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The Δ_{33} Resonance in Pion Nucleus Elastic, Single, and Double Charge Exchange Scattering

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The Δ_{33} resonance is strongly excited in pion-nucleon scattering, but there is clearly only a limited amount of information that can be learned in scattering the pion from an isolated nucleon. One learns that there is a resonance of mass 1232 MeV, width 115 MeV, and, if one is willing to introduce a dynamical model, something about the off-shell extension of the amplitude.[1] One stands to learn much more from pion-nucleus scattering because in this case the Δ_{33} resonance has an opportunity to scatter from nucleons, and how this occurs is not well understood. Nuclear theory must commit itself to a specific form for this interaction in order to do dynamical calculations, and nearly all subfields of nuclear physics would benefit from a specification of the Δ_{33}-nucleon (Δ-N) interaction that has empirical verification.

What do we know about the Δ-N interaction for pion-nucleus scattering? The isobar-hole model[2,3] was invented to deal directly with the Δ_{33}-nucleus dynamics, and in Ref. 3 a phenomenological determination of the isobar "shell-model" potential was attempted. The unknown dynamics δU_Δ is contained in a central isoscalar "spreading potential" of strength W_0 and a spin orbit potential

$$\delta U_0 = W_0 \rho + \text{spin-orbit} \quad . \tag{1}$$

The real part of $W_0\rho$ is measured relative to the nucleon-nucleus potential. One learns from this analysis that the isobar Δ_{33} is less bound in the center of the nucleus than a nucleon by about 20 MeV, and that the width of the isobar is increased by about 40 MeV by multiple interactions and absorption. The parameters were fit to a variety of light nuclei, where it is technically feasible to solve the theory.

Having learned that it is possible to study the isobar-nucleus interaction directly in pion-nucleus scattering, it is natural to ask how one might discover even more. A characterization of the isospin dependence of the isobar-nucleus interaction is one obvious lack in our understanding. I believe that there are great opportunities for exploiting the isospin degrees of freedom in future experiments at the meson factories to facilitate an understanding of this physics. From a more theoretical point of view, one would like to be able to calculate δU_Δ, including its isospin dependence, from an underlying dynamical model which is formulated in terms of the basic "effective" meson-baryon couplings, a few of which are shown in Fig. 1. Some salient properties of these couplings can be determined from models of quark-bag structure,[4] which raises the exciting possibilty of learning about these fundamental issues from pion scattering.

Most of the remarks of this talk will deal with our attempts at Los Alamos to build a theoretical framework to deal with these and other issues. To learn about the Δ-N isospin dependence in elastic scattering, it is clearly necessary to study carefully nuclei with $N > Z$, e. g., nuclei with a neutron excess. One can deal more directly with the isospin degrees of freedom in charge exchange reactions. Of the two varieties, single charge exchange (SCX) and double charge exchange (DCX), the latter is more useful for the present purposes because at least two nucleons must be struck in the process, which means the isobar-nuclear interaction contributes to leading order.

In building the theory, our decisions have been strongly influenced by the fact that there is an intriciate coupling between the Δ, π, and nuclear dynamics. To learn about any of these, one must make convincing arguments that the uncertainties in the others are under control. It makes most sense to begin building theories for nuclei near closed shell, to take advantage of the nuclear mean field models,[5] which in other contexts have proved quite successful. We have learned much about pion dynamics already in elastic scattering from various theoretical and experimental investigations, and we therefore want to incorporate this knowledge also.

We therefore have chosen a framework in which pion-nucleus elastic, SCX and DCX are calculated together in the same theory. We want to exploit the fact that the underlying strong interactions respect isospin invariance to a high degree of validity. Because SCX and DCX to isobaric analog states and elastic scattering are intimately connected by a symmetry, we are able to achieve a unified treatment of these processes for which the theoretical description is paticularly simple. Our second major demand is that the theory be microscopic and derivable in principle from the basic couplings shown in Fig. 1. By so doing we allow the results of the theory to make the strongest possible statements about fundamental physics.

In the remainder of this talk I want to describe in a bit more detail how we are building the theory and what we are learning from it. I will begin by describing the two complementary stages of development we are pursuing in which D. J. Ernst and E. R. Siciliano are major collaborators. Finally I will describe the status of a calculation of some important terms in this theory, which involve the "double delta" (Fig. (1b). This is a collaboration with E. Siciliano, H. Toki, and A. Wirzba.

