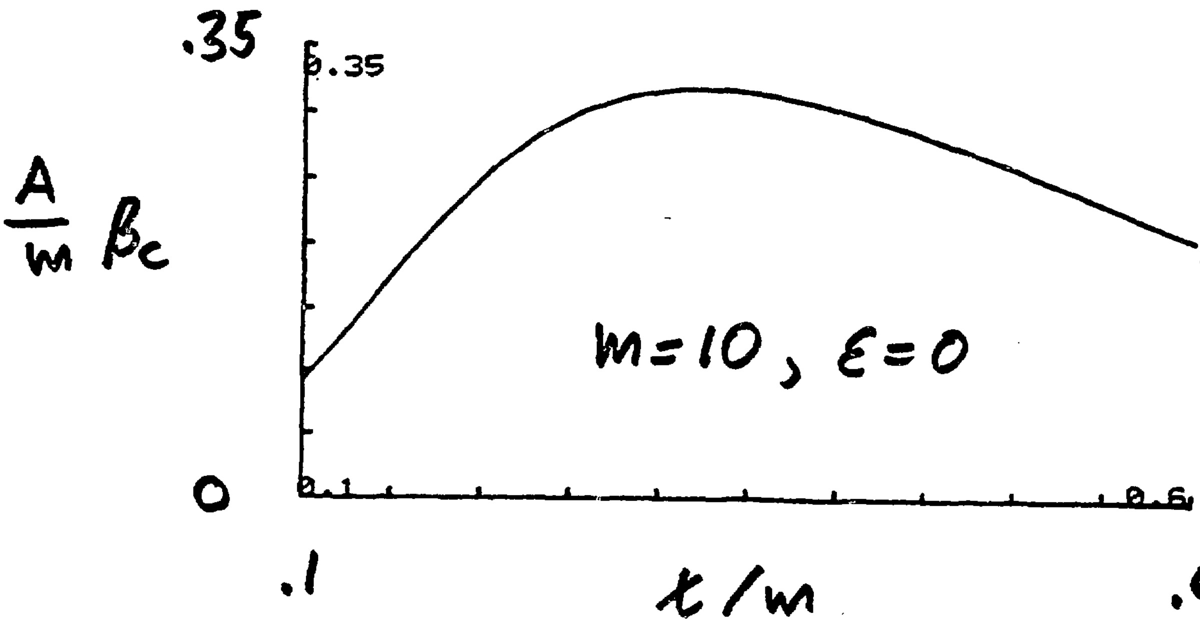
$$\frac{\iota}{m} \simeq 1 - \sqrt{\frac{1-\varepsilon^2}{1+k^2 r_0^2}}$$

Optimal Helical Axis Stellarator

(for equilibrium & stability)

$$\frac{\Delta R}{a_{hw}} = \beta \frac{A}{m}\left[\frac{1}{2m(1-\iota_n)^2} + \frac{1}{1-\varepsilon^2}\sqrt{\frac{1-\varepsilon^2}{\iota_n^2}-1}\,\right]$$

where $\iota_n = 1-\iota/m$.



$m=A=10 \Rightarrow \beta_c \doteq 30\%$ at $kr_0 \sim 1$ $(\iota/m \approx 1/3)$

Shape of flux surfaces $\rightarrow V''$.

Require V'' in 2nd stability regime.

Helical Vacuum Fields

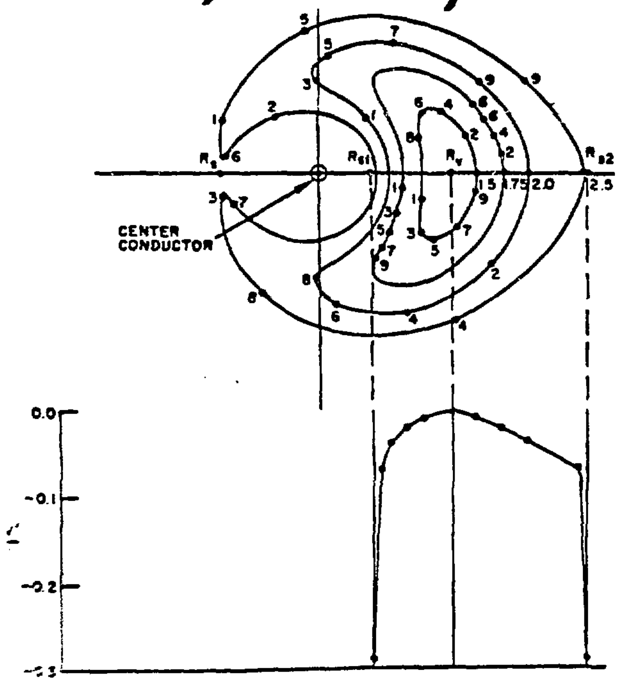
General Solution:

$$\psi = A\ln r + B r^2/2 + \sum_\ell A_\ell r \begin{Bmatrix} I'_\ell(\ell k r)\\ K'_\ell(\ell k r)\end{Bmatrix}\begin{Bmatrix}\cos \ell\zeta\\ \sin\ell\zeta\end{Bmatrix}$$

with $\zeta = \theta - kz$

I. $A \neq 0 \Rightarrow$ current at $r=0$

(Furth, Killeen, Rosenbluth, Coppi)



Sloth (heliac)

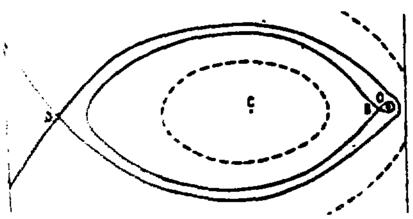
$$\psi = -\ln r + B\frac{r^2}{2} + A_1 r I_1'(kr)\cos\zeta$$

deep well

ι trade-off: $r_0 \lesssim a \Rightarrow k r_0 \lesssim m\frac{a}{R}$

deep well + large ι

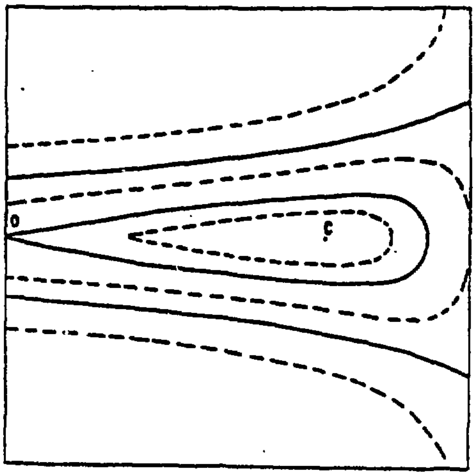
$\Rightarrow$ elongated surfaces



Snake (Tear-Drop)

$$\psi = \ln r + B r^2/2 + A_1 r I_1'(kr)\cos\zeta$$

II. $A=0$ (no conductor at $r=0$)

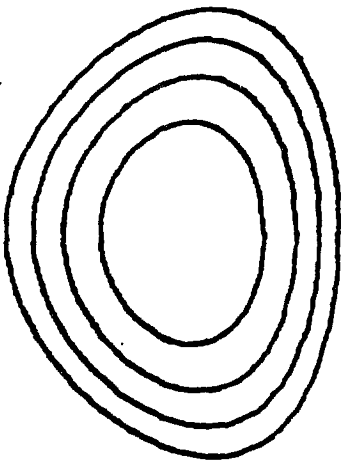


<u>McNamara, Whiteman, & Taylor</u>

$$\psi = B\frac{r^2}{2} + A_2 r I_2'(2kr)\cos 2\zeta + A_3 r I_3'(3kr)\cos 3\zeta$$

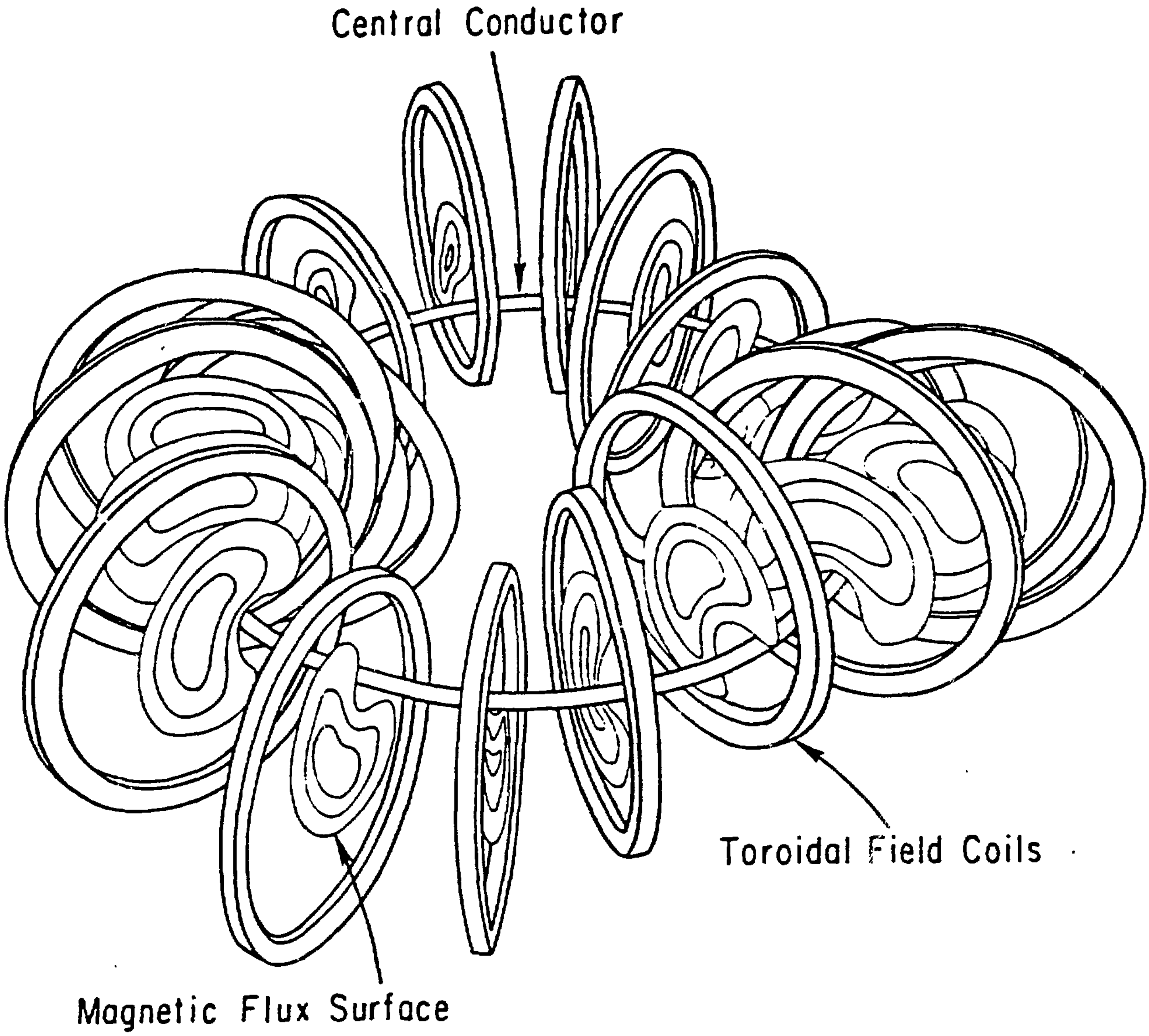
$$|kA_2| > B$$

$$V'(r_A)/V'(r_X) = (1+k^2r_A^2)/(1+k^2r_X^2)$$



<u>mixture of $\ell = 1,2,3$</u>

$$|A_1|, |A_2|, |A_3| \ll B$$


Central Conductor
Toroidal Field Coils
Magnetic Flux Surface

Heliac

Heliac Reference Design

m no. of periods 3

B magnetic field 1.17 T

R radius of toroidal ring 1.47 m

r_1 plasma radius along bean .72 m

r_2 plasma radius across bean .23 m

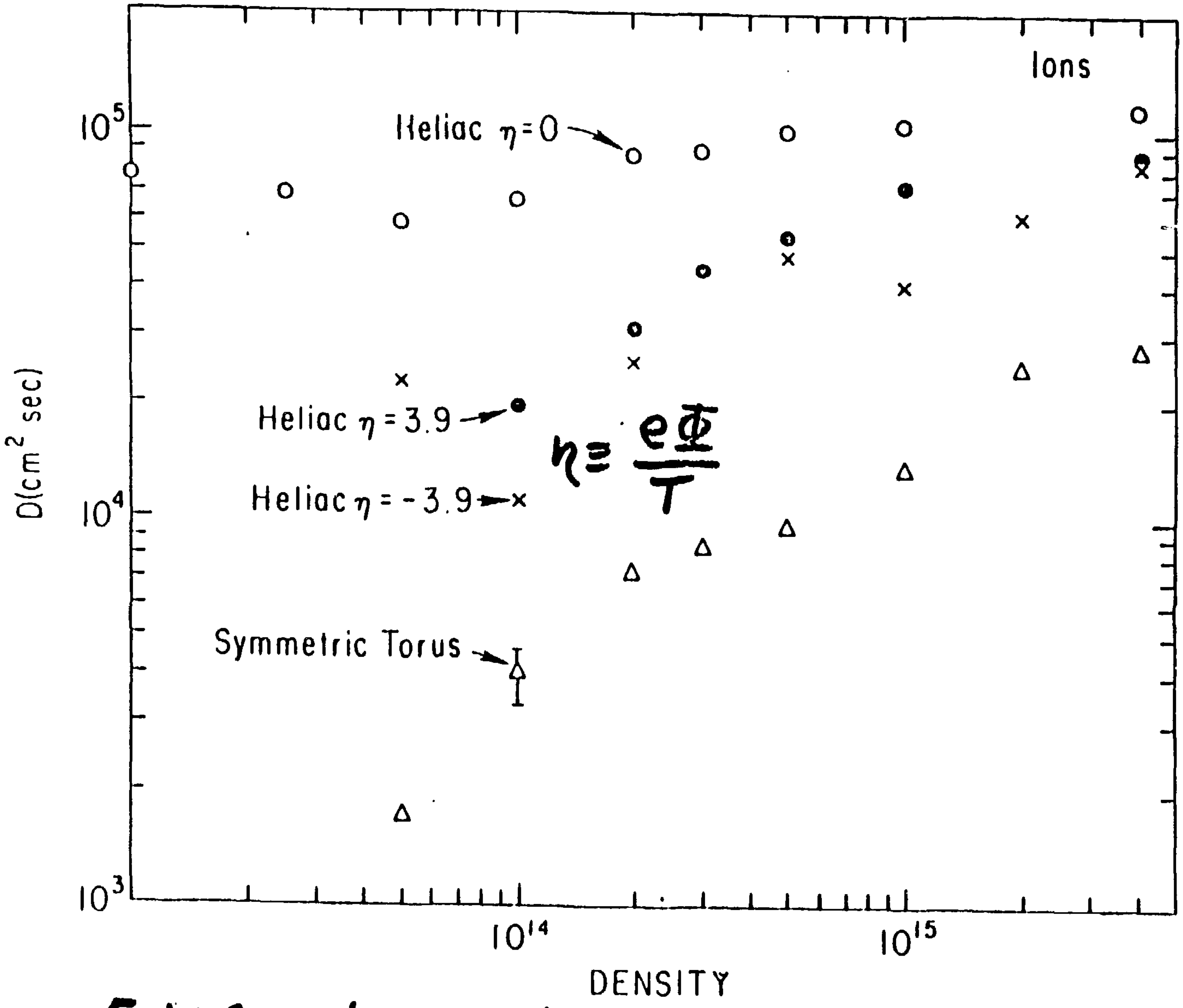
A aspect ratio ~5

$\beta \sim 15\%$

magnetic well ~ 2%

cost \$ 11.5 million (+\$2.3 million contingency)

Helioc Transport (Boozer + Petrovic) $T = 1$ keV



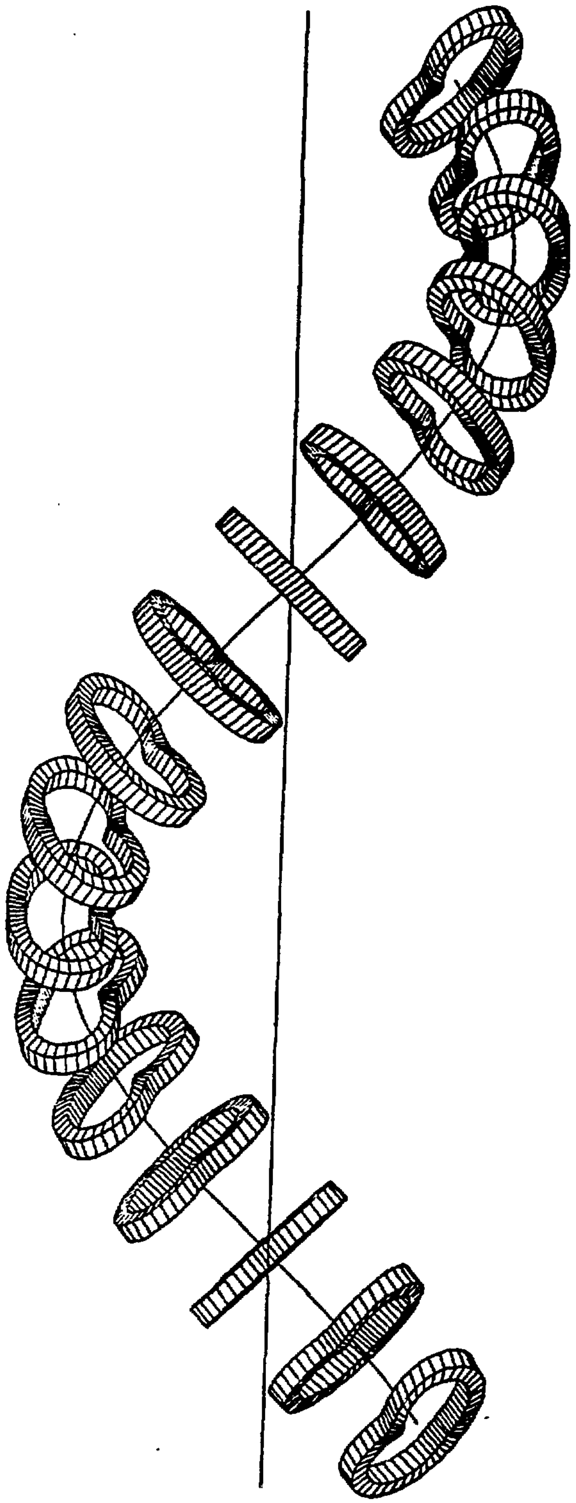
$\tau_{Ei} \sim 5$ ms at $n = 10^{14}$

Modified Heliacs

Yoshikawa - Toroidal field coils displaced in horizontal plane.

Mai, Seeger, Gibson, Chu - Indented toroidal field coils which don't link central conductor.

Stix - PDX + spiral shaped central conductor



$\left|\frac{1}{ka_c}\frac{\delta_3}{a_c}\right|$ required for well

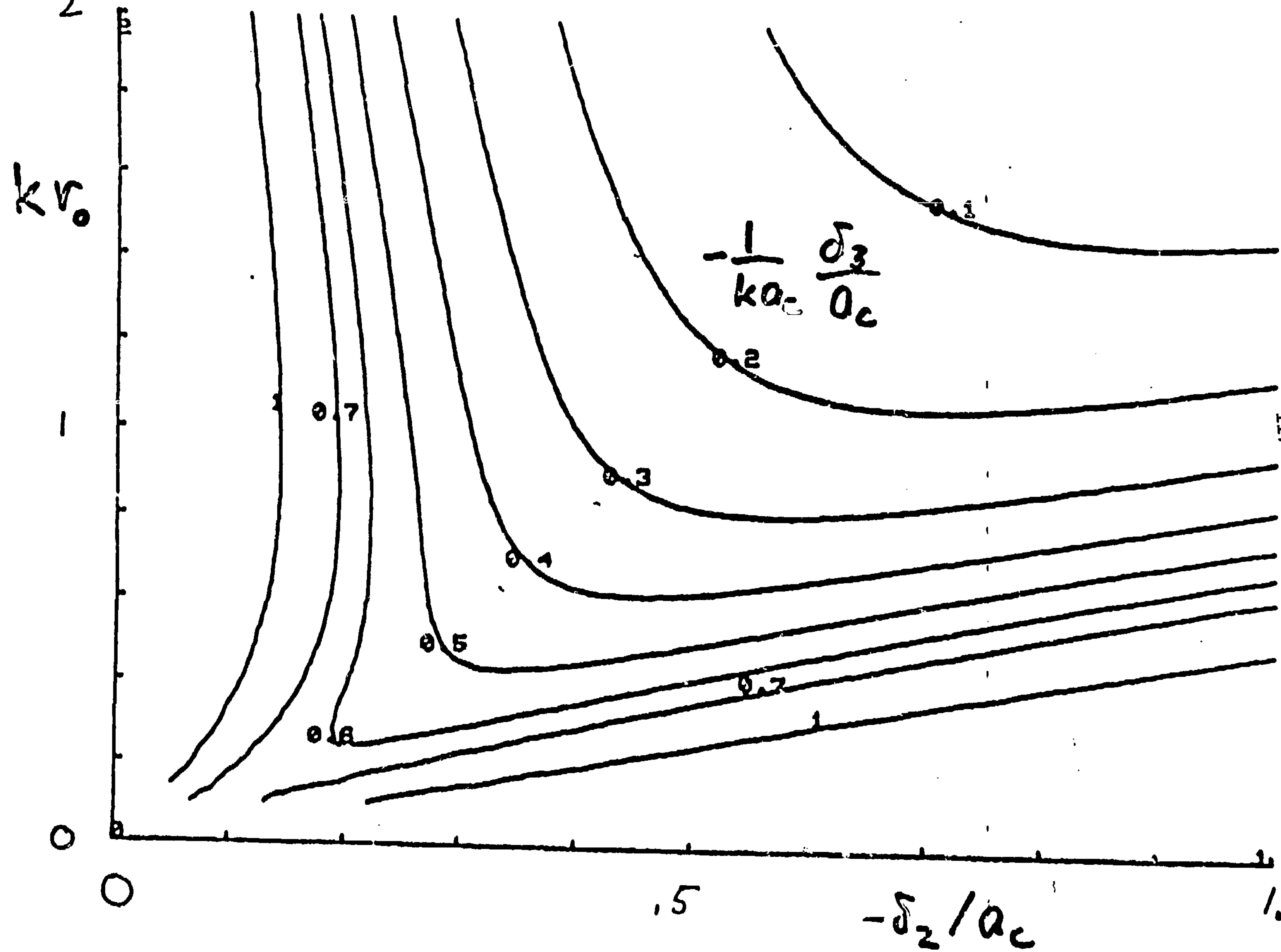
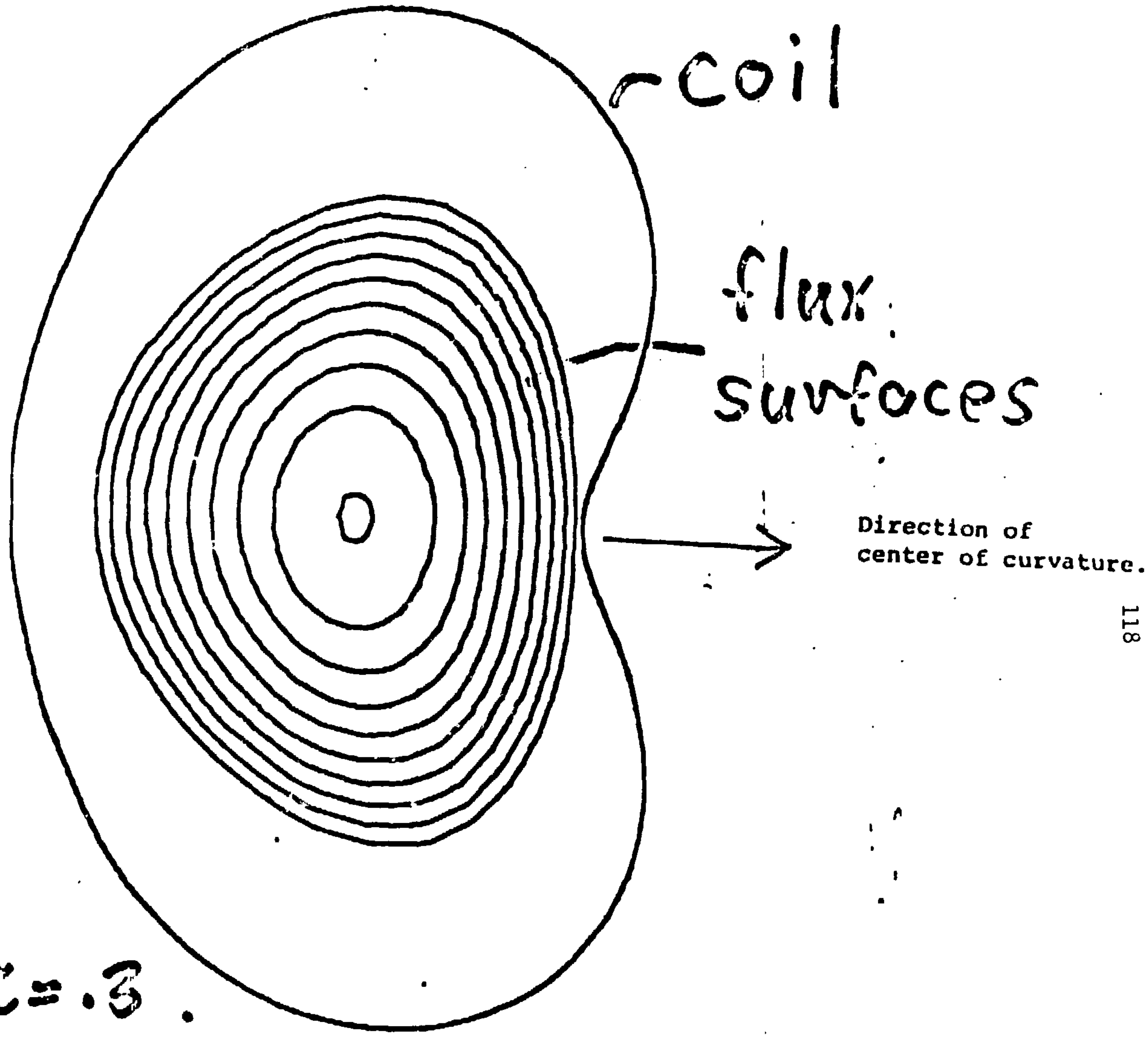


Figure shows outline of a non-interlocked planar coil and the magnetic surfaces a set of such coils would produce. In the illustrated case V" ≈ 0 and $\iota \approx 0.3$ per period.



<u>Configuration with a Well</u>, $\iota = .3$.

$k, r_0 = 1$, $\dfrac{\delta_2}{a_c} = -.3$, $\dfrac{\delta_3}{a_c} = -.1$.

expansion param.

$$\frac{a_c}{R_c} = \frac{1}{8}$$

Unlinked Coils - Other Recent Work

(Sherwood)

Yoshikawa - general issues

Shestakov + Mirin

vacuum field in toroidal geometry

currents on continuous shell

Mai - discrete, indented coils

Conclusions

Helical axis stellarators can have a magnetic well and large ι
$\Rightarrow$ large equilibrium and stability β.

Relatively simple coil systems are possible.

AESD

MODULAR HELIAC STELLARATOR†

L. P. MAI — WESTINGHOUSE, AESD

G. GIBSON — WESTINGHOUSE, AESD

T. K. CHU — PRINCETON PLASMA PHYSICS LABORATORY

†THIS WORK IS IN PART SUPPORTED BY THE U.S. DEPARTMENT OF ENERGY CONTRACT DE-AC02-76-CHO-3073.

THE FOURTH U.S. STELLARATOR WORKSHOP

APRIL 14-15, 1983

OAK RIDGE NATIONAL LABORATORY

Modular Heliac Stellarator.† L. P. MAI and G. GIBSON, Westinghouse Electric Corporation and T. K. CHU, Princeton University—

The classical heliac configuration[1] is generated by currents flowing in circular, planar TF coils whose centers are on a helical line twisting around a current-carrying, planar toroidal conductor. The current in the toroidal conductor provides a poloidal field. The TF coils act much like torsatron coils, providing the toroidal field as well as part of the poloidal field. One main engineering difficulty associated with heliac reactors is the interlocking of the TF and PF coils which makes their assembly and disassembly for maintenance purposes difficult.

Theoretical and computational results[2] *show that* for a prescribed (outermost) heliac magnetic surface, poloidally closed current loops on the surface can be found to generate the prescribed magnetic surface, without a toroidal current loop. These results suggest that in realistic geometries, heliac stellarator may be obtainable without a separate poloidal-field coil, thus achieving modularization of the heliac stellarator.

We have obtained toroidal heliac stellarator magnetic surfaces with a 1% magnetic well, without a toroidal current loop. This is accomplished by (1) using Pacman-shaped, planar TF coils[3] and (2) tilting the TF coils away from the constant-ϕ plane so that the normal vector to the plane of the TF coils is tangent to the local magnetic axis. These results are similar to the recent analytical results of straight heliac stellarators.[4]

†This work is in part supported by the U.S. Department of Energy Contract DE-AC02-76-CHO-3073.

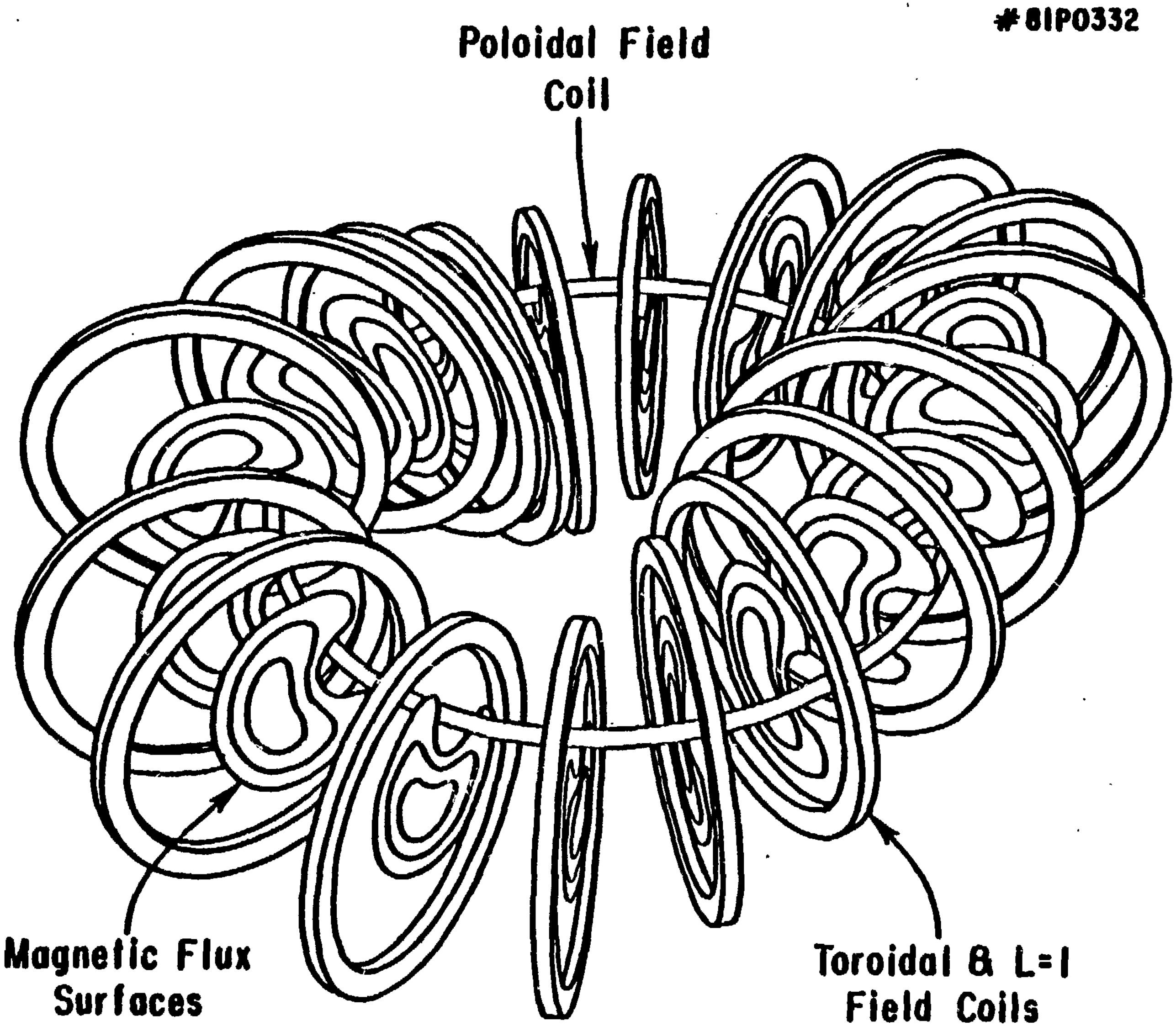
References

[1] A. H. Boozer et al., 9th International Conference on Plasma Physics and Controlled Nuclear Fusion, Paper IAEA-CN-41/Q-4, Baltimore (September 1982).

[2] A. A. Mirin and A. I. Shestakov, Bull. Am. Phys. Soc. 27, 1023 (1982); also Fourth International Stellarator Workshop, Cape May, N.J. (September 1982).

[3] C. Seeger, L. P. Mai, G. Gibson, Bull. Am. Phys. Soc. 27, 1018 (1982).

[4] A. Reiman and A. H. Boozer, Sherwood Meeting, Washington, D.C. (March 1983).



THE HELIAC EXPERIMENTAL MODELS CAN BE MADE FROM OLD TOKAMAK COILS.

PPPL ATC HELIAC (REFERENCE DESIGN) PARAMETERS

I. Toroidal Field Coils

Major Radius	R_o = 1.25 m
Field Periods	m= 2
Mean Coil Radius	a= 0.6485 m
Coil Oscillation Amplitude	d= 0.3125 m
Coil Current	I= 2.0×10^5 Amp/coil
Number of Coils	N= 24

II. Poloidal Field Coil

Mean Radius	a_p = 1.25 m
Coil Current	I_p = 2.96×10^5 Amp

III. Uniform Vertical Field

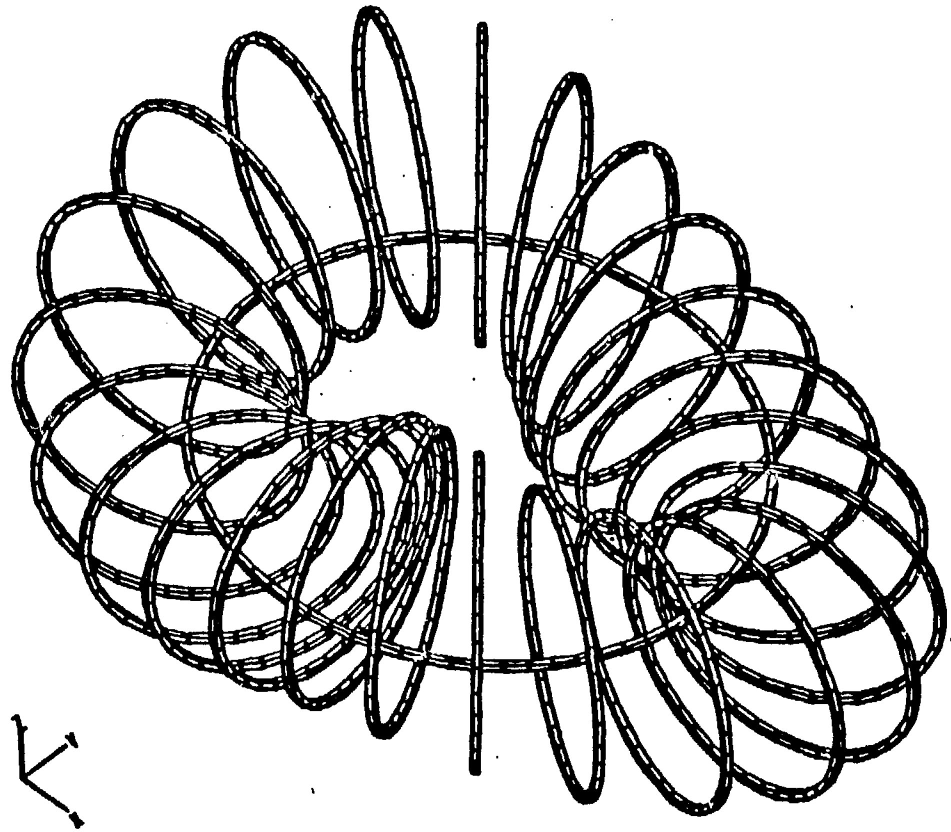
B_z = -0.025 Tesla

PPPL ATC REFERENCE HELIAC

1) **Coil Configuration**

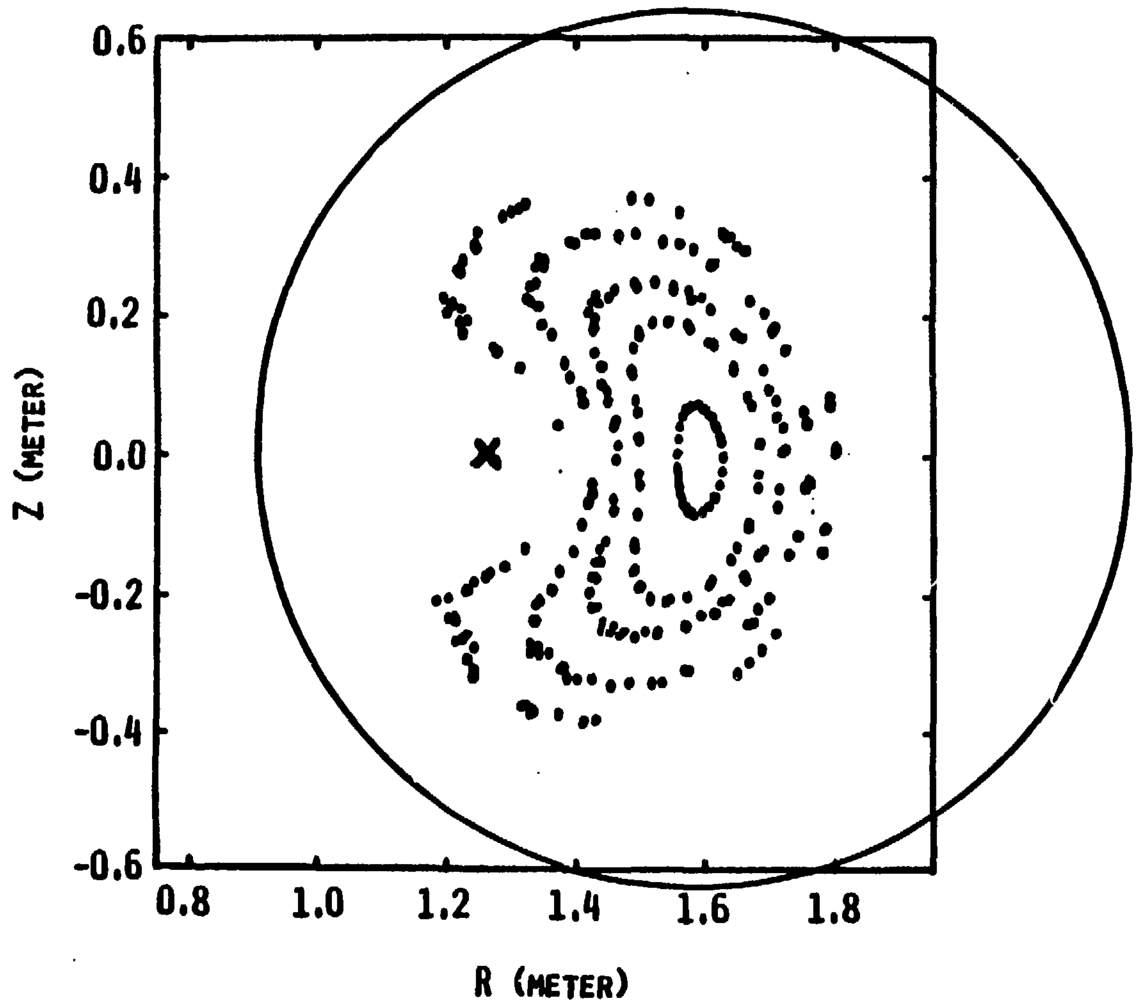
2) **Rotational Transform Profile**

3) **Magnetic Well Properties**

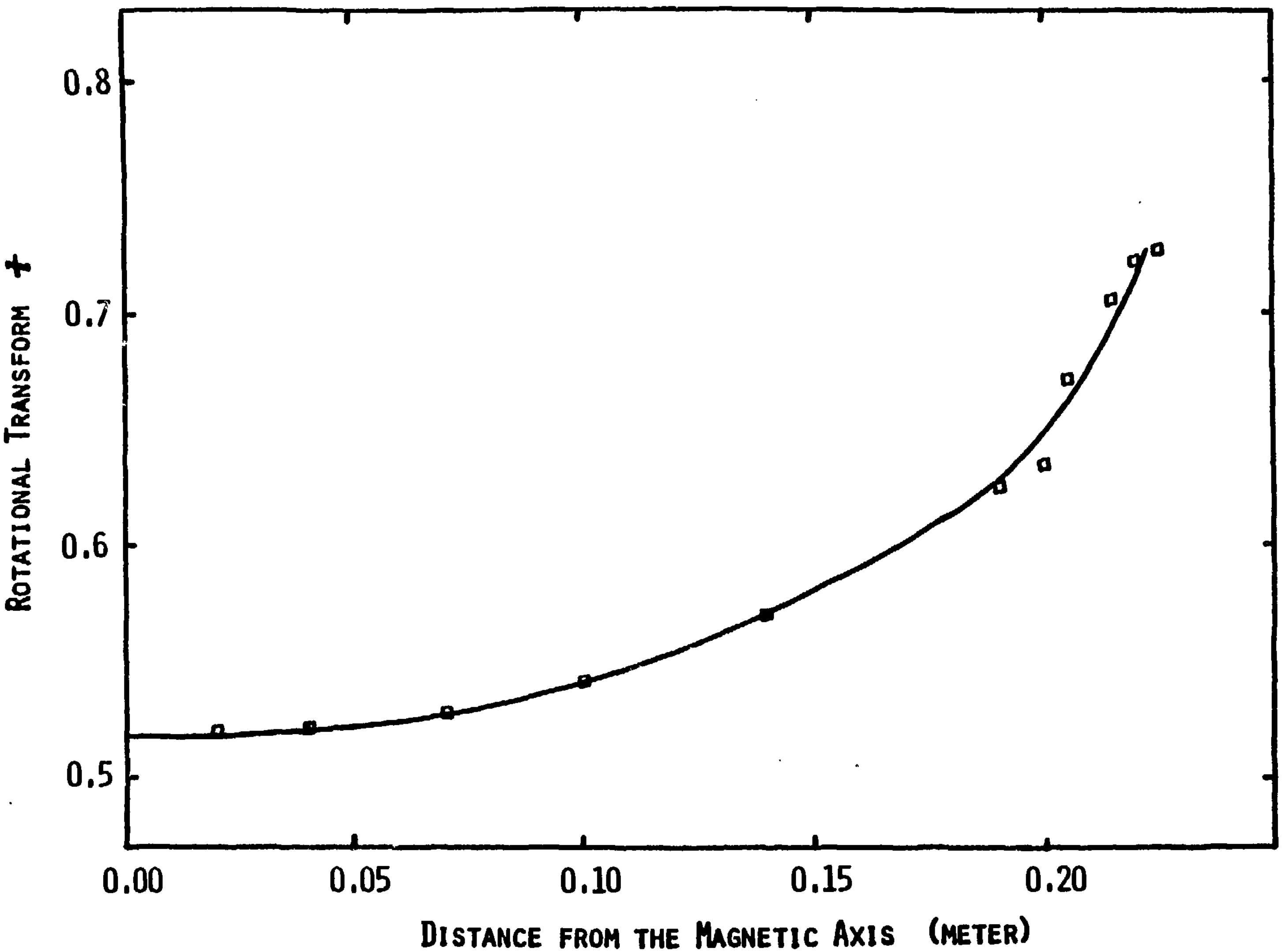


Coil Geometry of the PPPL ATC Heliac
(Reference Design)

