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RELATIVISTIC EFFECTS IN NUCLEAR MANY-BODY SYSTEMS

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Different approaches to the formulation of relativistic many-body dynamics yield different perspectives of nature and the magnitude of "relativistic effects". The effects of Lorentz invariance appear to be relatively unimportant. Important dynamical features of spinorial many-body formalisms are effects of subnuclear degrees of freedom which are represented in the many-body forces of the covariant nuclear Hamiltonian.

I. INTRODUCTION

This talk is an attempt to gain a perspective of the apparent success of recent "relativistic" many-body calculations.[1-5] Let me start by listing minimal requirements of quantum mechanics and Lorentz invariance. These requirements are:

1. There is a Hilbert space of states. States are represented by functions for which a positive scalar product is defined.
2. The spectrum of the energy operator is positive.
3. Lorentz transformations and translations (Poincaré group) are realized by unitary operators on the Hilbert space.
4. The generators of the space-time translations have the physical significance of momentum and energy.

These requirements are included in the axioms of an axiomatic formulation of field theory[6]. Ordinarily the Hilbert space consists of all square integrable functions of a complete set of dynamical variables, but this is not a necessary feature. Alternatively the physical Hilbert space may be a submanifold of functions embedded in a larger function space. The interactions appear in the metric with which the inner product of the Hilbert space is realized.[7] These two possibilities lead to two distinctly different approaches to the formulation of relativistic many-body dynamics, which I will discuss in Secs. III and IV. respectively. In Sec. II, I will give a capsule review of some key features of the non-relativistic many-body theory which provides the standard of comparison by which "relativistic

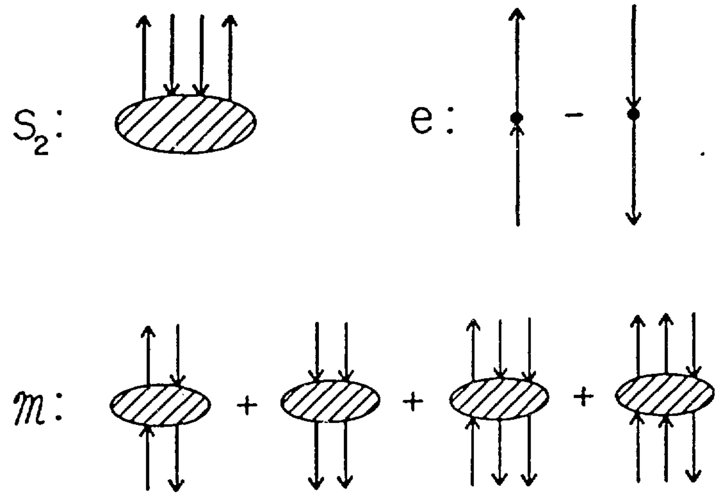

Fig. 1 Diagrams illustrating the operators S_2, e and $\mathcal{M}$.

Here Q is a projection operator that projects into particle states above the Fermi level, e contains the single particle and single hole energies and $\mathcal{M}$ contains particle-hole, hole-hole, particle-hole-hole and particle-particle-hole interactions. The operators e and are functionals of S_2. In the Brueckner approximation is neglected and Eq. (6) reduces to

$$E = \langle \Phi | H_0 | \Phi \rangle + \langle \Phi | G | \Phi \rangle \quad . \tag{10}$$

The purpose of this brief sketch is to emphasize some features of the norelativistic theory which should be kept in mind in comparing it to relativistic calculations. 1) The two- and three-body interactions in the many-body Hamiltonian are independent of the spectator nucleons. 2) The validity of Eq. (10) depends on the fact that the energy so calculated is demonstrably an approximation to the many-body Schrödinger equation, and that necessary improvements can be obtained from better approximations for $\mathcal{M}$. Calculations with two-body forces that fit scattering data have been carried out with sufficient accuracy to allow the conclusion that a realistic many-body Hamiltonian must include three-body forces.[10]

