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DIFFRACTION RADIATION PRODUCED BY A CHARGED PARTICLE PASSING NEAR OR THROUGH A DIELECTRIC SPHERE *

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We have derived expressions for the spectral distribution of diffraction radiation produced when a charged particle of constant velocity passes near or through a dielectric sphere of radius a. Our expressions, which are valid in the long wavelength limit $ka \ll 1$, describe the production of radiation as a function of the particle's impact parameter and energy and as a function of the dielectric property of the sphere. Our results reduce to forms similar to Rayleigh scattering of light when $ka \to 0$ and the impact parameter is large. Certain limiting cases of our expressions are found to be significantly different from the corresponding results previously published by other workers.

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Introduction

One potentially useful method of diagnosing intense charged particle beams is by observing the diffraction radiation (DR) produced when a beam passes near an object on an array of objects. Since such a diagnostic technique is noninterceptive, it could be done without disturbing the beam or any associated experiment.

As a step toward developing a DR diagnostic technique, we have derived expressions for the spectral density of DR produced by a relativistic charged particle passing by a dielectric sphere. We have chosen a sphere simply in order to compare our results in the nonrelativistic limit to those published previously. Actual diagnostic systems may involve other geometries.

DR is intimately connected with transition- and Cherenkov radiation. A complete solution of the electromagnetic field produced by a particle passing at constant velocity through or near an interface between media with different dielectric functions yields all three types of radiation fields, depending on the details of the particles trajectory. In this paper, we first investigate some features of DR produced by relativistic particles through the use of the Williams-Weizsäcker method of virtual quanta. Since this method is strictly valid only for impact parameters much greater than the sphere's radius, we next give a treatment which is valid for any impact parameter. The limiting cases of our results when $\beta \ll 1$ are then compared with previous nonrelativistic expressions, where $\beta = v/c$, the ratio of particle velocity to light velocity. We have considered only the dipole contribution to the radiation, thus our results represent the DR correctly only for ka << 1, where k is 2π divided by the wavelength of the radiation and **a** is the sphere's radius.

Diffraction Radiation Using the Method of Virtual Quanta

The system we are considering is shown in Fig. 1: a particle of velocity $\vec{v}$ passes by a dielectric sphere of radius **a** at an impact parameter b, with $\vec{v}$ along $\hat{z}$. The method of virtual quanta consists of dividing the calculation of an electromagnetic process occurring during the collision of a charged particle with particles or photons into two parts. First, the particle's electromagnetic field is replaced by an incoherent sum of photons. The spectral density of the photon field is just the modulus squared of the Fourier transform of the particle's fields. Then the processes is calculated by considering the corresponding process of photon scattering.[1,2]

The fields of a relativistic particle are[2] (see Fig. 1).

$$E_x(t) = q\gamma b(b^2 + \gamma^2 v^2 t^2)^{-3/2}, \quad B_y(t) = \beta E_x(t), \tag{1a}$$

and

$$E_z(t) = -q\gamma vt(b^2 + \gamma^2 v^2 t^2)^{-3/2}, \quad \gamma \equiv 1/(1 - \beta^2)^{1/2}. \tag{1b}$$

If $\beta \simeq 1$, the fields of (1a) are equivalent to those of a plane polarized pulse of radiation traveling along $\hat{z}$. If a fictitious magnetic field is associated with $E_z(t)$, a second pulse traveling along $\hat{x}$ can be thought of as replacing the effect of $E_z(t)$.[1,2] The frequency spectra of pulses 1 and 2 are just

$$\frac{dI_{1,2}}{d\omega}(\omega,b) = \frac{c}{2\pi}\left|E_{x,z}(b,\omega)\right|^2, \tag{2}$$

(energy per unit area per unit frequency interval)
where $E_{x,z}(b,\omega)$ represents the Fourier transform of $E_x(t)$ or $E_z(t)$. The explicit forms of the frequency spectra are[1,2]:

$$\frac{dI_1(\omega,b)}{d\omega} = I_0\, u^2\, K_1^2(u), \qquad (3)$$

$$\frac{dI_2(\omega,b)}{d\omega} = \frac{1}{\gamma^2}\, I_0\, u^2\, K_0^2(u), \quad u = \omega b/\gamma v, \qquad (4)$$

where $K_1(u)$ and $K_0(u)$ are modified Bessel functions and

$$I_0 = \frac{q^2}{\pi^2 c} \left(\frac{c}{v}\right)^2 \frac{1}{b^2} \qquad (5)$$

is the low frequency limit of $dI_1/d\omega$. Note that $dI_2/d\omega$ is smaller than $dI_1/d\omega$ by a factor γ^{-2}. A plot of Eq. (3) versus $\ln \omega$ would reveal[2] that $dI_1/d\omega$ contains all frequencies up to $\omega_{max} \sim 1/\Delta t$, where $\Delta t \sim b/\gamma v$ is the collision time. Similarly, $dI_2/d\omega$ looks like the modulus squared of the Fourier transform of one cycle of a sine wave of frequency $\omega \sim 1/\Delta t$, which is narrowly peaked around $\omega \sim \gamma v/b$.