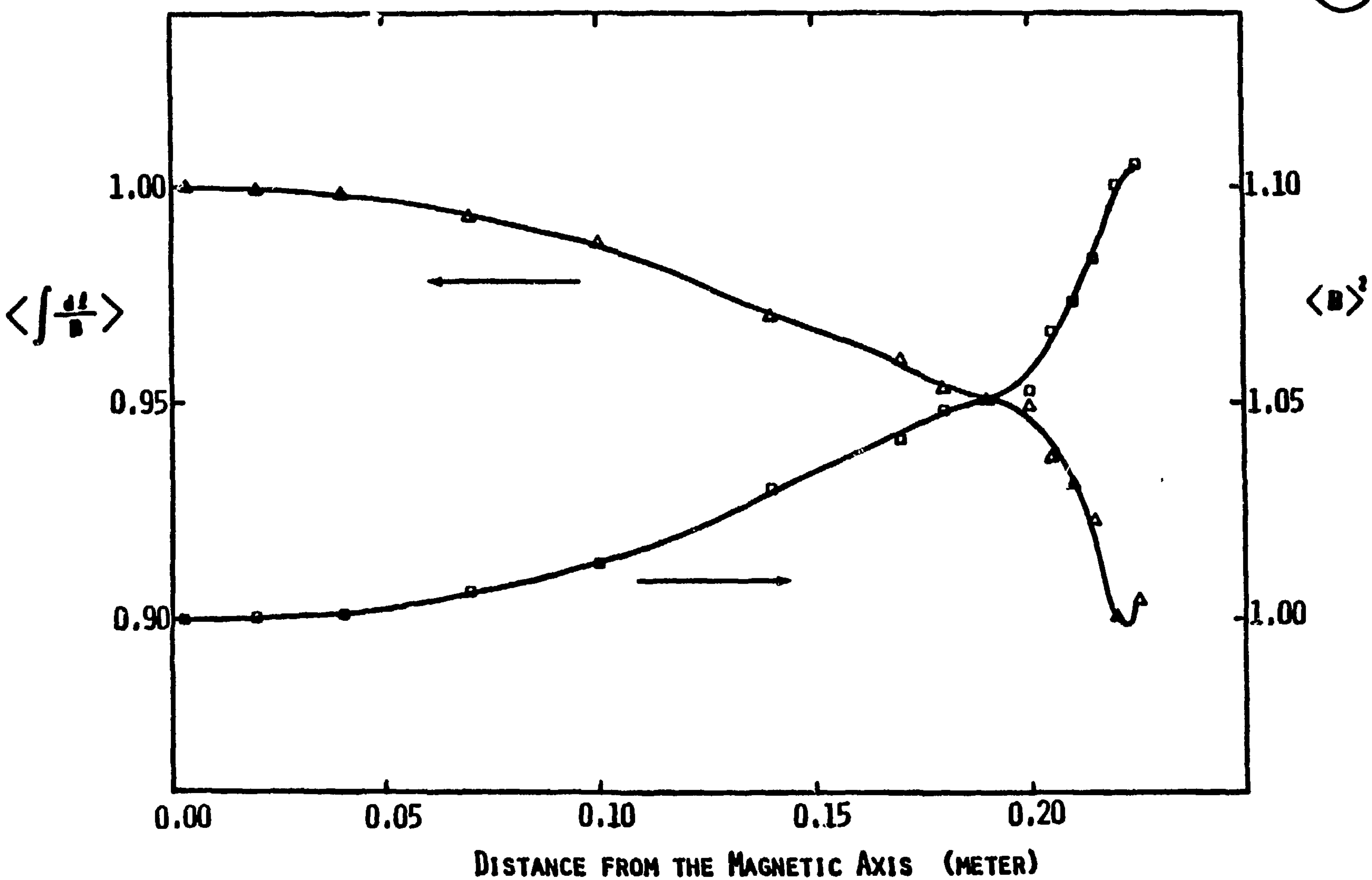
WARD



Magnetic Surfaces of the PPPL ATC Heliac (Reference Design)



Rotational Transform Profile of the PPPL ATC Heliac (Reference Design)



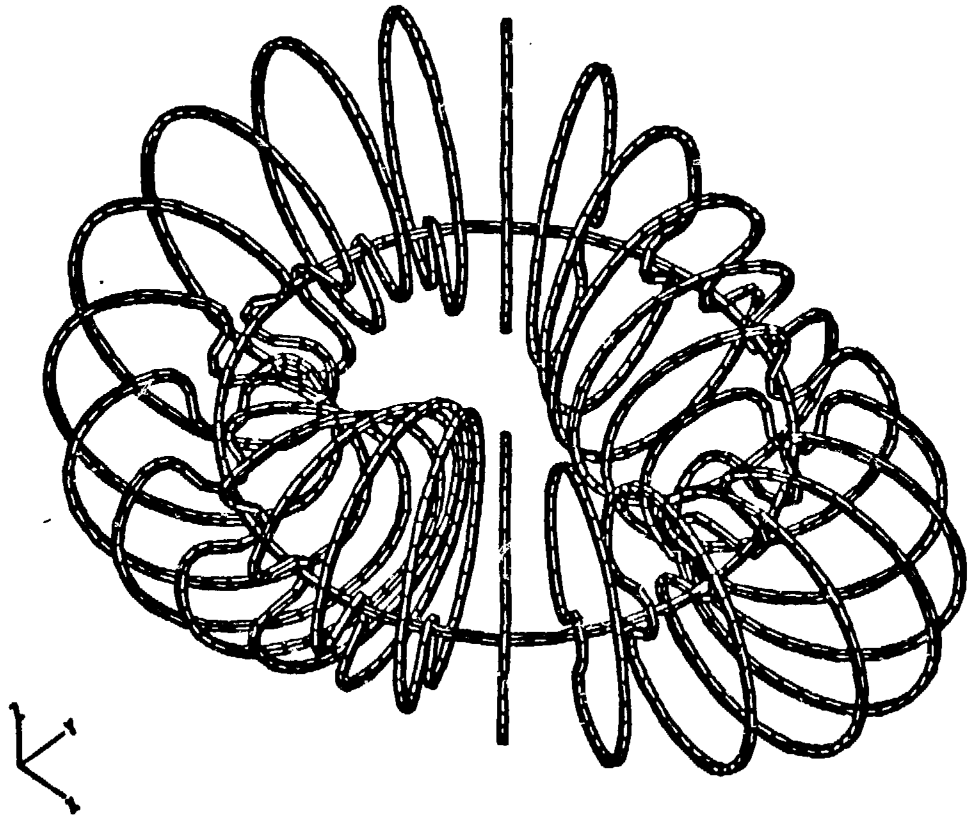
Magnetic Well Properties of the PPPL ATC Heliac
(Reference Design)

MODULAR HELIAC WITH "PACMAN" TF COILS

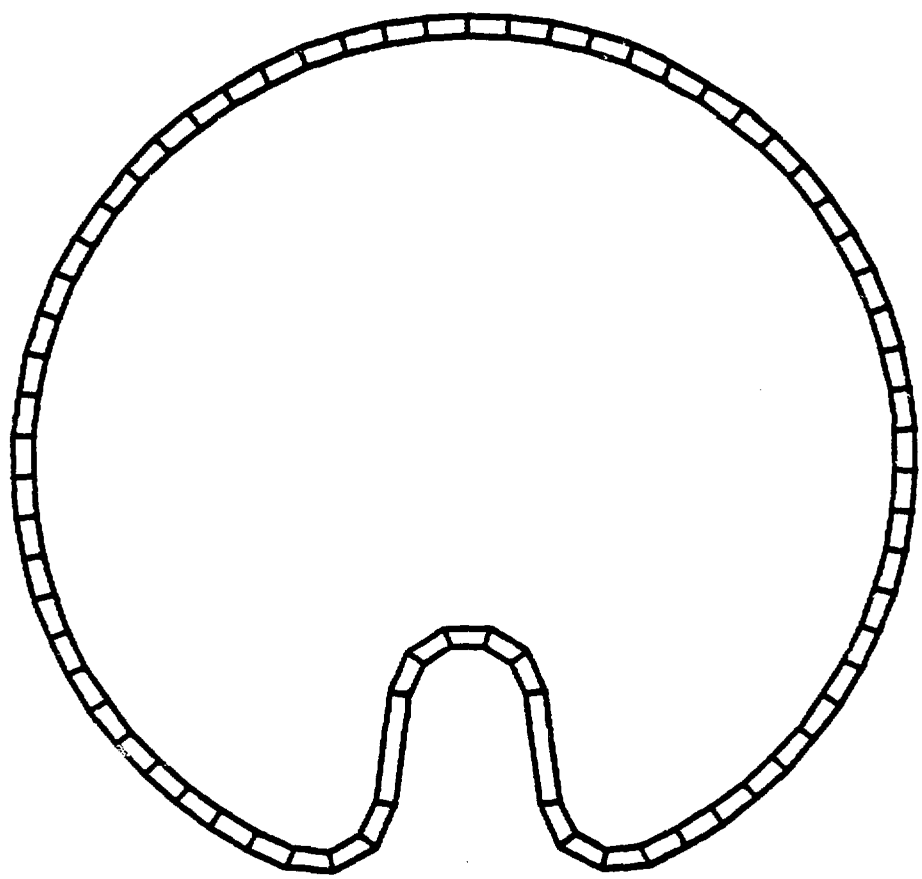
1) Coil Configuration

2) Rotational Transform Profile

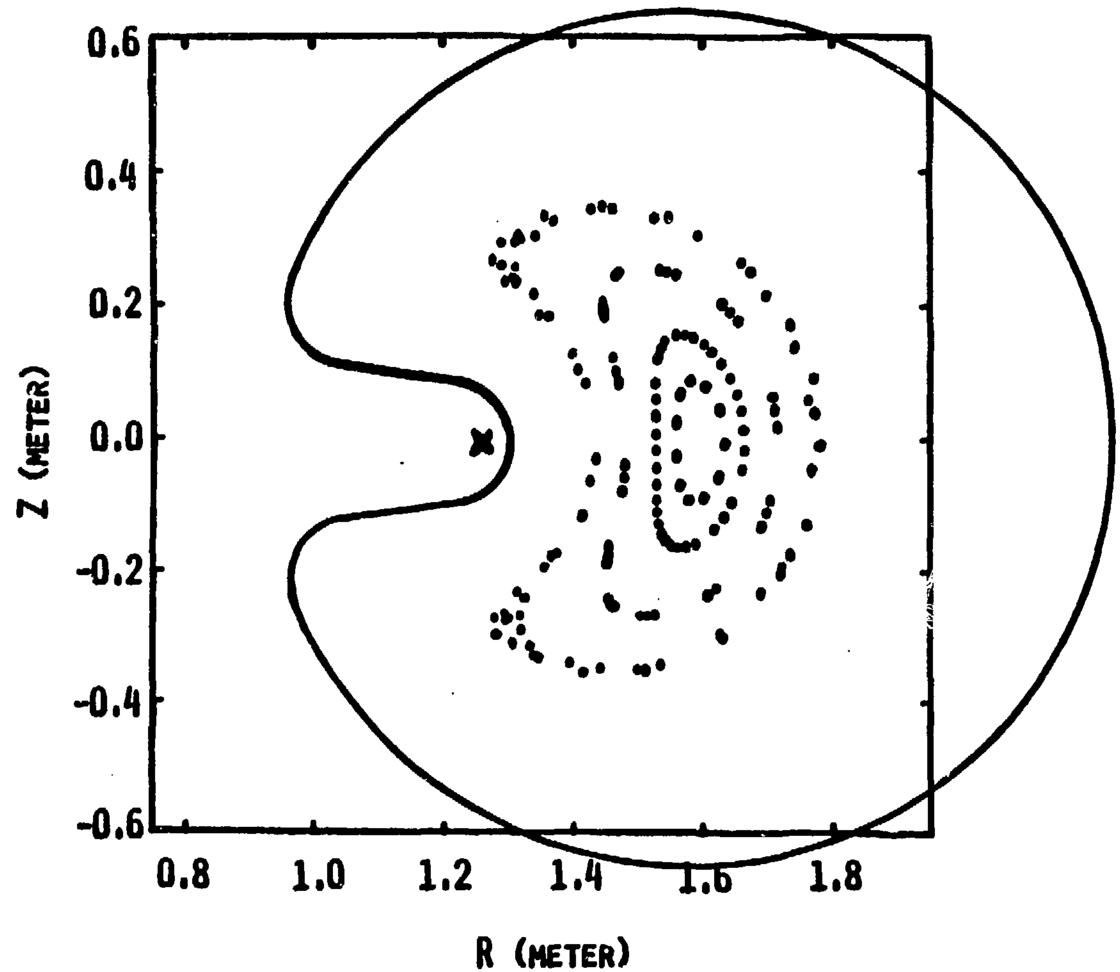
3) Magnetic Well Properties



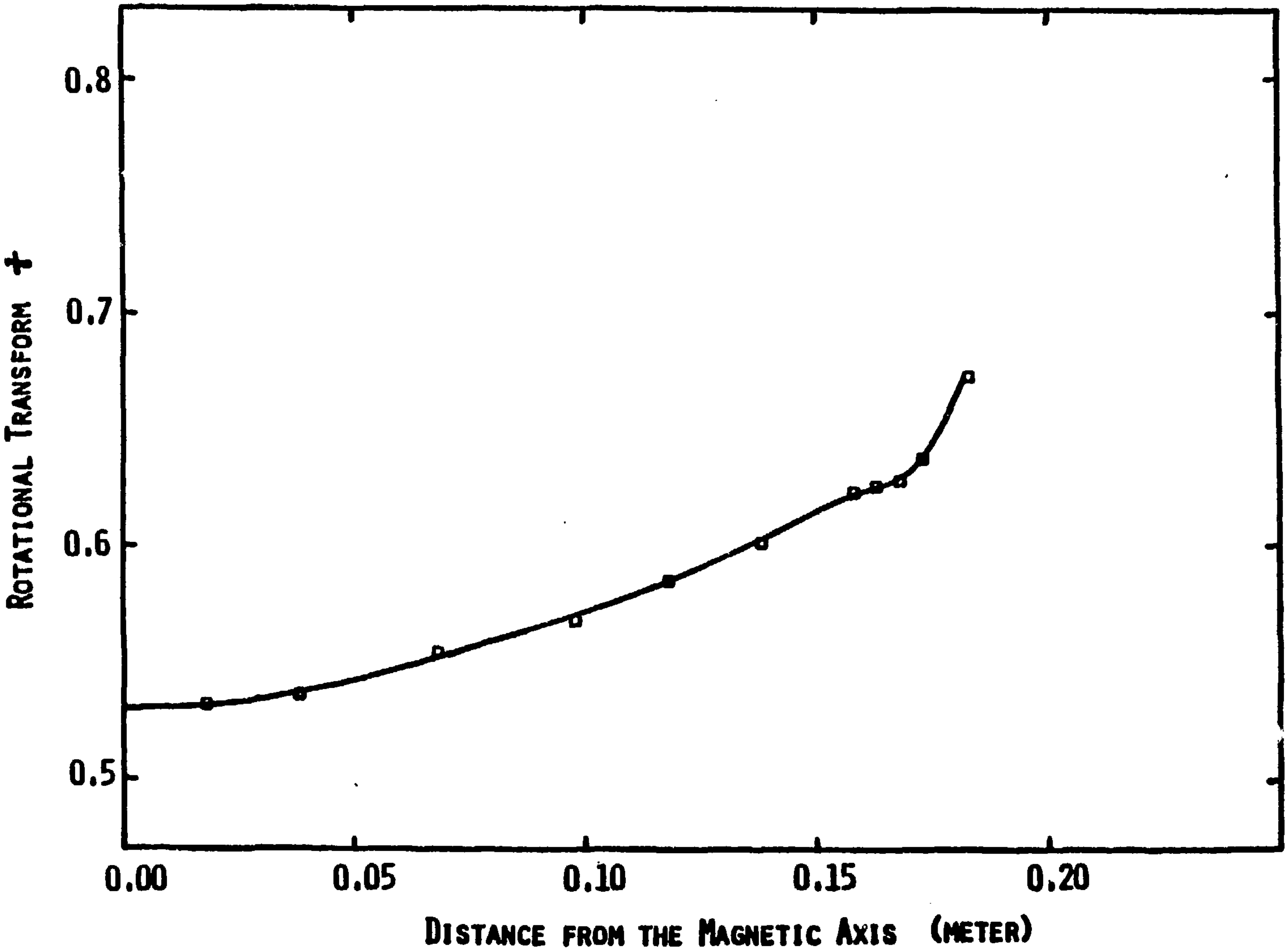
Coil Geometry of a Modular Heliac
with "Pacman" TF Coils



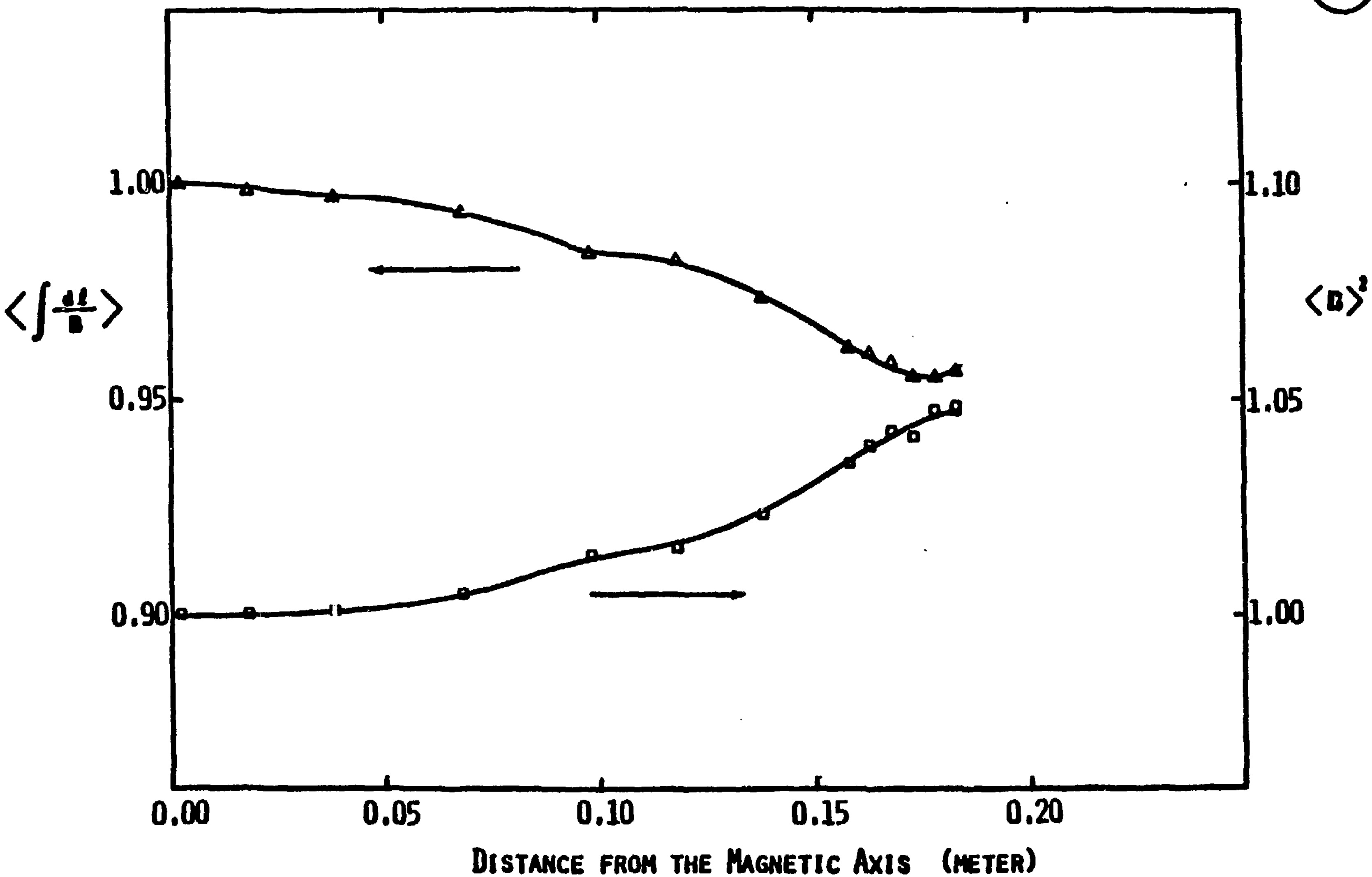
PACMAN COIL



Magnetic Surfaces of a Modular Heliac with "Pacman" TF Coils, in the plane of a TF Coil



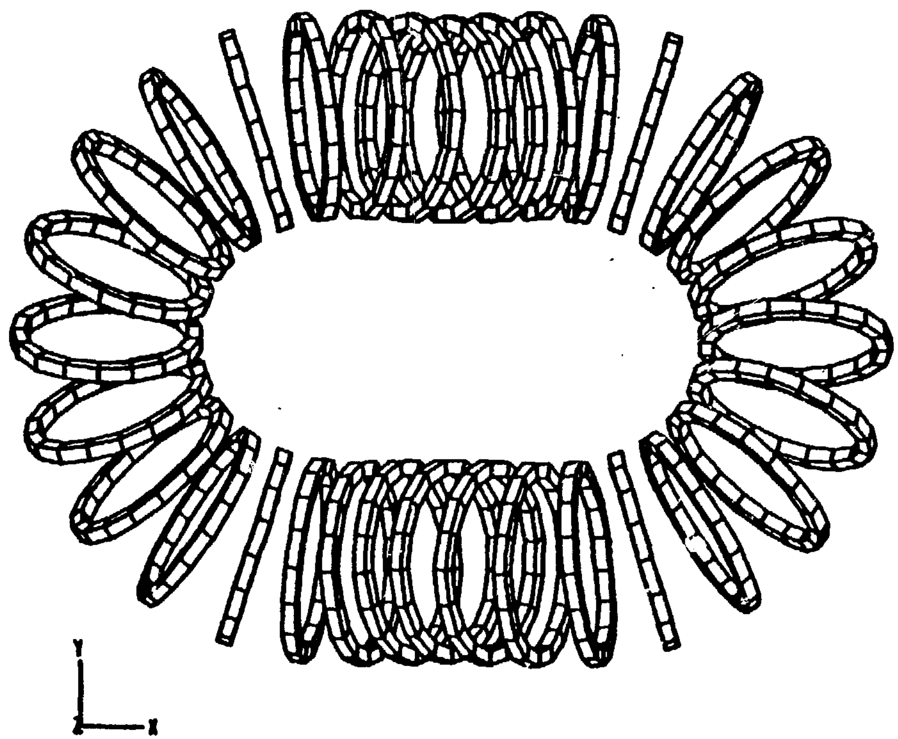
Rotational Transform Profile of a Modular Heliac with "Pacman" TF Coils, in the plane of a TF Coil



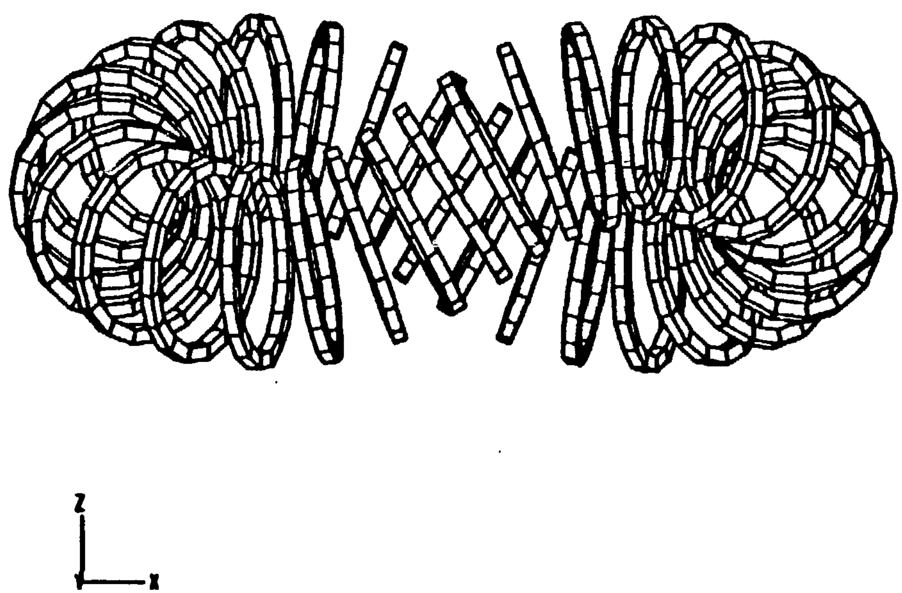
Magnetic Well Properties of a Modular Heliac with
"Pacman" TF Coils, in the plane of a TF Coil

Modular L=1 Stellarator Parameters
Circular TF Coils, No PF Coils

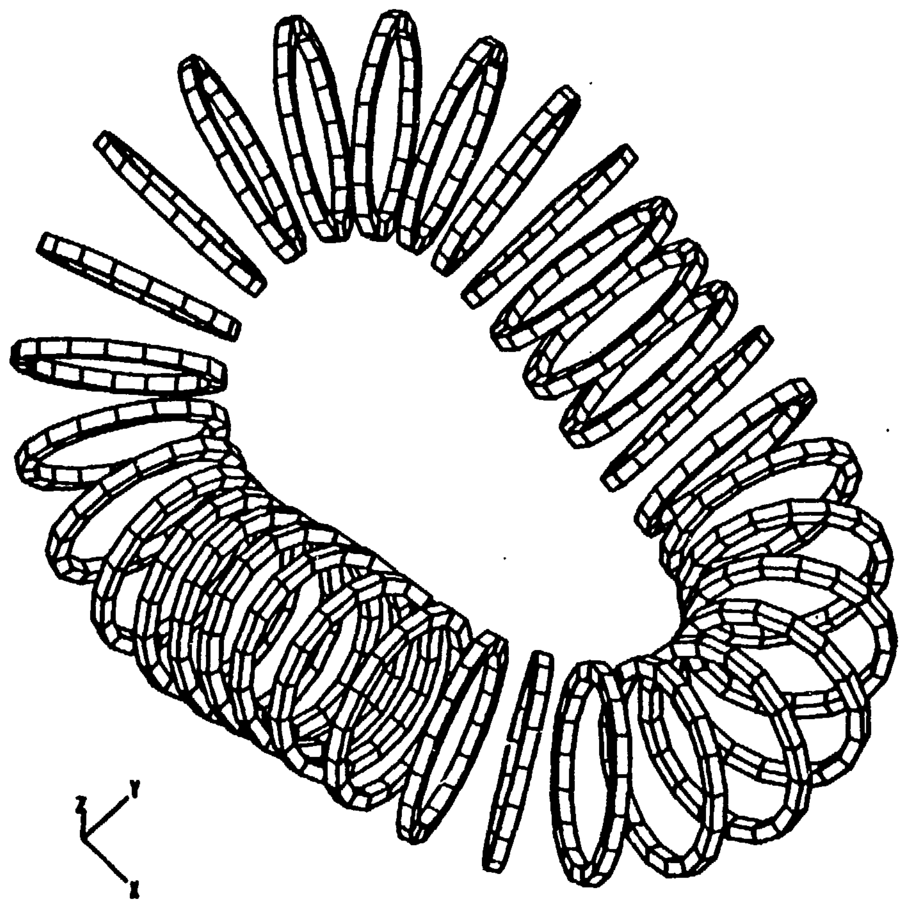
Major Radius	$R_0 = 2$ m
Helical Radius	$A_H = 0.5$ m
Mean Coil Radius	$A = 0.65$ m
Number of Coils	$N = 32$
Toroidal Periods	$m = 2$



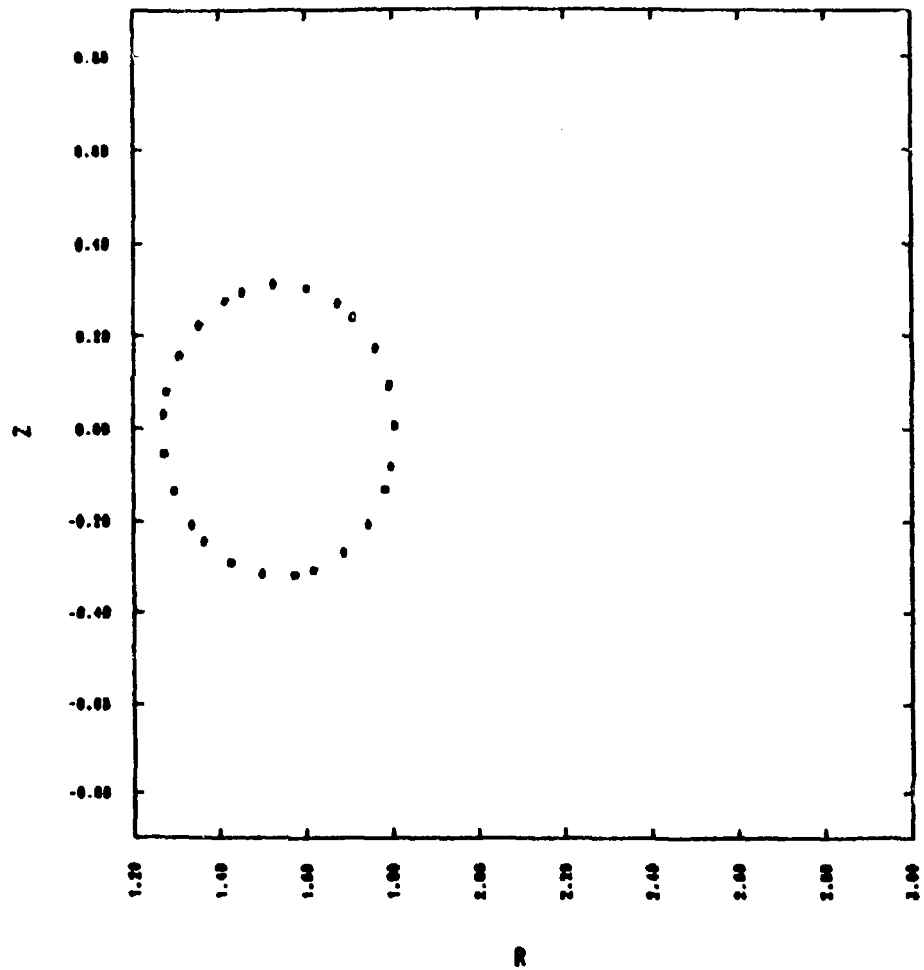
COIL GEOMETRY OF AN L=1 STELLARATOR WITH HELICAL SPATIAL AXIS



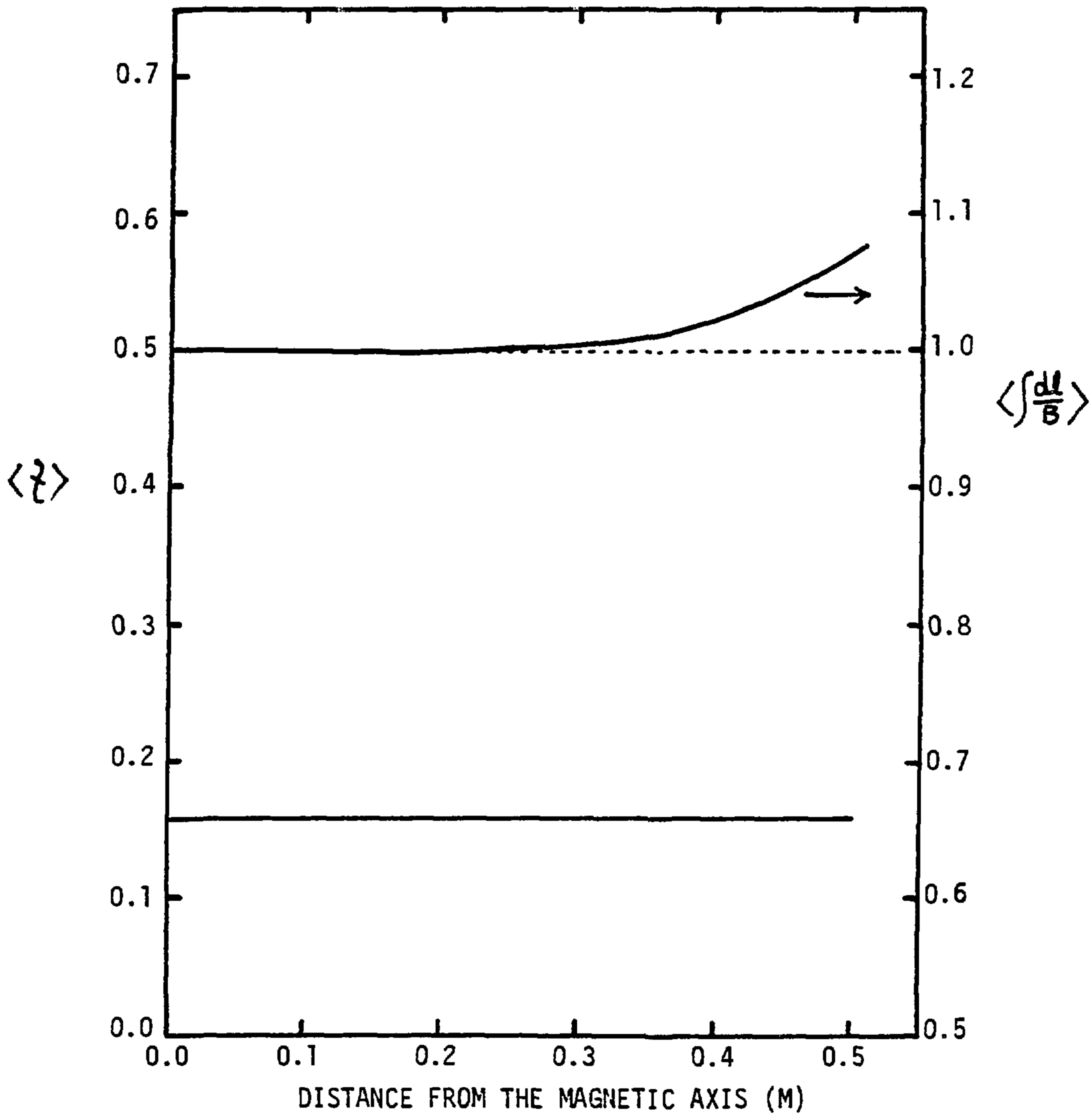
COIL GEOMETRY OF AN L=1 STELLARATOR WITH HELICAL SPATIAL AXIS



COIL GEOMETRY OF AN L=1 STELLARATOR WITH HELICAL SPATIAL AXIS



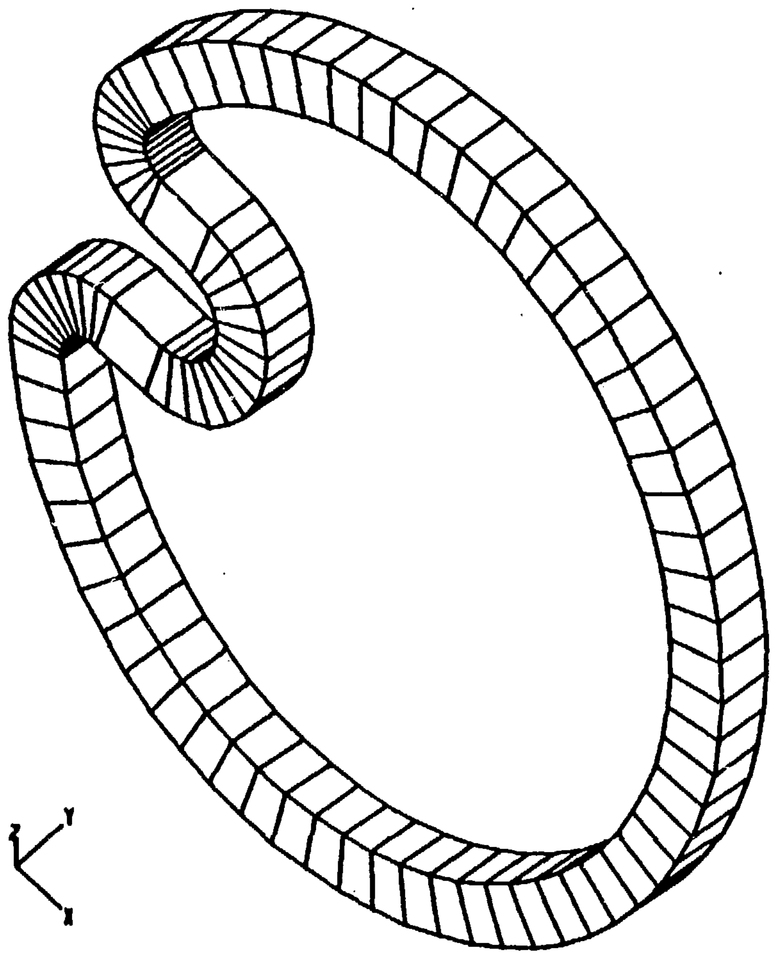
MAGNETIC SURFACE OF A MODULAR L=1 STELLARATOR WITH HELICAL SPATIAL AXIS AND CIRCULAR COILS; R_O = 2 M; A_H = 0.5 M; M = 2



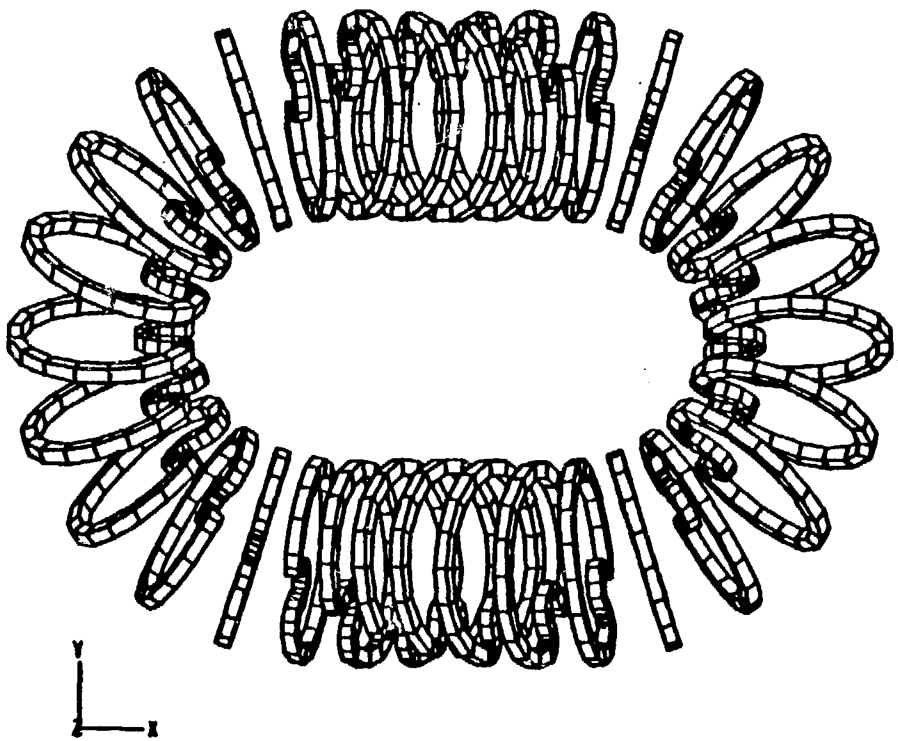
ROTATIONAL TRANSFORM AND $\int \frac{d\ell}{B}$ FOR CIRCULAR COILS AND HELICAL SPATIAL AXIS; NO PF COIL, R_0 = 2.0 M; HELICAL RADIUS = 0.5; M = 2

Parameters of Modular Heliac Stellarator
Pacman-Type TF Coils; No PF Coils

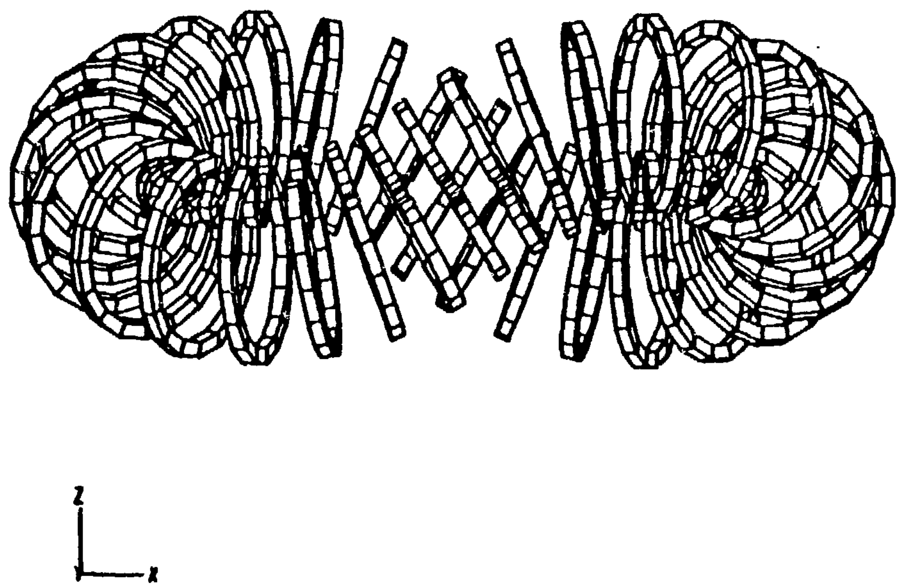
	Base	Variations
Major Radius (m)	2	2.5
Helical Radius (m)	0.5	0.4
Toroidal Periods	2	3
Number of Coils	32	32
Mean Coil Radius (m)	0.65	0.65