II. NONRELATIVISTIC MANY-BODY THEORY

Let $|\Phi\rangle$ be the ground state of a noninteracting Fermi gas, which satisfies the Schrödinger equation,

$$(H_0 - E_0)|\Phi\rangle = 0 \ , \quad (1)$$

where

$$H_0 = \sum_i \frac{p_i^2}{2m} \ . \quad (2)$$

The ground state $|\Psi\rangle$ of the complete Hamiltonian H,

$$H = \sum_i \frac{p_i^2}{2m} + \frac{1}{2}\sum_{ij} V_{ij} + \frac{1}{3!}\sum_{ijk} V_{ijk} \ , \quad (3)$$

normalized by $\langle\Phi|\Psi\rangle = 1$, can be expressed in the form[8]

$$|\Psi\rangle = \exp(\sum_n S_n)|\Phi\rangle \ , \quad (4)$$

where S_n is a linked operator that raises n nucleons above the Fermi level. The operators S_n are determined by coupled nonlinear equations that follow from the Schrödinger equation,

$$(H - E)|\Psi\rangle = 0 \ . \quad (5)$$

Only S_2 and S_3 are are needed to obtain the exact value of the energy E from the Hamiltonian (3). In the absence of three-body forces S_2 alone determines the energy E,

$$E = \langle\Phi|H_0|\Phi\rangle + \langle\Phi|V+VS_2|\Phi\rangle \ . \quad (6)$$

Formal elimination of the operators S_n for $n>2$ from the coupled cluster equations yields the Day equation[9] for S_2,

$$\{e(S_2) + QVQ + \mathcal{M}(S_2)\}S_2 + QV(1-Q) = 0 \ , \quad (7)$$

which is equivalent to

$$S_2 = -\frac{Q}{e}G(1-Q) + \frac{Q}{e}(1 - G\frac{Q}{e})\mathcal{M}S_2 \ , \quad (8)$$

where

$$G = (1 + V\frac{Q}{e})^{-1}V \ . \quad (9)$$

III. CANONICAL RELATIVISTIC QUANTUM MECHANICS

It is possible to satisfy the requirements listed in the Introduction within the framework of canonical quantum mechanics. This approach retains the salient features of conventional nuclear dynamics.[11-13]

The states of N nucleons are represented by square integrable functions $\phi(\vec{p}_1,\mu_1,\ldots,\vec{p}_N,\mu_N)$, $\mu_i = \pm\frac{1}{2}$. In the nonrelativistic theory the function space as well as the unitary representations of translations, rotations and Galilean boosts are the same with and without interactions. In the relativistic case the structure of the Poincaré group together with the assumption that the function space is independent of the interactions require that other transformations besides time translations depend on the interactions. This can easily be seen by the following argument. Since energy and momentum transform as a four-vector P, they must transform under the Lorentz transformation Λ according to

$$U^{\dagger}(\Lambda)P^{\mu}U(\Lambda) = \Lambda^{\mu}{}_{\nu}P^{\nu} \tag{11}$$

It follows that the generators of the rotationless Lorentz transformations, $\vec{K}$, must satisfy the commutation relations

$$[\vec{K},P^0] = i\,\vec{P} \quad \text{and} \quad [K_j,P_k] = i\delta_{jk}\,P^0\,. \tag{12}$$

If an interaction is added to P^0 then Eq. (12) requires that the interaction shows up in other generators as well. A subgroup of the Poincaré group may be chosen to be independent of the interactions and this choice of a kinematic subgroup leads to different forms of dynamics which are unitarily related to each other. The interaction terms in the Poincaré generators have simple covariance properties only for the kinematic transformations. If the Euclidean group (space translations and rotations) is kinematic, the dynamics is called "instant-form"[14], because the Euclidean group leaves the instant hyperplane t=0 invariant. For some purposes it is advantageous to include some Lorentz transformations in the kinematic subgroup. In the "front-form" dynamics[13-15] the kinematic subgroup leaves the light front $x_3+t=0$ invariant.