According to the method of virtual quanta, the frequency spectrum of radiation produced by the collision of a charged particle with a target is given by

$$\frac{dI_c(b,\omega)}{d\omega d\Omega} = \frac{d\sigma_p(\omega)}{d\Omega} \left[\frac{dI_1(b,\omega)}{d\omega} + \frac{dI_2(b,\omega)}{d\omega}\right]$$

$$[\text{energy/unit frequency-sr}], \qquad (6)$$

where $d\sigma_p(\omega)/d\Omega$ is the differential scattering cross section for scattering of a photon of frequency ω by the target, or more generally for any other process under consideration which has a counterpart in photon interactions. The differential cross section for the collision process is

$$\frac{d\sigma_c(\omega)}{d\Omega d(\hbar\omega)} = 2\pi \int_{b_{min}}^{\infty} \frac{dI_c}{d(\hbar\omega)d\Omega}\, b\, db \quad [\text{area/unit energy-sr}], \qquad (7)$$

in which the minimum impact parameter $b_{min} > 0$ must be introduced on physical grounds in order to avoid a divergent integral.

We now apply the above approach to the production of DR by a relativistic particle ($\gamma >> 1$) scattered by a dielectric sphere. We need the Rayleigh scattering cross section of a plane polarized wave when $ka << 1$. This is

$$\frac{d\sigma}{d\Omega} = \frac{k^4}{|\vec{E}_x|^2} \left| \hat{\varepsilon}^* \cdot \vec{p}(\omega) \right|^2, \tag{8}$$

where the polarization vector of the scattered wave is $\hat{\varepsilon}$, shown in Fig. 2, and

$$\vec{p}(\omega) = \frac{\varepsilon - 1}{\varepsilon + 2} a^3 \vec{E}_x(\omega) \tag{9}$$

is the Fourier transformed dipole moment induced in the sphere of dielectric constant ε by the incident photon with field amplitude $\vec{E}_x(\omega)$, provided $ka << 1$. Using (9) in (8) we have

$$\frac{d\sigma}{d\Omega} = k^4 a^6 \left|\frac{\varepsilon - 1}{\varepsilon + 2}\right|^2 \cos^2\theta, \tag{10}$$

where the angle θ is defined in Fig. 2.

If we neglect the dipole moment $p_z(\omega) \sim \gamma^{-1} p_x$, then, using Eq. (10) and the method of virtual quanta Eq. (6), we can immediately write down the following expression for DR:

$$\frac{dI}{d\omega d\Omega} = k^4 a^6 \left|\frac{\varepsilon-1}{\varepsilon+2}\right|^2 \cos^2\theta \; \{\pi^{-2} \frac{q^2}{c} \left(\frac{c}{v}\right)^2 \frac{1}{b^2} u^2 K_1^2(u)\} \tag{11a}$$

$$\xrightarrow[u << 1]{} \pi^{-2} \frac{q^2}{c} \frac{k^4 a^6}{\beta^2} \left|\frac{\varepsilon-1}{\varepsilon+2}\right|^2 \frac{\cos^2\theta}{b^2}, \; u = \frac{\omega b}{\gamma v}, \tag{11b}$$

where the term in brackets in (11a) is just $dI_1/d\omega$, Eq. (3). This procedure is valid as long as the motion of the displaced charge in the sphere is nonrelativistic and provided the field of the particle $\vec{E}_x(b,\omega)$ is essentially constant across the sphere[1], i.e. $\Delta\vec{E}_x/\vec{E}_x << 1$ where $\Delta\vec{E}_x$ is the change of the field across a diameter. The latter condition is satisfied for impact parameters $b \gg a$. Eqs. (11a) and (11b) show that for $u = \omega b/\gamma v \ll 1$, the spectral distribution is nearly independent of γ and that the $k^4 a^6$ dependence of Rayleigh scattering is obtained along with the $\cos^2\theta$ dependence of dipole radiation.