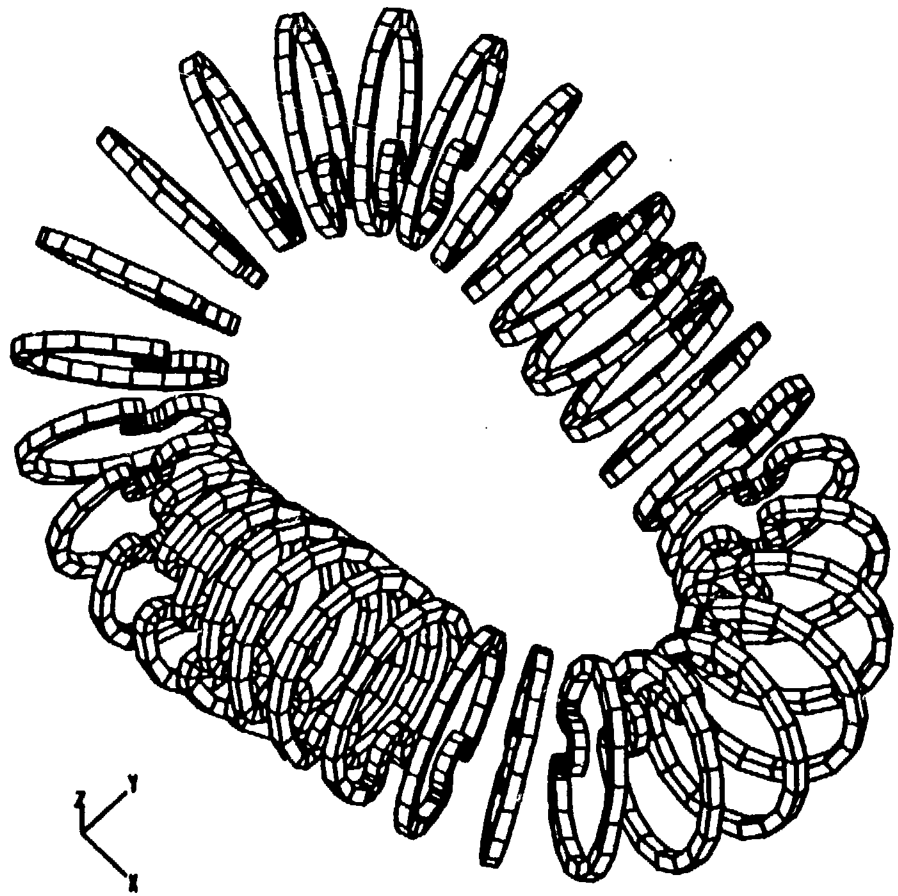
PACMAN COIL



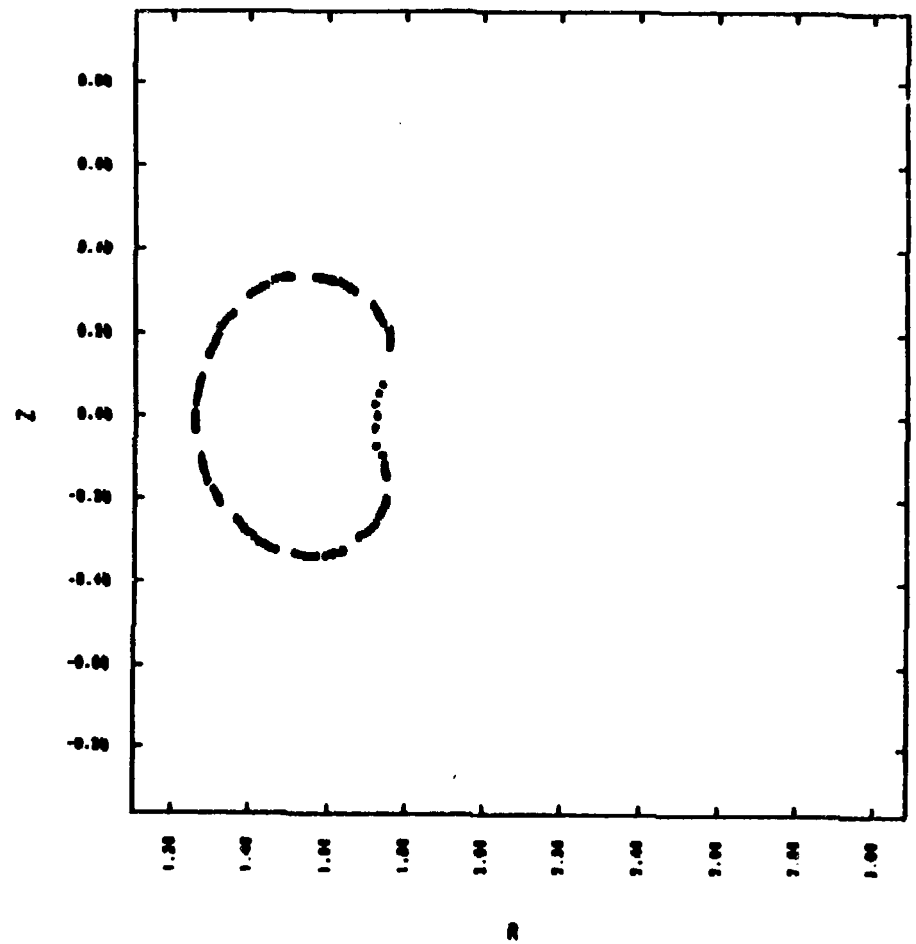
COIL GEOMETRY OF A MODULAR HELIAC WITH HELICAL SPATIAL AXIS



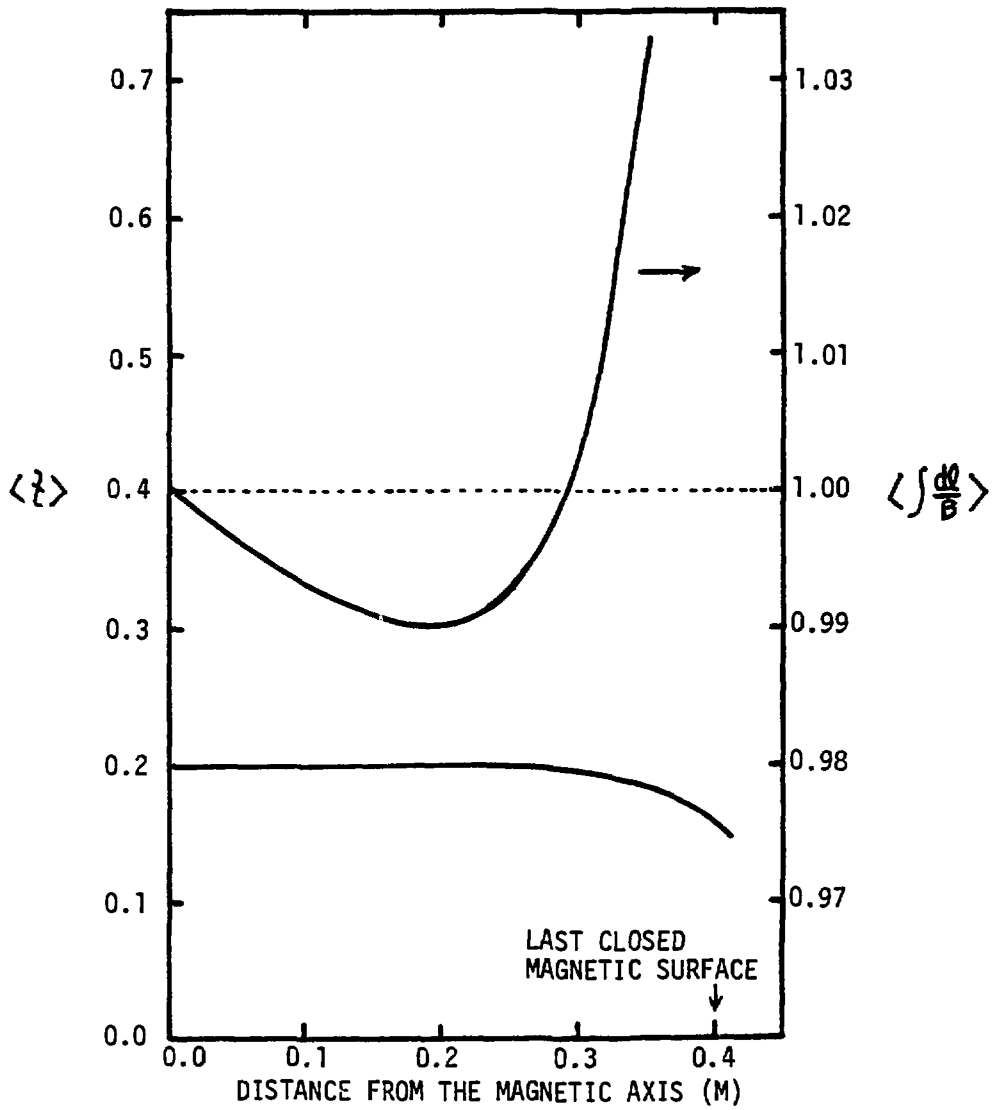
COIL GEOMETRY OF A MODULAR HELIAC WITH HELICAL SPATIAL AXIS



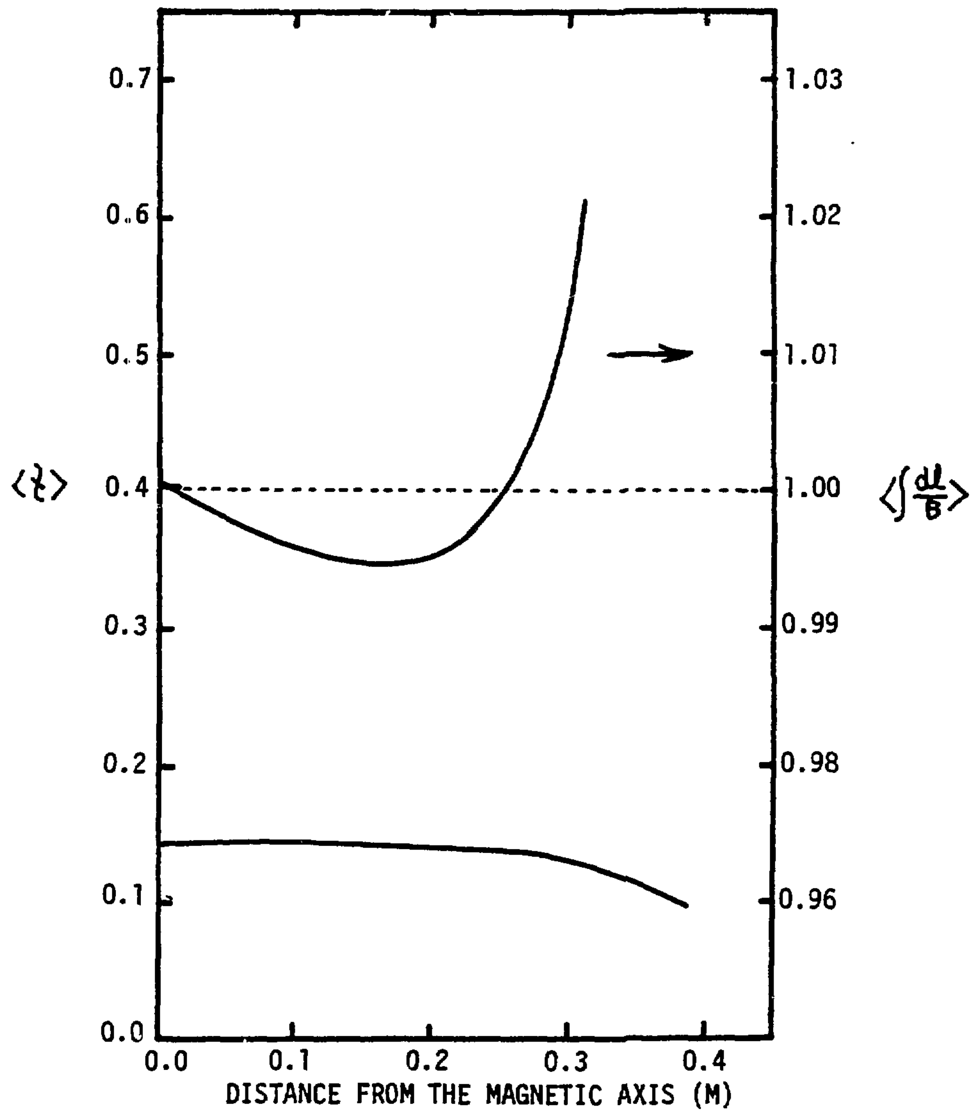
COIL GEOMETRY OF A MODULAR HELIAC WITH HELICAL SPATIAL AXIS



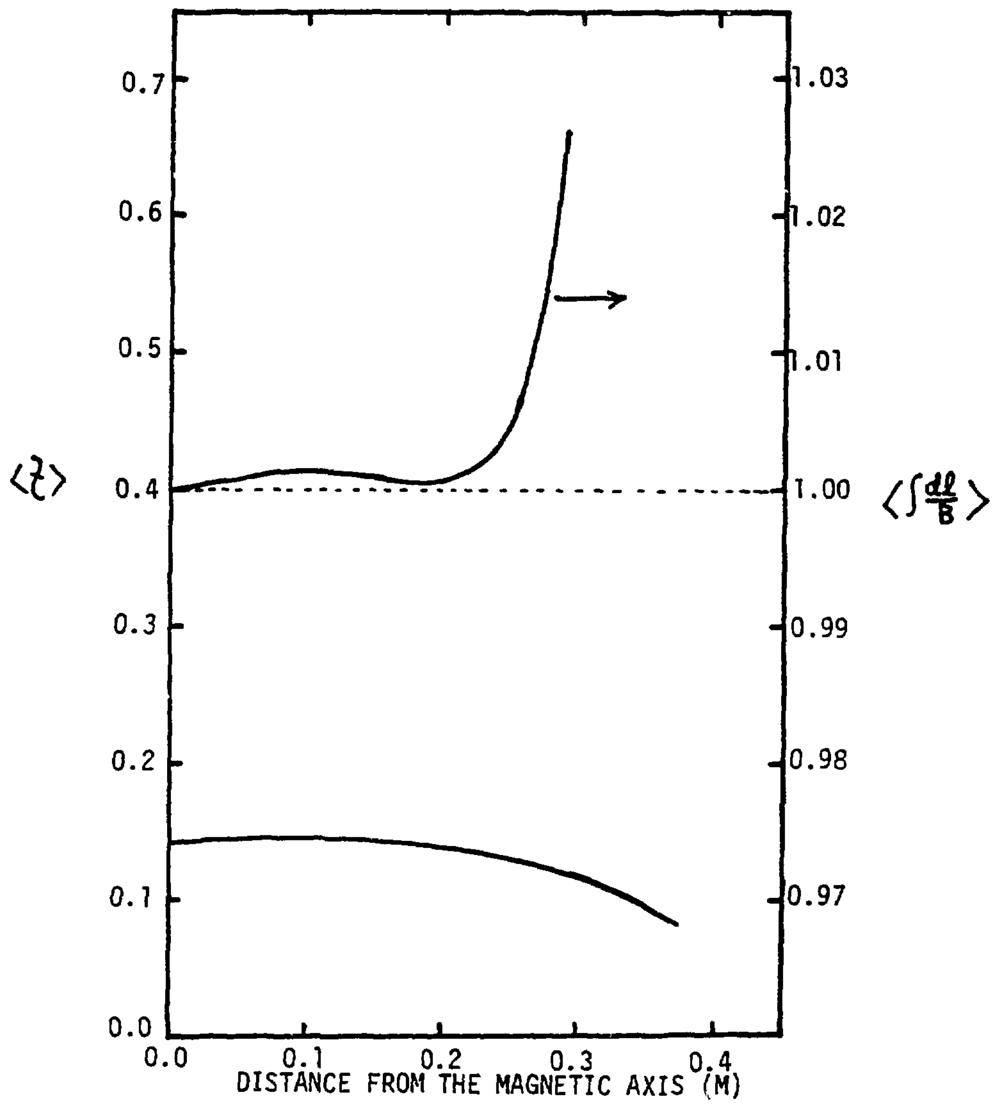
MAGNETIC SURFACE OF A MODULAR HELIAC WITH HELICAL SPATIAL AXIS AND PACMAN COILS; $R_O = 2$ M; $A_H = 0.5$ M; M = 2



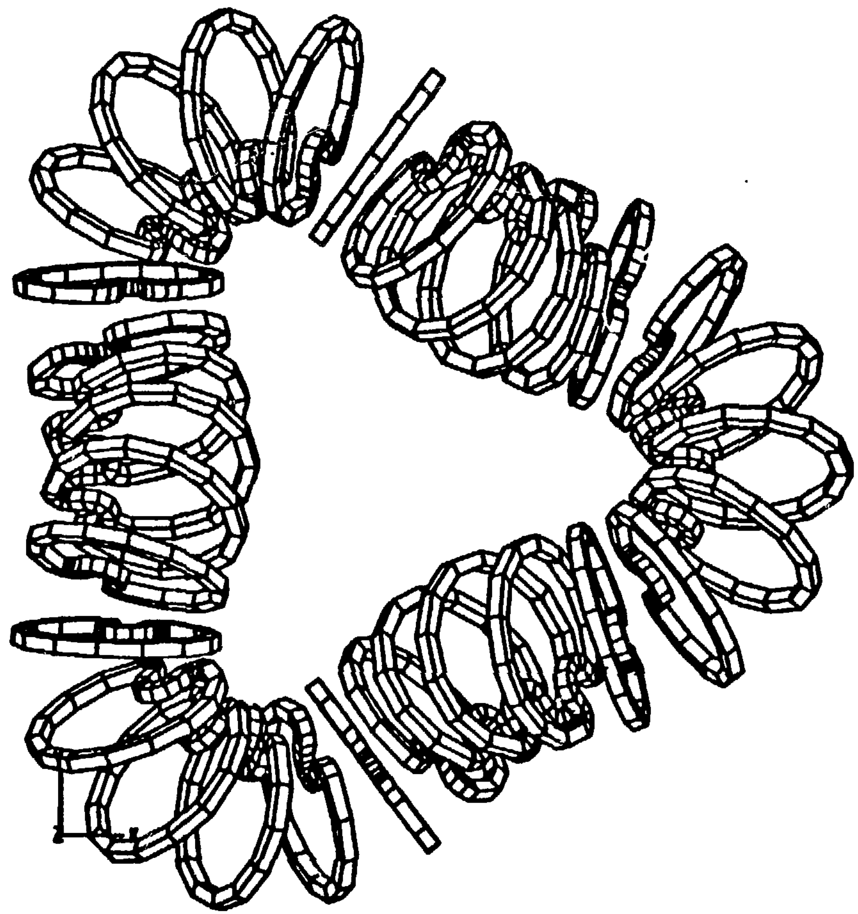
ROTATIONAL TRANSFORM AND $\int \frac{d\ell}{B}$ FOR PACKMAN TYPE COILS AND HELICAL SPATIAL AXIS; NO PF COIL; R_0 = 2.0 M; HELICAL RADIUS = 0.5; M = 2



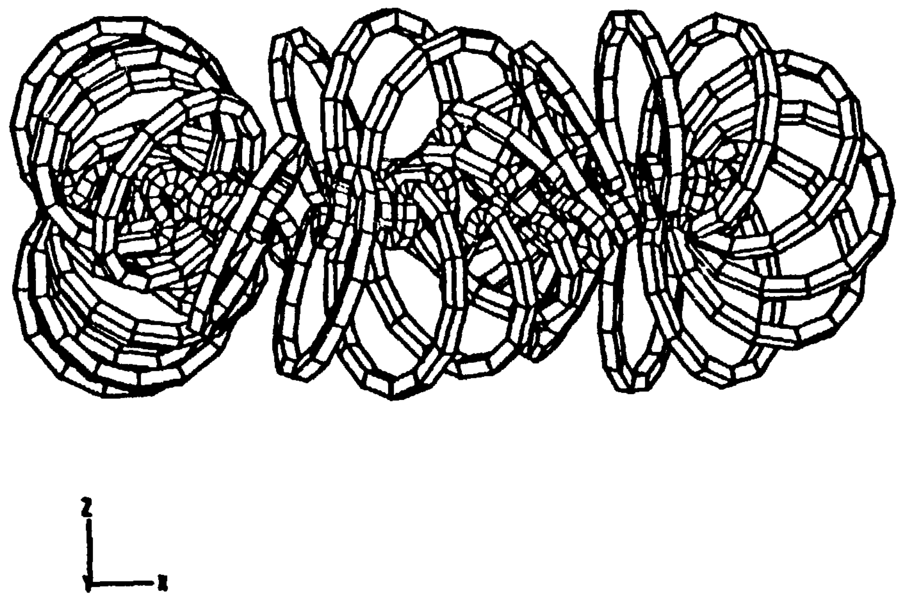
ROTATIONAL TRANSFORM AND $\int \frac{d\ell}{B}$ FOR PACMAN TYPE COILS AND HELICAL SPATIAL AXIS; NO PF COIL; R_0 = 2.5 M; HELICAL RADIUS = 0.5; M = 2



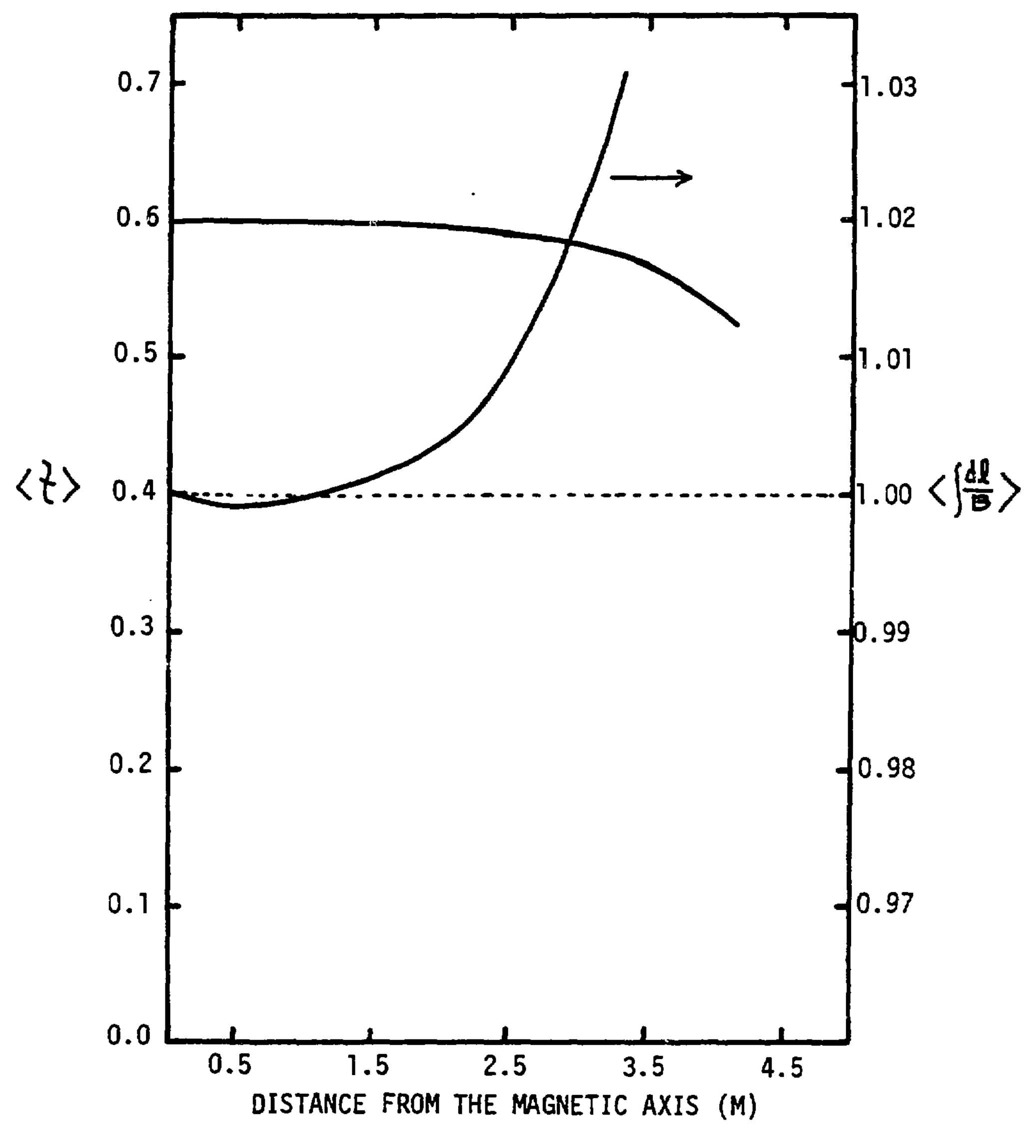
ROTATIONAL TRANSFORM AND $\int \frac{d\ell}{B}$ FOR PACMAN TYPE COILS AND HELICAL SPATIAL AXIS; NO PF COIL; R_0 = 2.0; HELICAL RADIUS = 0.4; M = 2



COIL GEOMETRY OF A MODULAR HELIAC WITH HELICAL SPATIAL AXIS (M=3)



COIL GEOMETRY OF A MODULAR HELIAC WITH HELICAL SPATIAL AXIS (M=3)



ROTATIONAL TRANSFORM AND $\int \frac{d\ell}{B}$ FOR PACMAN TYPE COILS AND HELICAL SPATIAL AXIS; NO PF COIL; R_0 = 2.0; HELICAL RADIUS = 0.5; M = 3

Conclusions

- The Base Case: With Circular Coils Centered on a Helical Axis and Interlocking PF Coil a Magnetic Well Depth of 10% and an Edge Transform of 0.73 (With Shear) Have Been Obtained.

- The Modular Heliac: With Pacman-Shaped Coils Centered on a Helical Axis and Non-Interlocking PF Coil a Magnetic Well Depth of 4% and an Edge Transform of 0.64 (With Shear) Have Been Obtained.

- L=1 Stellarator: With Circular Coils Centered on a Helical Axis and No PF Coil, No Magnetic Well and an Edge Transform of 0.16 (No Shear) Have Been Obtained.

- A New Modular Heliac: With Pacman Coils Centered on a Helical Axis and No PF Coil, Closed Magnetic Surfaces, a Well Depth of 1%, and a Rotational Transform of 0.1 (No Shear) Have Been Obtained.

- Results of Parameter Variations for the New Modular Heliac:

  1. If the Major Radius is Increased from 2 m to 2.5 m
     - the Well Depth Decreases from 1% to about 0.5%
     - $\bar{\iota}$ Drops From 0.2 to 0.14
  2. If the Helical Radius is Decreased From 0.5 m to 0.4 m
     - a Magnetic Hill Results
     - the plasma Volume Decreases
  3. If the Number of Toroidal Field Periods is Increased From M=2 to M=3
     - the Well is Negligibly Small
     - $\bar{\iota}$ Increases from 0.2 to 0.6
     - the Magnetic Ripple Increases

- Optimizations of the Volume Utilization, the Rotational Transform, Shear and Well Properties are Required.

FREE BOUNDARY FINITE BETA HELIAC EQUILIBRIA

O. Betancourt

Courant Inst. for Math. Sci.

New York University

New York, NY

FOURTH U. S. STELLARATOR WORKSHOP

OAK RIDGE, TENNESSEE

April 14-15, 1983

HAMILTONIAN $E_P - E_V = \text{MINIMUM}$

FREE BOUNDARY CONDITION

$$R_1 + iZ_1 = R_0 + iZ_0 + G(R_2 + iZ_2 - R_0 - iZ_0)$$

$$M\left[G, G_U, G_V\right] = \frac{1}{2}B_P^2 + P - \frac{1}{2}B_V^2 = 0$$

$$G_T = M + A(M_{G_U}M_U + M_{G_V}M_V)$$

WINDING LAW $B = \nabla\phi$

$$\phi = C_1V + C_2 \sin U + C_3 \tan^{-1}\frac{C_4 \sin U}{1 - C_4 \cos U}$$

$$R_2 + iZ_2 = E^{iV}\left[\Delta_1 + \Delta_0(1 + \Delta_2 \cos U)\, E^{i\Delta_3 \sin U}\right]$$

COIL

$$\phi - \phi^* = \text{CONST}$$

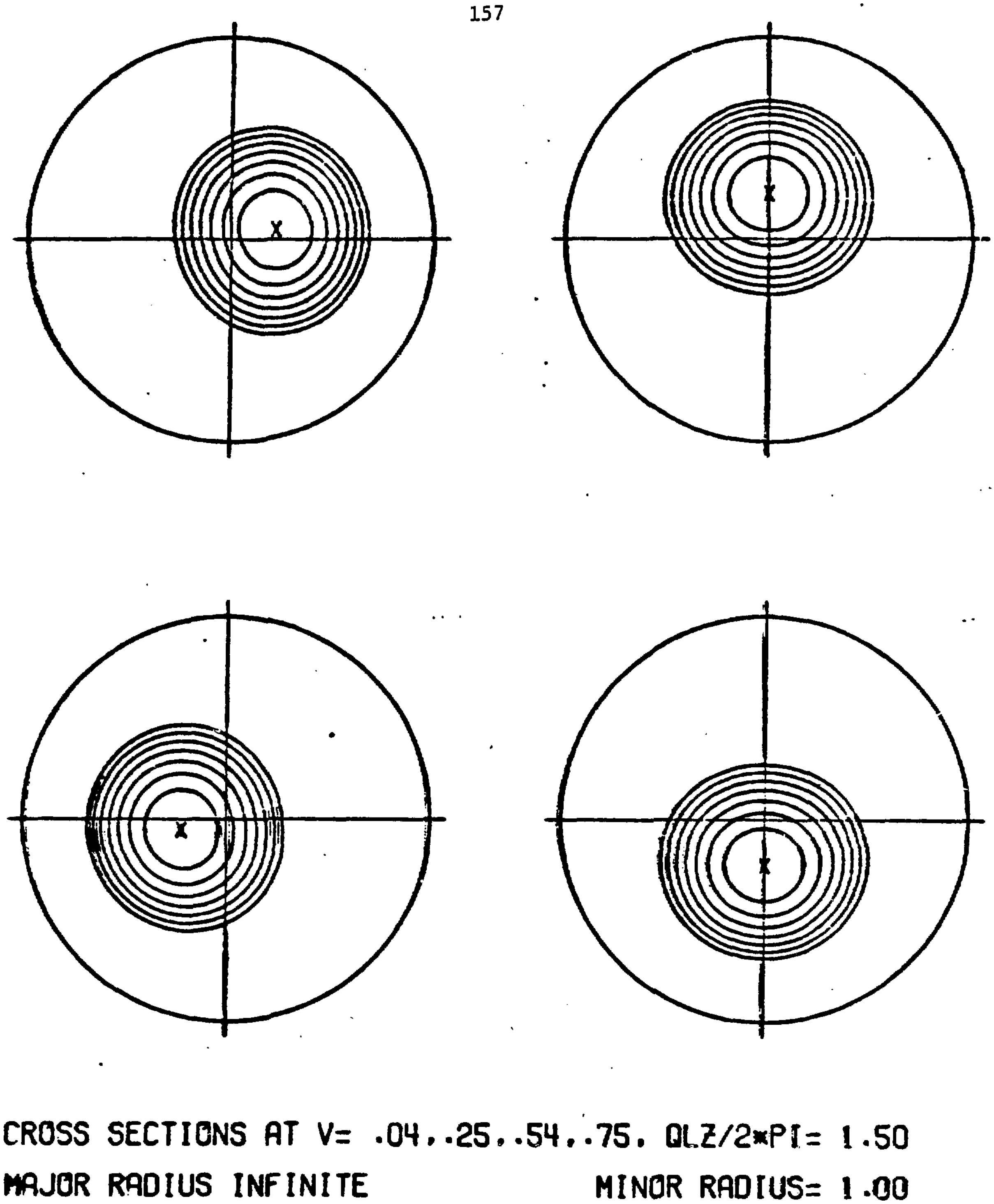
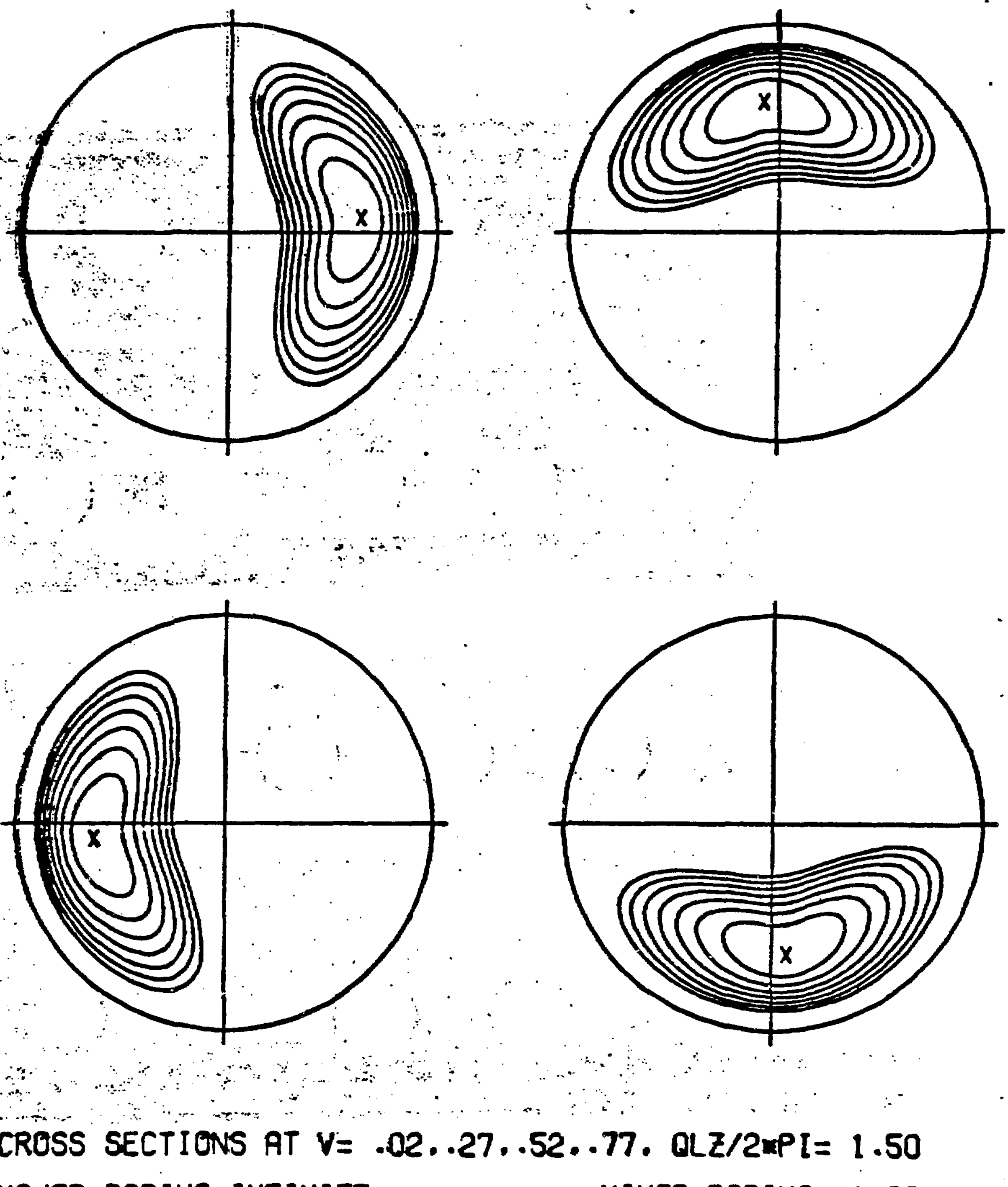


FIGURE 4. CROSS SECTIONS OF FLUX SURFACES.



CROSS SECTIONS AT V= .02,.27,.52,.77, QLZ/2*PI= 1.50

MAJOR RADIUS INFINITE MINOR RADIUS= 1.00

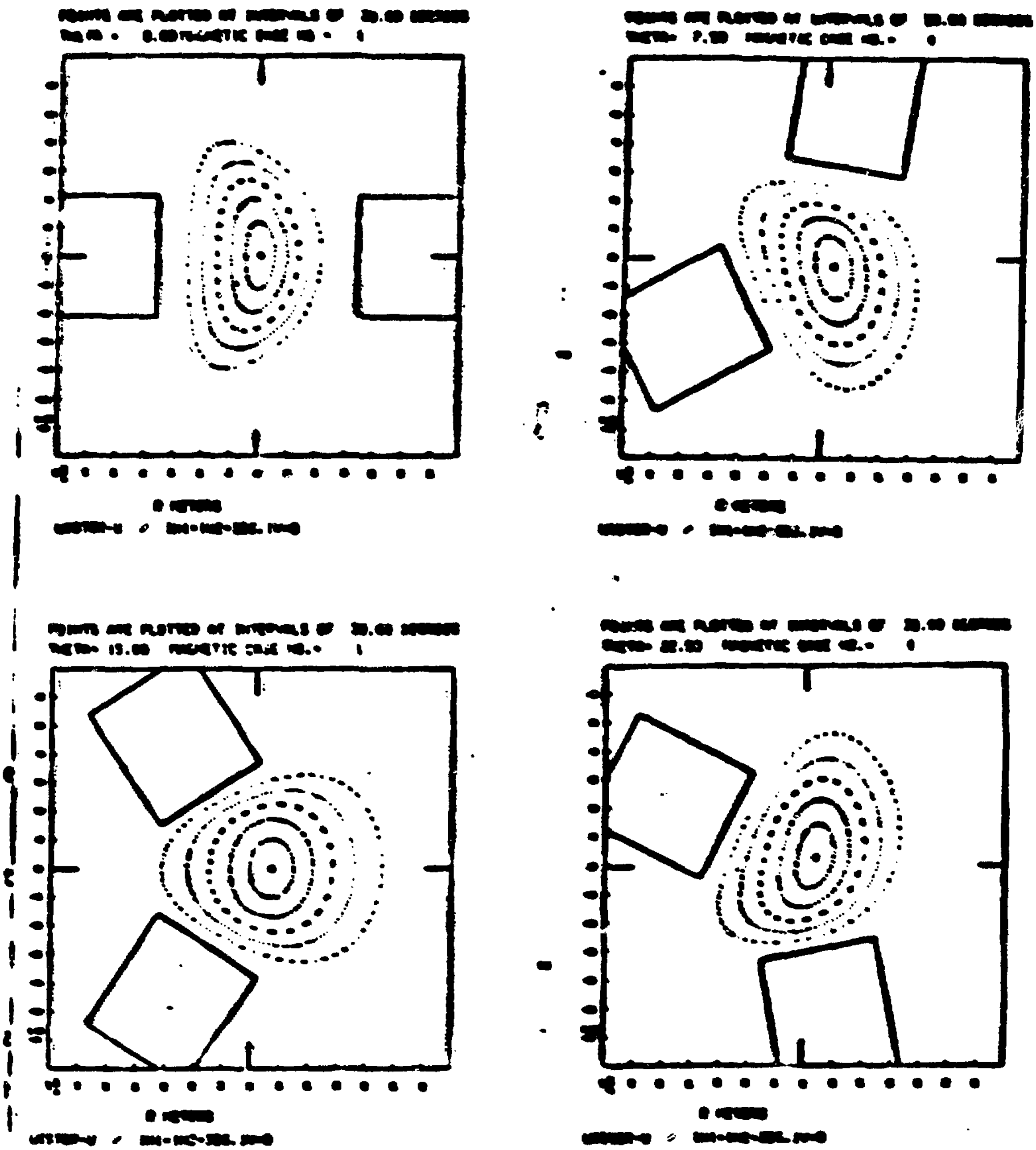
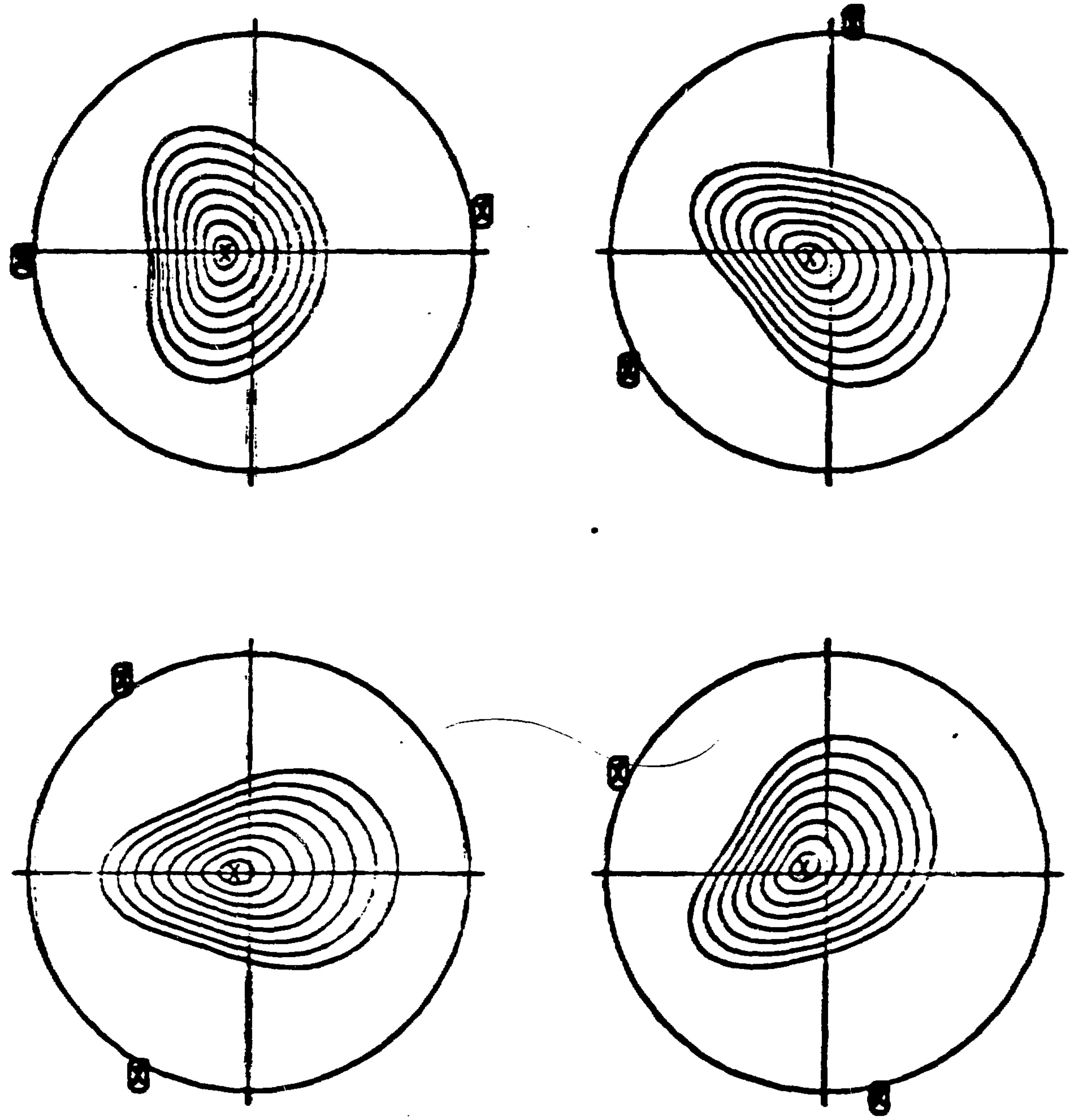
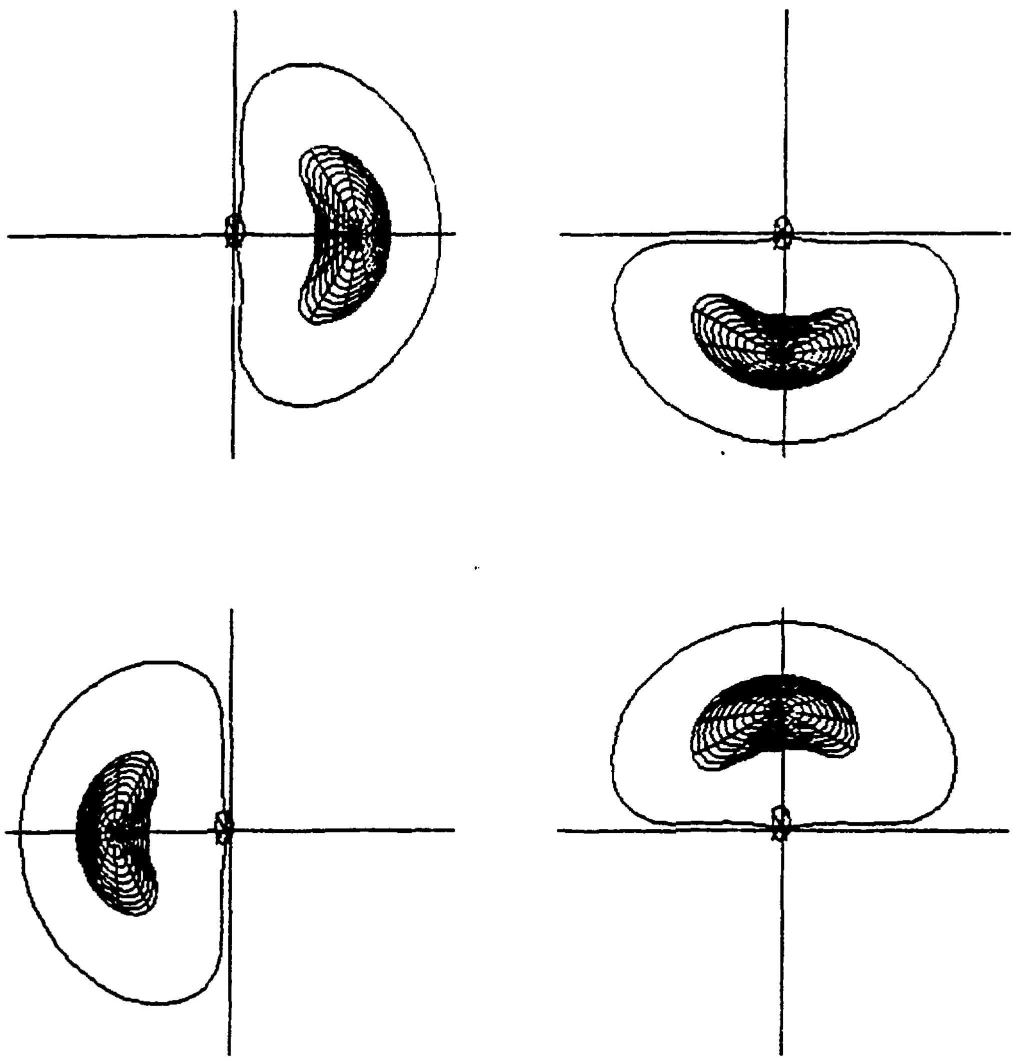


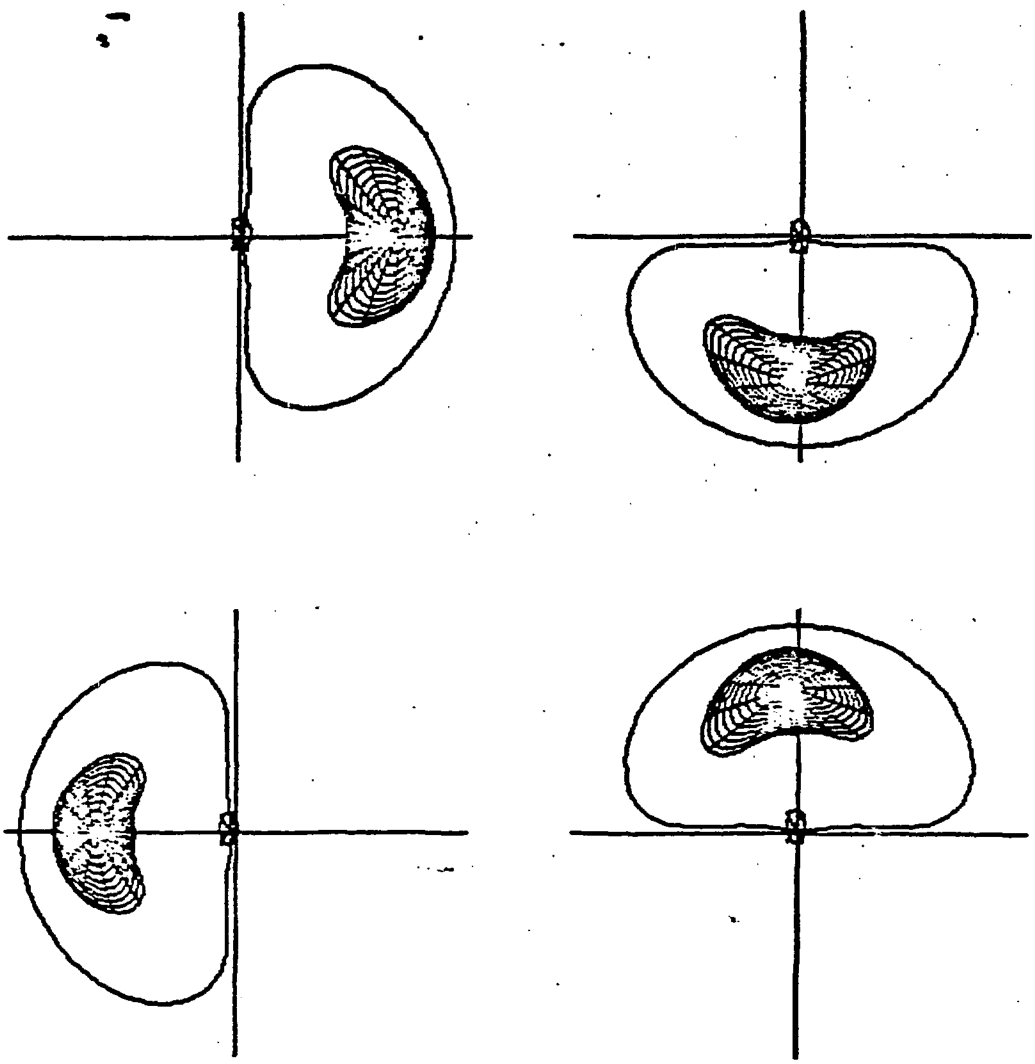
Fig. III.A.2. Computed magnetic surfaces for equal helical coil currents and no trim coil current.