Low energy pion-nucleus elastic scattering has been studied by Stricker, McManus, and Carr[6] in an optical model framework with great success. Their optical potential has the form

$$U = \nabla \cdot [\xi + \Delta\xi]\nabla - k^2[\xi + \Delta\xi] - \frac{1}{2}(p_1 - 1)\nabla^2\xi - \frac{1}{2}(p_2 - 1)\nabla^2\Delta\xi \quad , \tag{2}$$

where k is the momentum of the incident pion, $p_i = (1 + \omega/2iM)/(1 + \omega/AM)$ is a kinematical factor arising from the transformation from the pion-nucleus center-of-mass frame to the pion-nucleon center-of-mass frame (M = nucleon mass, $\omega^2 = k^2 + m_\pi^2$, A = number of nucleons) and where the quantities ξ and $\Delta\xi$ specify the dynamics in the p-waves of a pion relative to a cluster of one and two nucleons, respectively (ξ and $\Delta\xi$ are the same for s-waves).

We[7,8] have extended the theory of Ref. 6 to include an explicit dependence on the pion isospin operator $\vec{\phi}$ and the nucleus isospin operator $\vec{T}$. We therefore write

$$\xi = \xi_0 + \xi_1\vec{\phi} \cdot \vec{T} \quad , \tag{3}$$

where ξ_0 is the lowest order isoscalar and ξ_1 the lowest order isovector interaction term. Because the pion is an isovector meson and because two nucleons are str in describing $\Delta\xi$, $\Delta\xi$ may have in addition an "isotensor" term

$$\Delta\xi = \Delta\xi_0 + \Delta\xi_1 \vec{\phi} \cdot \vec{T} + \Delta\xi_2 (\vec{\phi} \cdot \vec{T})^2 \quad . \tag{4}$$

The operator form chosen here enables us to calculate elastic, SCX, and DCX to isobaric analog states in a unified framework.

The two stages of development of the theory aim at an integration of a correct microscopic description of the pion/Δ_{33} dynamics with a correct handling of the nuclear dynamics. Because the pion may be emitted and absorbed in single quanta, the following two basic tenets of traditional multiple scattering theory are invalid:

(1) the existence of an underlying two-body potential, and

(2) conservation of the number of projectile particles.

The first stage seeks to gain experience in the formulation of pion scattering in a framework which avoids these assumptions. For this we have examined multiple scattering based on the Chew-Wick static source field theory.[9] A few of the properties of our scattering theory which will be preserved in more comprehensive frameworks are:

(1) U may be derived from the couplings in Fig. 1 in terms of a well-defined diagrammatic cluster expansion.

(2) It is proper to embed U in the Klein-Gordon equation

$$(-\nabla^2 + m_\pi^2 + U)\psi = \omega^2\psi \quad . \tag{5}$$

(3) U (and the scattering T-matrix) are crossing symmetric in principle and this property may be preserved order by order in our expansion.

(4) Multipion intermediate states are naturally incorporated.

(5) Short range πN form factors are proper to use (the theory described above goes to the limit of zero ranged form factors).

We derive U from a diagrammatic spectator expansion in which the lowest order term $U^{(1)}$ is the sum of all diagrams for which a pion interacts with a single nucleon, and the second order term $U^{(2)}$ includes all diagrams for which a pion interacts with two nucleons, minus the iteration of the lowest order optical potential. Our $U^{(1)}$ is expressed in terms of the off-shell free pion-nucleon scattering amplitude in the usual way. Typical terms for $U^{(2)}$ are shown in Figs. 2 and 3. Fig. 2c is the iteration of $U^{(1)}$, which is subtracted to avoid double counting with Fig. 2a and 2b. The correction is nontrivial for the isotensor potential. The Pauli principle is incorporated through exchange terms, one example of which is shown in Fig. 1b. Generally speaking, all diagrams in U come in crossed and uncrossed form, but sometimes there is no distinction, i. e., Figs. 1a and 1d are the same in our theory because we do not distinguish time orderings.