In either form of relativistic dynamics it is possible to construct a nuclear many-body Hamiltonian of the general form,

$$H = \sum_i \sqrt{p_i^2+m^2} + \frac{1}{2} \sum_{ij} V_{ij} + \frac{1}{3!} \sum_{ijk} V_{ijk} + \cdots \quad . \tag{13}$$

The Hamiltonian (13) has qualitatively the same structure as the nonrelativistic many-body Hamiltonian. The main new feature is that the Lorentz invariance requires relations between the two-body interactions and the many-body interactions. For nuclear many-body sytems the required many-body forces are small and the dynamics is still dominated by two-body interactions. The relativistic theory formulated here clearly reduces to the conventional nuclear many-body dynamics when all momenta are small compared to the nucleon mass. There are no qualitative differences between relativistic and nonrelativistic dynamics. Quantitative effects have been found to be smaller than rough estimates would indicate.[16,17]

Canonical field theories of nucleons and mesons also belong to the class of dynamical models discussed in this Section. The Fock space is the same function space for free fields (free particles) as for interacting fields. The Lagrangian formulation automatically introduces the interactions into both the energy operator and the dynamical Lorentz transformations in either the instant[18] or the front form.[19] In this case the number of mesons and/or nucleon-antinucleon pairs is not Lorentz invariant. Any truncation of the theory to a definite number of particles destroys the Lorentz invariance. "Relativistic effects" that are effects of the suppressed antinucleon[20] and meson degrees of freedom show up as features of the nonrelativistic two- and three-body potentials. However the role of a canonical nucleon-antinucleon field is hard to reconcile with the quark structure of the nucleons and is not required either by Lorentz invariance or by the assumption that the underlying fundamental theory is a quantum field theory.

IV. HILBERT SPACES OF COVARIANT WAVE FUNCTIONS

Let us assume that the underlying theory is a field theory of quarks and glue in which there is a Poincaré invariant physical vacuum state $|0\rangle$. The Hilbert space of the physical states is generated by functionals of the fields acting on the vacuum. It is assumed that there exists a unitary representation $U(\Lambda,a)$ on this Hilbert space. Let $\bar{\psi}(x)$ be a covariant spinor functional of the fundamental fields that creates color singlet states of baryon number 1 when acting on the vacuum. The normalization of $\bar{\psi}(x)$ is fixed by the condition

$$\langle \vec{p} | \bar{\psi}(x) | 0 \rangle = (2\pi)^{-\frac{3}{2}} \bar{u}(\vec{p}) \, e^{-ip\cdot x} , \tag{14}$$

where $|0\rangle$ is the physical vacuum, $|p\rangle$ is a physical one-nucleon state with the normalization $\langle p' | p \rangle = \delta(\vec{p}\,' - \vec{p})$, and $u(\vec{p})$ is the positive-energy solution of the Dirac equation, $(i\gamma\cdot p+m)u(p)=0$, normalized according to $u^\dagger(\vec{p})u(\vec{p}) = 1$. The p and x dependences of the right hand side of (14) follow from the covariance of the field $\psi(x)$.

With any smooth test function f of n space-time points $x_1,\ldots,x_n$ and n spinor indices $\rho_1,\ldots,\rho_n$ we can associate a state of baryon number n by

$$|f\rangle = \sum_{\rho_1} \ldots \sum_{\rho_n} \int d^4x_1 \ldots \int d^4x_n \bar{\psi}_{\rho_1}(x_1) \ldots \bar{\psi}_{\rho_n}(x_n) |0\rangle \, f(x_n,\rho_n,\ldots,x_1,\rho_1), \tag{15}$$

In the following I will always suppress the spinor indices. The inner product of two vectors $|f\rangle$ and $|g\rangle$ implies an inner product of the functions f and g,

$$\langle g | f \rangle = \int d^4x_1' \ldots \int d^4x_n' \int d^4x_1 \ldots \int d^4x_n \bar{g}(x_1,\ldots,x_n)$$

$$W(x_n,\ldots,x_1;x_1',\ldots,x_n') \; f(x_n',\ldots,x_1') \; , \tag{16}$$

where

$$W(x_1,\ldots,x_n;x_n',\ldots x_1') \equiv \langle 0 | \psi(x_1) \ldots \psi(x_n) \bar{\psi}(x_n') \ldots \bar{\psi}(x_1') | 0 \rangle \tag{17}$$