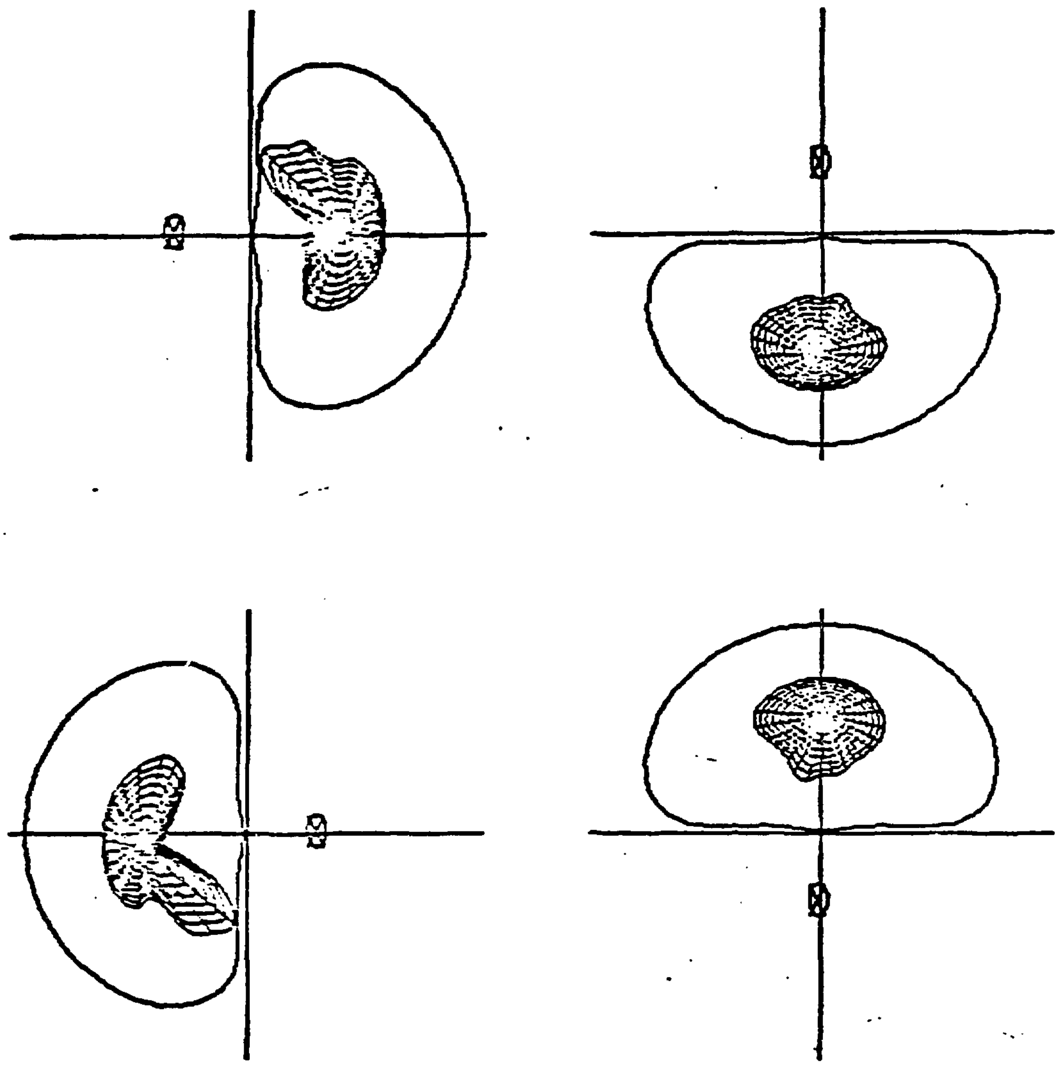
CROSS SECTIONS AT V= .02..27..52..77. 1/(EP*QLZ)= .42
MAJOR RADIUS= 5.00 MINOR RADIUS= 1.00



CROSS SECTIONS AT V= .00,.25,.50,.75, 1/(EP+QLZ)= 1.32
MAJOR RADIUS= 7.94 MINOR RADIUS= 1.00



CROSS SECTIONS AT V= .00,.25,.50,.75, 1/(EP*QLZ)= 1.32
MAJOR RADIUS= 7.94 MINOR RADIUS= 1.00



CROSS SECTIONS AT V= .00,.25,.50,.75, QLZ/2•PI= 1.32
MAJOR RADIUS INFINITE MINOR RADIUS= 1.00

TRANSPORT ANALYSIS OF THE PROPOSED HELIAC DEVICE HXI

G. Kuo-Petravic and A. Boozer
Plasma Physics Laboratory
Princeton University
Princeton, NJ

FOURTH U. S. STELLARATOR WORKSHOP

OAK RIDGE, TENNESSEE

April 14-15, 1983

TRANSPORT ANALYSIS OF THE PROPOSED HELIAC DEVICE HXi

Kuo-Petravic and Boozer

Number of Periods	3
Field	10 kG
Major Radius	1.5 m
Short minor radius	0.23 m
long minor radius	0.72 m
Transform (little shear)	~1.6
Design T	1 kev
Design n	10^{14}/cm³
Design β	8%

#83T0098

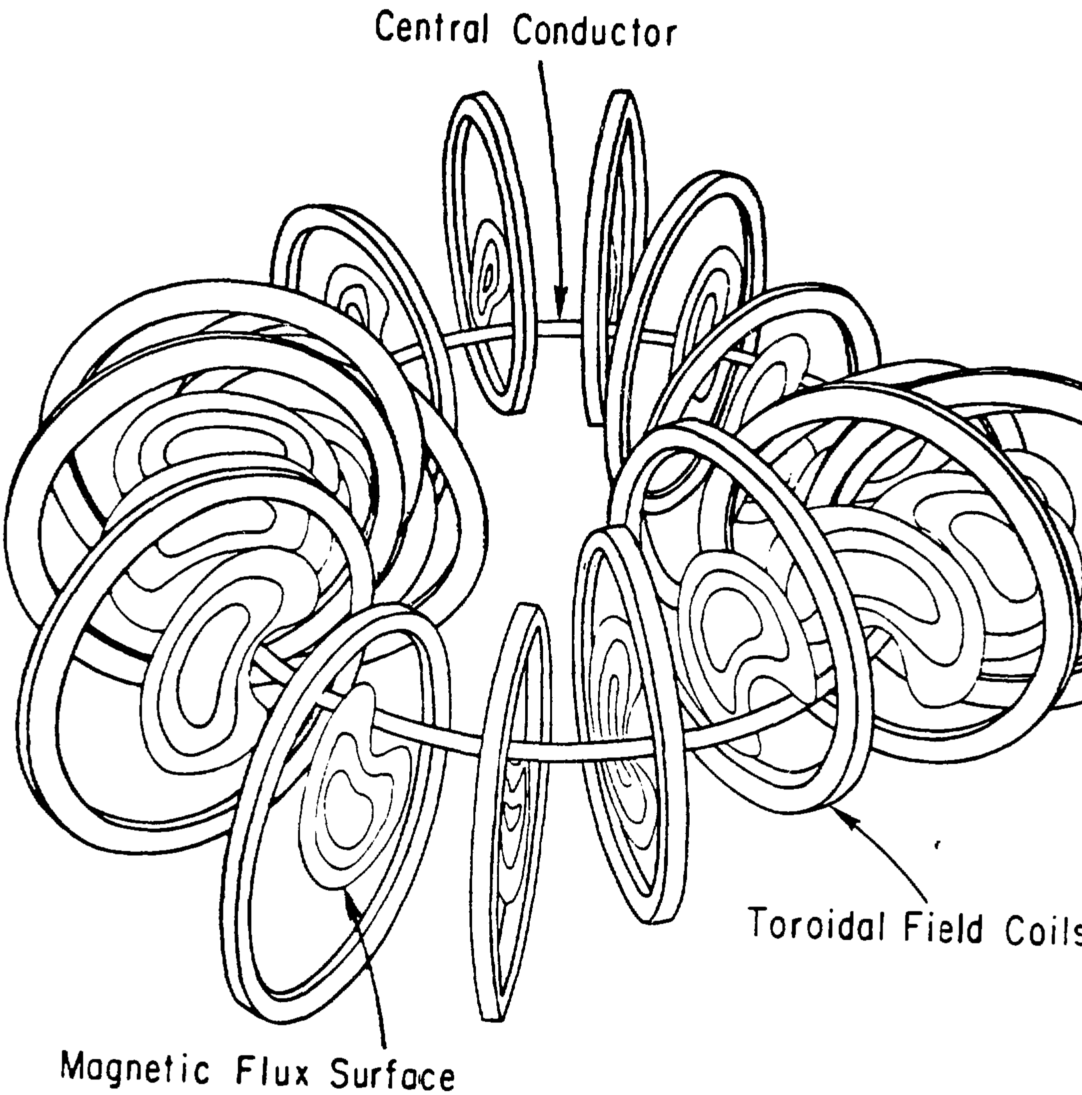

Central Conductor
Toroidal Field Coils
Magnetic Flux Surface

Fig 2

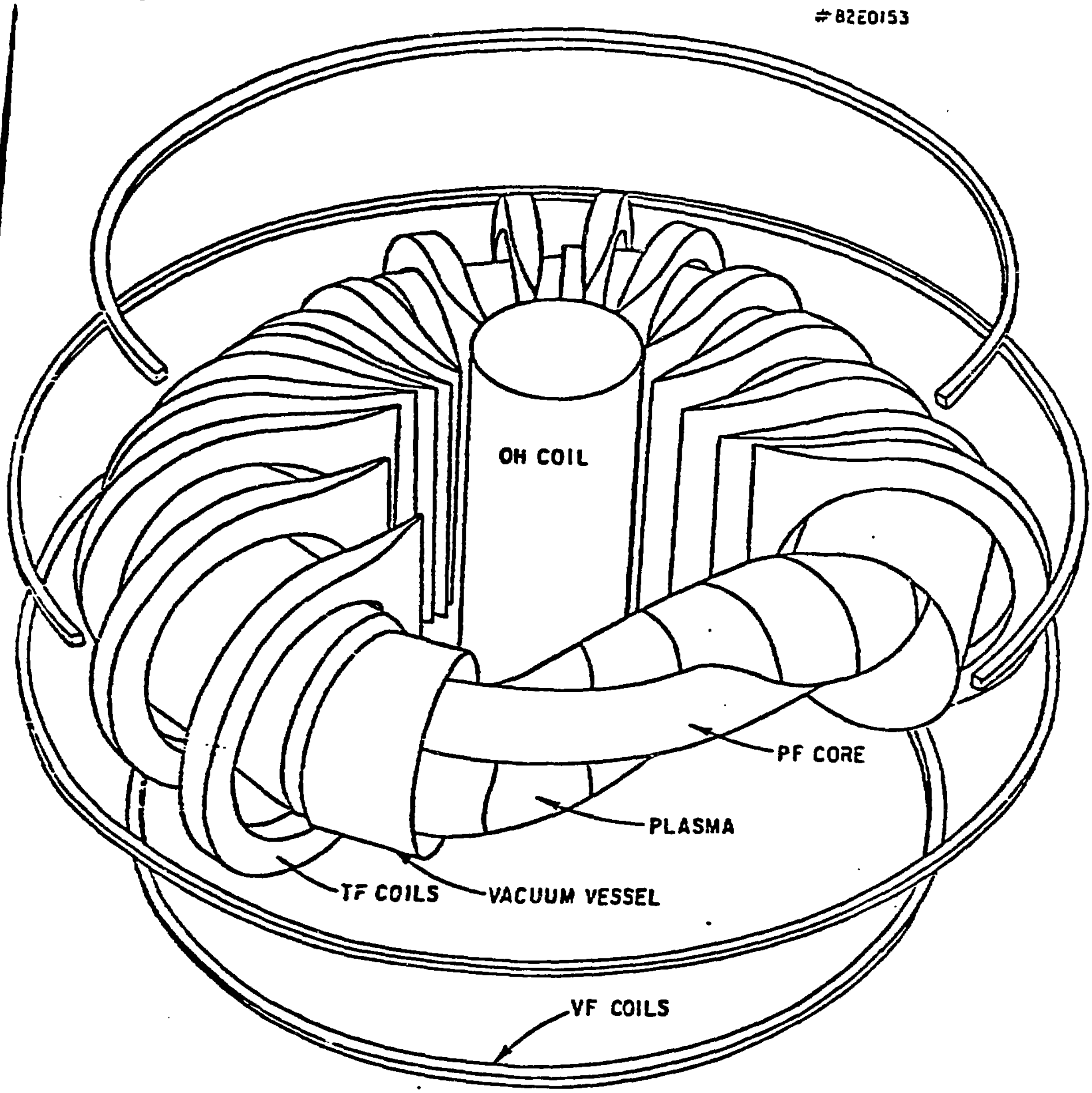


Figure 4.1-1

General Configuration of the HX-1

M=3 HELIAC

MONTE CARLO METHOD

1) Used Hamiltonian expression for drift motion in magnetic coordinates

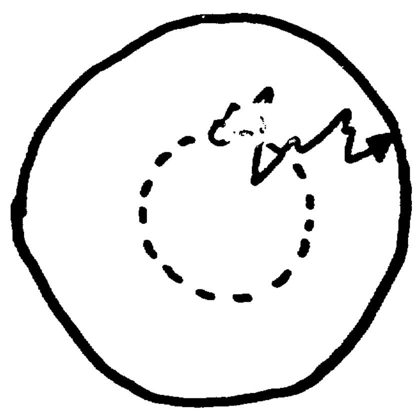
2) Field strength defined in Fourier decomposed form using vacuum field from wire filaments

$$B(\psi, \theta, \varphi) = B_0 \left[1 + \sum_{n,m}' a_{nm} \cos(n\varphi - m\theta)\right]$$

3) Have both pitch angle and energy scattering.

4) Imagine plasma volume filled with uniform density and temperature plasma.

5) Record time it takes $\frac{1}{e}$ of 64 particles launched deep in the plasma to leave.



6) Comparison with diffusion equation allows D to be calculated from $t_{1/2}$

$$\frac{\partial n_t}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left[r D \left(\frac{\partial n_t}{\partial r} + \frac{e\, n_t}{T}\frac{d\Phi}{dr} \right) \right]$$

n_t is the test particle density.

7) If one assumes

$$D(n,T) = \int_0^{E_m} \mathcal{D}(E)\, f_M\, d^3v$$

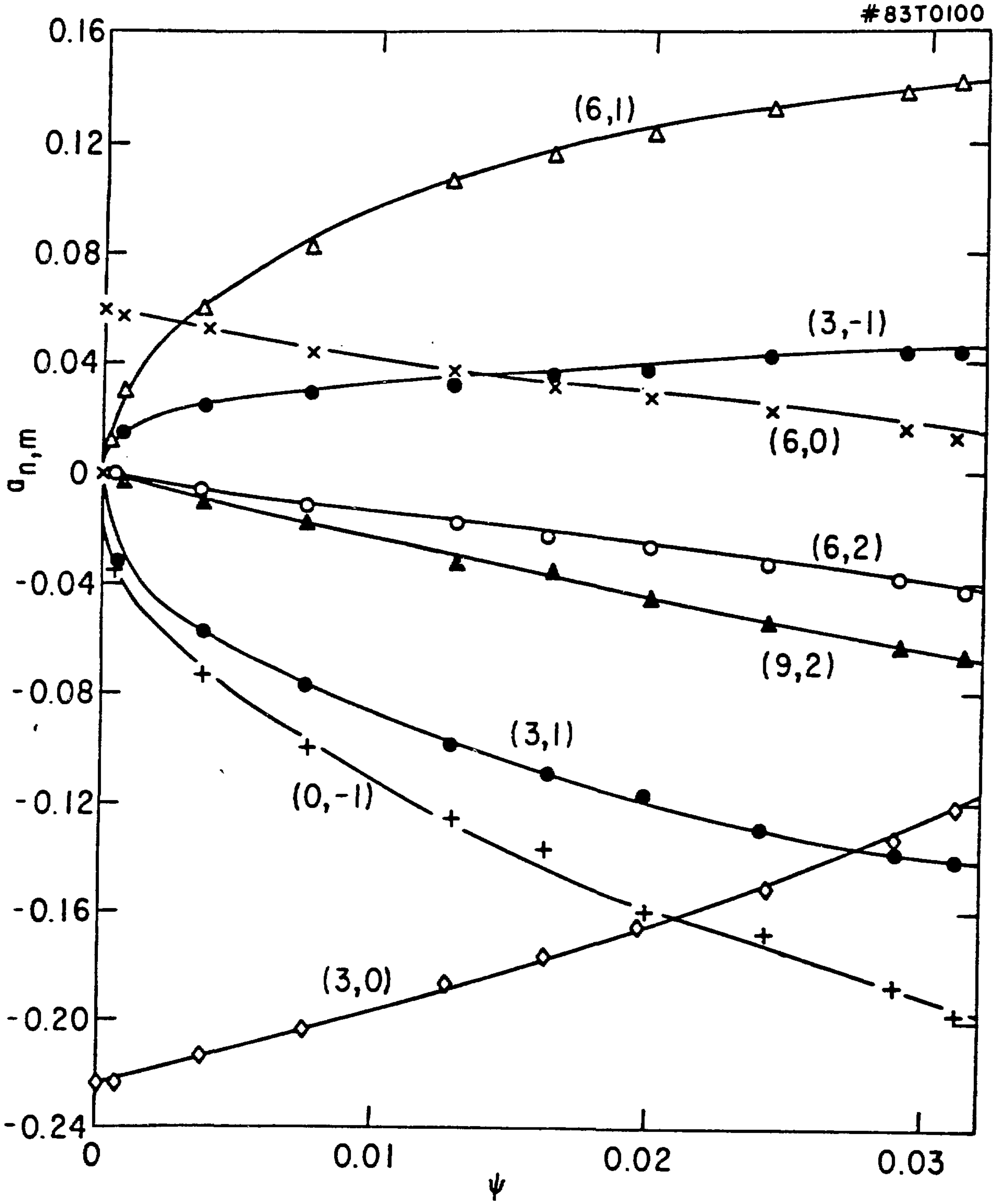
then one can obtain $\chi(n,T)$ from D.

$$q_t \simeq \left(\tfrac{3}{2} k_2 T + e\Phi\right)\Gamma + q$$

$$q = -\tfrac{3}{2}\,(k_2 D)\, n \frac{dT}{d\psi}$$

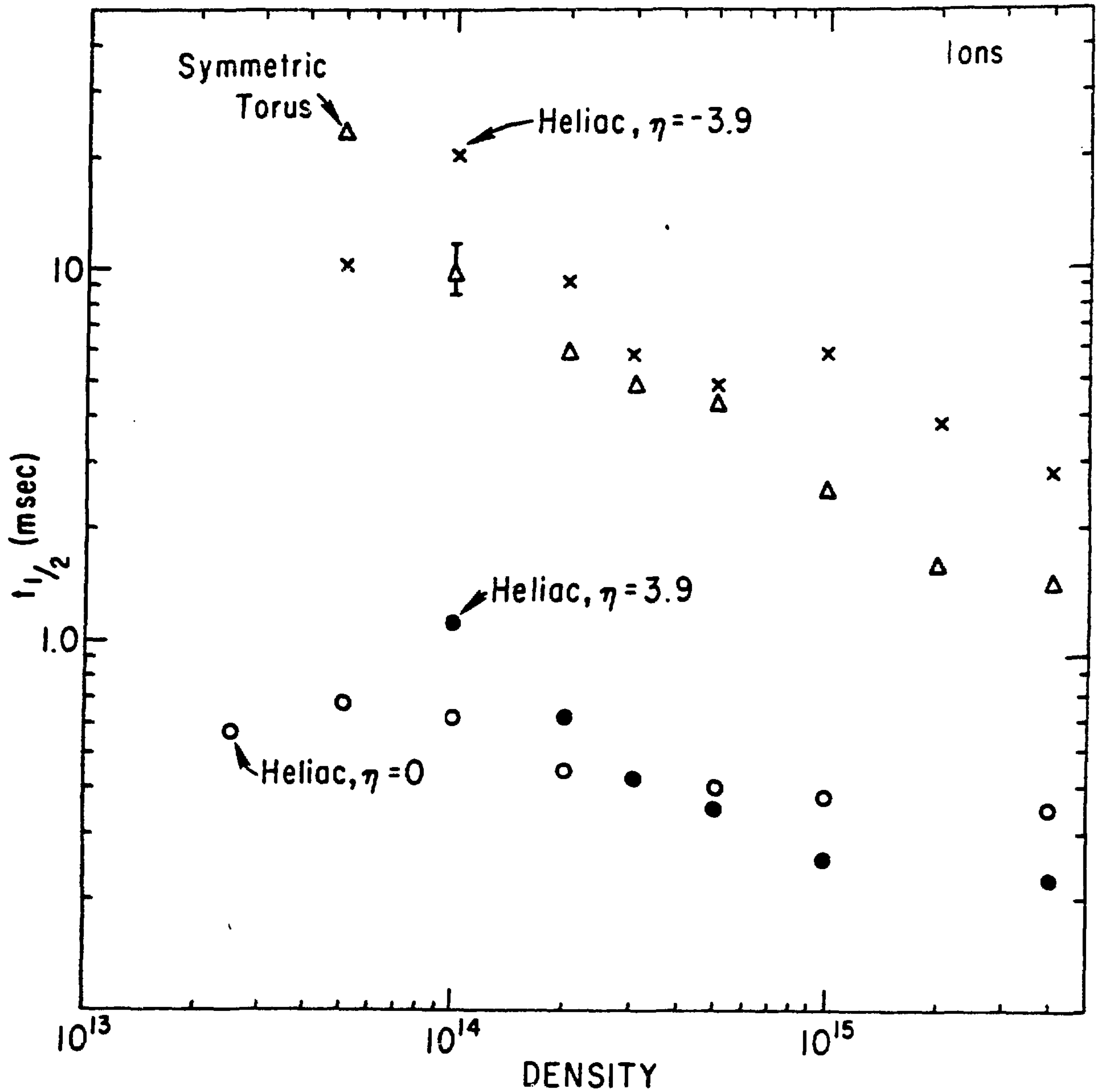
$$\chi(n,T) = k_2 D$$

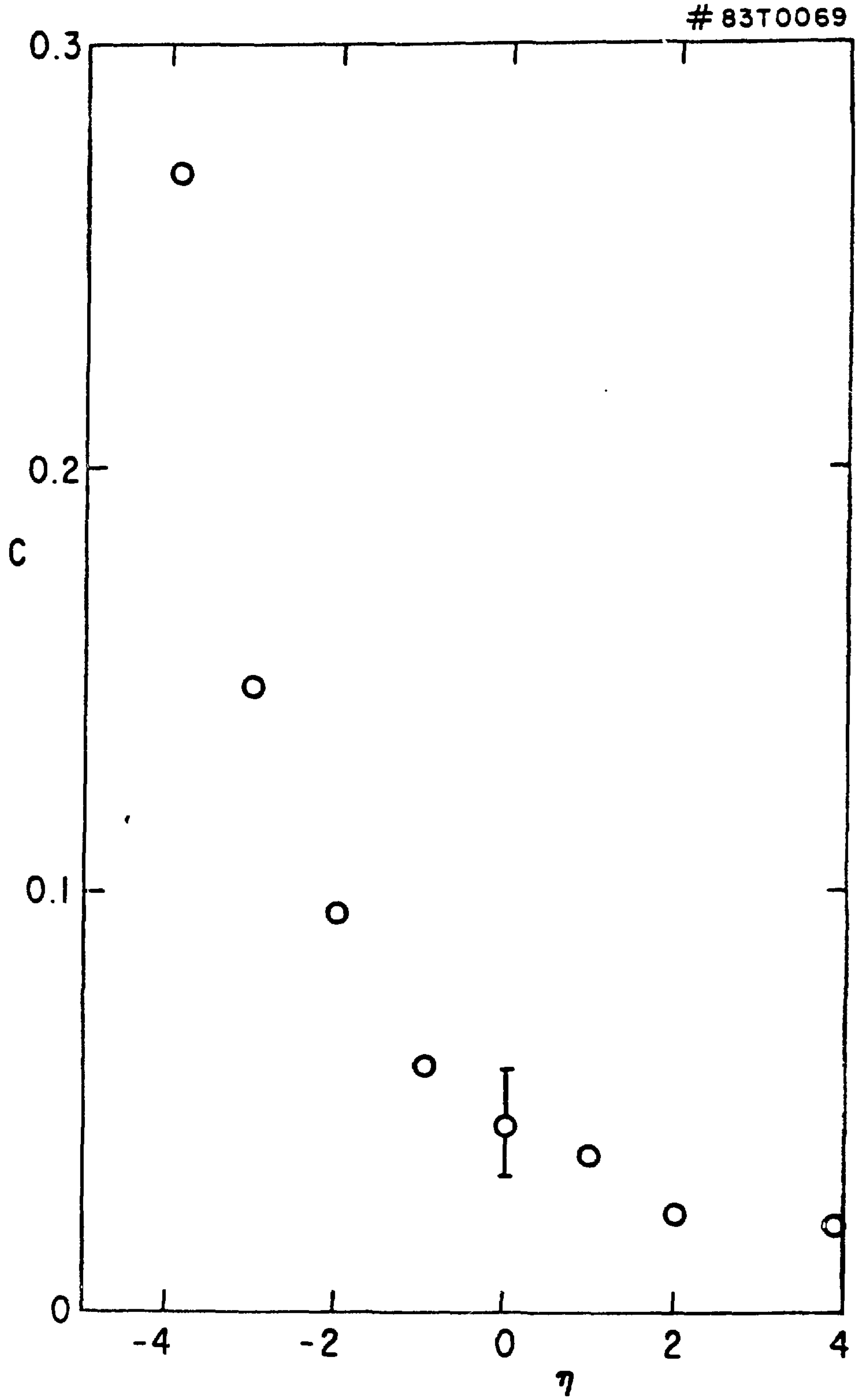
$$k_2 \approx 1.5$$



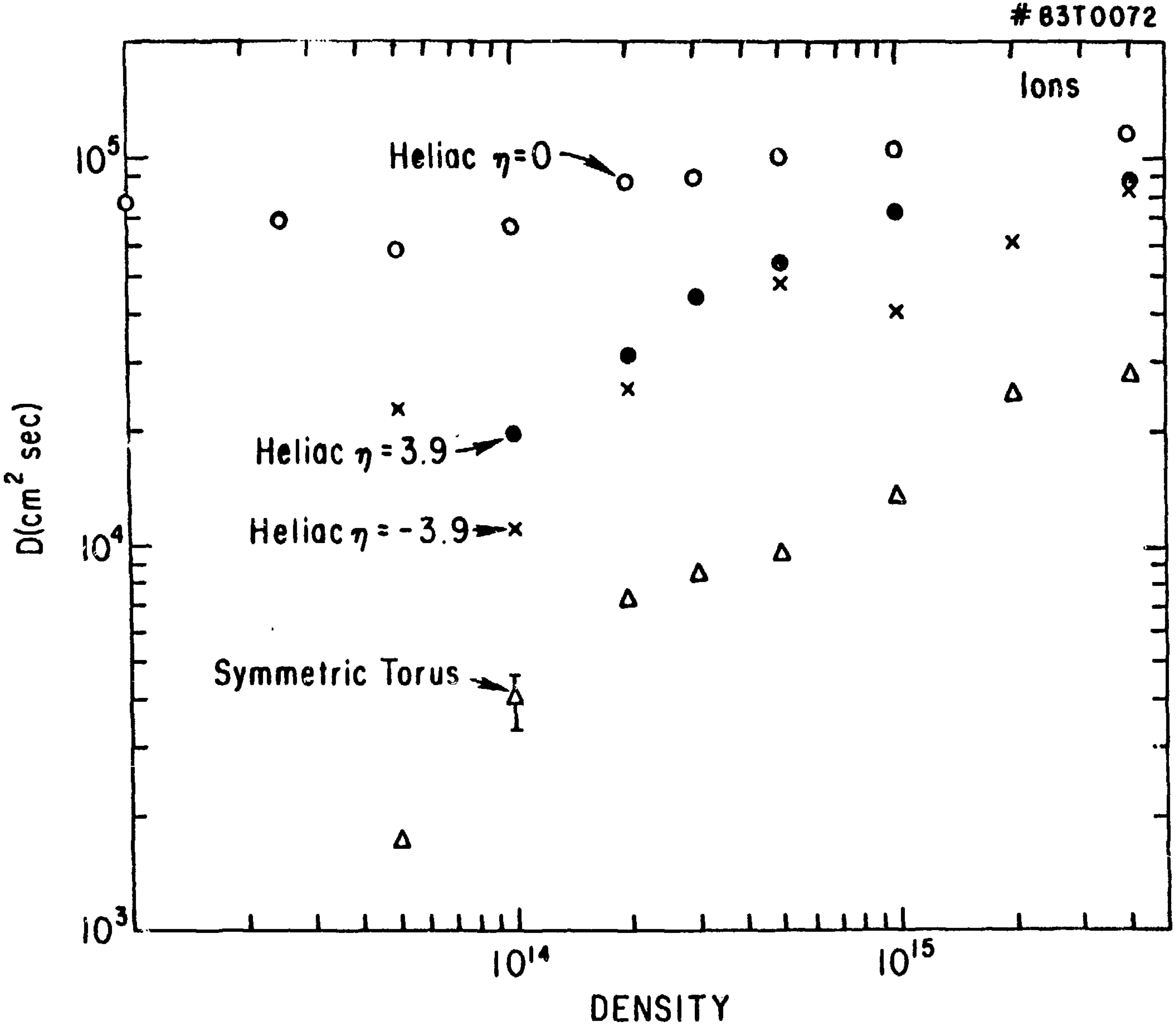
#83T0100
0.16
0.12
0.08
0.04
0
-0.04
-0.08
-0.12
-0.16
-0.20
-0.24
$a_{n,m}$
0
0.01
0.02
0.03
ψ
(6,1)
(3,-1)
(6,0)
(6,2)
(9,2)
(3,1)
(0,-1)
(3,0)

#83T0070

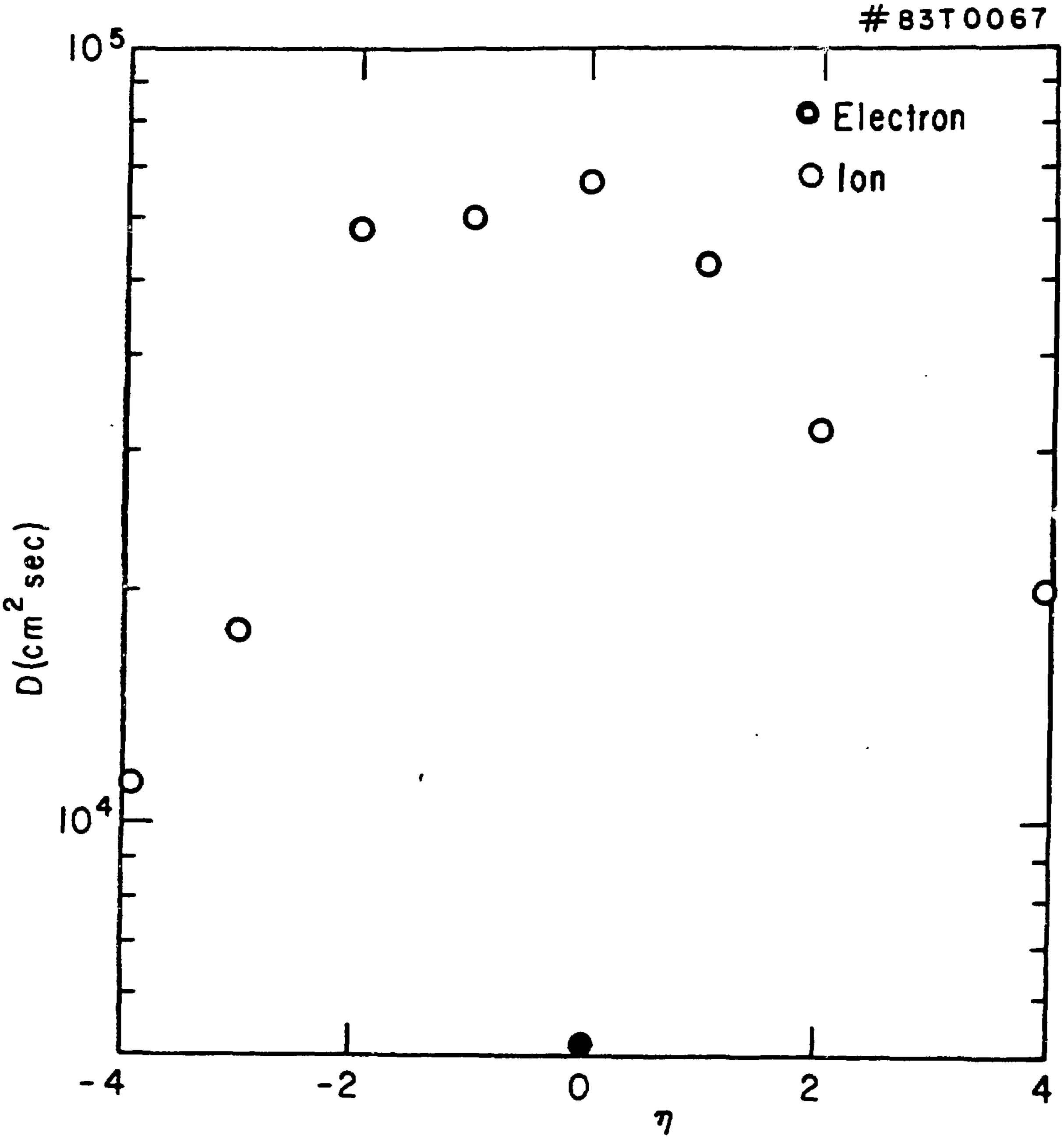

Ions
Symmetric Torus
Heliac, η = -3.9
Heliac, η = 3.9
Heliac, η = 0
$t_{1/2}$ (msec)
10
1.0
10^{13}
10^{14}
10^{15}
DENSITY



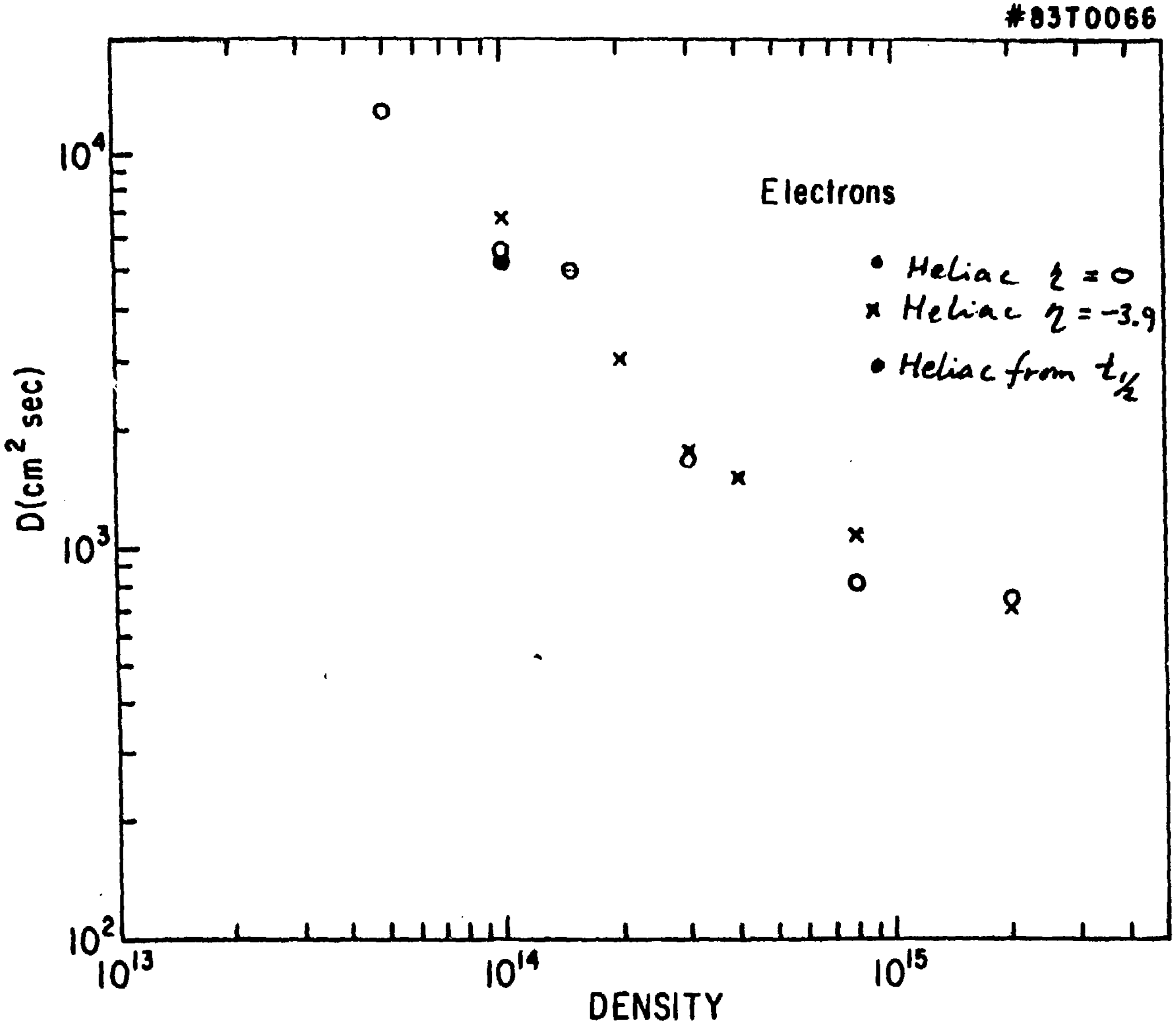
83T0069
0.3
0.2
0.1
0
C
-4
-2
0
2
4
η



#83T0072
Ions
Heliac η=0
Heliac η = 3.9
Heliac η = -3.9
Symmetric Torus
10^5
10^4
10^3
D(cm^2 sec)
10^{14}
10^{15}
DENSITY



#83T0067
Electron
Ion
10^5
10^4
D(cm² sec)
-4
-2
0
2
4
η


#83T0066
Electrons
• Heliac η = 0
x Heliac η = -3.9
• Heliac from t1/2
D(cm2 sec)
10^4
10^3
10^2
10^13
10^14
10^15
DENSITY

MODULAR COIL DESIGN & MAGNETIC SURFACES IN HELIAC

A. I. SHESTAKOV, A. A. MIRIN, & N. J. O'NEILL
NMFECC
LLNL

176

THE INTERIOR OF THE HELIAC IS THE TOROIDAL DOMAIN SWEPT OUT BY THE CROSS-SECTION X_s AS IT IS WOUND ABOUT THE CIRCLE WITH RADIUS a BY THE WINDING LAW $\theta(\phi)$. (SEE FIG 1.).

FIGURE 2 SHOWS A HELIAC WITH A SMALL ASPECT RATIO & 2 HELICAL PERIODS PER TOROIDAL TRANSIT

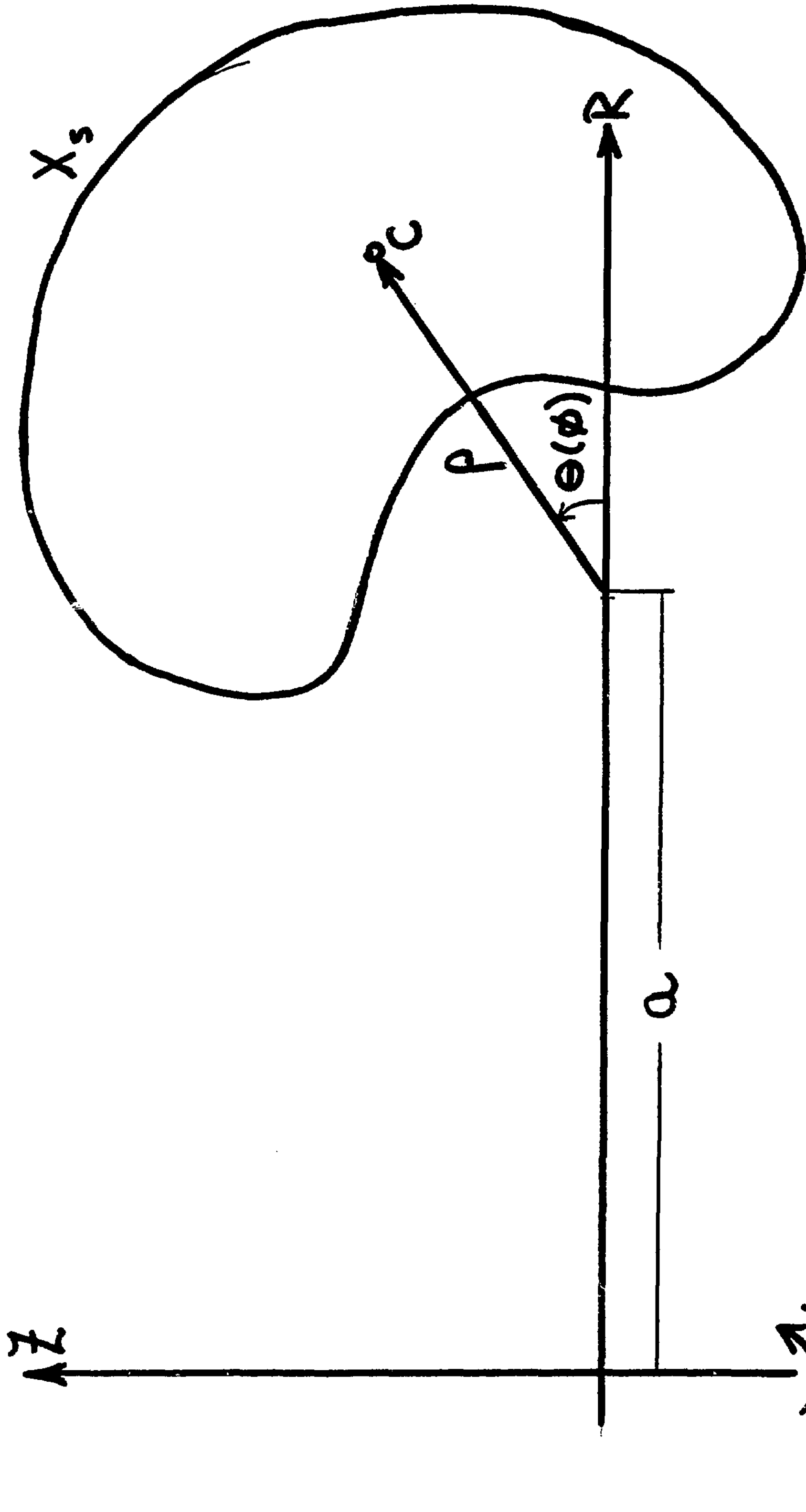


Fig. 1

a = MAJOR RADIUS
ρ = WINDING RADIUS
$\theta(\phi)$ = WINDING LAW

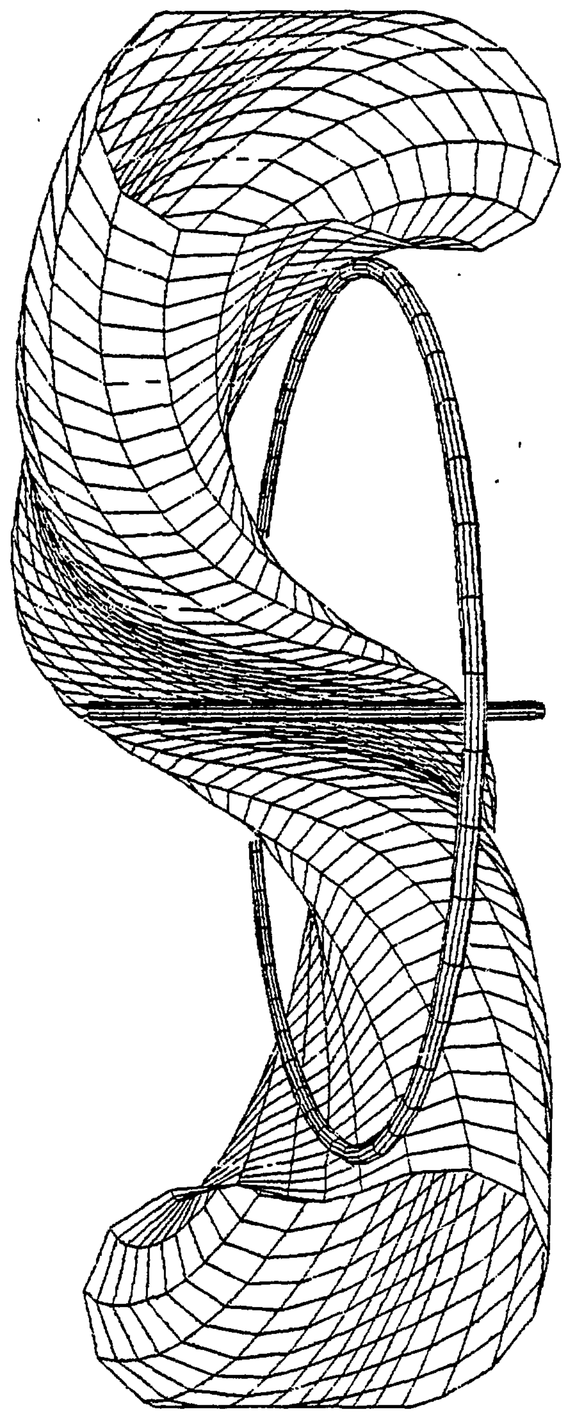


FIGURE 2.

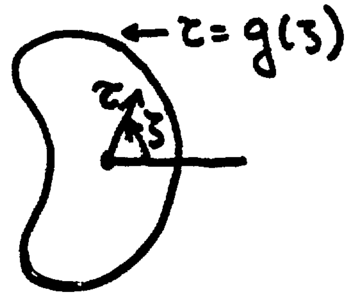
THE NUMERICAL METHOD DESCRIBED IN REF. [1] IS USED TO SOLVE $\nabla^2 \psi = 0$ INSIDE THE HELIAC. THE POTENTIAL ψ IS MULTIPLE VALUED IN ϕ, THE TOROIDAL ANGLE, $\psi(\bar{x}(\phi+2\pi)) - \psi(\bar{x}(\phi)) = 1$. FURTHERMORE, $\partial\psi/\partial n = 0$. THE VACUUM FIELD, $\bar{B} = \nabla\psi$, IS GENERATED BY SURFACE CURRENTS. THESE CAN BE SHOWN TO COINCIDE WITH CONTOUR LINES OF ψ ON THE BDRY. FIGURE 3Q3 SHOWS THESE SURFACE CURRENTS DISCRETIZED INTO RINGS.