One of our main findings is that isospin invariance imposes a rather strong constraint on the form of $\Delta\xi$, giving rise to a rather special "global" form.[7] One expects that $\Delta\xi_0$, $\Delta\xi_1$, and $\Delta\xi_2$ will depend on ρ^2, $\Delta\rho^2$, and $\rho\Delta\rho$, where ρ is the total nuclear density and $\Delta\rho$ is the valence neutron density. We find that isospin invariance allows these densities to be combined with four numbers $\lambda_i^{(2)}$, which may depend strongly on energy, but are in practice very weakly dependent on N, Z, and A. The object of the theory is then to calculate these four numbers, in terms of which scattering <u>throughout the periodic</u> table should be predicted at a given energy.

The terms shown in Fig. 3 are perhaps of more theoretical interest than those in Fig. 2. Figure 3a includes the multiple reflection and true absorption of the pion and is generally believed to account for the collision broadening of the Δ_{33} as determined experimentally in the isobar-hole model. A calculation of these terms[10,11] made in the static theory, employing a self-consistent procedure, gives about the right sign and magnitude of the terms in the resonance region. Figure 4 shows a comparison[11] of the spreading potential determined in Ref. 3 with the earlier calculation of Ref. 10. The results of Ref. 2b support the conclusion that the spreading potential W_0 should arise from these terms. Microscopic evaluations of these terms for charge exchange have not been made, but the importance of doing so is heightened by their apparent importance in the isoscalar channels.

Figure 3(b), (c), and (d) have achieved a high level of interest lately based in large part on measurements of double charge exchange cross sections. These terms are quite puzzling and for this reason I will postpone the discussion of these to the end of the talk.

Let me now turn to a discussion of some experimental results in the region of the (3,3) resonance and indicate the extent to which they can be understood in the theory at a very elementary level of application. Elastic scattering has been studied in a theory very similar to ours by Cottingame and Holtkamp,[12] and they found a remarkably simple result, namely that a systematic reproduction of elastic scattering throughout the periodic table in an energy interval about the (3,3) resonance can be accomplished if the pion-nucleon scattering amplitude is evaluated at an energy shifted <u>downward</u> by 20-30 MeV. A theoretical description of this shift depends upon the details of the Δ_{33} propagation and interactions in the medium and I will shortly show how we propose to include this in our theory microscopically.

Extensive measurements[13] of Sc o isobaric analog states have shown that 0^{o} cross sections follow a very simple law

$$\sigma(0^{o}) \sim (N-Z)/A^{4/3} \quad , \tag{6}$$

which holds within a factor of two. Similarily, DCX[14] follows the law

$$\sigma(0^{o}) \sim (N-Z)(N-Z-1)/A^{10/3} \quad . \tag{7}$$

Examples of this are shown in Figs. 5 and 6. These trends are reproduced by the theory if very simple scaling densities

$$\rho_N/\rho_P = N/Z \tag{8}$$

are utilized and if all second order terms are dropped.[15]

The fact that the theory reproduces trends seen in the data in many different experiments is taken as encouragement to take the approach seriously at the next level, namely encouragement to include more realistic descriptions of nuclear densities and to see what new can be said about Δ-N dynamics. As guidance for these studies, we find the following two results[8] to be of help:

(1) Our global form for $U^{(2)}$ shows that the second order terms tend <u>not</u> to affect the relative (N,Z,A) dependence of cross sections, with only one exception: the sequential terms lead to a relative enhancement of DCX cross sections for $T \approx 1$.

(2) Including more realistic densities can have large effects on the relative (N,Z,A) dependence of SCX and DCX cross sections.

We expect that these systematics will help us separate the uncertainties in nuclear structure from those of Δ-N dynamics.

Now let me turn to the problem of determining the effective energy of the Δ_{33} resonance in pion-nucleus scattering. I will discuss the initial results of our attempts to incorporate nuclear dynamics at a microscopic level,[16] which so far has focused only on the lowest order optical potential. Let me emphasize that the necessity for as careful a treatment as possible of the lowest order optical potential arises from the rapid energy variation of the Δ_{33} resonance and our desire to separate the uninteresting kinematic aspect of Δ_{33} propagation from the Δ-N dynamics, which are specified by the higher order terms in U.