Extension of the Method to Any Value of Impact Parameter

The results of the previous section were restricted to impact parameters $b \gg a$ in order for the method of virtual quanta to be applicable. In order to extend the calculation to the range of impact parameters $b \lesssim a$, we proceed as follows: first we determine the induced dipole moment of the sphere during the passage of the charaged particle; then we use this result in the standard expression for the differential frequency spectrum for dipole radiation,

$$\frac{d^2 I}{d\omega d\Omega} = \frac{ck^4}{2\pi} \left| \hat{n} \times [\hat{n} \times \vec{p}\ (\omega)] \right|^2. \quad (12)$$

Instead of (9), we determine the dipole moment of the sphere from

$$\vec{p}(\omega) = \frac{3}{4\pi} \frac{\varepsilon-1}{\varepsilon+2} \int \vec{E}\ (\omega, \vec{r} - \vec{r}\,')\ d\vec{r}\,' = p_x \hat{x} - i p_z \hat{z}, \quad (13)$$

which is based on the long wavelength approximation.

Using (13) in (12) we obtain

$$\frac{d^2 I}{d\omega d\Omega} = \frac{ck^4}{2\pi} [p_x^2 + p_z^2] \left|\cos^2\alpha - \sin^2\theta\right|, \ \alpha \equiv \tan^{-1}(p_z/p_x), \quad (14)$$

where θ is the angle of the unit propagation vector $\hat{n}$ with respect to $\hat{z}$.

For $b > a$, (13) yields

$$p_{x,z} = 3 \frac{\varepsilon-1}{\varepsilon+2} a^3 \frac{j_1(ka)}{ka} E_{x,z}(b,\omega) \xrightarrow[ka \to 0]{} \frac{\varepsilon-1}{\varepsilon+2} a^3 E_{x,z}(b,\omega). \quad (15)$$

Note that for $ka \to 0$, p_x becomes identical to the form (9) of Rayleigh scattering. With the result (15), the frequency spectrum becomes

$$\frac{d^2I}{d\omega d\Omega} = \frac{q^2}{c} \left(\frac{3}{\pi}\right)^2 \left(\frac{1}{\gamma\beta^2}\right)^2 \left|\frac{\varepsilon-1}{\varepsilon-2}\right|^2 (ka)^6 \left[\frac{j_1(ka)}{ka}\right]^2$$

$$\times \left[K_1^2(u) + \frac{1}{\gamma^2} K_0^2(u)\right] \left|\cos^2\alpha - \sin^2\theta\right|, \quad b > a. \qquad (16)$$

Note that $\cos\alpha \to 1$ as $\gamma \to \infty$ and the last factor in (16) becomes $\cos^2\theta$ (see Fig. 2).

We have also obtained an approximate expression for the frequency spectrum when $b < a$, which is more cumbersome than (16). For lack of space we give here only the limiting case $b = 0$:

$$\lim_{b \to 0} \frac{d^2I}{d\omega d\Omega} = \frac{q^2}{c} \left(\frac{3}{4\pi}\right)^2 \left(\frac{1}{\gamma^2\beta^2}\right)^2 \left|\frac{\varepsilon-1}{\varepsilon+2}\right|^2 (ka)^6$$

$$\times \left[j_0\left(\frac{ka}{\beta}\right) - 4 \ln \left(\frac{ka\ e^{0.5772}}{2\gamma\beta}\right) j_1 \left(\frac{ka}{\beta}\right)/(ka/\beta)\right]^2 \sin^2\theta. \qquad (17)$$

From symmetry, $p_x = 0$ when $b = 0$ and (17) is the result of the contribution from p_z alone.

The Method of Image Charges for Nonrelativistic Particles

We have used the method of image charges to obtain expressions for the spectral density of DR produced by a nonrelativistic particle passing either near a perfectly conducting sphere ($b > a$) or through it along a diameter ($b = 0$). In the latter case, the result for the total spectral density of dipole radiation was obtained in closed form with no approximations. The result, after integration over 4π steradians, is

$$\frac{dI(b=0)}{d\omega} = \frac{8}{3\pi}\frac{q^2}{c}(ka)^2 \lfloor \cos x - (x+\frac{1}{x})\sin x + x^2\,\mathrm{Ci}(x)\rfloor^2, \quad x \equiv ka/\beta. \qquad (18)$$

The nonrelativistic limit of Eq. (17) integrated over solid angle is in good agreement with the above result when $x \ll 1$, if in (17), $\varepsilon \to -\infty$, $\gamma \to 1$, and the small argument forms $j_0(x) \to 1$ and $j_1(x)/x \to \frac{1}{3}$, for $x \ll 1$ are used. Similarly, for $b > a$, the image charge method was found to agree with the result (16) above when $\varepsilon \to -\infty$, $\gamma \to 1$, and $ka \ll 1$.