1. SHESTAKOV & MIRIN, PROC. $10^{\underline{TH}}$ NUM. SIM. PLAS., SAN DIEGO, CA. '83

THE NUMERICALLY EVALUATED Ψ IS GIVEN TO A FIELD LINE INTEGRATOR, REF. [2]. THIS PROGRAM PLOTS THE MAGNETIC SURFACES, COMPUTES $\int dl/B$ & THE TRANSFORM ON EACH SURFACE.

2. O'NEILL & MIRIN, TUBE, LIBRIS ABS. P81, MFECC.

IN THE SEARCH FOR CONFIGURATIONS WITH FAVOURABLE MAGNETIC WELLS (I.E. $\int d\ell / B$ MONOTONICALLY DECREASING OUTWARD FROM THE MAGNETIC AXIS) WE VARY:



1. THE CROSS SECTION $g(\zeta)$:
2. THE MAJOR RADIUS, a.
3. THE WINDING RADIUS, ρ.
4. THE WINDING LAW, $\theta(\phi)$.

TYPICALLY, $g(\zeta) = 1 + a_1 \cos\zeta + a_2 \cos 2\zeta + \cdots$

$$\theta(\phi) = k\phi + \varepsilon \sin k\phi$$

WHERE k = INTEGER IS THE NO. OF HELICAL PERIODS IN THE TORUS, & ε MODULATES HOW MUCH MORE TIME IS SPENT OUTSIDE $(\varepsilon < 0)$, OR INSIDE $(\varepsilon > 0)$, THE MAJOR RADIUS.

RESULTS

IT IS POSSIBLE TO CONSTRUCT HELIACS WITH ~1% WELLS USING ONLY POLOIDAL CURRENT LOOPS. THE CROSS SECTION INFLUENCES THE WELL DEPTH. OTHER WAYS TO CREATE OR DEEPEN A WELL ARE TO:

1. INCREASE a, MAJOR RADIUS.
2. INCREASE ρ, WINDING RADIUS.
3. VARY $\theta(\phi)$, THE WINDING LAW.

   IF $\theta(\phi) = k\phi + \varepsilon \sin k\phi$ IT IS BETTER TO:

   a. DECREASE k

   b. SET $\varepsilon < 0$, (THE LARGER $|\varepsilon|$ THE BETTER).

IN 3a, THE EFFECT IS TO HAVE FEWER HELICAL PERIODS. IN 3b, THE HELIAC SPENDS MORE TIME OUTSIDE THE MAJOR RADIUS

WE PRESENT THE FOLLOWING CASES & THEIR WELL DEPTH:

V19: X_s: $g(\zeta) = 1 - .073\cos\zeta - .3165\cos 2\zeta + .1193\cos 3\zeta$

P: $a = 9$, $\rho = 1.2$, $\theta(\phi) = 3\phi - .25\sin 3\phi$

~.57% WELL

Q: $a = 9$, $\rho = 1.3$, $\theta(\phi) = 3\phi - .25\sin 3\phi$

~ .72% WELL

S: $a = 9$, $\rho = 1.2$, $\theta(\phi) = 2\phi - .25\sin 2\phi$

~ 1.26% WELL

V23: X_s: $g(\zeta) = 1 - .1225\cos\zeta - .3625\cos 2\zeta + .1905\cos 3\zeta$

V: $a = 9$, $\rho = 1.3$, $\theta(\phi) = 3\phi - .25\sin 3\phi$

~.86% WELL

W: $a = 9$, $\rho = 1.2$, $\theta(\phi) = 2\phi - .25\sin 2\phi$

~ 1.14% WELL

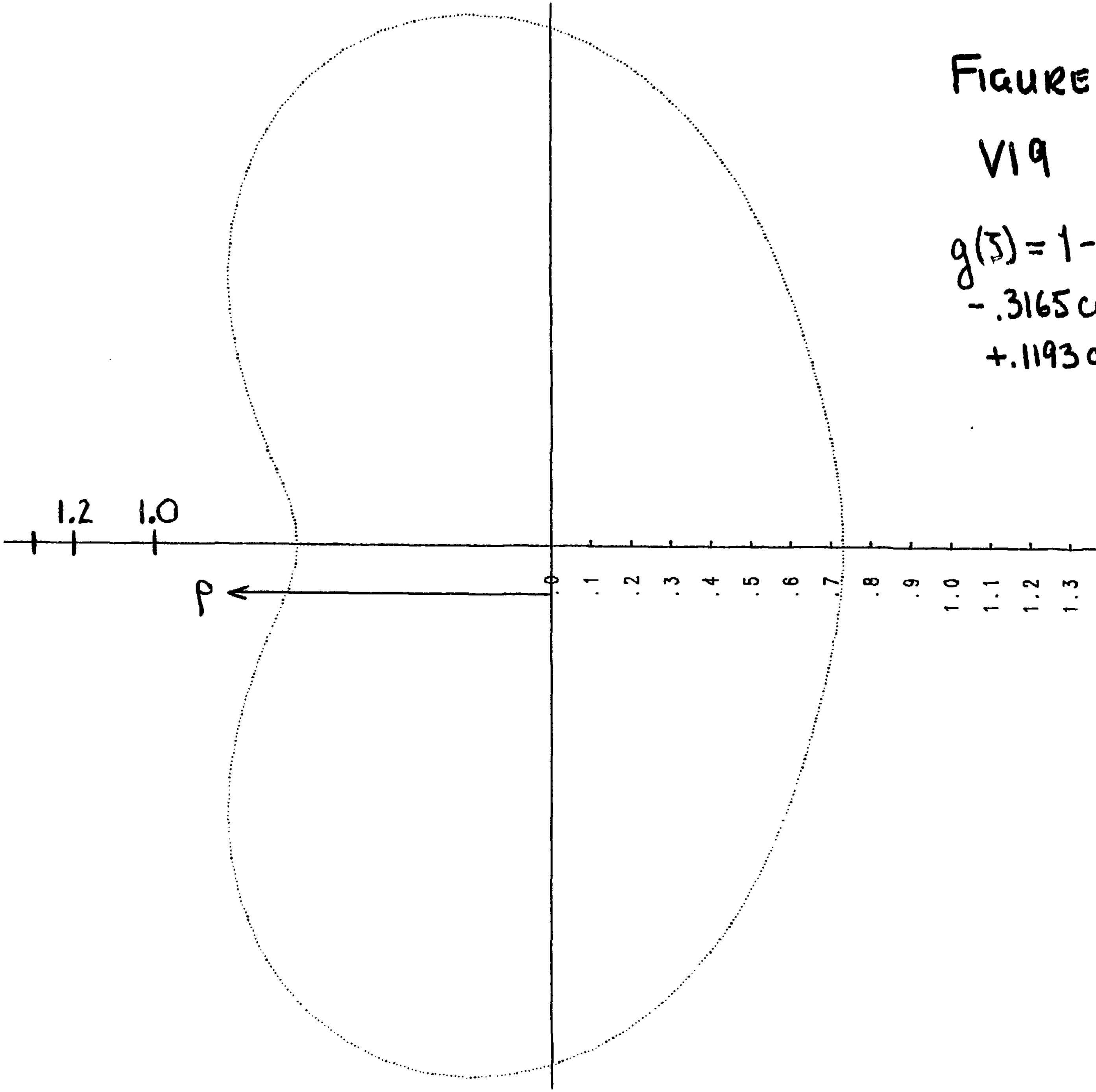


FIGURE 3.

V19 X-SECTION

$g(\zeta) = 1 - .073\cos\zeta - .3165\cos 2\zeta + .1193\cos 3\zeta$

283

FIGURES 3Q3,4,5

TOP, SIDE & CUTAWAY VIEWS OF V19Q CONFIGURATION.

TOROIDAL SURFACE IS THE HELIAC BOUNDARY.

THE MESH ON THE SURFACE SHOWS CONSTANT POLOIDAL AND TORDIDAL ANGLES.

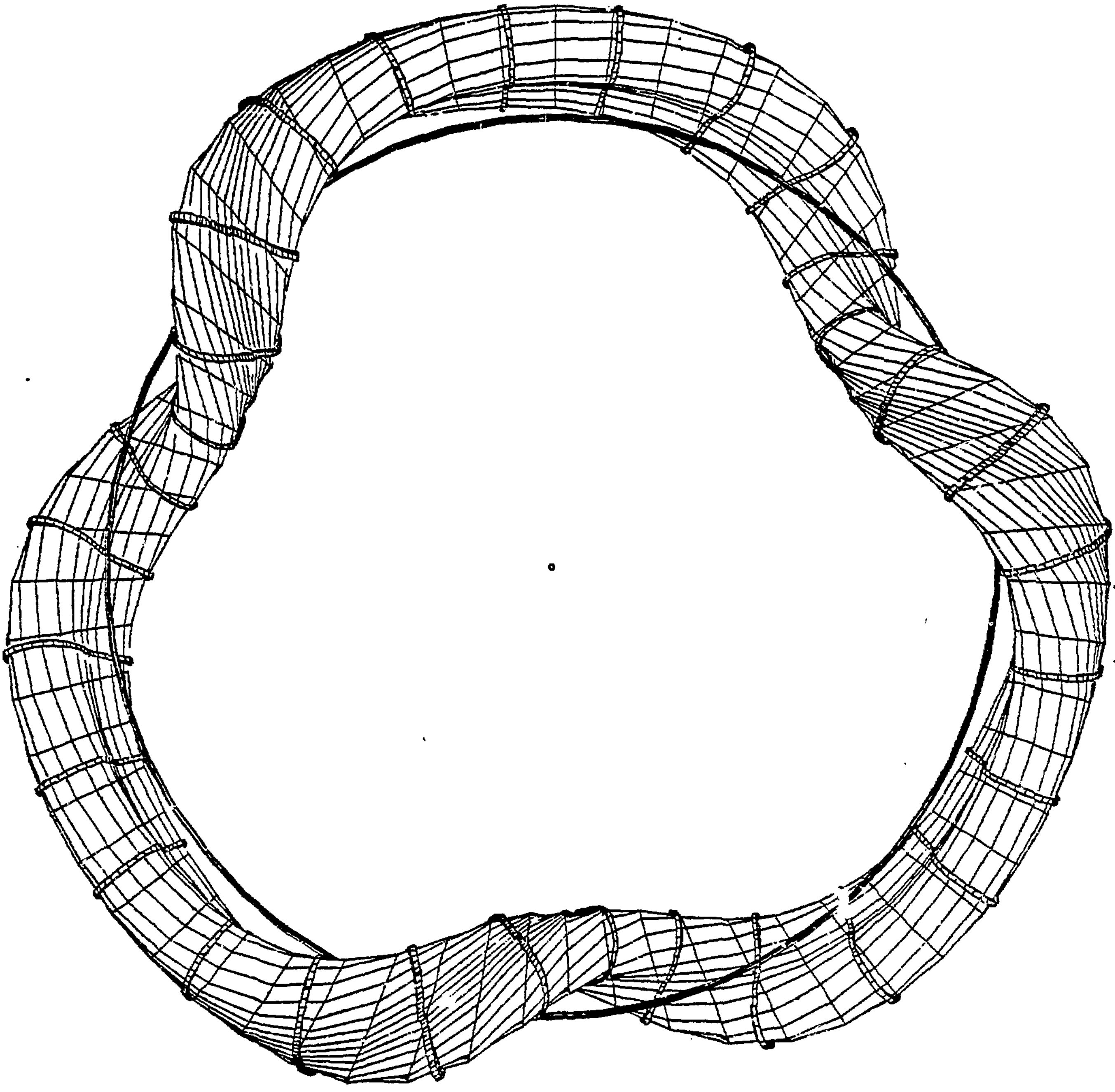
THE CLOSED RINGS ENCIRCLING THE TORUS ARE POTENTIAL (HENCE CURRENT) CONTOURS.

THE MAJOR AND CIRCULAR AXES ARE FOR REFERENCE ONLY. THEY ARE <u>NOT</u> CONDUCTORS.

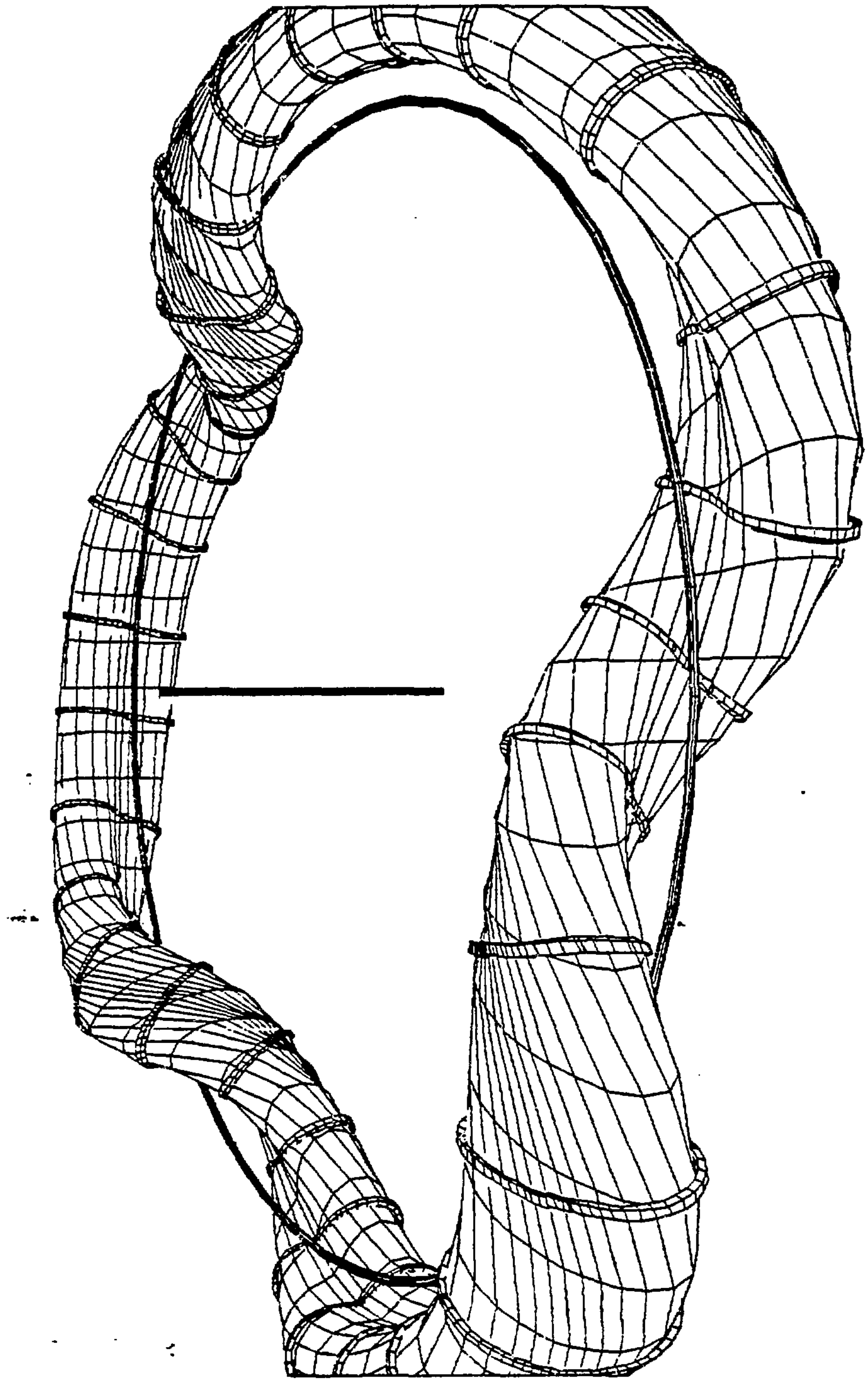
FIGURE 3S3

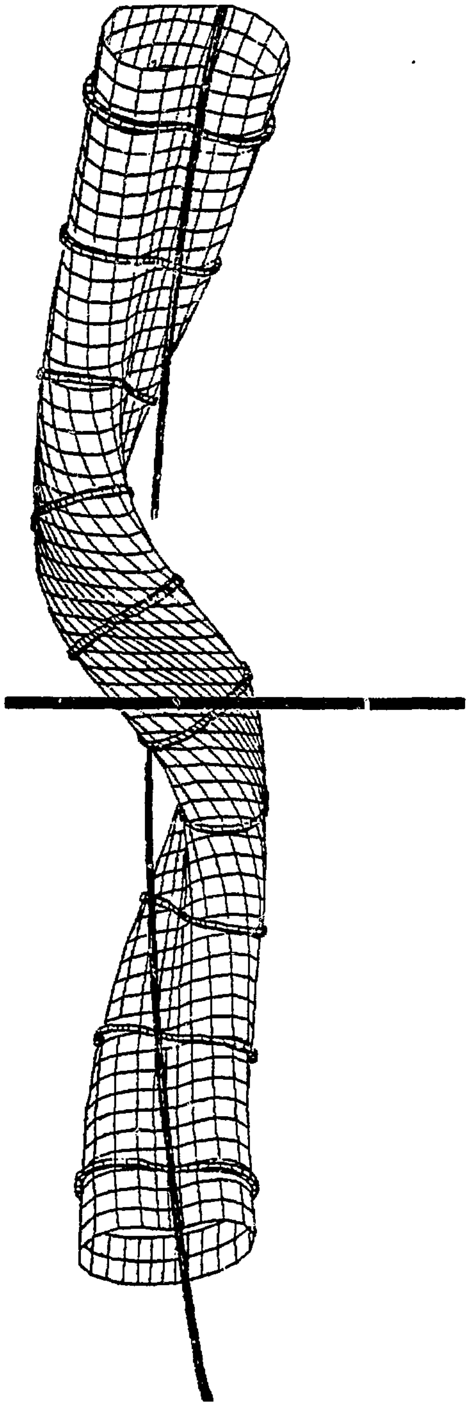
CUTAWAY VIEW OF V19S CONFIGURATION.

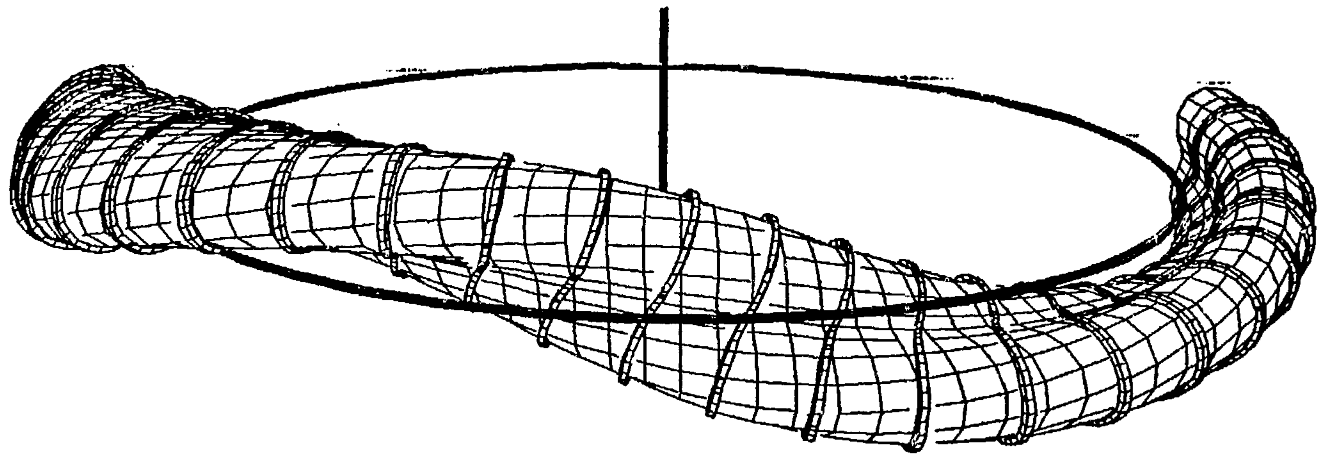
3Q3



3Q4

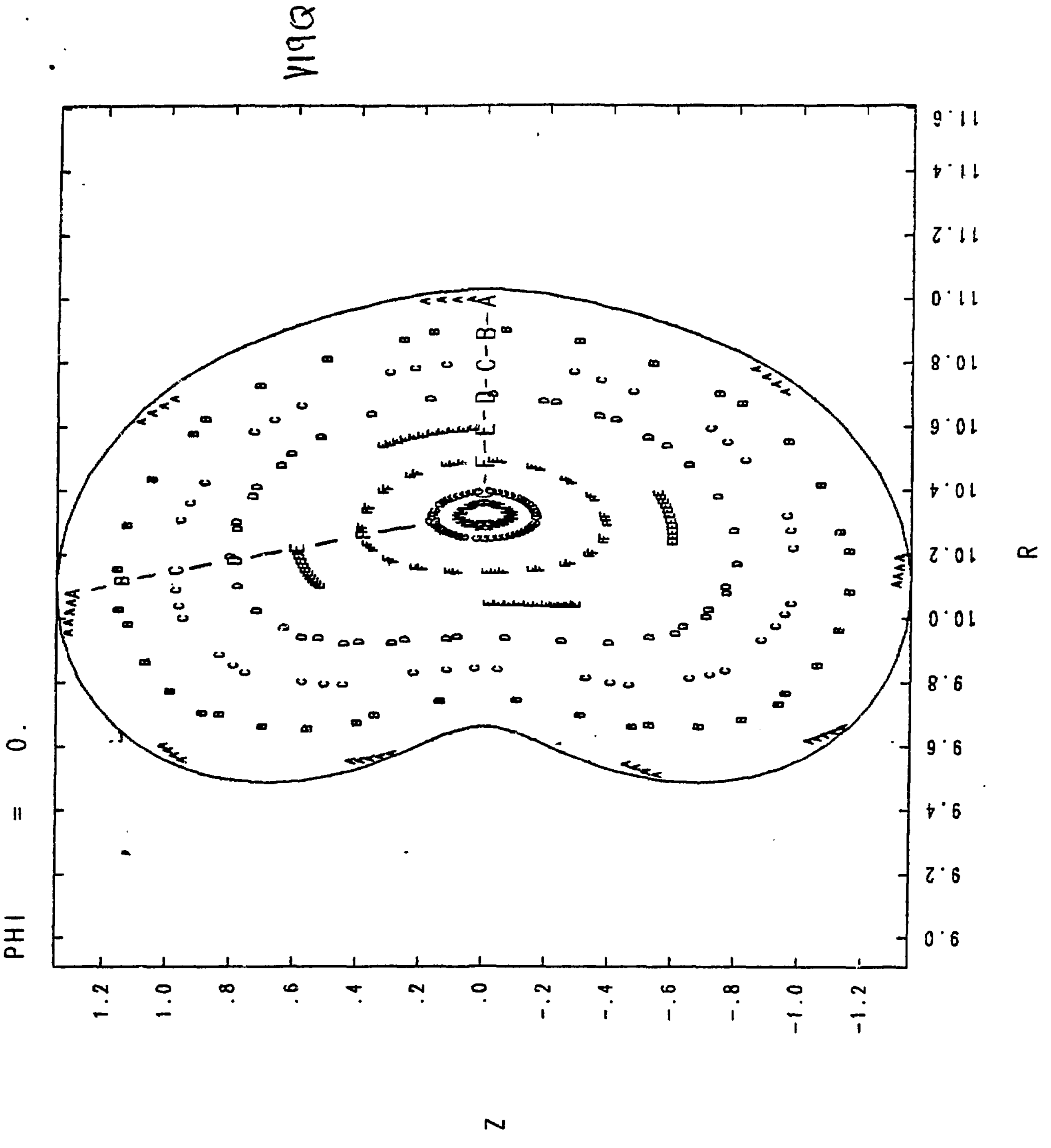






GRATON

353


PHI = 0.
R
Z

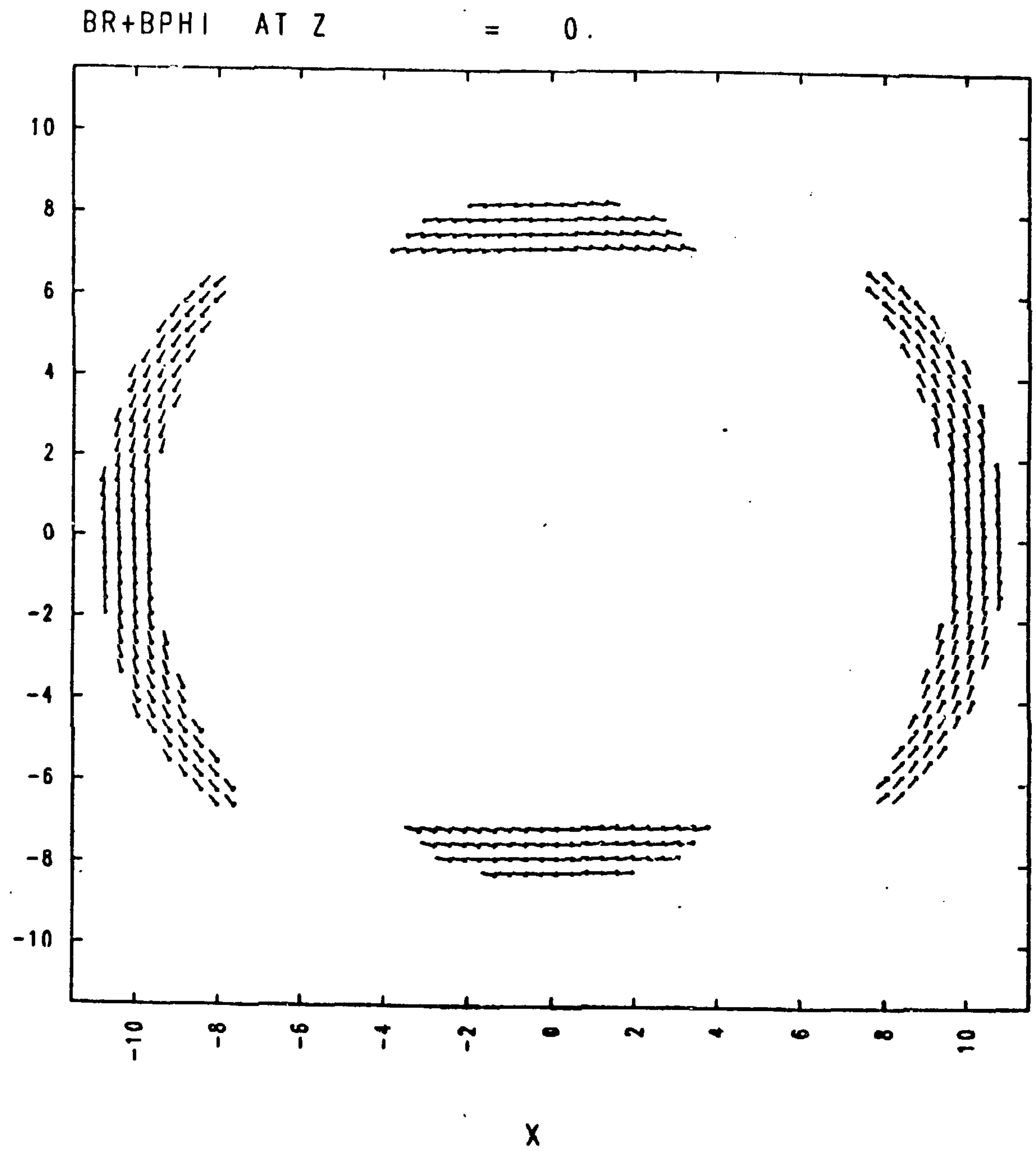
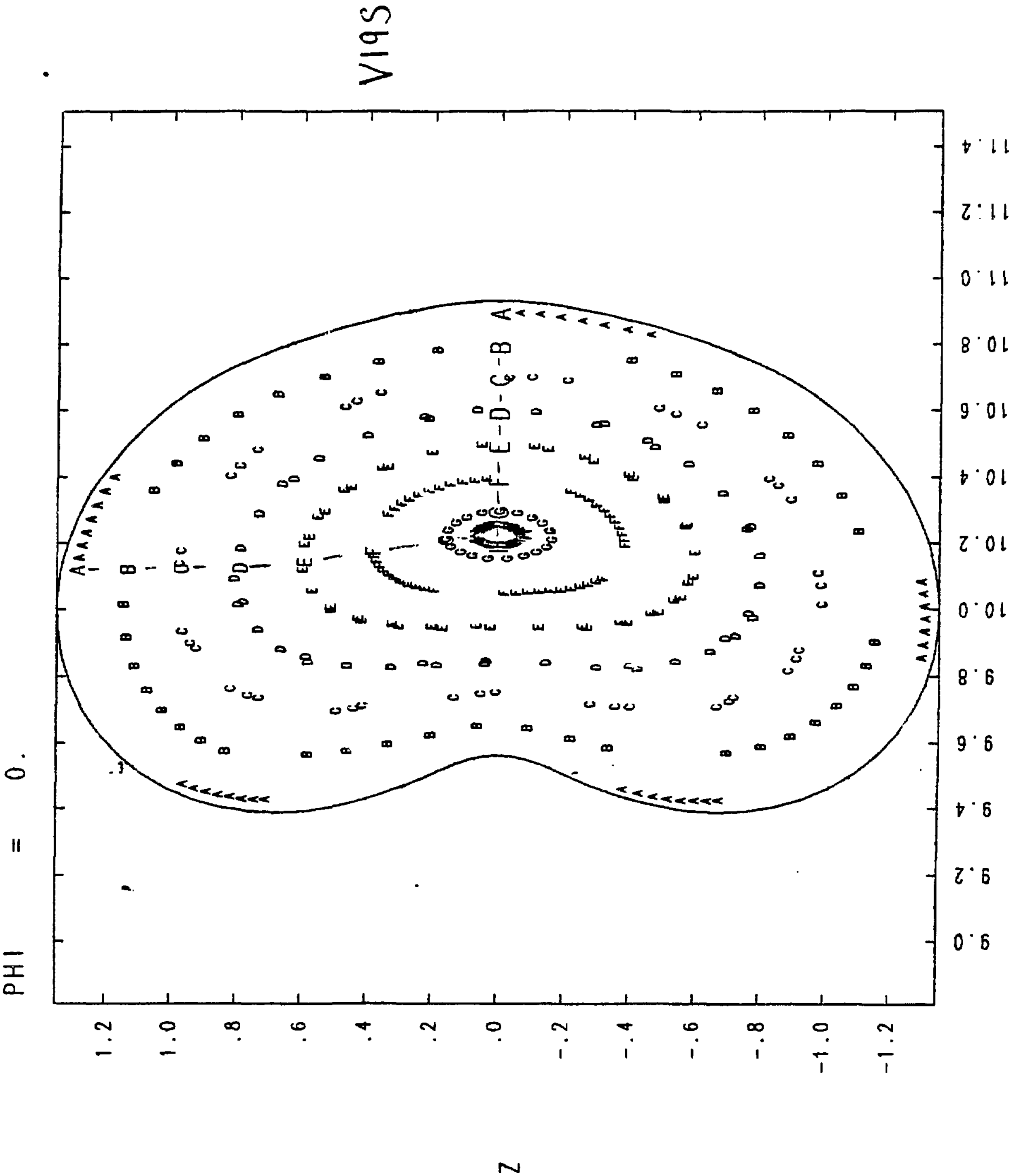
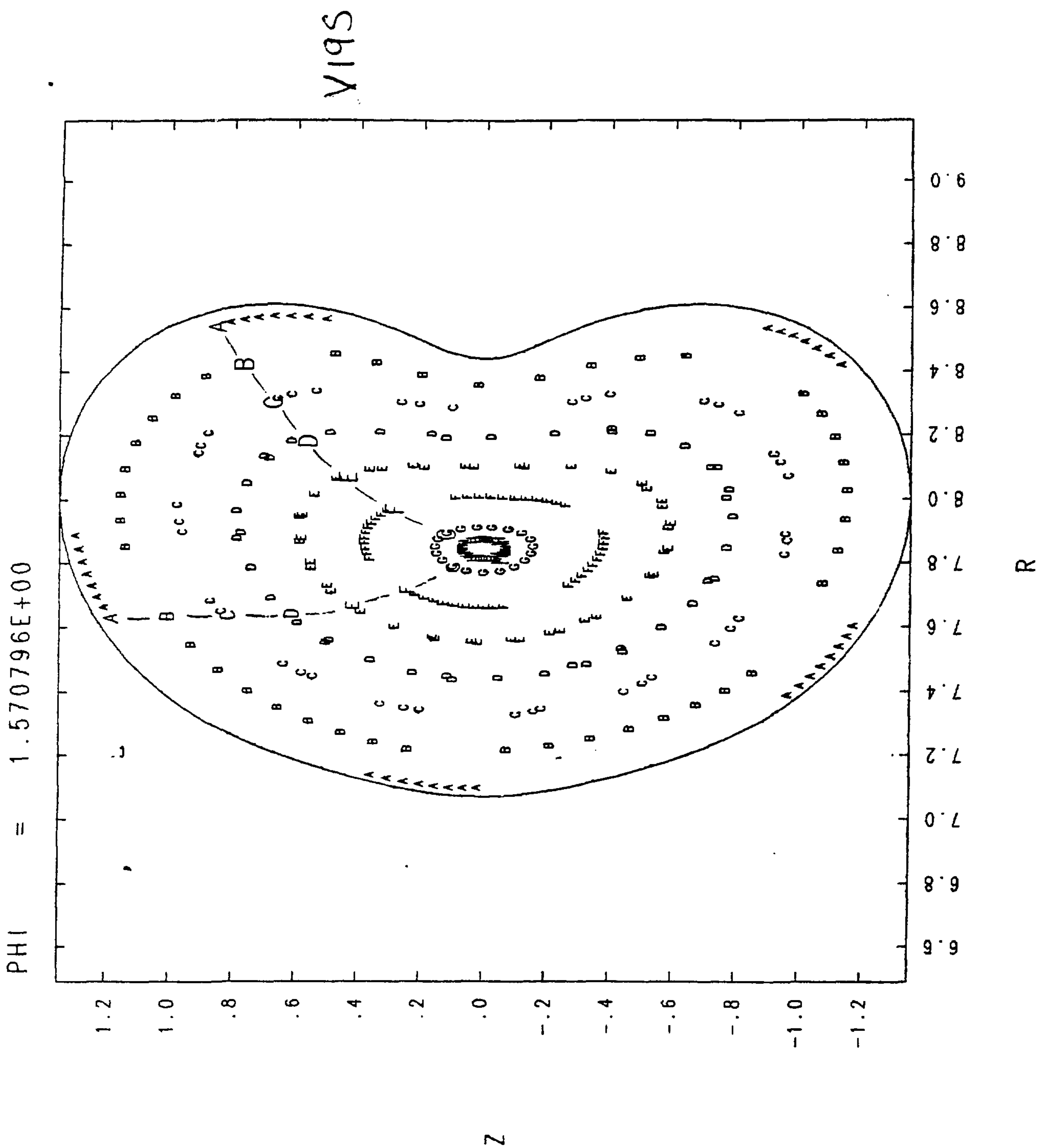
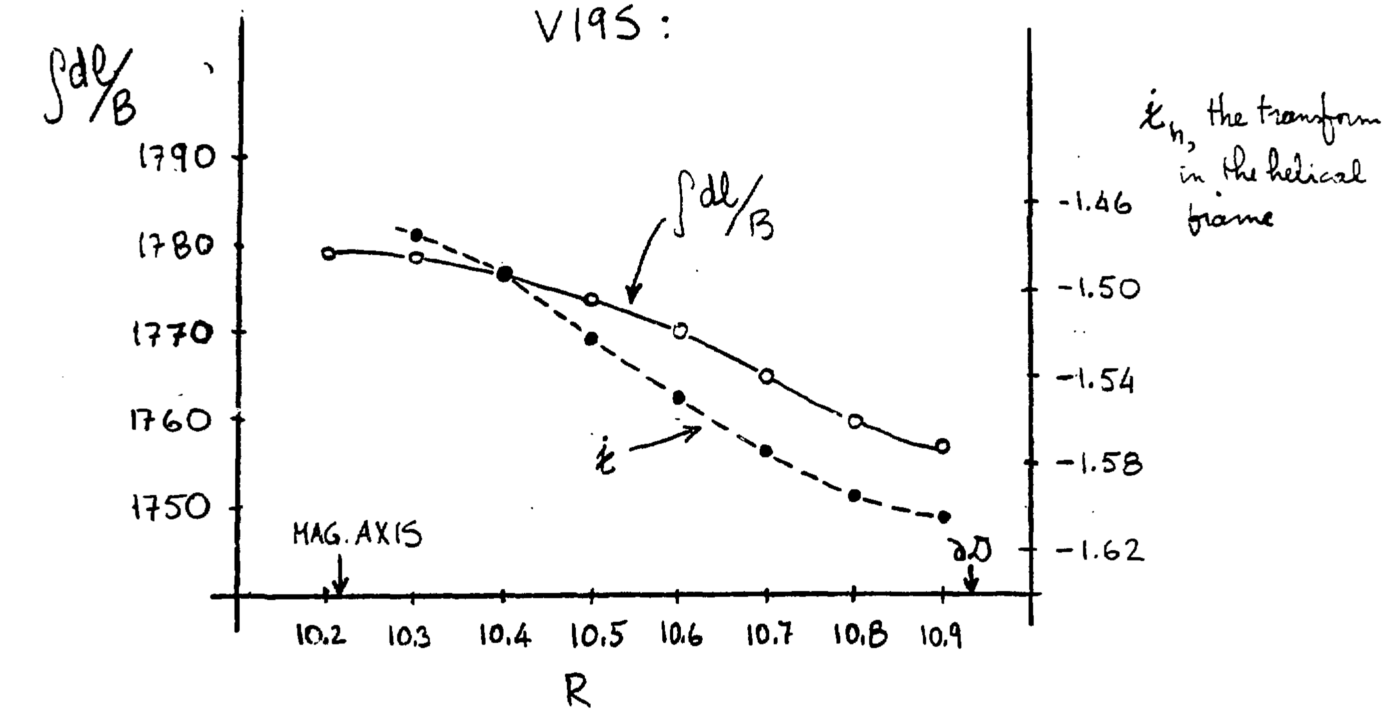


FIGURE 3.51
INTERSECTION
OF HELIAC WITH
Z=0 PLANE

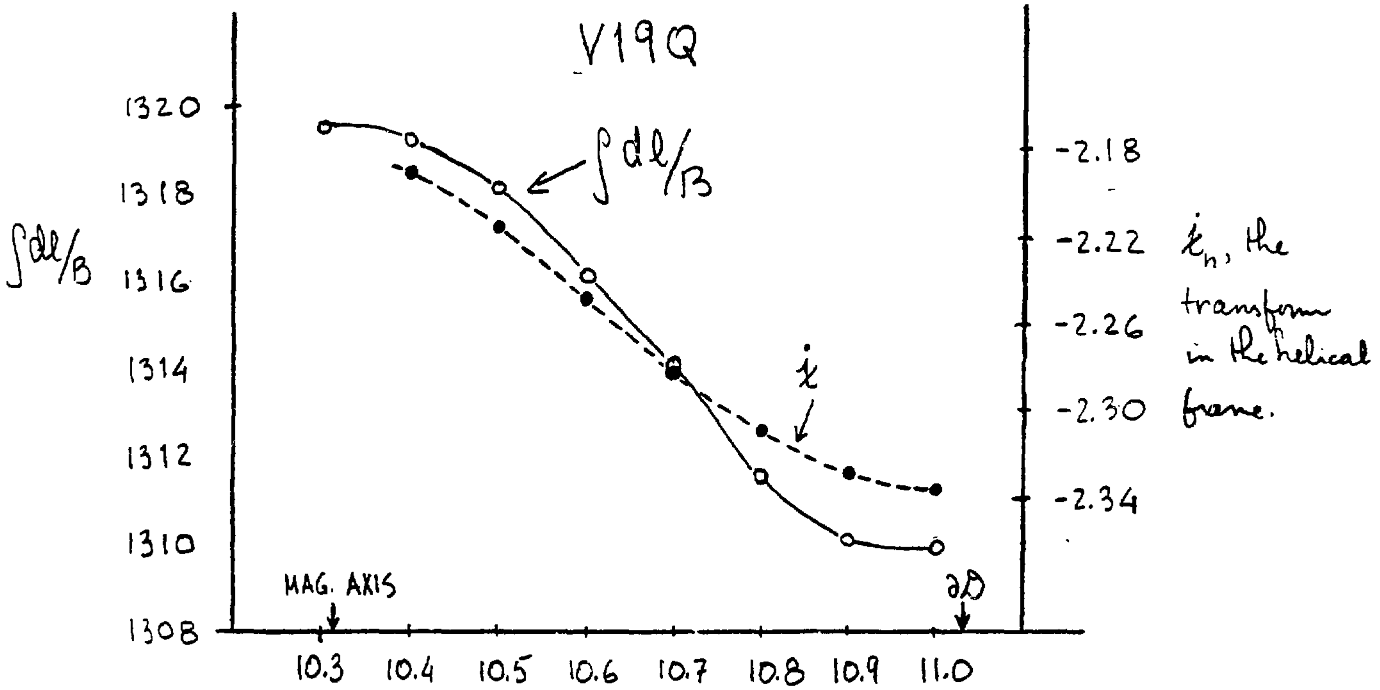

V19S
PHI = 0.
R
Z


Y195
PHI = 1.570796E+00
R
Z
6.6
6.8
7.0
7.2
7.4
7.6
7.8
8.0
8.2
8.4
8.6
8.8
9.0
1.2
1.0
.8
.6
.4
.2
.0
-.2
-.4
-.6
-.8
-1.0
-1.2



WELL DEPTH: 1.3% , $\iota \sim -1.5$,

$\Delta\iota/\iota \sim 9\%$



WELL DEPTH: ~.73% , $\iota \sim -2.26$.

$\Delta\iota/\iota \sim 6\%$

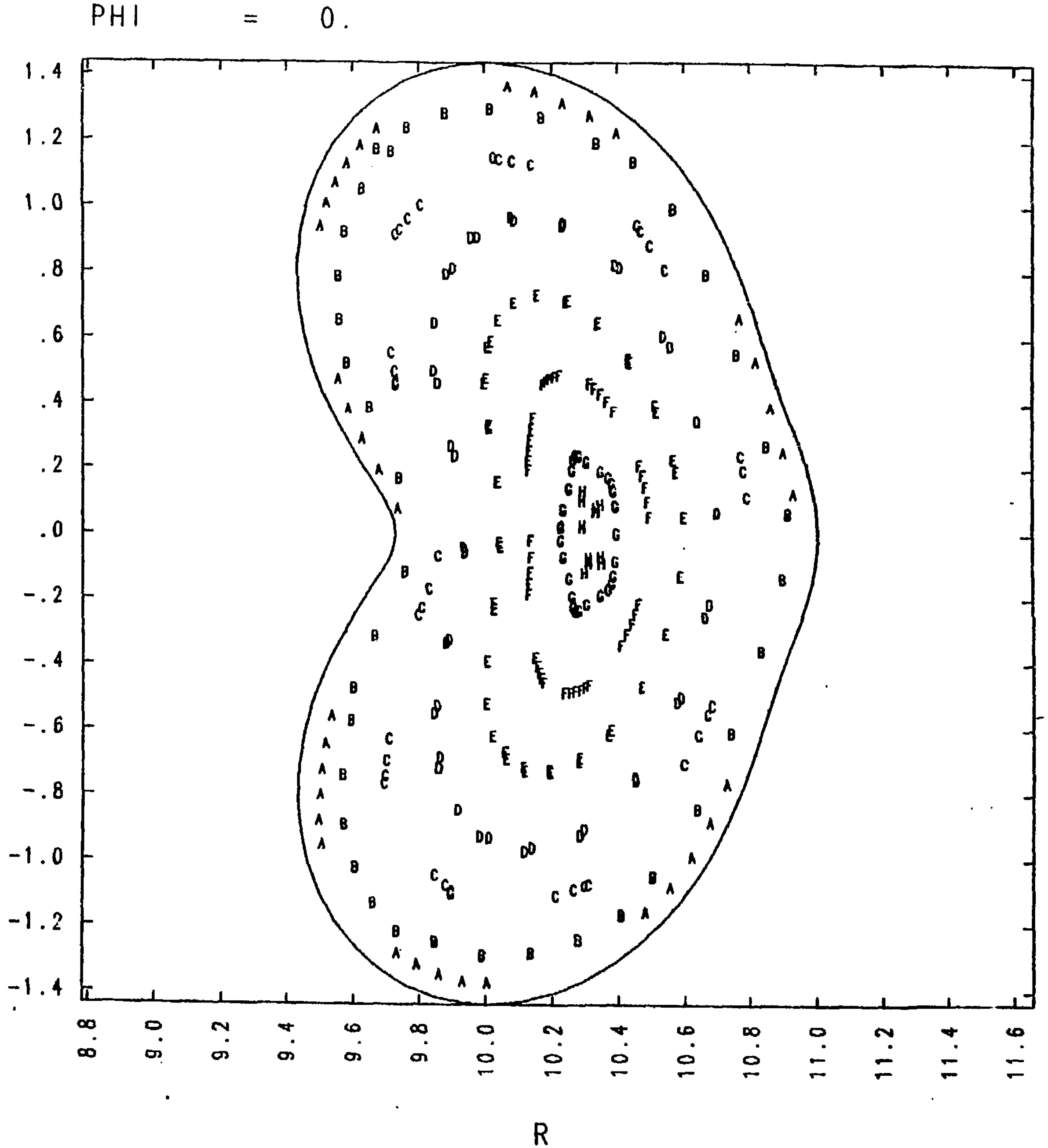
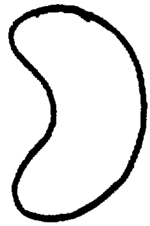


FIGURE 4V2
FIELD LINE
TRACING

FUTURE DIRECTIONS:

THE NUMERICAL PROGRAM HAS BEEN MODIFIED TO ALLOW THE STUDY OF CROSS-SECTIONS REQUIRING MANY HARMONICS

E.G. :  'D' SHAPES OR  'KIDNEY' SHAPES

THE EFFECTS THAT THESE HAVE ON ι & $\int dl/B$ WILL BE STUDIED

CONCLUSION

REALISTIC CONFIGURATIONS WITH ~ 3-5% WELLS SHOULD BE ATTAINABLE. HOWEVER, IT SEEMS UNLIKELY THAT THESE WILL HIGH SHEAR WHEN CONSTRUCTED WITH ONLY CLOSED POLOIDAL CURRENT LOOPS.

