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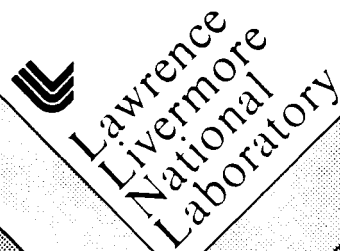
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NONLINEAR HYDRODYNAMICAL INSTABILITIES,  
TRANSITIONS, AND COHERENT FEATURES

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**FINAL REPORT****NONLINEAR HYDRODYNAMICAL INSTABILITIES,  
TRANSITIONS, AND COHERENT FEATURES**

Philip S. Marcus, Principal Investigator

The purpose of the research funded by this grant was to explore strongly nonlinear processes in hydrodynamical flows. To study the flows, large-scale numerical computations were required. Furthermore, each flow that was studied numerically was also examined in a laboratory experiment by another research group (not funded by this grant). Generally, the laboratory experiments were done after the numerical calculations were completed for the purpose of confirming the results and for exploring other parts of parameter space. Although all of the phenomena that we looked at were rich in nonlinear behavior, they were all bound together with a common thread: they exhibited temporally chaotic behavior while maintaining (or in some instances forming) coherent spatial structure.

The monies obtained by this grant were primarily used to support a post-doctoral research associate, Dr. Arthur Or. A secondary use of the monies was to defray some research costs (mostly publication charges) of the principal investigator, Philip S. Marcus (with no salary support). The funded work included studies of thermal convection, rotating Couette flows, modeling of two-dimensional turbulence, vortex dynamics, and the Red Spot of Jupiter. Eight papers supported by this grant have been published, submitted, or are about to be submitted for publication. We summarize our main results:

**Rotating Couette flows**

Experiments show that the laminar flow between two differentially rotating cylinders (Taylor-Couette flow) forms stable patterns that are almost periodic in the axial direction. (Mathematically, the periodicity is exact as the size of the apparatus goes to infinity). Experiments show further that there is a band of axial wavelengths for which the flow is stable (when the outer cylinder is held stationary). At the present time, no analytic theory correctly predicts this band. We computed the flows using an equilibrium finder that converged whether or not the flow was stable. Using these equilibria we determined a flow's linear stability and if it was unstable we computed the most unstable (quickly growing) eigenmodes and determined what new stable equilibrium the original flow evolved to.

The numerical code used in this calculation was modified to allow the study of the non-axisymmetric, spiral flows which can occur when the inner and outer cylinders of the Couette apparatus rotate in opposite directions. The spiral flows are of current interest for two reasons. (1) At low Reynolds numbers there is competition among the spirals with different pitch angles that leads to chaos. (2) Unlike most flows, it appears from the laboratory experiments that spiral flows do not have a band of stable axial wavelengths but instead have a *unique* axial

wavelength. The reasons for the uniqueness and the numerical experiments used in finding it were discussed in our WORK IN PROGRESS REPORT of June 1986 and are not repeated here. We have been the first to compute the nonlinear spiral flows and also the first to compute the "ribbon" flows which are related to spiral flows and compete with them. From our numerical simulations we have been able to show that these flows are not centrifugally driven like the usual, axisymmetric Taylor vortex flows, but are in fact viscously dominated. Exactly what role the curvature of the cylindrical boundaries plays in these flows is still not clear. We had planned on getting a quantitative understanding of the role of curvature by using the well-known "thin-gap" equation of DiPrima which is a useful approximation to the full equations that govern Couette flow because it allows the curvature effects to be separated from the centrifugal effects. The "thin-gap" equation was successfully used by DiPrima in calculations of axisymmetric Taylor-vortex flows. However, we (including graduate student (K. Coughlin) have been able to show that contrary to the findings of previous workers, the "thin-gap" equations are ill-posed for non-axisymmetric flows. We have modified the "thin-gap" equation and our new equation is well posed.

### Coherent Features in quasi-geostrophic flows

Using numerical techniques that are similar to those used in the Taylor-Couette code we have written a program that simulates the flow in a rapidly rotating cylindrical annulus with a rigid upper surface and a radially sloping bottom boundary (with no differential rotation between the radial boundaries). The two-dimensional, quasi-geostrophic approximation to the equations of motion that govern this flow are the same as the approximation for a rapidly rotating planetary atmosphere. The purpose of the study was to examine two-dimensional vortex dynamics when the vortices are embedded in a background, shearing, zonal flow. Our experiments showed the coalescence of patches of vorticity whose sign was the same as the background shear and the break up of vortical patches with the opposite sign. We showed that many of the properties of vortex mergers can be understood from linear theory and from the *ad hoc* hypothesis that the energy in the coherent part of the flow is minimized. When random forcing and dissipation were added to our numerical simulations we found that coherent spots superposed on spatial chaos spontaneously form. During the last year of this funded research we examined a simplified set of equations that corresponded to a gas of vortices on a lattice. These equations shared many of the self-organizing properties of the Navier-Stokes equations, but due to the discrete number of states they were easier to study. We also examined the properties of vortices in flows where the shear of the background zonal flow changes sign. We found that the stability of vortices depends on the sign of the shear of the *local* background zonal flow. Moreover, zones with opposite shear can stably co-exist in the same flow, with each zone containing its own vortices that interact and merge with each other but not with those in neighboring zones.

Realizing that real flows are not two-dimensional, we explored how the third dimension affects vortices. In particular we have looked at how it would affect the Red Spot of Jupiter. We were interested in determining whether coherent spots of vorticity would still exist, how they would be confined to finite extent in the vertical direction, whether they would be stable, whether they would form spontaneously from random initial conditions, and whether three-dimensionality would break the degeneracy that exists in the two-dimensional quasi-geostrophic approx-

imation between vortices that rotate in the same direction as the planet (cyclonic) and those that rotate in the opposite direction (anti-cyclonic). The observations of Jupiter show that the long-lived coherent vortices including the Red Spot are anti-cyclonic. We found that if a two-dimensional anti-cyclonic vortex slowly loses angular momentum (say due to radiation by Rossby waves), it creates a secondary flow that is three-dimensional: upward in the top half and downward in the bottom half of the vortex. A cyclonic spot produces vertical motions in the opposite direction. If the vortex lies in a stably stratified, sub-adiabatic atmosphere, this vertical motion makes the center of the upper half of an anti-cyclonic vortex cooler than the surrounding gas and the center of the lower half warmer. (This is consistent with observations of the Red Spot from Voyager.) This temperature gradient in the horizontal direction must be balanced by a vertical gradient of the horizontal velocity due to the thermal wind equation. (The latter equation is the extension of the Taylor-Proudman theorem to baroclinic flows.) The net result is that the vertical flow and thermal wind equation break the degeneracy between cyclonic and anti-cyclonic vortices. The thermal wind equation forces the horizontal winds of an anti-cyclonic vortex to decrease exponentially in magnitude as one goes away from the mid-plane of the vortex. Thus an anti-cyclonic vortex is naturally confined in the vertical direction and the horizontal velocities go smoothly to zero above and below the vortex. Cyclonic vortices have their horizontally velocities exponentially increase away from their mid-planes, are unbounded, and are therefore not allowed by the boundary conditions.