In contrast to our results, if one takes the limiting case of Porgorzelski and Yeh[3], corresponding to our Eq. (17), one obtains

$$\lim_{b \to 0} \frac{d^2I}{d\omega d\Omega} = \frac{q^2}{c}\frac{18}{\pi}\left|\frac{\varepsilon-1}{\varepsilon-2}\right|^2 (ka)^2\, j_0^2(ka/\beta)\,\sin^2\theta, \qquad (19)$$

which doesn't have the $(ka)^6$ dependence which appears in Eqs. (17) and (18) for $ka \ll 1$. Yet another expression for the spectral density at zero impact parameter is cited in the review by Bass and Yakovenko[4]. That expression, strangely enough, gives a nonzero result as $a \to 0$, unlike (17), (18), or (19) above.

Numerical Results

In Figs. 3 and 4 we plot the spectral density of DR in units of q^2/c as a function of impact parameter in units of sphere radii for electrons with energy 0.1, 1 and 10 MeV, with $ka = 0.1$ and $\varepsilon = 2.0$. In Fig. 3, the observation angle $\theta = \pi$, i.e. we are observing backscattered radiation, while in Fig. 4 $\theta = \pi/2$.

In Fig. 5 we plot the DR spectral density versus impact parameter for three different values of ka: $ka = 1.0$, 0.5 and 0.1. The observation angle is $\theta = \pi$ and the electron energy is 100 MeV.

In Fig. 6, we illustrate the angular dependence of DR for three impact parameters: $b = 0$, $b = a$, and $b = 5a$, at an energy of 0.1 MeV and with $ka = 0.1$. Note that for $b = 0$, the radiation pattern is that of a single dipole oriented along the $\hat{z}$-axis, while for $b > a$, the dipole oriented along the $\hat{x}$ -axis also contributes.

In Fig. 7, we show the angular dependence for an impact parameter $b = a$ at 0.1 and 100 MeV with $ka = 0.1$. The contribution from the dipole induced along $\hat{z}$, which peaks at $\theta = \pi/2$ for 0.1 MeV, has disappeared at 100 MeV because of the γ^{-2} weighting which appears in Eq. (16).

Summary

We have examined the problem of the production of diffraction radiation (DR) by a charged particle passing near or through a dielectric sphere from three closely related points of view: the method of virtual quanta, the radiation of induced dipole moments, and the method of images. In a sense, all three methods are similar in that they all can be considered as a determination of radiation produced by the dipoles induced in the sphere by a passing particle. We illustrated the connection between Rayleigh scattering of light and the method of virtual quanta for the case of $ka \ll 1$, large impact parameters, and relativistic particles. We then gave a more general formula, valid for any impact parameter and any energy, provided $ka \ll 1$. The general formula may be considered as resulting from a generalization of the method of virtual quanta to a situation where the particle fields vary appreciably across the sphere. Finally, the image charge method was used to determine the radiation from the dipole induced in a perfectly conducting sphere by a nonrelativistic particle. Our relativistic expressions agree with the image charge results in the nonrelativistic limit. In contrast, we found that the literature contains at least two different expressions for the same

physical situation of diffraction radiation produced by a nonrelativistic particle incident on a sphere, neither of which agrees with the present results.