BALLOONING MODE CODE IN STELLARATORS

H. L. Berk,
M. N. Rosenbluth
INSTITUTE FOR FUSION STUDIES
Austin, Texas

and

L. Shohet
Univ. of Wisconsin
Madison, WI

FOURTH U. S. STELLARATOR WORKSHOP
Oak Ridge, Tennessee
April 14-15, 1983

Ballooning Mode Code in Stellarators

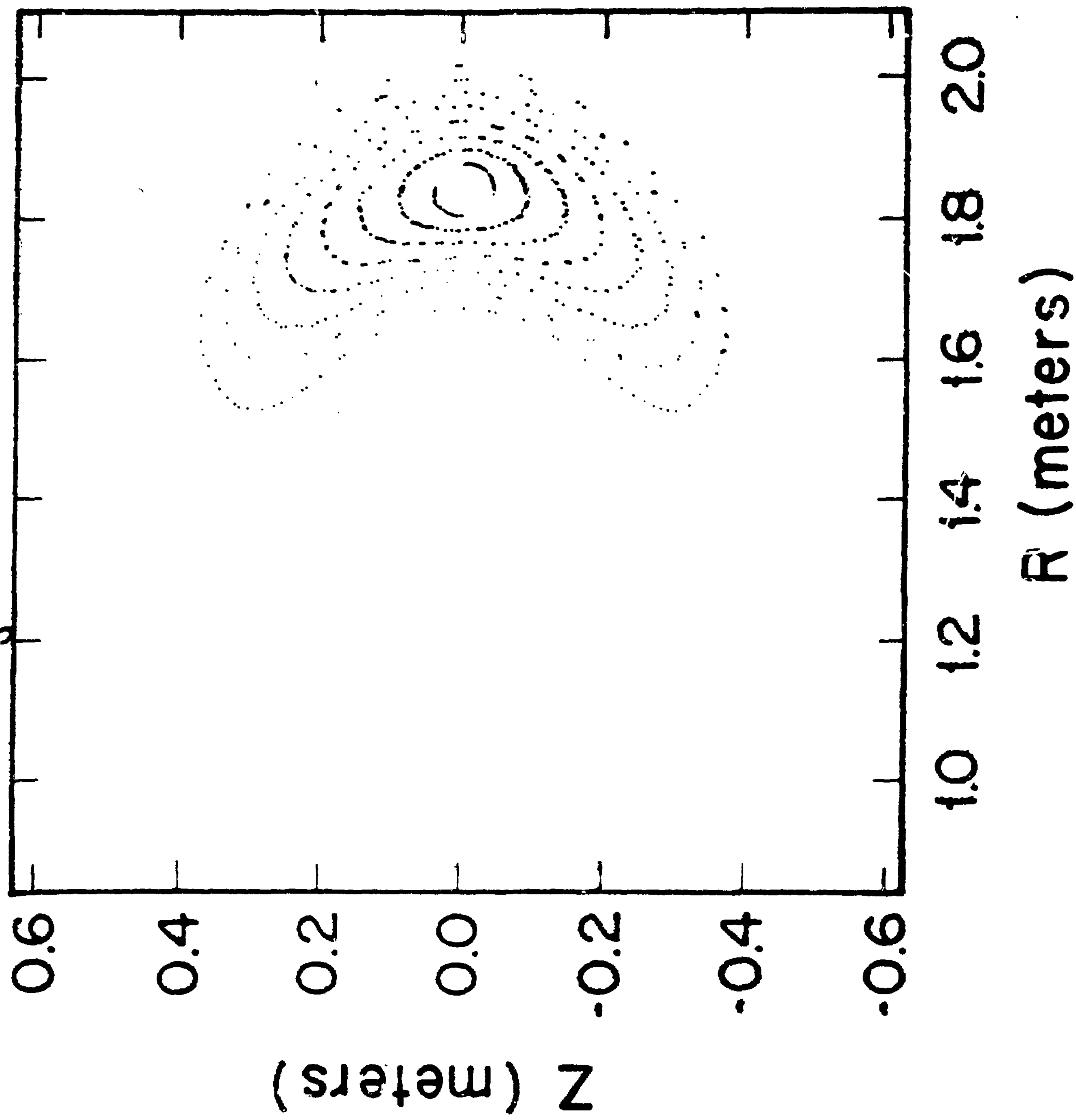
H. L. Berk,
M. N. Rosenbluth
I. F. S. Austin, Texas

L. Shohet
Univ. of Wisco

Motivation:

Given a vacuum current configuration, and a field line solver using Biot Savart Law, how can one analyze stability!

Magnetic Surface in Heliac



Ballooning Mode Equation

From eikonal expression!!

$$\frac{d}{ds}\frac{k_\perp^2(s)}{B(s)}\frac{d}{ds}\xi(s) + \frac{2}{B^3}\left(\underset{\sim}{k}_\perp \times \underset{\sim}{b}\cdot\underset{\sim}{\nabla} p\right)\left(\underset{\sim}{k}_\perp \times \underset{\sim}{b}\cdot\underset{\sim}{\chi}\right)\xi = 0$$

This equation is short wave limit of MHD, whose asymptotic solution leads to Mercier condition as necessary for stability:

$$\underset{\sim}{k}_\perp = \underset{\sim}{\nabla} S(\alpha,\beta) = \underset{\sim}{\nabla}\alpha\frac{\partial S}{\partial\alpha} + \underset{\sim}{\nabla}\beta\frac{\partial S}{\partial\beta}$$

$$\underset{\sim}{B} = \underset{\sim}{\nabla}\alpha \times \underset{\sim}{\nabla}\beta$$

Hence:

$$\underset{\sim}{\nabla}\beta = \Lambda\underset{\sim}{\nabla}\alpha + \underset{\sim}{B}\times\underset{\sim}{\nabla}\alpha / |\nabla\alpha|^2$$

where Λ must satisfy:

$$\boxed{\frac{d\Lambda}{ds} = -\frac{B}{|\nabla\alpha|^4}\left(\underset{\sim}{b}\times\underset{\sim}{\nabla}\alpha\right)\cdot\underset{\sim}{\nabla}\times\left(\underset{\sim}{b}\times\underset{\sim}{\nabla}\alpha\right)}$$

If we let $\frac{\partial S}{\partial \alpha} = 0$ (equivalent to some choice of intial Λ) we obtain:

$$\frac{d}{ds}\frac{|\nabla\beta|^2}{B}\frac{d\xi}{ds} + \frac{2}{B^2}\frac{\partial p}{\partial\alpha}(\underset{\sim}{\nabla}\beta \times \underset{\sim}{b} \cdot \underset{\sim}{\kappa})\xi = 0$$

$$\frac{d\Lambda}{ds} = -\frac{B}{|\nabla\alpha|^4}(\underset{\sim}{b} \times \underset{\sim}{\nabla}\alpha) \cdot \underset{\sim}{\nabla} \times (\underset{\sim}{b} \times \underset{\sim}{\nabla}\alpha)$$

$$\underset{\sim}{\nabla}\beta = \Lambda\underset{\sim}{\nabla}\alpha + \underset{\sim}{B} \times \underset{\sim}{\nabla}\alpha / |\nabla\alpha|^2$$

Difficulty of applying these equations is in determining $\underset{\sim}{\nabla}\alpha$, which requires knowing the flux surface, a fairly detailed & complicated calculation in 3-D.

Easy way To calculate $\underset{\sim}{\nabla}\alpha$

$$d\underset{\sim}{r} = d\phi\, \underset{\sim}{e}_1 + d\alpha\, \underset{\sim}{e}_2 + d\beta\, \underset{\sim}{e}_3$$

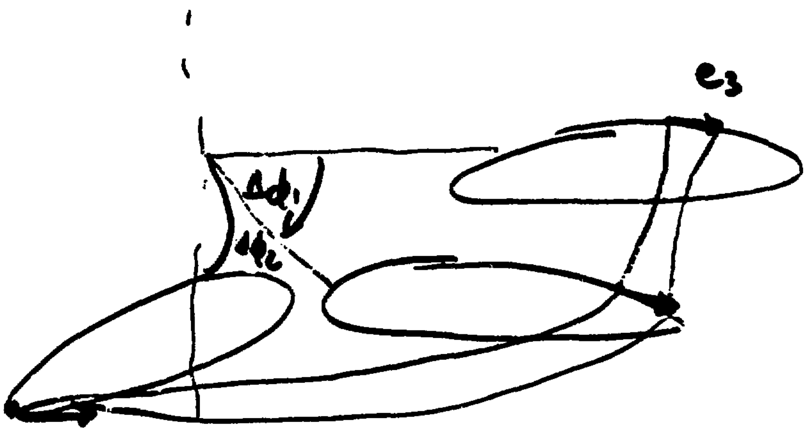
contravariant basis vectors

$$\underset{\sim}{e}_3 = \frac{\underset{\sim}{\nabla}\phi \times \underset{\sim}{\nabla}\alpha}{\underset{\sim}{\nabla}\phi \cdot \underset{\sim}{\nabla}\alpha \times \underset{\sim}{\nabla}\beta}, \text{ etc.}$$

ϕ is usual toroidal angle

We find $\cancel{\nabla}\alpha = \underset{\sim}{e}_3 \times \underset{\sim}{B}$

$\underset{\sim}{e}_3$ is very easily constructed



$\underset{\sim}{e}_3$ is just the distance between two neighboring field lines on the same flux surface at fixed ϕ.

Ballooning Mode Equations in Terms of $\underset{\sim}{e}_3$

$$\frac{d}{d\phi}\left[\frac{B_\phi}{B^2 R}\left(\frac{1}{|\underset{\sim}{e}_3\times\underset{\sim}{b}|^2}+B^2\Lambda^2|e_3\times\underset{\sim}{b}|\right)\frac{d}{d\phi}\xi\right.$$

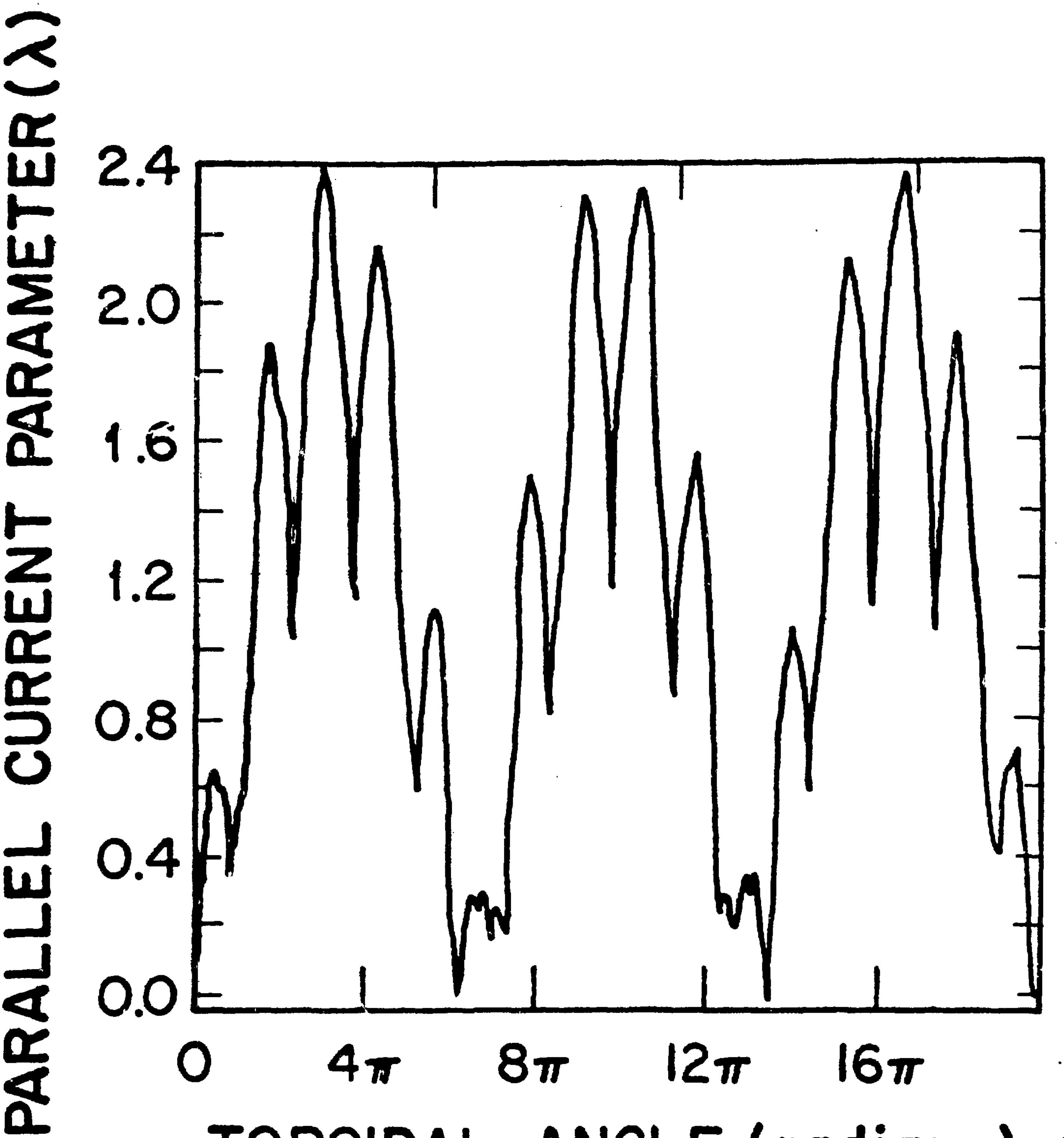
$$\left.+2\frac{\partial p}{\partial\alpha}\frac{R}{B_\phi}\left(\frac{\underset{\sim}{e}_3\times\underset{\sim}{b}\cdot\underset{\sim}{x}}{B|\underset{\sim}{e}_3\times\underset{\sim}{b}|^2}-\Lambda\,\underset{\sim}{e}_3\cdot\underset{\sim}{x}\right)\xi\right.=0$$

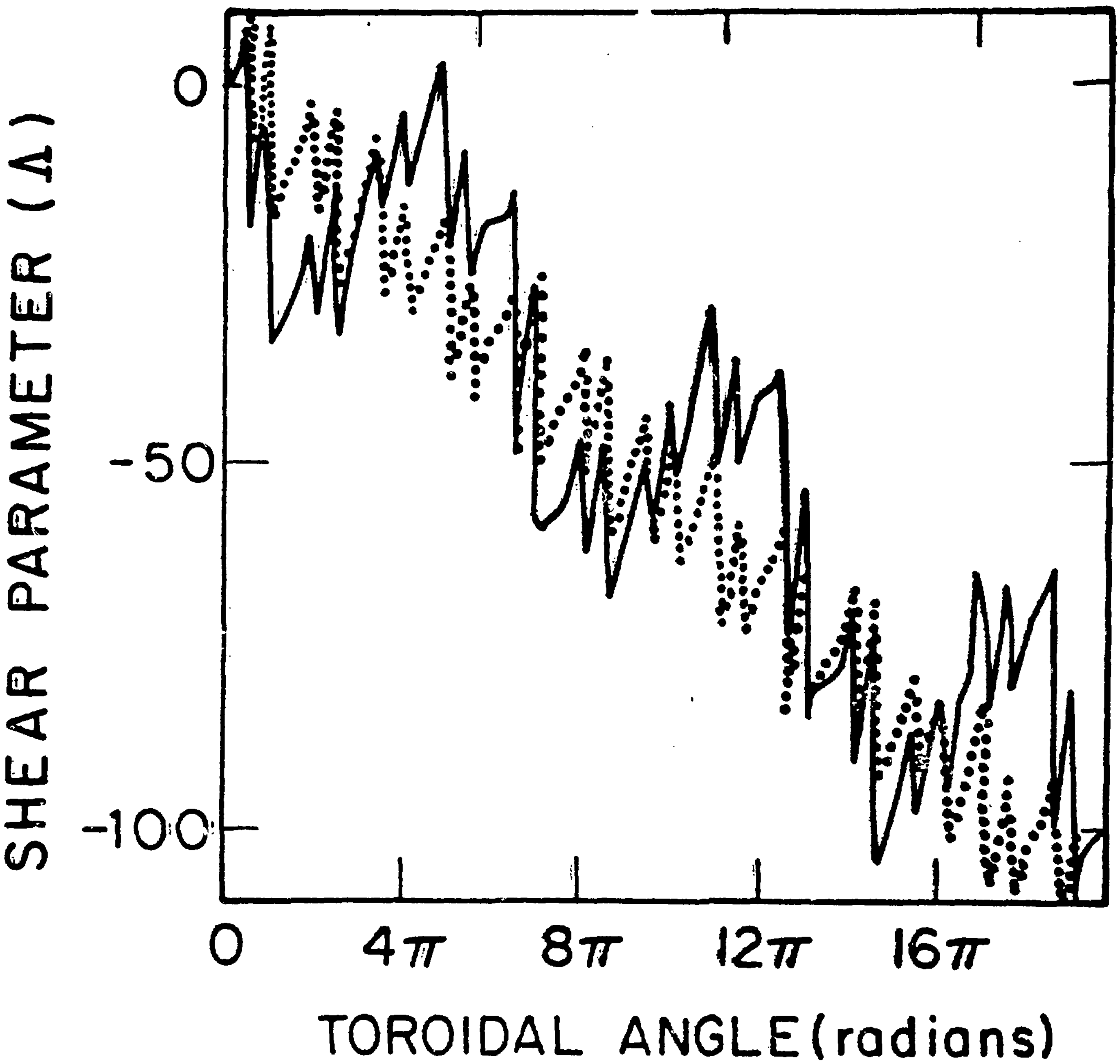
$$\frac{d\Lambda}{d\phi}=\frac{R}{B_\phi}\left\{\frac{[\underset{\sim}{e}_3-(\underset{\sim}{e}_3\cdot\underset{\sim}{b})\underset{\sim}{b}]\cdot[\underset{\sim}{b}\cdot\underset{\sim}{\nabla}(\underset{\sim}{e}_3\times\underset{\sim}{b})-(\underset{\sim}{e}_3\times\underset{\sim}{b})\cdot\underset{\sim}{\nabla}\underset{\sim}{b}]}{|\underset{\sim}{e}_3\times\underset{\sim}{b}|^4}\right\}$$

Note: $$\underset{\sim}{j}=\frac{\partial p}{\partial\alpha}\left[2\lambda\underset{\sim}{B}+(\underset{\sim}{B}\times\underset{\sim}{\nabla}\alpha)/B^2\right]$$

From $\underset{\sim}{\nabla}\cdot\underset{\sim}{j}=0$

$$\lambda=\int_0^\phi d\phi\,\frac{R}{B_\phi}\,\underset{\sim}{e}_3\cdot\underset{\sim}{x}$$


PARALLEL CURRENT PARAMETER(λ)
0.0
0.4
0.8
1.2
1.6
2.0
2.4
0
4π
8π
12π
16π
TOROIDAL ANGLE (radians)



SHEAR PARAMETER (Λ)
0
-50
-100
0
4π
8π
12π
16π
TOROIDAL ANGLE(radians)

Mercier Condition

Asymptotic evaluation of Ballooning Mode Equation Demands

$$\xi = \Lambda^{\nu}\left(\xi_0 + \xi_1/\Lambda + \xi_2/\Lambda + \cdots\right)$$

An asymptotic analysis leads to a recursion relation

$$\nu^2 + \nu + D = 0$$

Stability Demands

~~$0 > D - 1/4 =$~~

$$1/4 < D =$$

$$\left(\frac{\overline{G}}{\Lambda}\right) \lim_{\phi \to \infty} \frac{1}{\Lambda(\phi)} \int_0^{\phi} \frac{d\ell}{B_\phi} 2 \frac{\partial p}{\partial \alpha}$$

$$\cdot \left[\frac{e_3 \times b \cdot \kappa}{|e_3 \times b|^2} + \frac{B_\phi}{R}\frac{d\Lambda}{d\phi}(\lambda - \langle\lambda\rangle) + 2\frac{\partial p}{\partial \alpha}\frac{(\lambda^2 - \langle\lambda\rangle}{|e_3 \times b|^2}\right.$$

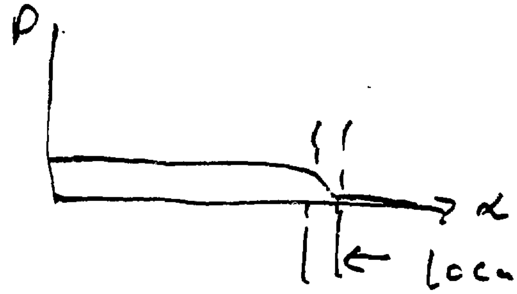
intrinsically destabili term:

Definitions

$$G = \int_0^{\phi} \frac{d\phi \, B}{|\underset{\sim}{e}_3 \times \underset{\sim}{b}|^2 B_\phi}$$

$$\langle \lambda \rangle = \int_0^{\phi} \frac{d\phi \, B \, \lambda}{B_\phi \, |e_3 \times \underset{\sim}{b}|^2} \Big/ \int_0^{\phi} \frac{d\phi \, B}{B_\phi \, |e_3 \times b|^2}$$

Iteration for small sharp pressure gradient



← localized pressure gradient

$$\underset{\sim}{\nabla} \alpha = \underset{\sim}{\nabla} \alpha_0 + \underset{\sim}{\nabla} \alpha_1 , \quad \underset{\sim}{\nabla} \beta = \underset{\sim}{\nabla} \beta_0 + \underset{\sim}{\nabla} \beta_1$$

Then:

$$\underset{\sim}{\nabla} \times \left[\underset{\sim}{\nabla} \alpha_0 \times \underset{\sim}{\nabla} \beta_1 + \underset{\sim}{\nabla} \alpha_1 \times \underset{\sim}{\nabla} \beta_0 \right] = J$$

$$= \frac{\partial p}{\partial \alpha} \left[\frac{\underset{\sim}{B}_0 \times \underset{\sim}{\nabla} \alpha_0}{B_0^2} + 2 \lambda_0 \underset{\sim}{B}_0 \right]$$

Then we find

$$\underset{\sim}{\nabla}\alpha_1 \simeq -\frac{p}{B^2}\nabla\alpha_0 \equiv \text{can be neglected at low beta:}$$

$$\underset{\sim}{\nabla}\beta_1 \simeq \gamma_{\beta\alpha}\underset{\sim}{\nabla}\alpha_0$$

with $\displaystyle \frac{d\gamma_{\beta\alpha}}{ds} = -j_{\parallel}B/|\nabla\alpha|$

With this result ballooning mode equation remains the same, except

$$\frac{d\Lambda}{d\phi} = \frac{d\Lambda_0}{d\phi} - 2\frac{\partial p}{\partial\alpha}\frac{(\lambda-\lambda_0)}{|e_3\times\underset{\sim}{b}|^2}\frac{B}{B_\phi} \equiv \frac{d}{d\phi}(\Lambda_0+\delta\Lambda)$$

Mercier criterion becomes:

$$D = \lim_{\phi\to\infty}\frac{G}{(\Lambda_0+\delta\Lambda)^2}\,2\frac{\partial p}{\partial\alpha}$$

$$\cdot\int_0^{\phi} d\phi\left[\frac{\underset{\sim}{e_3}\times\underset{\sim}{b}\cdot\underset{\sim}{\partial\varepsilon}B}{B_\phi\underset{\sim}{B}\,|e_3\times b|^2} + \frac{d\Lambda_0}{d\phi}(\lambda-\langle\lambda\rangle)\right]$$

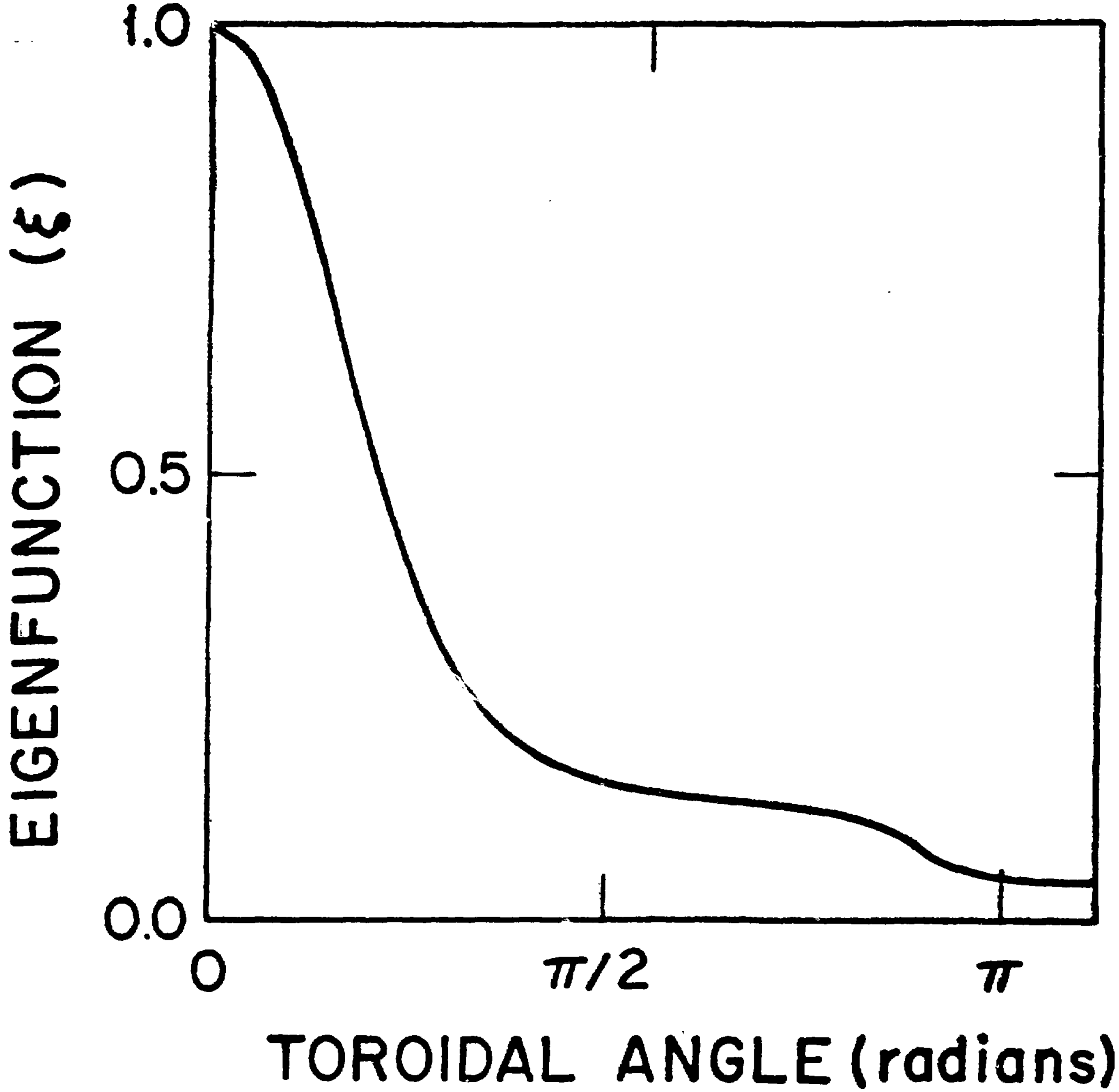
Note: Intrinsically destabilizing term has vanished!!
(Also found by Shafranov (1965)).

Numerical Results:

1. Proto-Cleo has magnetic well
2. Wistor-U has magnetic hill with low beta threshold determined by Mercier codition
3. Heliac - Mercier stable but ballooning unstable mode found:

Comment of Shafranov's conjecture

1. If shear increasing radially Mercier condition suffices for stability
2. If shear decreases ballooning possible (In tokamak sets barrier to second stability regime, although Mercier stable).
3. If Shafranov correct, establishing a magnetic well suffices for stability to high beta (although equilibrium may be problem)
4. Contrary result found for Heliac


1.0
0.5
0.0
0
π/2
π
EIGENFUNCTION (ξ)
TOROIDAL ANGLE (radians)

BALLOONING STABILITY OF 3-D STELLARATOR EQUILIBRIA*

W. A. COOPER
ORNL

ACKNOWLEDGEMENTS:
T. C. HENDER
B. A. CARRERAS

*Research sponsored by the Office of Fusion Energy, OER, U. S. Department of Energy under contract W-7405-eng-26 with the Union Carbide Corporation.

BALLOONING MODE EQUATION FOR 3-D TOROIDAL EQUILIBRIA WITH SHEAR

$$(\vec{B}\cdot\vec{\nabla})\left[\frac{k_\perp^2}{B^2}(\vec{B}\cdot\vec{\nabla})\hat{\xi}\right]+\left[\frac{2\vec{B}\times\vec{k}_\perp\cdot\vec{\kappa}}{B^2}\frac{dp}{d\psi}-\frac{\rho_M\gamma^2k_\perp^2}{B^2}\right]\hat{\xi}=0$$

$$\vec{B}=\vec{\nabla}\varphi\times\vec{\nabla}\psi-q\,\vec{\nabla}\theta\times\vec{\nabla}\psi$$

$$\vec{k}_\perp=\vec{\nabla}\varphi-q\vec{\nabla}\theta-\frac{dq}{d\rho}(\theta-\theta_k)\vec{\nabla}\rho$$

$2\pi\psi =$ toroidal flux

BECAUSE $q=q(\rho)$

$$\vec{B}=\vec{\nabla}\alpha\times\vec{\nabla}\psi\ ;\ \alpha=\varphi-q\theta$$

THEN:

$$\vec{B}\cdot\vec{\nabla}=\left(\frac{d\psi}{d\rho}\right)(\vec{\nabla}\varphi\times\vec{\nabla}\rho\cdot\vec{\nabla}\theta)\,\frac{\partial}{\partial\theta}\bigg|_{\rho,\,\alpha=\varphi-q\theta}$$

THE BALLOONING MODE EQUATION REDUCES TO

$$\left.\frac{\partial \hat{\eta}}{\partial \theta}\right|_{\rho,\, \alpha=\varphi-q\theta} = C_1 \hat{\xi}$$

$$\left.\frac{\partial \hat{\xi}}{\partial \theta}\right|_{\rho,\, \alpha=\varphi-q\theta} = C_2 \hat{\eta}$$

WHERE THE COEFFICIENTS C_1 AND C_2 ARE

$$C_1 = \frac{\sqrt{g}}{\left(\frac{d\psi}{d\rho}\right)} \left\{ -\frac{\beta_h}{2B^4}\frac{dp}{d\rho}\left[\left(\beta_h \frac{dp}{d\rho} + \frac{\partial B^2}{\partial \rho}\right)\bar{K}_\rho + \frac{\partial B^2}{\partial \theta}\bar{K}_\theta + \frac{\partial B^2}{\partial \varphi}\bar{K}_\varphi\right] + \frac{\rho_m \gamma^2 k_\perp^2}{B^2} \right\}$$

AND

$$C_2 = \frac{\sqrt{g}}{\left(\frac{d\psi}{d\rho}\right)} \frac{B^2}{k_\perp^2}$$

WHERE

$$\bar{K}_\rho = \frac{\bar{B}\times \bar{k}_\perp \cdot \bar{\nabla}\rho}{\left(\frac{d\psi}{d\rho}\right)} \; ; \; \bar{K}_\theta = \frac{\bar{B}\times \bar{k}_\perp \cdot \bar{\nabla}\theta}{\left(\frac{d\psi}{d\rho}\right)} \; ; \; \bar{K}_\varphi = \frac{\bar{B}\times \bar{k}_\perp \cdot \bar{\nabla}\varphi}{\left(\frac{d\psi}{d\rho}\right)}$$

$$\sqrt{g} = \left(\bar{\nabla}\psi \times \bar{\nabla}\rho \cdot \bar{\nabla}\theta\right)^{-1}$$

IT IS CONVENIENT TO NORMALISE

$$P \rightarrow P_0 \quad \ldots \text{(P ON AXIS)}$$

$$B \rightarrow \Psi_{EDGE}\, A^2 / R_0^2$$

A = ASPECT RATIO ; R_0 = MAJOR RADIUS

THEN

$$\beta_h = \frac{2 P_0 R_0^4}{\Psi_{EDGE}^2 A^4}$$

AND THE GROWTH RATE γ IS MEASURED

IN UNITS OF

$$\left[\frac{\Psi_{EDGE} R_0^3}{\rho_M^{1/2} A^3}\right]$$

CODE STRUCTURE

TO RECONSTRUCT THE EQUILIBRIUM QUANTITIES THAT MAKE UP C_1 AND C_2, THE FOLLOWING INFORMATION IS REQUIRED FROM AN EQUILIBRIUM CODE:

PARAMETERS... R_0, A, L = NUMBER OF FIELD PERIODS

P_0, NUMBER OF FLUX CONTOURS.

FLUX SURFACE QUANTITIES...

$$\frac{d\Phi}{d\rho} \text{ or } \frac{d\Psi}{d\rho} \; ; \quad \frac{dp}{d\rho} \quad ; \; q(\rho) \quad ; \; \rho$$

$2\pi\Phi$ = TOROIDAL FLUX

AND THE FOURIER AMPLITUDES OF $\sqrt{g}$ AND THE METRIC ELEMENTS g^{ij} (i.e., $g^{\rho\rho} = \nabla\rho\cdot\nabla\rho$), AS WELL THE TOROIDAL AND POLOIDAL MODES NEEDED TO RECONSTRUCT $\sqrt{g}$ AND g^{ij}.

FOR EXAMPLE

$$\bar{K}_\theta = \frac{1}{\rho}\left\{ g^{\rho\theta}\left[g^{\varphi\varphi} - \frac{q}{\rho} g^{\theta\varphi} - \frac{dq}{d\rho}(\theta-\theta_k)\left(g^{\rho\varphi} - \frac{q}{\rho} g^{\rho\theta}\right)\right] - \left(g^{\theta\varphi} - \frac{q}{\rho} g^{\theta\theta}\right)\left[g^{\rho\varphi} - \frac{dq}{d\rho}(\theta-\theta_k) g^{\rho\rho}\right]\right\}$$

WHERE, FOR EXAMPLE,

$$g^{\theta\varphi} \equiv \rho \vec{\nabla}\theta \cdot \vec{\nabla}\varphi = \sum_{m,n} g^{\theta\varphi}_{mn}(\rho) \cos(m\theta - Ln\varphi)$$

THE BALLOONING EQUATION IS SOLVED ON EACH FIELD LINE OF EACH FLUX SURFACE

TO PICK A FIELD LINE, WE CHOOSE $\alpha = \alpha_0$ SUCH THAT $0 \leq \alpha_0 < \frac{2\pi}{L}$

THEN THE ANGLE φ IS DETERMINED BY

$$\varphi = \alpha_0 + q\theta$$

NUMERICAL METHOD

WE USE A SHOOTING APPROACH

IN PRINCIPLE WE CAN SHOOT FROM EITHER

a) $\Theta = +\infty$ TO $\Theta = -\infty$

OR

b) $\Theta = -\infty$ TO $\Theta = +\infty$

WITH BOUNDARY CONDITIONS

$$\hat{\xi}(\Theta = \pm\infty) = 0$$

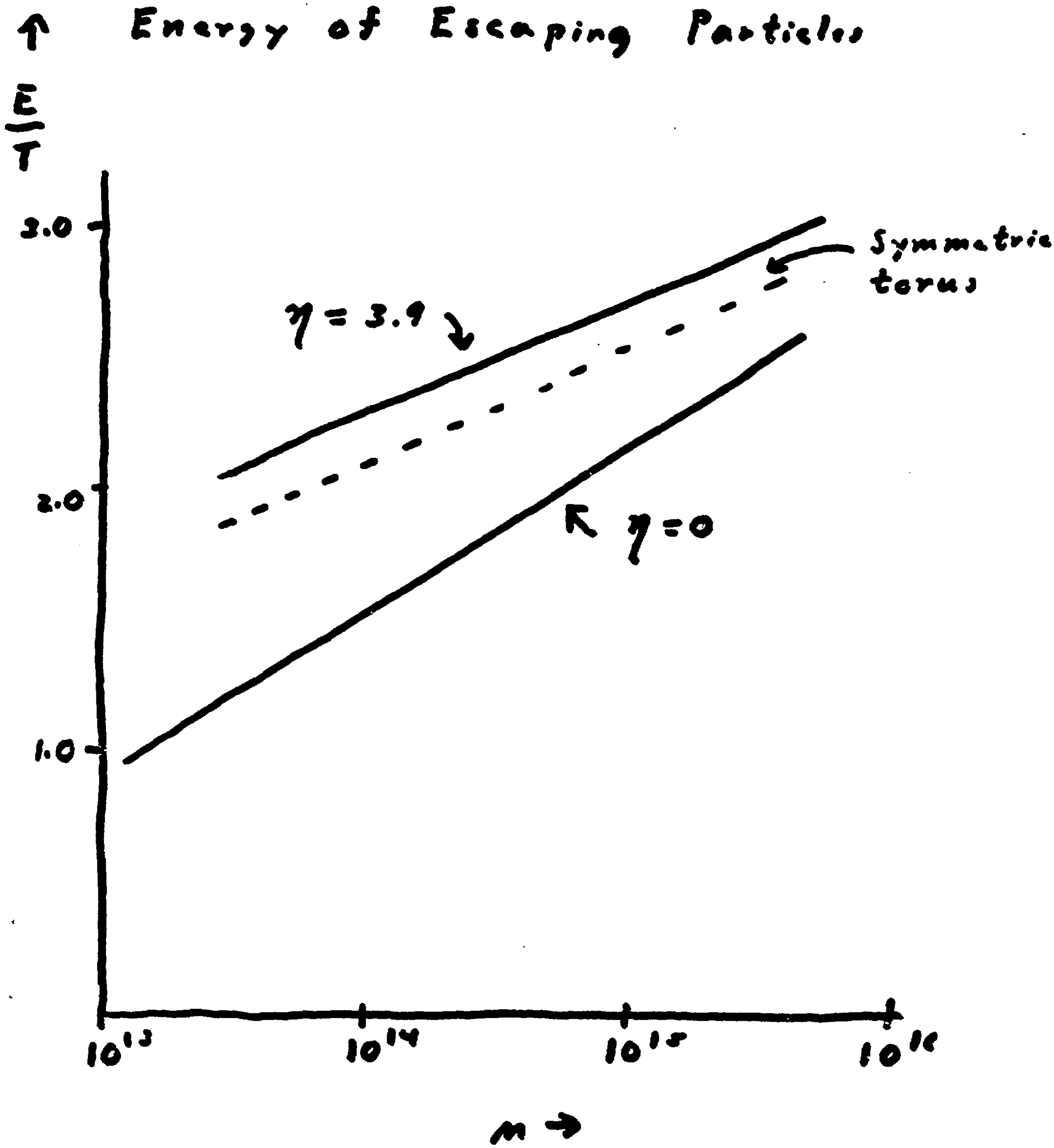
BUT

CASE a) NUMERICALLY UNSTABLE

$$0 \geq \Theta \geq -\infty$$

CASE b) NUMERICALLY UNSTABLE

$$0 \leq \Theta \leq \infty$$


Energy of Escaping Particles
E/T
3.0
2.0
1.0
η = 3.9
Symmetric torus
η = 0
10^13
10^14
10^15
10^16
n

ADVANTAGE:

SCHEME IS NUMERICALLY STABLE

DISADVANTAGE:

NO OBVIOUS WAY TO PICK Q_c THAT CORRESPONDS TO MOST UNSTABLE EIGENFUNCTION

INSTEAD WE SHOOT FROM

$\Theta = \Theta_F \to \pm\infty$

WITH B.C. : $\hat{\xi}(\Theta_F) = 0$

$$\frac{\partial \hat{\xi}}{\partial \Theta}(\Theta_F) = \mp \delta$$

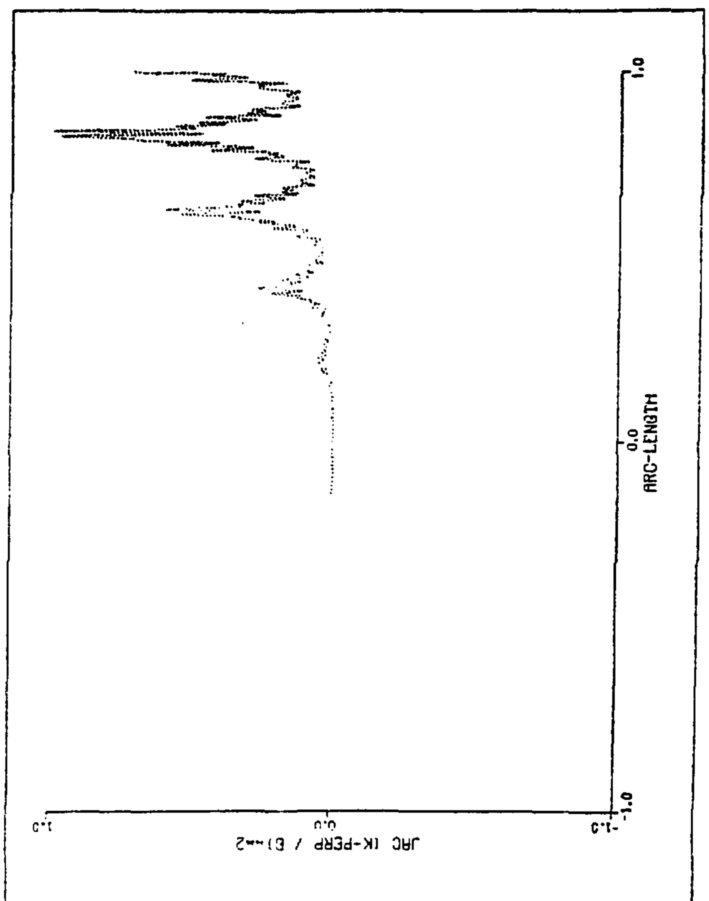
TO SOME $\Theta = \Theta_c$ AND ITERATE ON γ^2 TO SATISFY THE CONDITION

$$\frac{\partial \hat{\xi}}{\partial \Theta}(\Theta_c) \simeq 0$$

APPLICATION TO THE
VACUUM FIELDS OF A
12-FIELD PERIOD TORSATRON
CONFIGURATION WITH $\beta_0 \simeq 5\%$

(COURTESY OF TIM HENDER)