The inner product $\langle f | f \rangle$, by construction, cannot be negative, but it does vanish for many functions. We have thus a Hilbert space of equivalence classes of covariant functions, where the metric is given by the "quasi-Wightman function" W. Because of the covariance properties of the field $\psi(x)$ the inner product of the functions g and f defined in Eq. (16) is invariant under the transformation

$$f(x_1,\ldots,x_n) \rightarrow S(\Lambda_1^{-1}) \ldots S(\Lambda_n^{-1}) f(\Lambda x_1 - a,\ldots,\Lambda x_n - a) \tag{18}$$

of the functions f and g. The transformation (18) is therefore a unitary representation of the Poincaré group on the function space.

It is easy to see that for free fields we recover exactly the Hilbert space of Sec. III. Instead of introducing interactions by modifying the representations of the Poincaré group as in Sec. III one may try to construct dynamical models by generating a manifestly covariant inner product of the wave functions that depends on the interactions. For two particles (e.g. two nucleons or a nucleon and a nucleus) this can be done easily by modifying the mass-shell constraints of free particles with covariant quasipotentials.[11,21-24] In the limit in which the mass of one of the two particles tends to infinity the two-body equation reduces to the Dirac equation with a one-body potential and the physical Hilbert space is spanned by the positive energy solutions of that equation. It should be clear from the preceding derivation that covariant spinor wave functions need not be interpreted in terms of a nucleon-antinucleon Fock space. The "small components" of phenomenological Dirac wave functions need not be associated with antinucleon probabilities.

The generalization of this approach to many-body systems is not straightforward. The problem of obtaining the correct cluster properties remains unsolved. In particular there is so far no model of relativistic many-body dynamics which would justify "relativistic" Brueckner calculations.

The dynamical role of the Hilbert-space metric may still be important for systems where Lorentz invariance is unimportant. The spinorial shell model can serve as an illustration of this point. Consider the Hamiltonian

$$H = \sum_i \Lambda_{i+}(\phi_V,\phi_S)[\vec{\alpha}\cdot\vec{p} + \phi_V + \beta(m+\phi_S)]_i \Lambda_{i+}(\phi_V,\phi_S) + \frac{1}{2}\sum_{ij} \Lambda_{i+}(\phi_V,\phi_S)\Lambda_{j+}(\phi_V,\phi_S)U_{ij}\Lambda_{i+}(\phi_V,\phi_S)\Lambda_{j+}(\phi_V,\phi_S) \ , \quad (19)$$

where $\Lambda_+(\phi_V,\phi_S)$ is the projector into the positive-energy states of

$$h_1 \equiv \alpha\cdot p + \phi_V + \beta(m+\phi_S) \ , \quad (20)$$

ϕ_V,ϕ_S are shell-model potentials and U_{ij} is a residual two-body interaction. The physical Hilbert space is spanned by products of the positive energy eigenfunctions of h_1. The residual interaction U_{ij} can be (and must be) chosen such that the spectrum of H is positive. This model satisfies all the minimal requirements listed in the introduction except Lorentz invariance. For physical reasons such a spinorial shell model might well compare favorably with conventional shell models.

V. CONCLUSIONS

Requirements of relativistic invariance leave a very large amount of freedom in the formulation of dynamical nuclear models: The conventional nonrelativistic nuclear many-body dynamics can be generalized to satisfy the requirements of Poincaré invariance without altering the space of functions which represent the states. On the other hand quantum field theory suggests models in which the metric of the Hilbert space is determined by the dynamics. This approach naturally accommodates Dirac-spinor wave functions. While antinucleons, composed of antiquarks, exist in the underlying field theory the "small components" of the model wave functions are not directly associated with antinucleon degrees of freedom.

The effects of Lorentz invariance on low-energy nuclear many-body systems appear to be relatively unimportant. "Relativistic effects" are primarily effects of subnucleon degrees of freedom, but they are not convincing evidence for a dominant role of any particular degree of freedom.

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