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Fig. 1

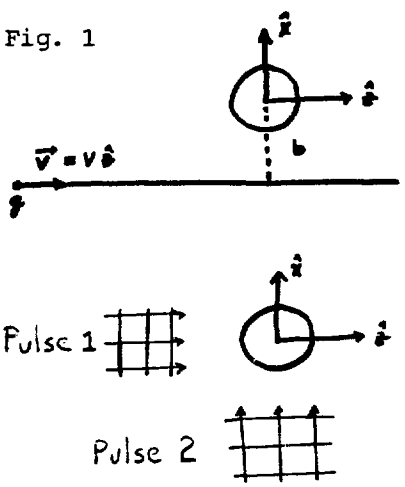
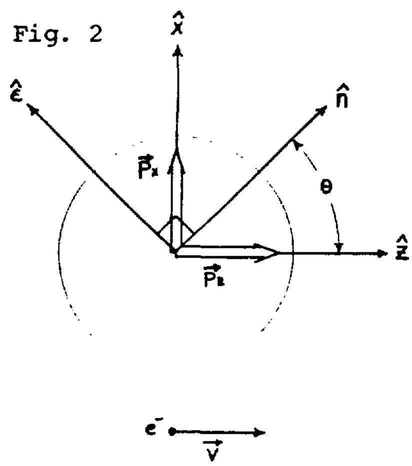
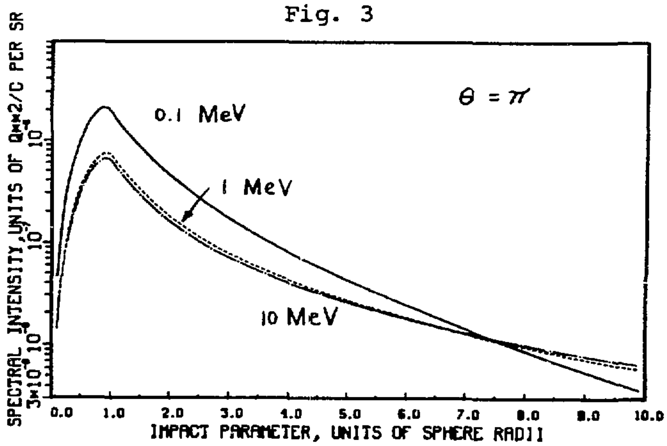


Fig. 2



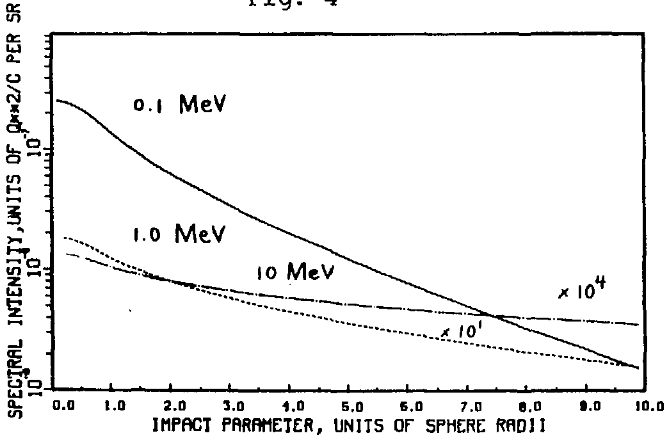
DIFFRACTION RADIATION FROM A SPHERE

Fig. 3



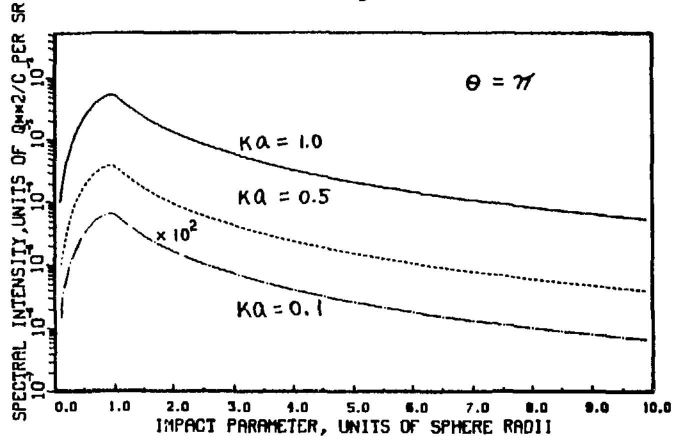
DIFFRACTION RADIATION AT 90 DEG. FROM FORWARD

Fig. 4



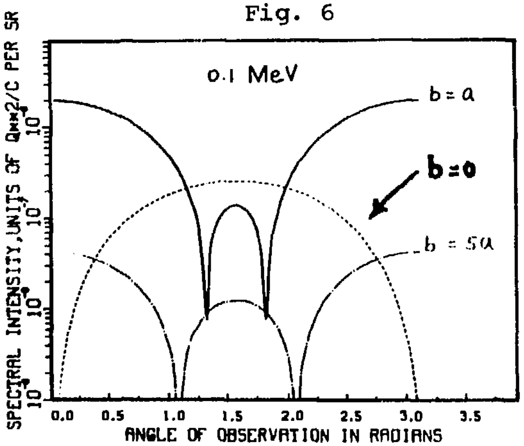
RADIATION FROM A 100 MEV ELECTRON FOR KA=0.1, 0.5, AND 1.0

Fig. 5



ANGULAR DEPENDENCE OF THE RADIATION

Fig. 6



ANGULAR DEPENDENCE FOR 0.1 AND 100 MEV

Fig. 7