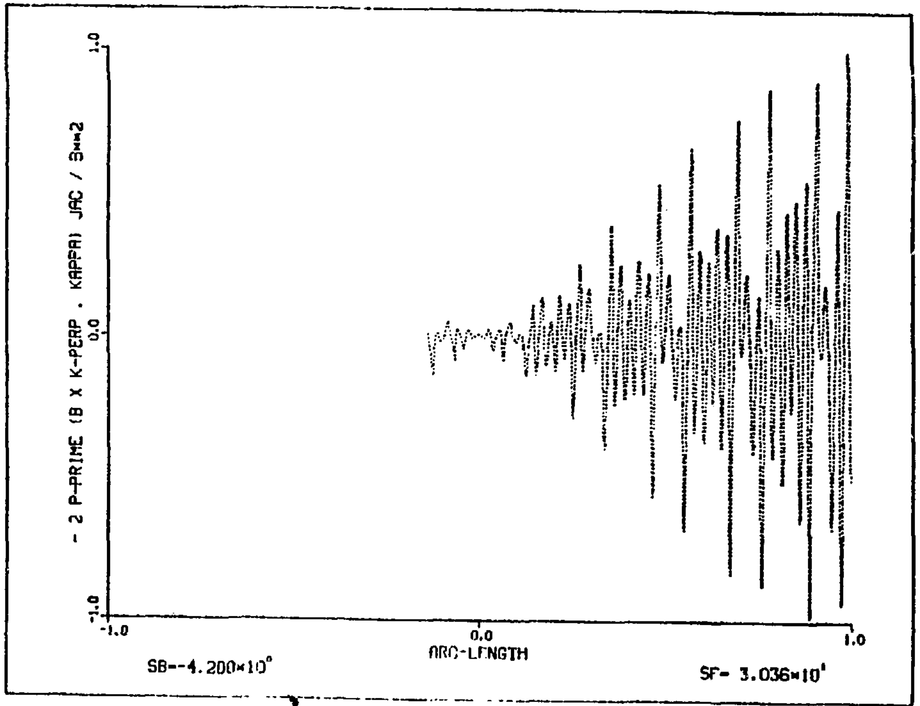
FIELD LINE BENDING TERM
VERSUS POLOIDAL ANGLE



C-12

$\rho=0.8$

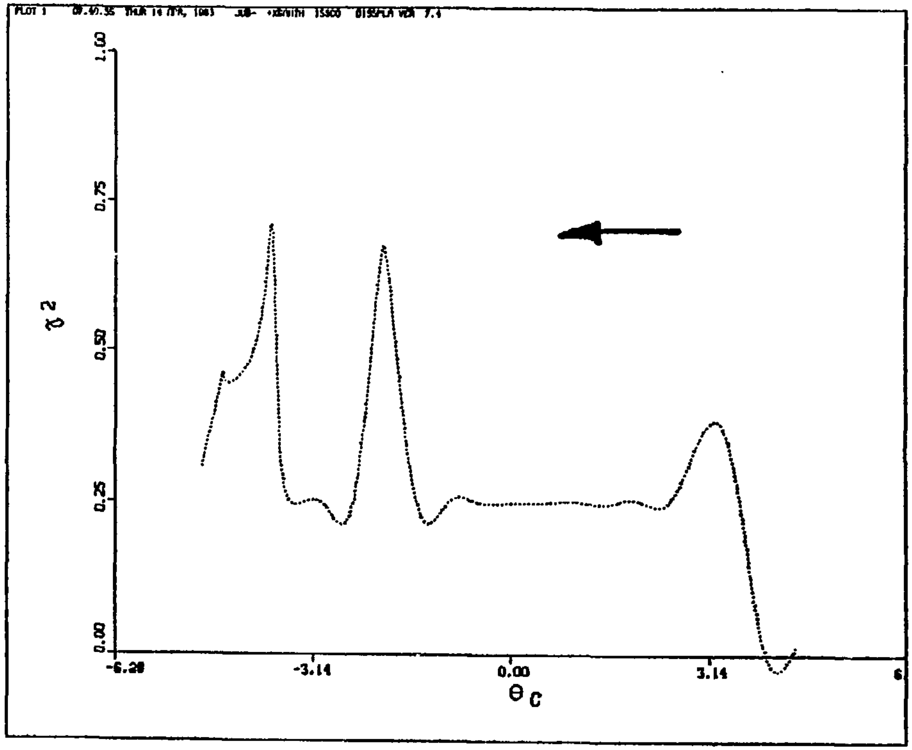
DRIVING TERM VERSUS
POLOIDAL ANGLE



$\rho = 0.8$

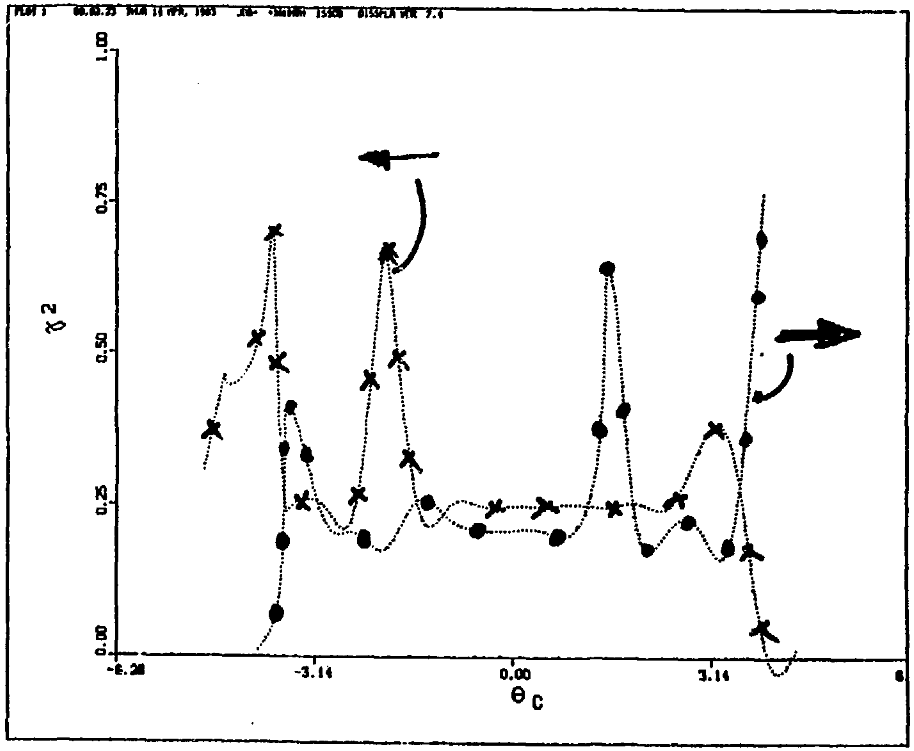
C-12

GROWTH RATE SQUARED VERSUS Θ_c FOR FIELD LINE WITH $\rho = 0.8$ AND $\alpha_0 = 0.42$



SHOT FROM $\Theta_F \rightarrow +\infty$

GROWTH RATE SQUARED VERSUS Θ_c FOR FIELD LINE WITH $\rho = 0.8$ AND $q_0 = 0.42$

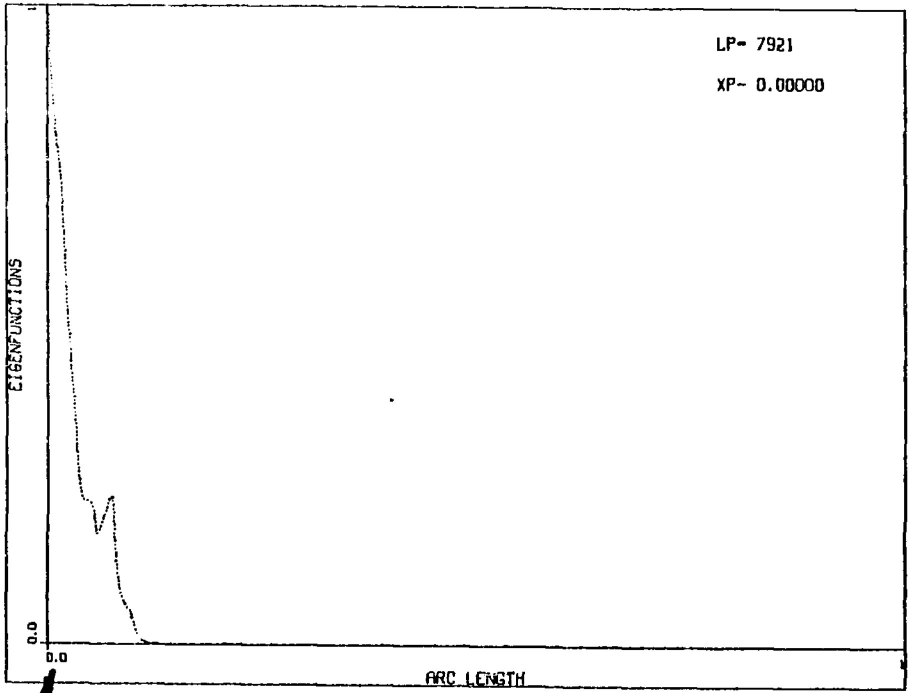


● SHOT FROM $\Theta_F \to -\infty$

✱ SHOT FROM $\Theta_F \to +\infty$

$\hat{\xi}(\theta)$ VERSUS Θ

$\rho = 0.8 \qquad \alpha_0 = 0.42$

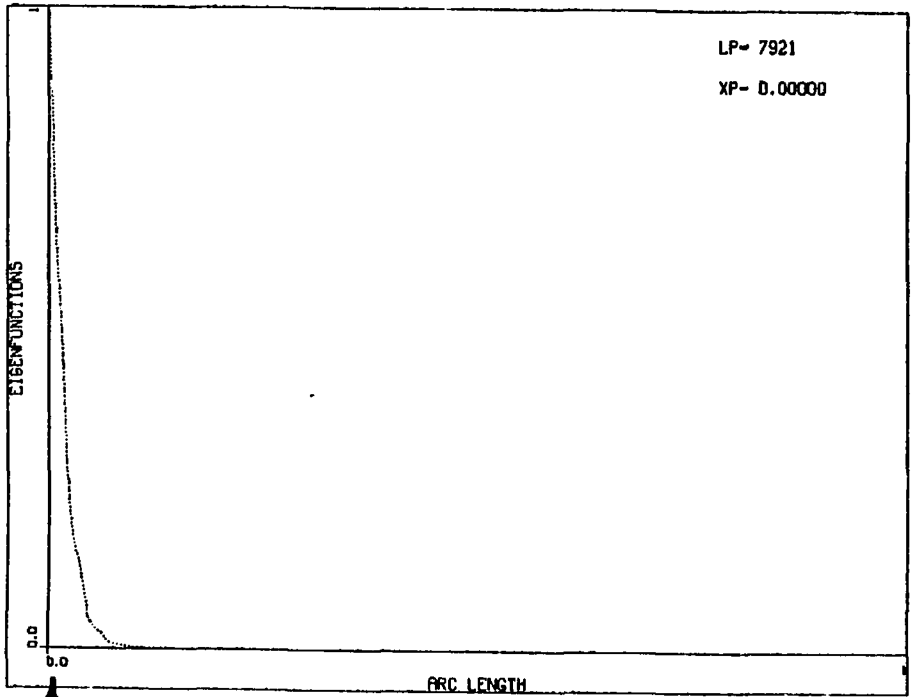


$\Theta_c \simeq 3.9$

SHOT FROM $\Theta_F \to +\infty$

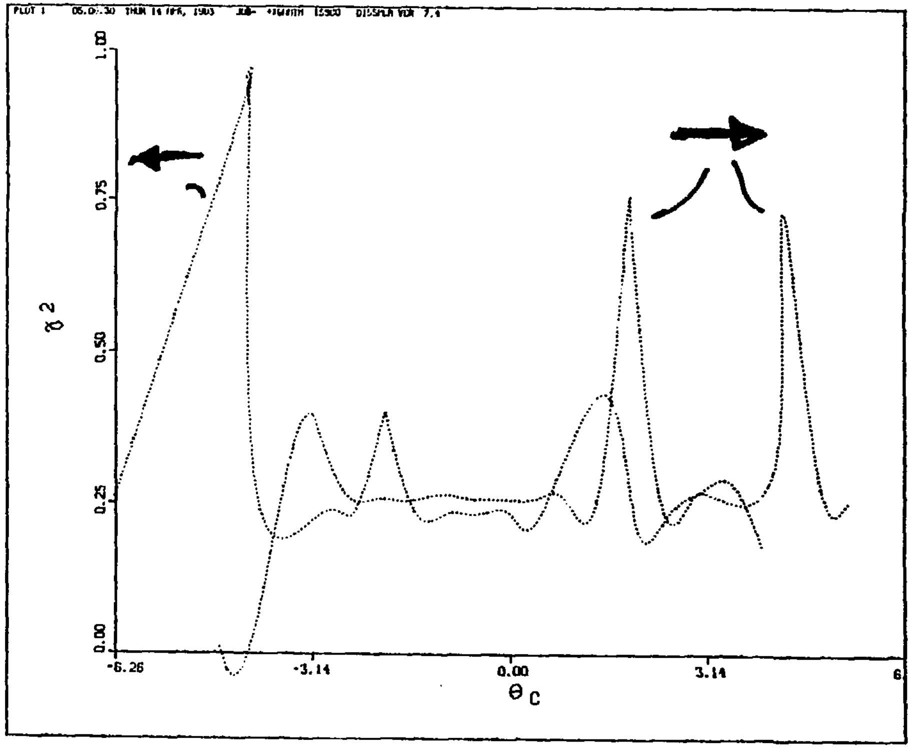
$\hat{\xi}(\Theta)$ VERSUS Θ

$\rho = 0.8 \quad a_0 = 0.42$



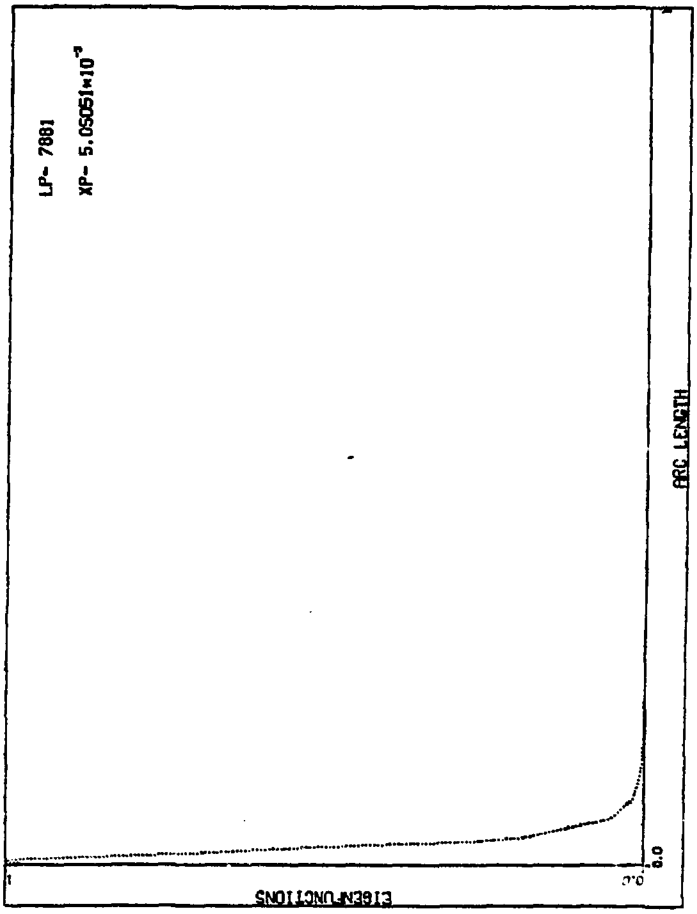
$\Theta_c \simeq 3.9$ SHOT FROM $\Theta_F \to \infty$

GROWTH RATE SQUARED VERSUS Θ_c FOR FIELD LINE WITH $\rho = 0.8$ AND $\alpha_0 = 0.21$



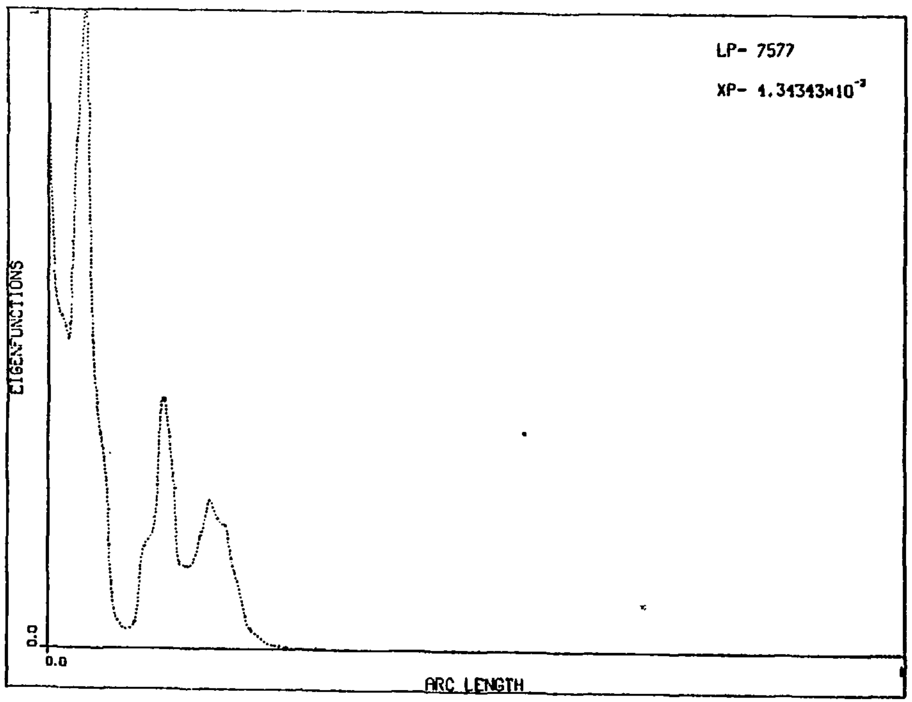
$\hat{\xi}(\Theta)$ VERSUS Θ

$\rho = 0.8$ $\alpha_0 = 0.21$



$\hat{\xi}(\theta)$ VERSUS θ

$\rho = 0.8$ $\qquad a_0 = 0.21$



SUMMARY

- WE HAVE DEVELOPED A SHOOTING CODE TO INVESTIGATE THE BALLOONING STABILITY OF 3-D TOROIDAL EQUILIBRIA WITH SHEAR.

- LACK OF SYMMETRY FORCES THE REQUIREMENT TO UNDERTAKE A LARGE NUMBER OF CALCULATIONS TO DETERMINE THE CORRECT EIGENVALUE AND EIGENFUNCTION EVEN ON ONE MAGNETIC FIELD LINE.

- CONSEQUENTLY A SHOOTING APPROACH MAY NOT BE THE OPTIMAL MANNER TO SOLVE THIS PROBLEM.

"COMPACT" STELLARATORS ?*

J. H. Harris
B. F. Masden
& ATF Design Team

Fourth US Stellarator Workshop

14-15 April 1983

*Research sponsored by the Office of Fusion Energy, OER, U. S. Department of Energy under contract W-7405-eng-26 with the Union Carbide Corporation.

- EXISTING STELLARATOR REACTOR DESIGNS HAVE BEEN STRONGLY CRITICIZED FOR BEING TOO BIG:

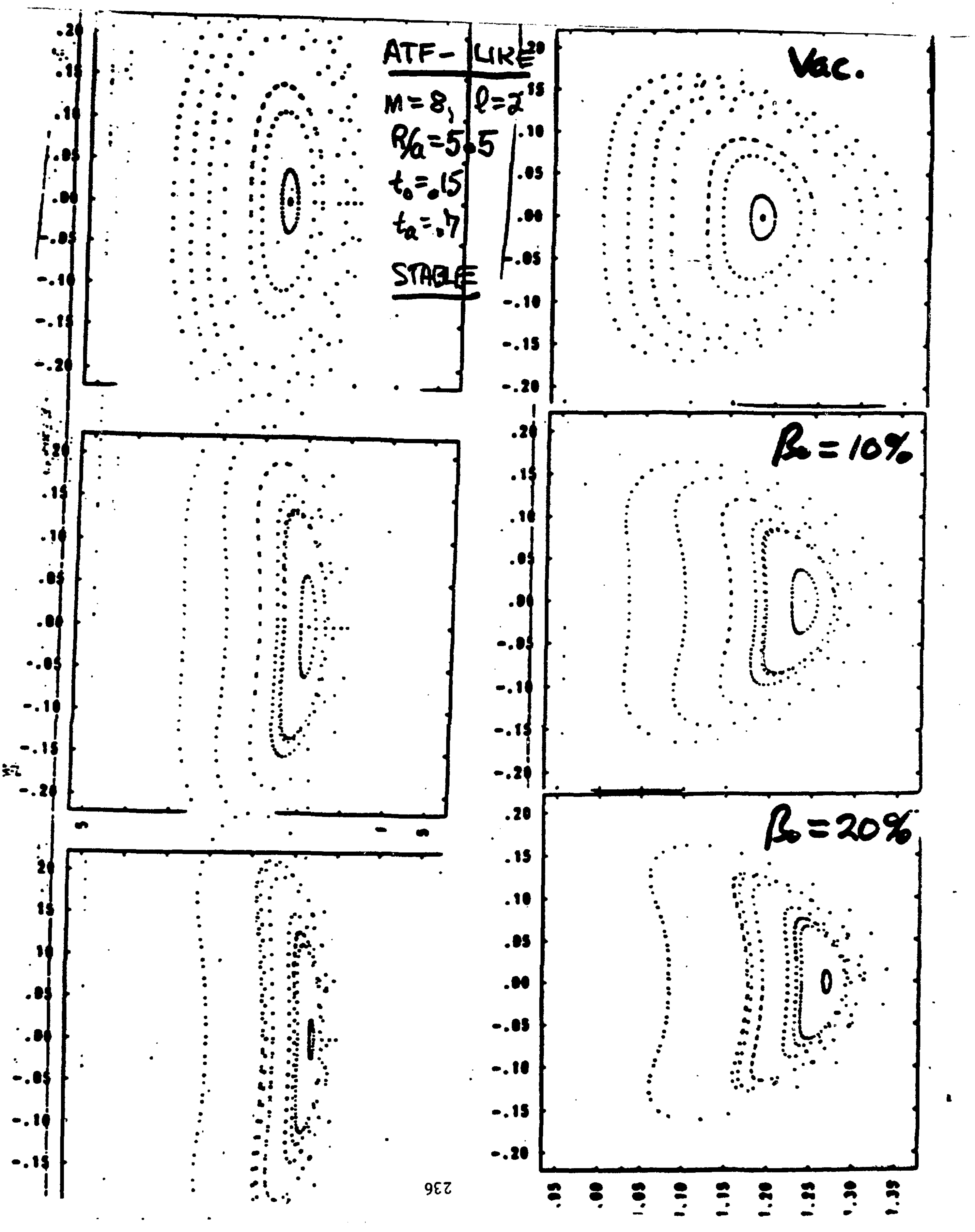
$$\left.\begin{array}{l} R \geq 20\ m \\ R/a \geq 10 \end{array}\right\} \text{high initial cost}$$

- GOOD EVIDENCE THAT "LOW" R/a HELPS STABILITY (ATF studies, NYU HELIAC studies)

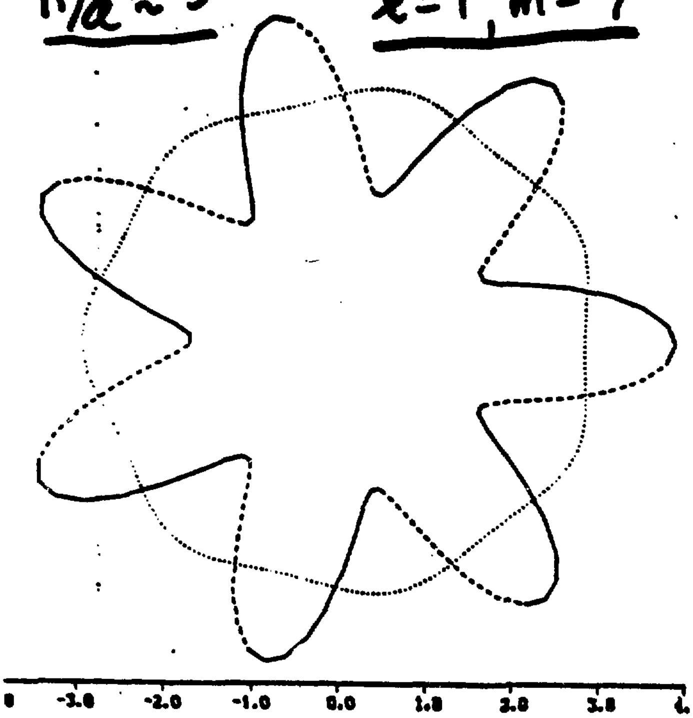
- PROBLEMS:
  - EQUILIBRIUM → COIL DESIGN
  - TRANSPORT → OPTIMIZATION INCL. E-FIELD, FINITE-β EFFECTS
  - MAY HAVE TO ACCEPT COMPROMISES (NOT AN "IDEAL" STELL.)
  - SOLUTIONS TO THESE PROBLEMS → <u>CO</u>MPACT <u>ST</u>ELLARATOR

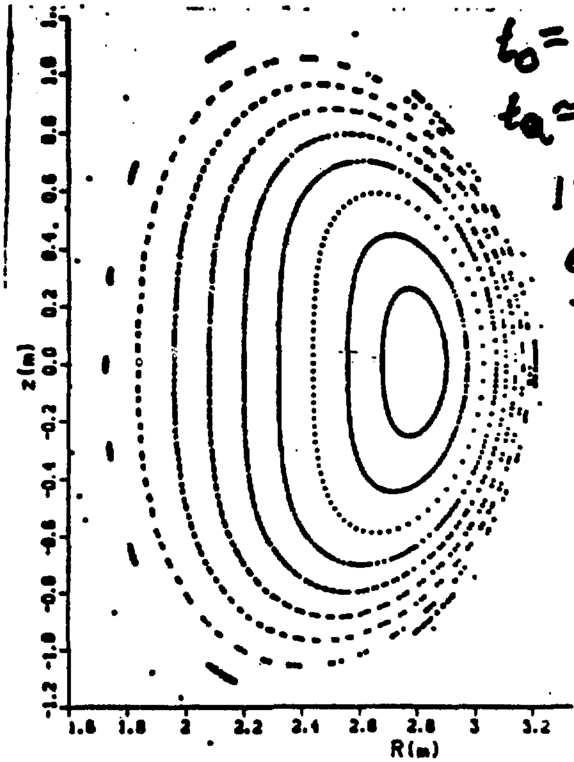
- DREAM: $\left.\begin{array}{l} R/a \rightarrow 3\text{-}5 \\ R_{reactor} \rightarrow 6\text{-}10m \end{array}\right]$ a very tough job!

- LIKELY FEATURES:
  - "some" helical axis → ι_0
  - more "open" coil structure (modular units not as simple as in high R/a → may need to extract blanket)
  - T/S hybrid (lower ϵ_H) if pure stell. fails


ATF- LIKE
M=8, ℓ=2
R/a=5.5
ɩ0=.15
ɩa=.7
STABLE
Vac.
β0=10%
β0=20%

$R/a \approx 3$ $\quad$ $\ell = 1, m = 7$



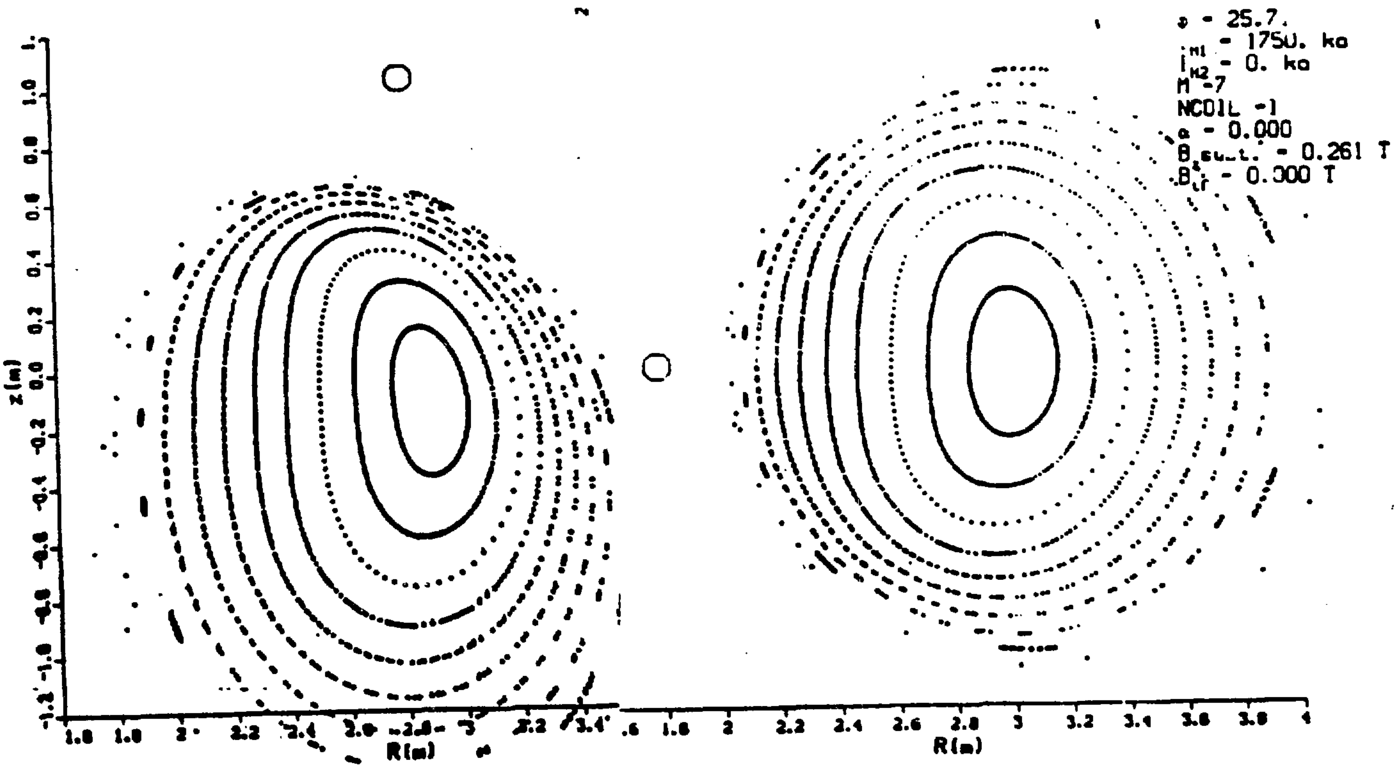


$\iota_0 = .25$

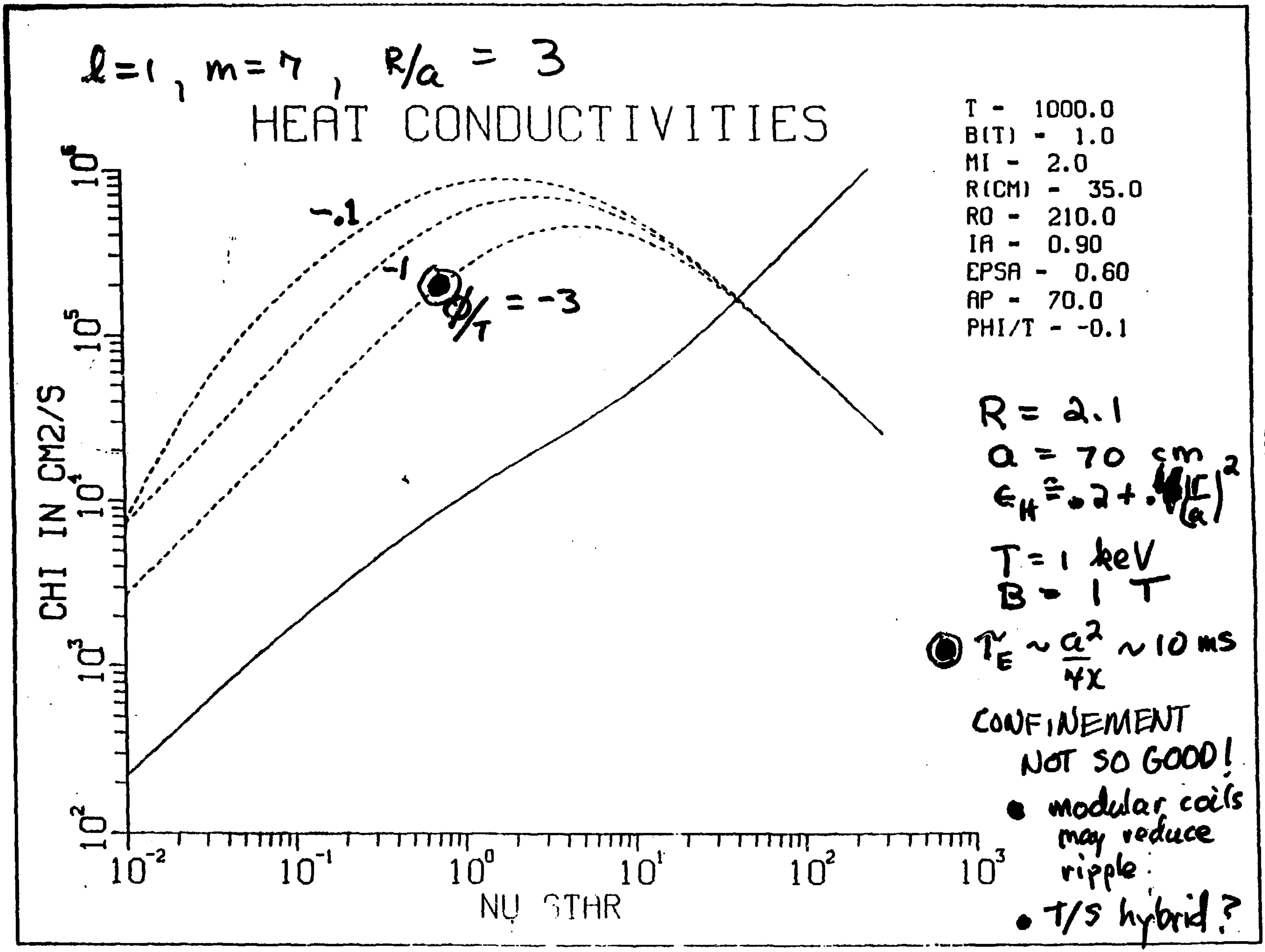
$\iota_a \simeq .7$

1% well

$\left.\frac{\epsilon_H}{\epsilon_t}\right|_{\text{most of plasma}} \sim 2.5$



IF ACCEPT LOWER $\iota \Rightarrow R/a \to 2$



l=1, m=7, R/a = 3
HEAT CONDUCTIVITIES
T - 1000.0
B(T) - 1.0
MI - 2.0
R(CM) - 35.0
RO - 210.0
IA - 0.90
EPSA - 0.60
AP - 70.0
PHI/T - -0.1
-.1
-1
Φ/T = -3
CHI IN CM2/S
NU STHR
R = 2.1
a = 70 cm
T = 1 keV
B = 1 T
τ_E ~ a²/4χ ~ 10 ms
CONFINEMENT NOT SO GOOD!
• modular coils may reduce ripple.
• T/S hybrid?

STEEPEST DESCENT MOMENT METHOD FOR THREE-DIMENSIONAL MHD EQUILIBRIA*

by

S. P. HIRSHMAN

J. C. WHITSON

OAK RIDGE NATIONAL LABORATORY

SHERWOOD MTG - 3/83

*Research sponsored by the Office of Fusion Energy, OER, U. S. Department of Energy under contract W-7405-eng-26 with the Union Carbide Corporation.

MOMENTS MOTIVATION

- MOMENTS ARE THE FOURIER AMPLITUDES OF THE SPATIAL COORDINATES (R, ϕ, Z) IN FLUX COORDINATE BASIS

- EXPERIENCE WITH 2-D MOMENTS CODE AT ORNL INDICATES FACTORS $\gtrsim 10$ FASTER RUNNING TIMES OVER S.O.R. GRIDDED CODES

- SPEED OF MOMENTS CODE ADEQUATE FOR "ON-LINE" DATA ANALYSIS, COUPLING TO TRANSPORT MODULES, ETC.

SOME NUMERICAL ADVANTAGES OF MOMENTS METHOD

- GREATLY SIMPLIFIES DIFFERENCING OF EQUILIBRIUM EQUATIONS IN SPACE (ONLY 1D DIFFERENCES IN ρ)

- STABLE ITERATION TIME STEP EASY TO DETERMINE IN 1D

- TRIGONOMETRIC INTERPOLATION PRODUCES SMOOTH EQUILIBRIUM PROFILES WITH MINIMUM NUMBERS OF ANGLE MESH POINTS

HISTORICAL SURVEY OF MOMENTS-RELATED ENDEAVORS

- 1978 : BAUER, BETANCOURT, GARABEDIAN:
  3-D LAGRANGIAN-COORDINATE CODE (REAL SPACE - NO MOMENTS) - UTILIZE DESCENT METHOD

- 1981 : LAO, ET. AL.
  INTRODUCE LAGRANGIAN VARIATIONAL (MOMENT) PRINCIPLE FOR 2-D EQUILIBRIA

- 1982: BHATTACHARJEE, ET. AL.
  GENERALIZE LAO, ET. AL., TO 3-D ; δL PRINCIPLE NOT SUITABLE FOR STEEPEST DESCENT

- 1982^{+} : ZAKHAROV, PUSTOVITOV, SHAFRANOV:
  MERGE METHOD OF AVERAGING WITH MOMENT APPROACH

GOAL OF MOMENTS METHOD

- PERIODICITY OF TORUS $\Rightarrow$ MOMENT EXPANSION

$$R(\rho,\theta,\zeta) = \sum R_{mn}(\rho)\, e^{i(m\theta - n\zeta)}$$

$$Z(\rho,\theta,\zeta) = \sum Z_{mn}(\rho)\, e^{i(m\theta - n\zeta)}$$

- DETERMINE (FINITE) SET OF MOMENTS (R_{mn}, Z_{mn}) SO MHD EQUILIBRIUM IS SATISFIED (APPROXIMATELY):

$$\vec{F} \equiv \nabla p - \vec{J} \times \vec{B} = 0$$

$$\mu_0 \vec{J} = \nabla \times \vec{B}$$

$$\nabla \cdot \vec{B} = 0$$

- THIS IS AN "INVERSE" SOLUTION

$$\underset{\text{dependent}}{(R, \phi, Z)} \longrightarrow \underset{\text{independent}}{(\rho, \theta, \zeta)}$$

MAGNETIC FIELD REPRESENTATION: THE NEED FOR ANGLE RENORMALIZATION

- IN 2-D

$$\vec{B} = \underbrace{\nabla\zeta \times \nabla\chi}_{\vec{B}_P} + \underbrace{F\,\nabla\zeta}_{\vec{B}_T}$$

- IN 3-D: CLEBSCH (CONTRAVARIANT)

$$\vec{B} = \nabla\zeta \times \nabla\chi + \nabla\Phi \times \nabla\theta^*$$

$(\zeta, \theta^*) \rightarrow$ straight field lines:

$$\left.\begin{aligned} B^{\theta^*} &\equiv \vec{B}\cdot\nabla\theta^* = \chi'/\sqrt{g} \\ B^{\zeta} &\equiv \vec{B}\cdot\nabla\zeta = \Phi'/\sqrt{g} \end{aligned}\right\rangle \quad \frac{B^{\theta^*}}{B^{\zeta}} = \iota(\rho)$$

- IN 2-D, θ ABSENT (EXPLICITLY); REQUIREMENT OF CONVERGENT SERIES FOR R, Z (IN θ, ζ) CONFLICTS WITH STRAIGHT FIELD LINES;

$$\underbrace{\theta^*}_{\text{MAGNETIC}} = \underbrace{\theta}_{\text{GEOMETRIC}} + \underbrace{\lambda(\rho, \theta, \zeta)}_{\text{PERIODIC}}$$

ENERGY PRINCIPLE IN INVERSE COORDINATES: INTRODUCING THE MOMENT EXPANSION

- KRUSKAL - KULSRUD PROVED

$$W = \int \left[\frac{B^2}{2\mu_0} + \frac{p}{\gamma - 1} \right] dV$$

IS STATIONARY FOR FLUX AND MASS CONSERVING VARIATIONS OF $\vec{B}$ AND p

$$\delta \vec{B} = \nabla \times \vec{\xi} \times \vec{B}$$

$$\delta \rho = -\nabla \cdot \vec{\xi} \rho$$

$$p = \rho^{\gamma} \qquad \Rightarrow \quad \delta W = 0 \Rightarrow \vec{F}_{MHD} = 0$$

- SCALAR INVARIANCE OF W

CAN EVALUATE W DIRECTLY IN FLUX COORDINATES AND THEN PERFORM VARIATION → INVERSE EQUILIBRIUM

$$\vec{x} \equiv (R, \phi, Z) = \vec{x}(\rho, \theta, \zeta)$$

RECALL: $\vec{x}$ is dependent

(ρ, θ, ζ) is independent

Variation of W in Inverse Coordinates

$$W_{\text{inverse}} = \int \left[\frac{B^i B_i}{2\mu_0} + \frac{M(\rho)(V')^{-\gamma}}{\gamma - 1} \right] d\theta\, d\zeta\, d\rho \sqrt{g}$$

where:

$$p(\rho) = M(\rho)(V')^{-\gamma} \qquad \text{(adiabatic law)}$$

$$V' = \iint d\theta\, d\zeta \sqrt{g} \qquad \text{(differential volume)}$$

$$B_i = g_{i\theta} B^\theta + g_{i\zeta} B^\zeta \qquad \text{(covariant components of B)}$$

$$g_{ij} = R_i R_j + Z_i Z_j + R^2 \delta_{i\zeta} \qquad \text{(metric elements for } \zeta = \phi \text{)}$$

$$R_\rho = \frac{\partial R}{\partial \rho}, \text{ etc.}$$

$$\sqrt{g} = R\,\tau; \qquad \text{(3D Jacobian)}$$

$$\tau = R_\theta Z_\rho - R_\rho Z_\theta \qquad \text{(2D Jacobian)}$$

- let $R(\rho, \theta, \zeta) = R(\rho, \theta, \zeta; t)$; take

$$\frac{dW}{dt} = \frac{\partial W}{\partial R}\frac{dR}{dt} + \frac{\partial W}{\partial Z}\frac{dZ}{dt} + \frac{\partial W}{\partial \lambda}\frac{d\lambda}{dt}$$

CALCULATION OF INVERSE EQUILIBRIUM FORCES

- $$\frac{dW}{dt} = -\int_V \left(F^R \frac{dR}{dt} + F^Z \frac{dZ}{dt} + F^\lambda \frac{d\lambda}{dt} \right)$$

  + surface terms

- The forces F^j conjugate to $x^j = (R, Z, \lambda)$ are co-variant components of MHD residual force $\vec{F}$:

$$F^R = \vec{e}^R \cdot \vec{F} \qquad \vec{e}^R = \sqrt{g}\, \nabla\phi \times \nabla Z$$

$$F^Z = \vec{e}^Z \cdot \vec{F} \qquad \vec{e}^Z = \sqrt{g}\, \nabla R \times \nabla\phi$$

$$(1+\lambda_\theta) F^\lambda = \vec{e}^\theta \cdot \vec{F} \qquad \vec{e}^\theta = \sqrt{g}\, \nabla J \times \nabla\rho$$

$$\sim \vec{J} \cdot \nabla\rho$$

- SUBSTITUTE MOMENT EXPANSIONS FOR R, Z AND λ INTO $\dot{W}$ TO OBTAIN CONJUGATE FORCES IN FOURIER SPACE

- $$\frac{dW}{dt} = -\int d\rho \left[\dot{R}_{mn} (F^R_{mn})^* + \dot{Z}_{mn} (F^Z_{mn})^* + \dot{\lambda}_{mn} (F^\lambda_{mn})^* \right]$$

- F^j_{mn} ARE TRANSFORMS OF F^j:

$$F^j_{mn}(\rho) = \iint d\theta\, d\zeta\, e^{i(m\theta - n\zeta)} F^j$$

- IN EQUILIBRIUM, $\dot{W} = 0 \Rightarrow$

$$\boxed{F^j_{mn}(\rho) = 0}$$

- EXCEPT FOR $p = M^\gamma$ AND F^λ, F^j_{mn} CALCULATED HERE AGREE WITH BHATTACHARJEE'S RESULTS

APPLICATIONS OF VARIATIONAL PRINCIPLE

- TRUNCATION OF FINITE SERIES EXPANSION IN NATURAL, MOST CONVERGENT WAY (RITZ METHOD)

- YIELDS FORCES F^j IN A CONSERVATIVE FORM BEST FOR NUMERICAL DIFFERENCING:

$$F^R = \frac{\partial}{\partial\rho}(RZ_\theta P) - \frac{\partial}{\partial\theta}(RZ_\rho P) + \frac{\partial}{\partial\theta}(B^\theta b_R) + \frac{\partial}{\partial\zeta}(B^\zeta b_R) + \frac{\sqrt{g}}{R}\left[P - \frac{(RB^\zeta)^2}{\mu_0}\right]$$

$$P = p + B^2/2\mu_0$$

- ENERGY PRINCIPLE ($W > 0$ for $\gamma > 1$) SUGGESTS DESCENT ALGORITHM FOR SOLVING LARGE NOS. OF F^j_{mn} SIMULTANEOUSLY — (CAN'T USE $\delta\mathcal{L}$ PRINCIPLE HERE)

DIRECT (JACOBIAN INVERSION) TECHNIQUES BECOME INAPPROPRIATE FOR LARGE NOS. OF MODES IN 3D

DESCENT ALGORITHM

- CHOOSE "PATH" IN PHASE SPACE OF $(R_{mn}, Z_{mn}, \lambda_{mn})$ ALONG WHICH W DECREASES MONOTONICALLY:

$$\frac{dR_{mn}}{dt} = F^{R}_{mn} \quad ; \quad \frac{dZ_{mn}}{dt} = F^{Z}_{mn}$$

$$\frac{d\lambda_{mn}}{dt} = F^{\lambda}_{mn}$$

THEN:

$$\frac{dW}{dt} = -\int d\rho \left[|F^{R}_{mn}|^2 + |F^{Z}_{mn}|^2 + |F^{\lambda}_{mn}|^2 \right]$$

$$\leq 0$$

$$\Rightarrow \quad \dot{W} = 0 \quad \text{iff} \quad F^{j}_{mn} = 0 \quad \forall\, m, n, j$$

- TECHNICAL NOTE (REF. 8, 8, & G)

IN PRACTICE, MUST ACCELERATE CONVERGENCE BY PUTTING IN CRITICALLY DAMPED WAVES:

$$\ddot{R}_{mn} + \frac{1}{\tau} \dot{R}_{mn} = F^{R}_{mn}$$

$$\vdots$$

ANALYTIC EXAMPLE
IN 2D: SOLOV'EV EQUILIBRIUM

- ILLUSTRATE NEED FOR ANGLE RENORMALIZATION ($\lambda \neq 0$)

SOLOV'EV SOLUTION:

$$\rho^2 = \frac{\beta_1}{\chi_0^2}\left[Z^2 R_0^2 + \frac{\beta_0}{8\beta_1}(R^2 - R_m^2)^2 \right]$$

where:

$$\chi' = 2\rho\chi_0$$

$$F^2 = R_0^2(1 - 4\beta_1\rho^2) \qquad [B_T^2(0) = 1]$$

$$p = \beta_0(1-\rho^2)$$

- In $u = R^2$, Z COORDINATES, THIS IS AN ELLIPSE:

$$\left.\begin{aligned} u &= R_m^2\left(1 - \frac{\rho}{a}\cos\theta\right) \\ Z &= \kappa\frac{\rho}{a}\sin\theta \end{aligned}\right\} \text{Geometric Solution}$$

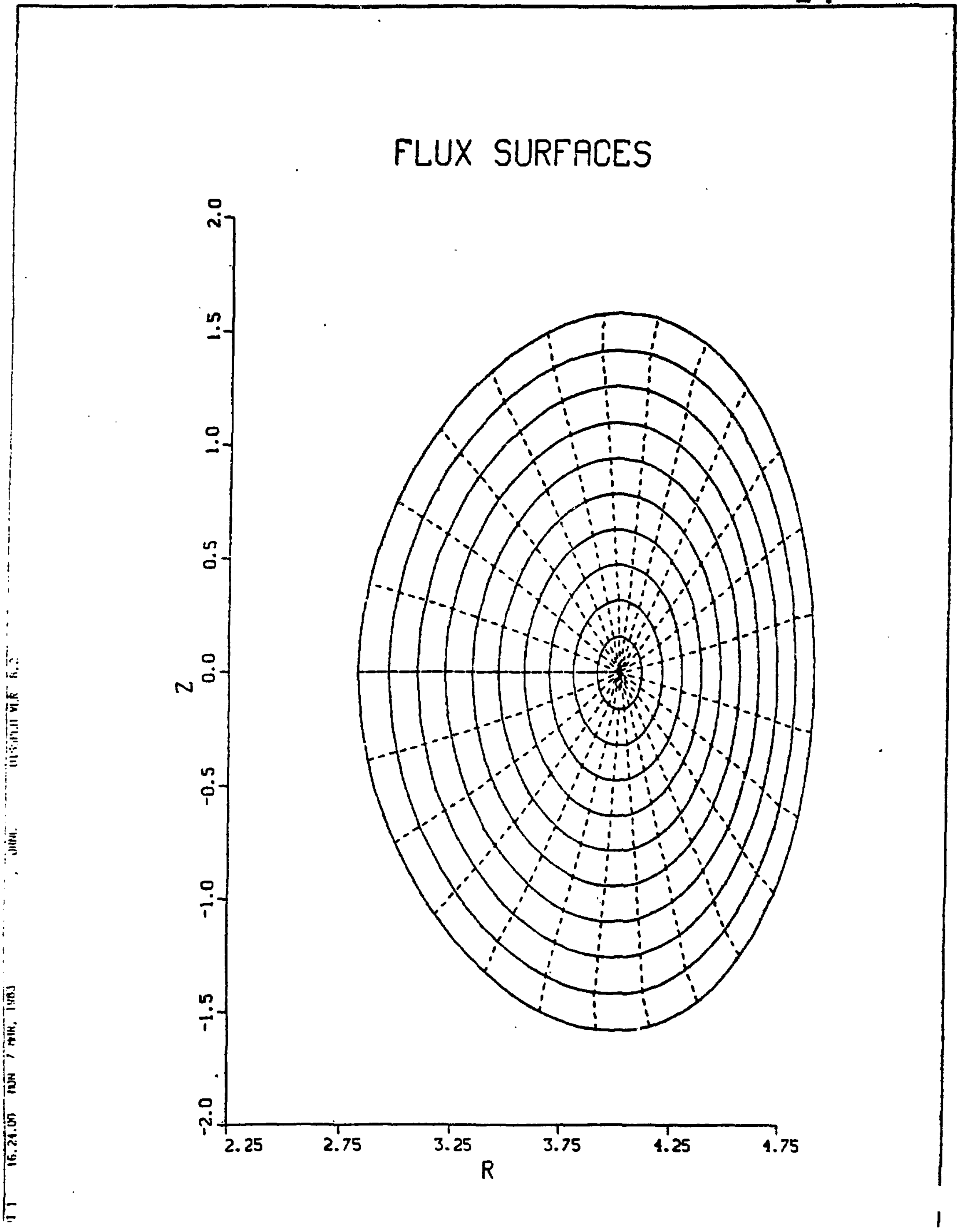
where $a^{-1} = \chi_0(8/\beta_0)^{1/2} R_m^{-2}$

$$\kappa = \frac{R_m^2}{R_0}\left(\frac{\beta_0}{8\beta_1}\right)^{1/2}$$

Solov'ev Equilibrium

$R^2 = 16(1 - \frac{1}{2}\rho\cos\theta)$

$Z =$ [illegible]$\rho\sin\theta$



- Can evaluate Jacobian

$$\sqrt{g} = \tfrac{1}{2}(u_\theta z' - u' z_\theta)$$

$$= \frac{\chi_0^2}{R_0}\left(\frac{2}{\beta_0\beta_1}\right)^{1/2}\rho \quad \sim \rho$$

- Obviously, in this representation, the neglect of λ is incorrect, since

$$J\cdot\nabla\rho = 0 \quad \Rightarrow \quad \frac{\partial}{\partial\theta}\frac{R^2}{\sqrt{g}} = 0$$

but

$$\frac{\sqrt{g}}{R^2} \sim \frac{\rho}{1-\frac{\rho}{a}\cos\theta} \qquad : \text{not independent of } \theta$$

- Paradox resolved :

$$J\cdot\nabla\rho = 0 \quad \Rightarrow \quad \frac{\partial}{\partial\theta}\left[\frac{R^2}{\sqrt{g}}(1+\lambda_\theta)\right] = 0$$

$$\Rightarrow \quad \lambda_\theta = \frac{\left(1-\frac{\rho^2}{a^2}\right)^{1/2}}{1-\frac{\rho}{a}\cos\theta} - 1$$

- IN THIS REPRESENTATION, CAN COMPUTE FOURIER EXPANSION COEFFICIENTS OF BOUNDARY ANALYTICALLY :

$$Z_b = \frac{\sqrt{10}}{2} \sin\theta \qquad \left(R_0 = 4; \ \beta_0 = \frac{1}{8}, \ \beta_1 = \frac{1}{40}\right)$$

i.e. $$Z_{bn} = \frac{\sqrt{10}}{2} \delta_{n1}$$

$$R_0 = \frac{1}{\pi} \int_0^\pi 4\left(1 - \frac{1}{2}\cos\theta\right)^{1/2}$$

$$= \frac{8}{\pi} \sqrt{\frac{3}{2}}\, E\left(\frac{2}{3}\right) \doteq 3.933$$

$$R_n = \frac{2}{\pi} \int_0^\pi 4\cos n\theta \left(1 - \frac{1}{2}\cos\theta\right)^{1/2}$$

$R_1 = -1.0259$

$$\left.\begin{aligned} R_2 &= -6.809\ 10^{-2} \\ R_3 &= -9.08\ 10^{-3} \\ R_4 &= -1.52\ 10^{-3} \end{aligned}\right\} \text{note exponential drop-off}$$

$\vdots$

- MAIN CONCLUSION:

RAPID CONVERGENCE OF GEOMETRIC SERIES IN θ

IS GENERAL INCOMPATIBLE WITH STRAIGHT MAGNETIC

FIELD LINES

- IN DESCENT ALGORITHM, λ IS NECESSARY

TO ALLOW θ^* LINES TO "SLIP" AT BOUNDARY:

$$R(\rho, \theta^*) = R^*_n(\rho) \cos n\theta^*$$

$$\Rightarrow \dot{R} = \underbrace{\dot{R}^*_n(\rho) \cos n\theta^*}_{=0 \text{ at bdy}} + \underbrace{\frac{\partial R}{\partial \theta}}_{\text{\textless change } \neq 0} \dot{\lambda}$$

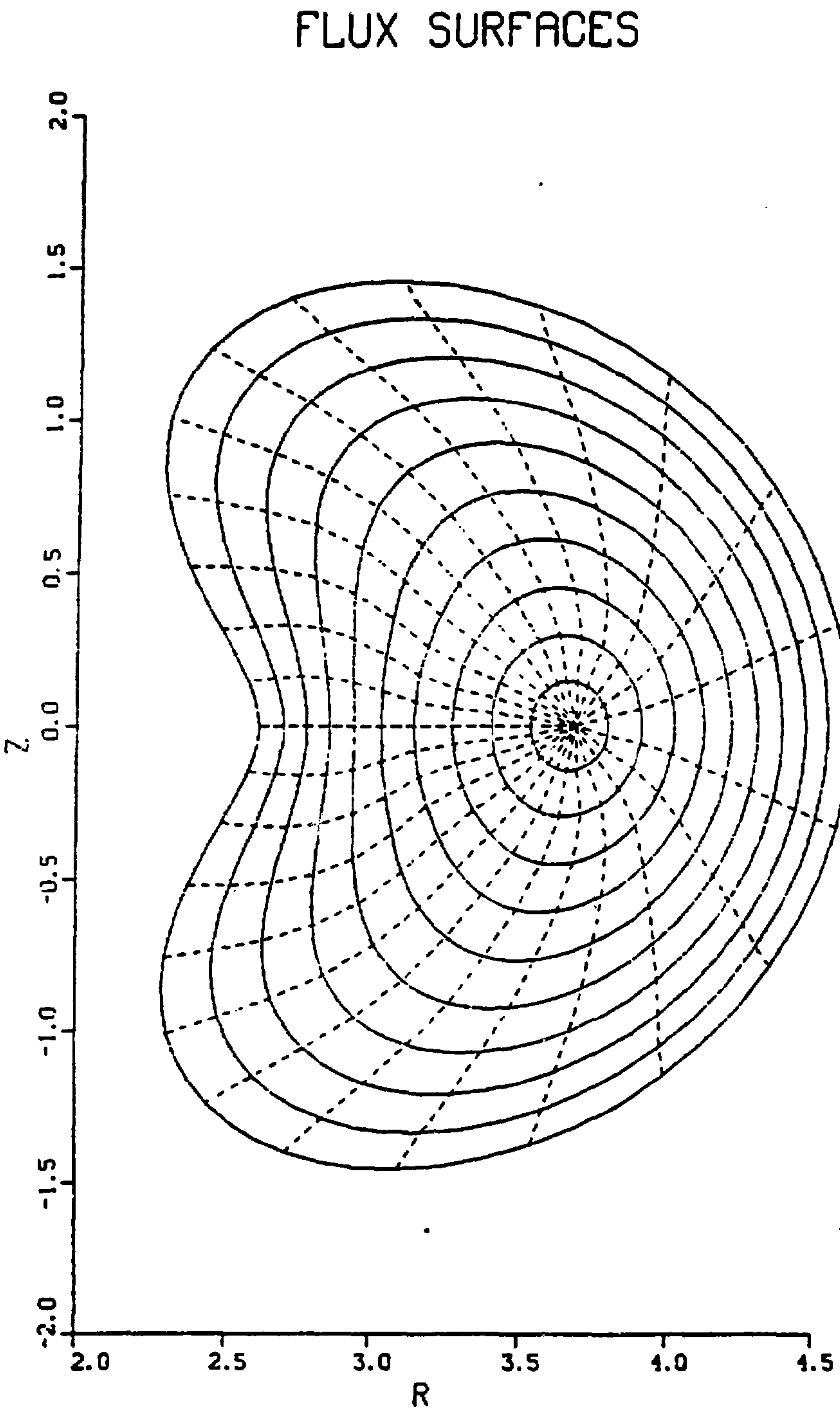
(doesn't change shape of bdy)

$n_R = 4$
$n_z = 3$
$n_v = 3$ $\langle\beta\rangle = 2\%$

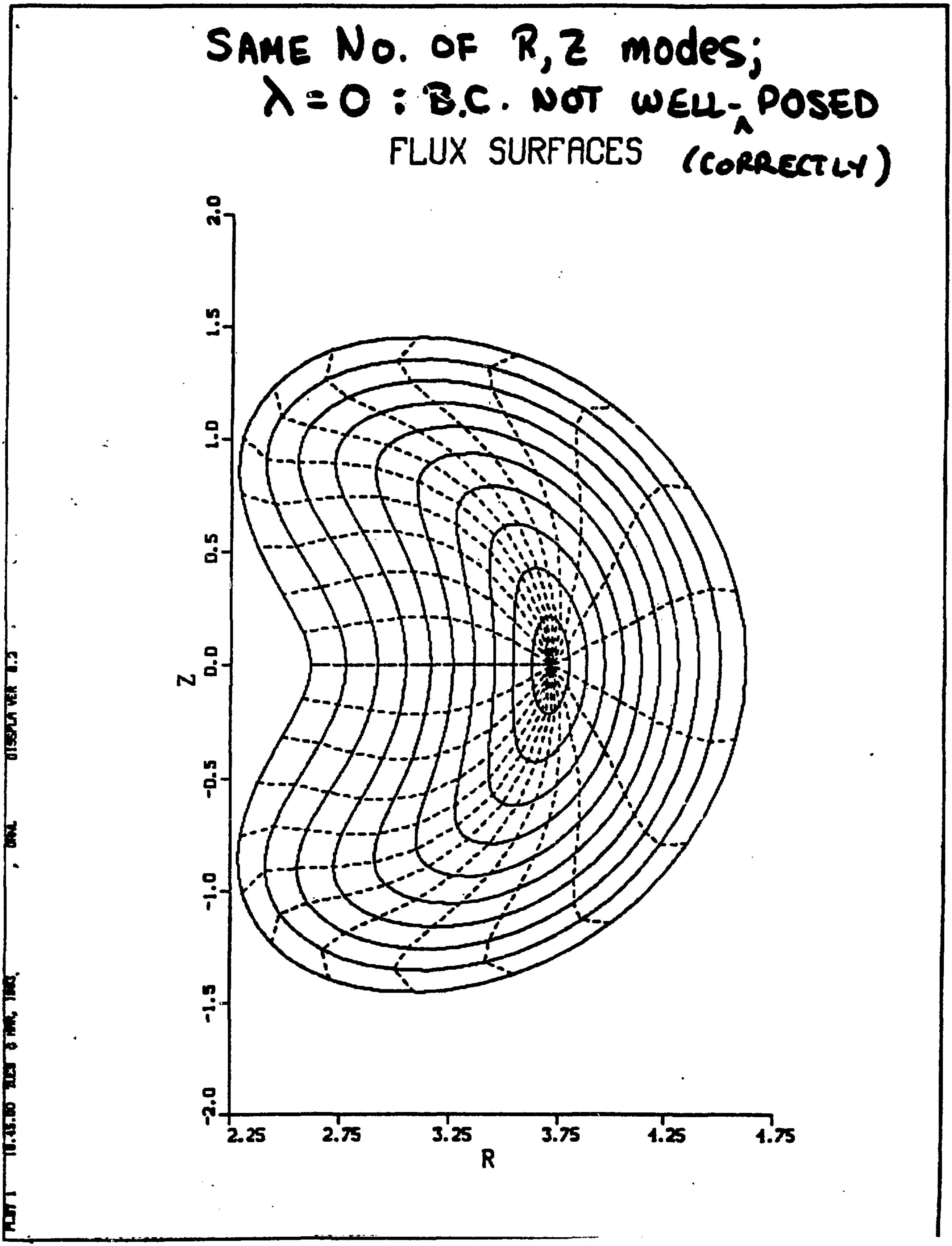
Case 1
Hi - β bean
3/16/83

$R_b = 3.065 - \cos\theta + .55\cos 2\theta$

$Z_b = 1.3\sin\theta - .36\sin 2\theta$



hi β Bean $n_R = 4$
3/7/83 Case 3 $n_Z = 3$; $n_\lambda = 0$



ATF Test Case

- $\iota \sim \frac{1}{3} + \frac{2}{3}\rho^2$ (External transform)

  $A \sim 7$

- ATF is $\ell_p = 2$, $N_T = 12$ coil device

  For our test, used $N_T = 2$ (not the same pitch)

- Outer Boundary:

  $\underbrace{\text{Rotating Ellipse}}_{\text{helical part}} + \underbrace{\text{Fixed "Egg" Shape}}_{\text{toroidal part}}$

$$R_b = 2.05 - 0.025\cos 2\theta - 0.275\cos(\theta - \phi_N) + 0.075\cos(\theta + \phi_N) + 0.0125\left[\cos 2(\theta - \phi_N) + \cos 2(\theta + \phi_N)\right]$$

$$Z_b = -0.0135\sin 2\theta + 0.315\sin(\theta - \phi_N) + 0.065\sin(\theta + \phi_N) + 0.00675\left[\sin 2(\theta - \phi_N) + \sin 2(\theta + \phi_N)\right]$$

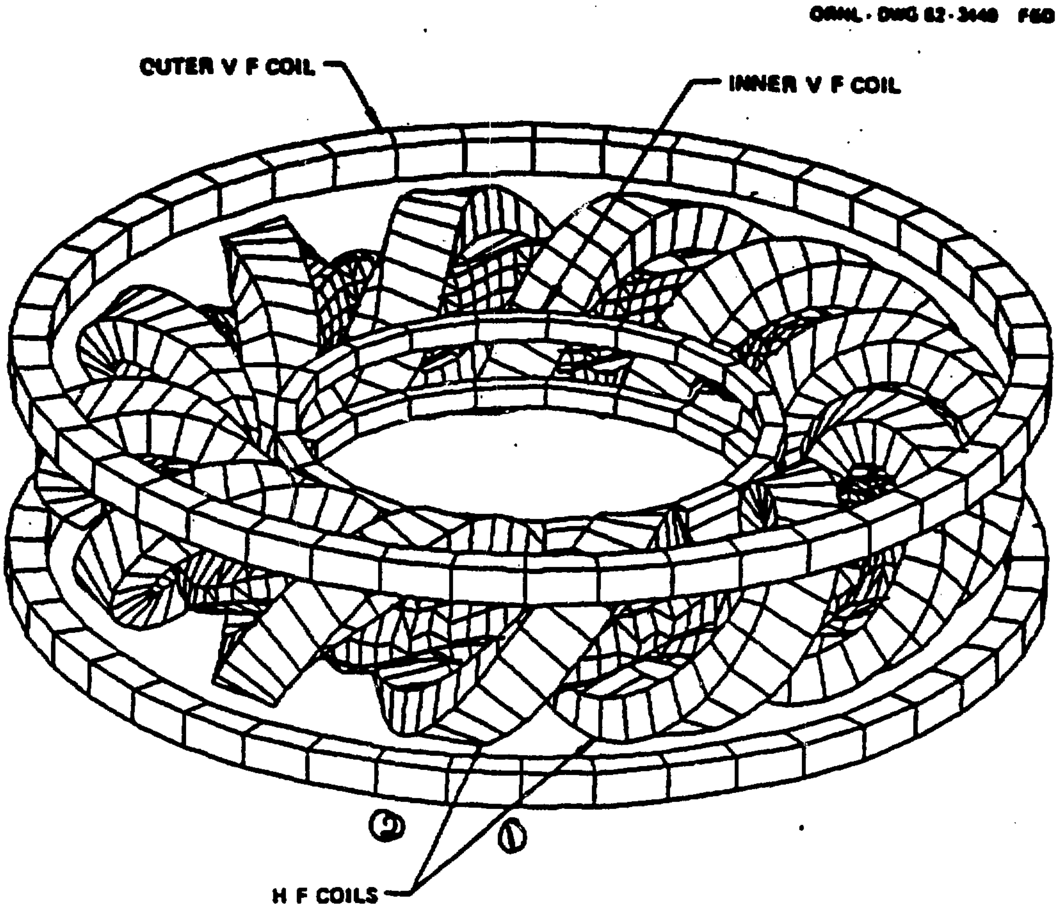


Fig. 1.2.1. The ATF-1 coil set, showing the last closed flux surface inside the HF coils.

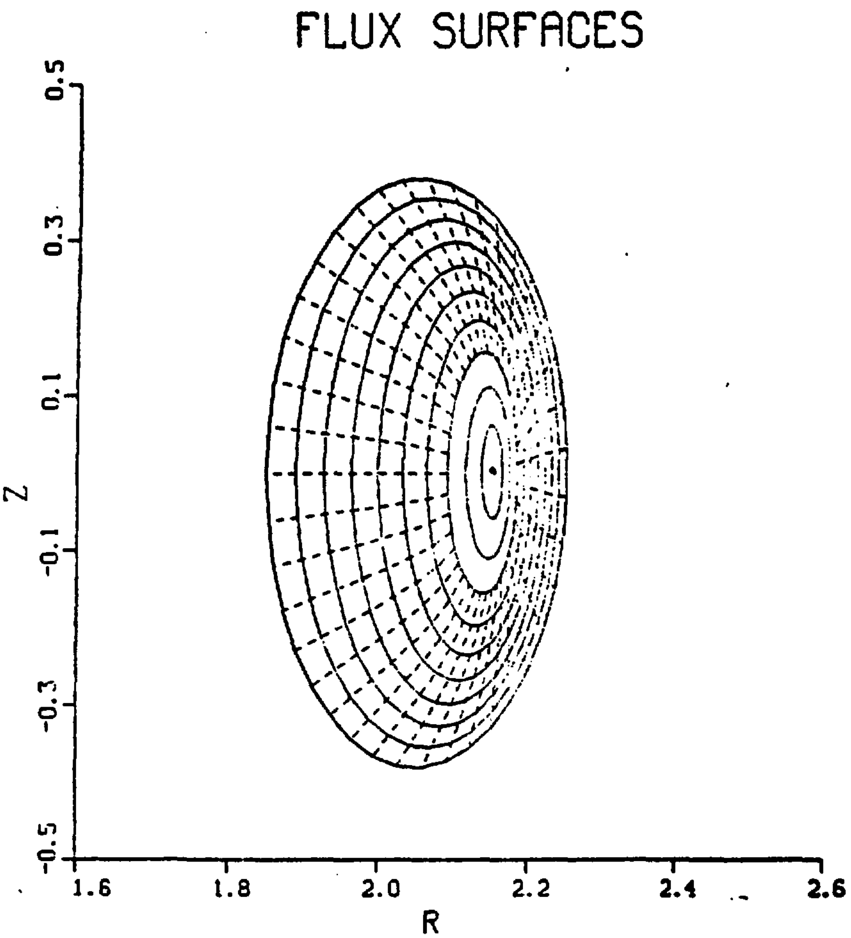
$m = \ell = 2$

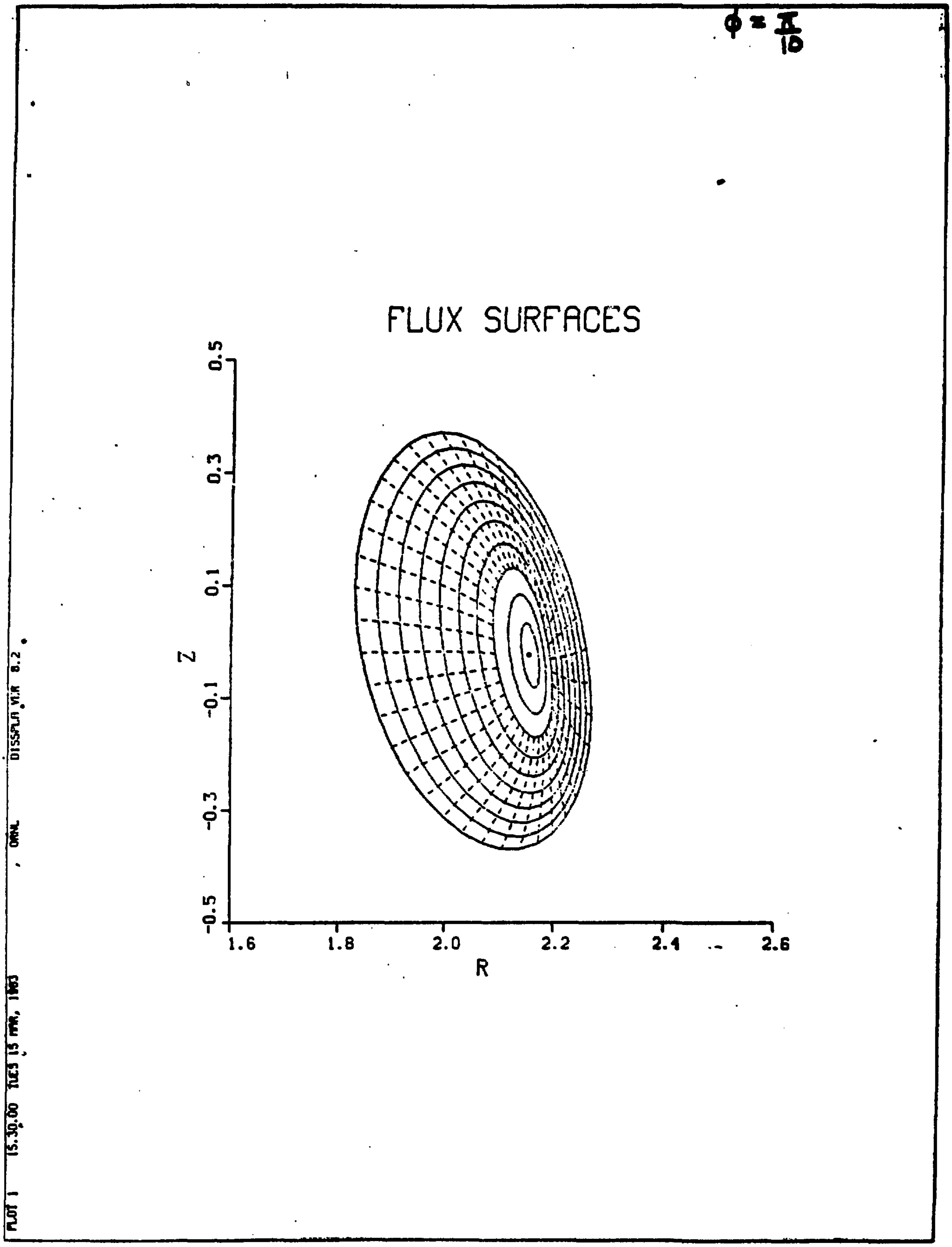
$N = 12$

- RUN TOOK 160 sec. on cray

$q = [1/(.33 + .67(r^2))]$

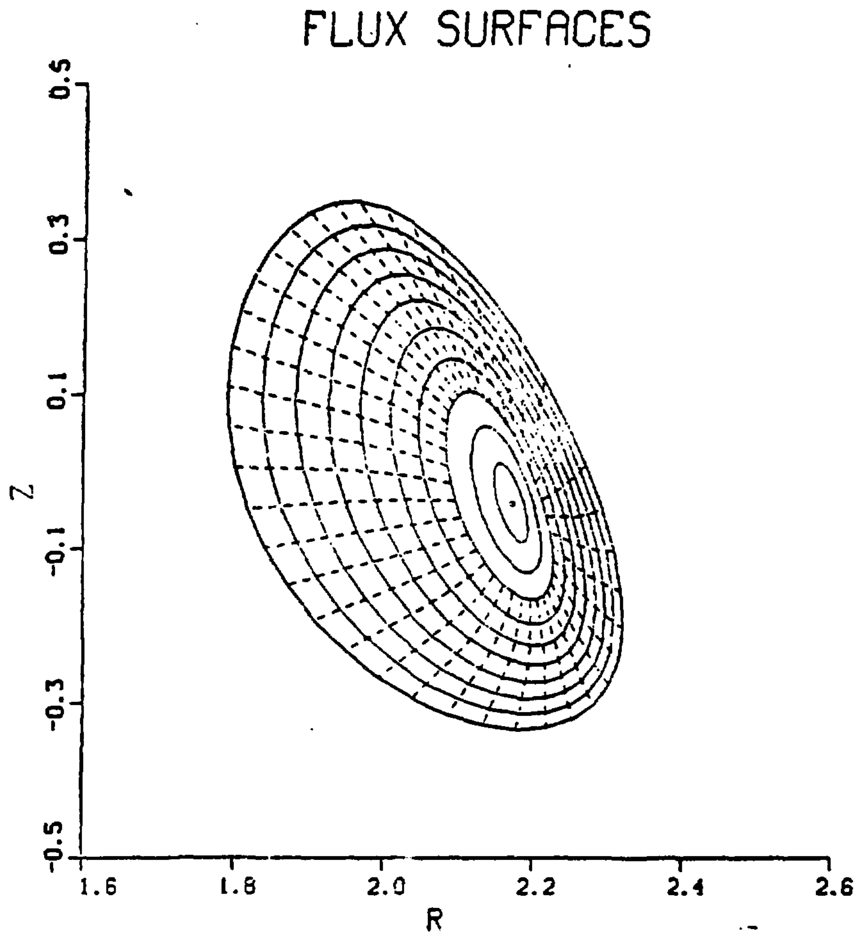
$\phi = 0$





$\phi = \frac{\pi}{10}$
FLUX SURFACES
0.5
0.3
0.1
Z
-0.1
-0.3
-0.5
1.6
1.8
2.0
2.2
2.4
2.6
R
PLOT 1 15.30.00 TUES 15 MAR, 1983 , ORNL DISSPLA VER 8.2

$\phi = \frac{2\pi}{10}$



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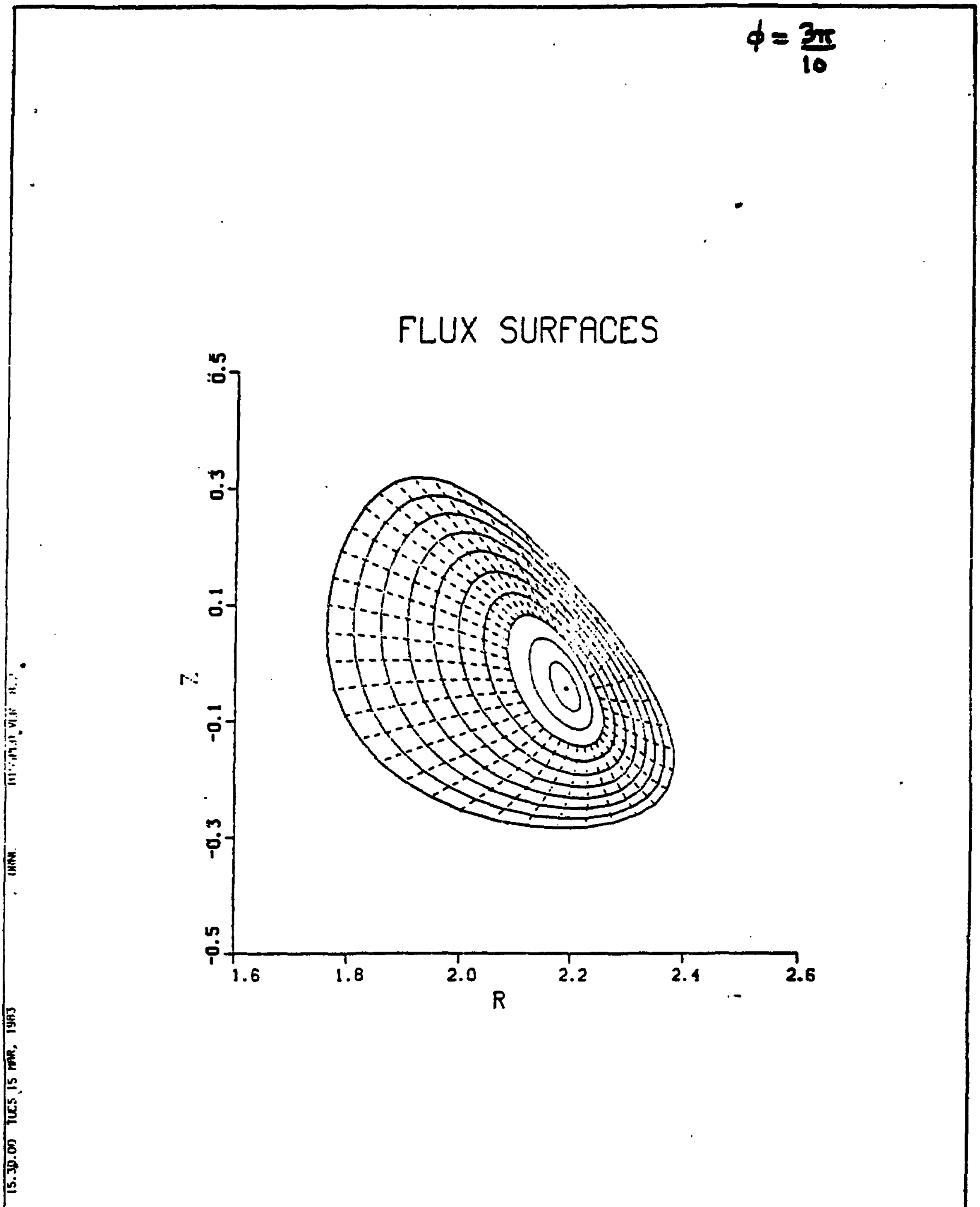
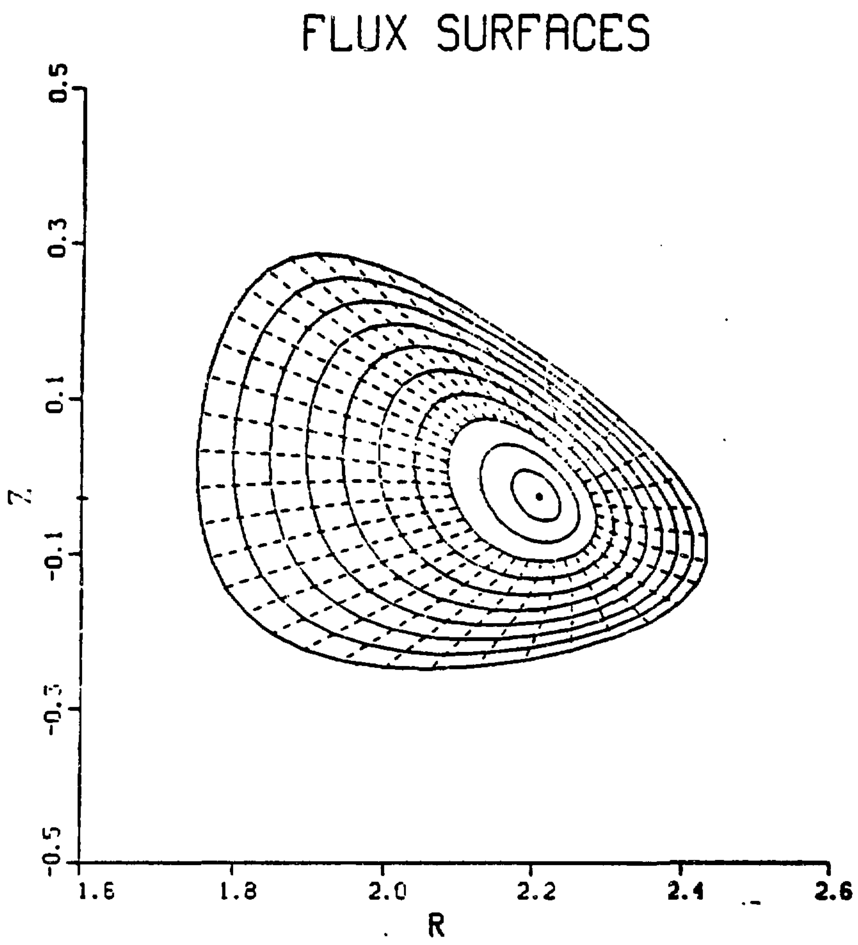


$\phi = \frac{3\pi}{10}$
FLUX SURFACES
Z
R
0.5
0.3
0.1
-0.1
-0.3
-0.5
1.6
1.8
2.0
2.2
2.4
2.6
PLOT 1
15.30.09 TUES 15 MAR, 1983

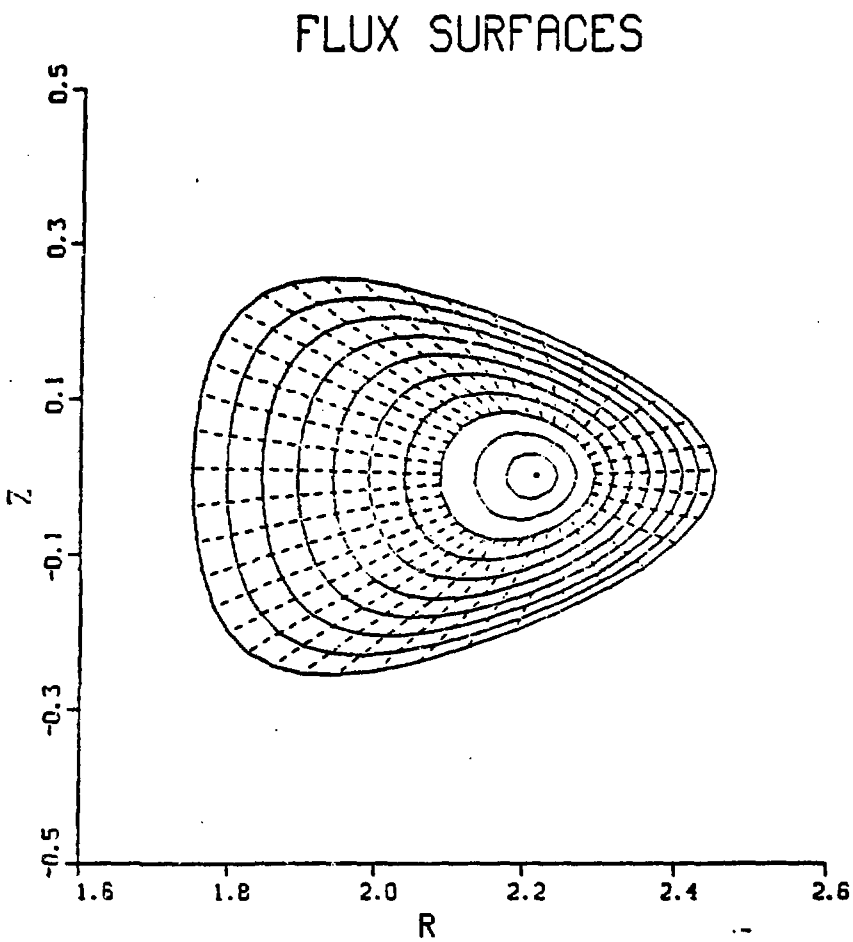
$\langle\beta\rangle \sim 5\%$ $\quad t = \frac{1}{3} + \frac{2}{3}\rho^2$

$\phi = \frac{4\pi}{10}$



$\langle\beta\rangle \sim 3\%$
$\iota = \frac{1}{3} + \frac{2}{3}\rho^2$

$\phi = \frac{5\pi}{10}$



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CONCLUSIONS

- 3D MOMENTS CODE UNDER DEVELOPMENT SHOWS PROMISING INDICATIONS OF A SIGNIFICANT REDUCTION IN RUNNING TIME COMPARED WITH GRIDDED CODES

- FUTURE TRENDS

  CONSIDERING SOLVING λ EQN. BY MATRIX INVERSION AT EACH RADIAL POINT (UNCOUPLED)

AUTHOR INDEX

LIST OF ATTENDEES

Fourth U. S. Stellarator Workshop

Oak Ridge, Tennessee

April 14-15, 1983

Dr. D. B. Batchelor
Oak Ridge National Laboratory
P. O. Box Y
Oak Ridge, Tennessee 37830

Dr. H. L. Berk
Institute for Fusion Studies
University of Texas
Austin, TX 78712

Dr. O. Betancourt
Courant Inst. for Math. Sci.
New York University
251 Mercer St.
New York, NY 10012

Dr. A. H. Boozer
Plasma Physics Laboratory
Princeton University
P. O. Box 451
Princeton, NJ 08544

Dr. J. D. Callen
Nuclear Engineering Dept.
University of Wisconsin
Madison, WI 53706

Dr. B. A. Carreras
Oak Ridge National Laboratory
P. O. Box Y
Oak Ridge, TN 37830

Dr. John Cary
Institute for Fusion Studies
University of Texas
Austin, TX 78712

Dr. L. A. Charlton
Oak Ridge National Laboratory
P.O. Box Y
Oak Ridge, TN 37830

Dr. E. Chrien
Los Alamos National Laboratory
P. O. Box 1663
Los Alamos, NM 87545

Dr. T. K. Chu
Plasma Physics Laboratory
Princeton University
P. O. Box 451
Princeton, NJ 08544

Dr. W. A. Cooper
Oak Ridge National Laboratory
P. O. Box Y
Oak Ridge, TN 37830

Dr. R. A. Dory
Oak Ridge National Laboratory
P. O. Box Y
Oak Ridge, TN 37830

Dr. W. F. Dove
Office of Fusion Energy, OER
U. S. Department of Energy
M.S. G-256
Washington, DC 20545

Dr. A. G. Engelhardt
Los Alamos National Laboratory
MS F646
P. O. Box 1663
Los Alamos, NM 87545

Dr. R. H. Fowler
Oak Ridge National Laboratory
P. O. Box Y
Oak Ridge, TN 37830

Dr. L. Garcia
Oak Ridge National Laboratory
P. O. Box Y
Oak Ridge, TN 37830

Dr. M. Gouge
U. S. Department of Energy
Oak Ridge Operations
Oak Ridge, TN 37830

Dr. J. H. Harris
Oak Ridge National Laboratory
P. O. Box Y
Oak Ridge, TN 37830

Dr. D. E. Hastings
Oak Ridge National Laboratory
P.O. Box Y
Oak Ridge, TN 37830

Dr. C. L. Hedrick
Oak Ridge National Laboratory
P.O. Box Y
Oak Ridge, TN 37830

Dr. T. C. Hender
Oak Ridge National Laboratory
P. O. Box Y
Oak Ridge, TN 37830

Dr. S. Hiroe
Oak Ridge National Laboratory
P. O. Box Y
Oak Ridge, TN 37830

Dr. S. P. Hirshman
Oak Ridge National Laboratory
P. O. Box Y
Oak Ridge, TN 37830

Dr. D. J. Hoffman
Oak Ridge National Laboratory
P.O. Box Y
Oak Ridge, TN 37830

Dr. W. A. Houlberg
Oak Ridge National Laboratory
P. O. Box Y
Oak Ridge, TN 37830

Dr. J. L. Johnson
Plasma Physics Laboratory
Princeton Unviversity
P. O. Box 451
Princeton, NJ 08544

Dr. R. K. Linford
Los Alamos National Laboratory
P. O. Box 1663
Los Alamos, NM 87545

Dr. J. F. Lyon
Oak Ridge National Laboratory
P.O. Box Y
Oak Ridge, TN 37830

Dr. L. P. Mai
Westinghouse, Fusion Engineering
AESD
Box 158
Madison, PA 15663

Dr. A. A. Mirin
Lawrence Livermore National Laboratory
P. O. Box 5509
Livermore, CA 94550

Dr. R. Moses
Los Alamos National Laboratory
P. O. Box 1663
Los Alamos, NM 87545

Dr. E. Oktay
Office of Fusion Energy, OER
U. S. Department of Energy
M. S. G-256
Washington, DC 20545

Dr. L. W. Owen
Oak Ridge National Laboratory
P. O. Box Y
Oak Ridge, TN 37830

Dr. P. Politzer
NW 16-286
Mass. Institute of Technology
Cambridge, MA 02139

Dr. A. Reiman
Plasma Physics Laboratory
Princeton University
P. O. Box 451
Princeton, NJ 08544

Dr. K. V. Roberts
Inst. for Fusion Studies
University of Texas
Austin, TX 78712

Dr. J. A. Rome
Oak Ridge National Laboratory
P. O. Box Y
Oak Ridge, TN 37830

Dr. A. Schlüter
Plasma Physics Laboratory
Princeton University
P. O. Box 451
Princeton, NJ 08544

Dr. K. C. Shaing
Oak Ridge National Laboratory
P.O. Box Y
Oak Ridge, TN 37830

Dr. J. Sheffield
Oak Ridge National Laboratory
P. O. Box Y
Oak Ridge, TN 37830

Dr. A. I. Shestakov
Lawrence Livermore Natl. Lab.
L-561
P.O. Box 5509
Livermore, CA 94550

Dr. D. J. Sigmar
Oak Ridge National Laboratory
P. O. Box Y
Oak Ridge, TN 37830

Dr. D. A. Spong
Oak Ridge National Laboratory
P. O. Box Y
Oak Ridge, TN 37830

Dr. N. A. Uckan
Oak Ridge National Laboratory
P.O. Box Y
Oak Ridge, TN 37830

Dr. K. Weimer
Plasma Physics Laboratory
Princeton University
P. O. Box 451
Princeton, NJ 08544

Dr. H Weitzner
Courant Inst. of Math. Sci.
New York University
251 Mercer St.
New York, NY 10012

IV US STELLARATOR WORKSHOP

FED DESIGN CENTER OAK RIDGE

A G E N D A

THURSDAY APRIL 14 1983

0900-0910	R. A. DORY	MEETING ARRANGEMENTS
0910-0320	J. SHEFFIELD	WELCOME
0920-0930	T. K. CHU	SPECIAL PRESENTATION
0930-1010	J. F. LYON	ATF-1 STATUS
1010-1030		DISCUSSION AND COFFEE BREAK
1030-1100	K. C. SHAING	NEOCLASSICAL TRANSPORT IN A MULTIPLE HELICITY TORUS
1100-1130	J. A. ROME	MONTE-CARLO TRANSPORT IN ATF
1130-1200	W. A. HOULBERG	TRANSPORT MODELING - TOWARD SELF-CONSISTENT DETERMINATION OF THE RADIAL ELECTRICAL FIELD
1200-1230	J. D. CALLEN	STATUS OF TRANSPORT ANALYSIS
1230-200	LUNCH	
200-240	A. REIMAN	REVIEW OF HELICAL AXIS RESEARCH
240-300		DISCUSSION AND COFFEE
300-330	L. P. MAI	MODULAR HELICAL STELLARATOR
330-400	O. BETANCOURT	FREE BOUNDARY FINITE BETA HELIAC EQUILIBRIA
400-430	A. BOOZER	TRANSPORT ANALYSIS OF A SMALL HELIAC STELLARATOR
430-500	A. I. SHESTAKOV	SEARCHING FOR MODULAR COILS IN HELIAC
630-800		RECEPTION AT DESIGN CENTER

FRIDAY APRIL 15 1983

0900-1000	PANEL	DISCUSSION OF STELLARATOR PROGRAM
1000-1015		COFFEE
1015-1045	H. L. BERK	BALLOONING MODE CALCULATIONS IN STELLARATOR
1045-1115	W. A. COOPER	BALLOONING STABILITY OF 3D STELLARATOR EQUILIBRIA
1115-1145	J. H. HARRIS	COMPACT STELLARATORS (?)
1145-1215	S. P. HIRSHMAN	3D EQUILIBRIA USING A STEEPEST DESCENT MOMENTS METHOD
1215-1230	ANYONE	CLOSING REMARKS
1230		ADJOURNMENT