Credit for the widespread awareness of the importance for carefully handling the kinematics of Δ_{33} propagation is due to the proponents of the isobar-hole model. However, this same publicity has generated the perception that a microscopic treatment of pion scattering requires complicated and time-consuming numerical calculations. We believe that this is not the case, and in particular that the isobar-hole model is not necessary. Our alternative within the optical model is made possible by the work of D. Ernst, G. Miller, and D. Weiss.[17]

When the formulation of the scattering dynamics of $U^{(1)}$ is redone allowing for a microscopic treatment of isobar recoil and interaction, $U^{(1)}$ may be expressed[16] in terms of the Feynman diagram shown in Fig. 7. In addition to the contributions shown there are other background terms, which are small in the resonance region. In the isobar model the "Δ_{33} term" may be expressed in the form

$$V \frac{1}{\omega - M_\Delta - [\frac{p^2}{2M_\Delta} + U_\Delta] + i\Gamma/2} V \quad , \qquad (9)$$

where V's are appropriate N-Δ vertex functions, ω is the "starting energy" evaluated in terms of the nuclear Hartree-Fock eigenvalues and incident pion energy, and where $\underline{P}$ is the recoil momentum of the Δ_{33}. The quantity $\frac{P^2}{2M_\Delta} + U_\Delta$ is the "isobar propagation and interaction," which has been the object of intense study in isobar-hole models. The Ernst, Miller, and Weiss technique permits the nonlocalities arising from the V's and isobar propagation to be handled efficiently in momentum space.

The U_Δ plays a bit more general role in our theory than in the isobar-hole model. In our approach U_Δ, which we refer to as the "dispersive correction,"[16] plays the dual role of establishing a single particle basis for the Δ_{33} and simultaneously cancelling some important higher order terms in the theory. The terms that are important and should be cancelled remain to be decided on the basis of physics considerations as they become more clearly defined. Thus, we have no unique lowest order potential and we have a "dial" in the theory that can be used to enhance the convergence of the expansion for the U.

The dispersive corrections are very important and are quite often neglected. They are important because they largely compensate for the binding potential of the nucleus, which enters the starting energy with the opposite sign. One can see this in the isobar-hole model. Dropping the spin-orbit force for simplicity, we have

$$U_\Delta = U_N + W_0 \rho \quad , \tag{10}$$

where U_N is the nucleon-nucleus potential. The main point is that U_N "sticks out" farther in the nucleus than the density ρ (due to the finite range of the NN potential) so that even though the Δ_{33} tends to be produced in the far tail of the nucleus at resonance (in a region centered about the 10% density point), the Δ_{33} still has the opportunity to experience a substantial attraction. The region in which the Δ_{33} is formed is expected to be not too much different from that mapped out by the product of the nucleon density and the square of the pion wave-function. The average attraction is then $\bar{U}_\Delta$ where

$$\bar{U}_{\Delta} = \int |\psi_{\pi}(r)|^2 \rho(r) U(r) dr / \int |\psi_{\pi}(r)|^2 \rho(r) dr \quad , \tag{11}$$

which is plotted in Fig. 8 as a function of energy. The solid line corresponds to U_N and the dashed line to the full U in Eq. (10). The effect of the spreading potential is sufficiently small to be included perturbatively in a second order term in the optical potential.

Figure 9 shows a calculation of pion scattering with different approximations in the treatment of the energy denominator of Eq. (9). All curves employ the results of Negele's Hartree-Fock theory. The solid curve corresponds to the full calcuation with $\bar{U} = -24$ MeV and with ω including the HF single particle binding. The resonance is correctly described, as evidenced by the depth of the first diffraction minimum being correctly described. If one omits $\bar{U}_{\Delta}$, as in the dashed curve, the resonance is pushed too far up in energy, and if one omits both the $\bar{U}_{\Delta}$ and the nucleon potential (dot-dashed curve), the minimum becomes too deep. The fact that the solid curve sits above the data[18] in the region of the secondary maxima means that the diffuseness of the optical potential is too sharp; the spreading potential W_0 is expected to reduce the imaginary part of the optical potential in the nuclear interior[19] in this energy region, which would have the desired effect. Weakening the optical potential would also make the nucleus appear somewhat smaller, which would improve the positioning of the minima. In the near future we will be able to study these second order effects quantitatively in our extended theory.

Let me now restate the main points and move to a discussion of the double delta effects. We believe that in order to learn about Δ-N interactions we must treat the nuclear structure and kinematic aspects of pion scattering as carefully as possible. Information about the interesting Δ-N interaction is then carried explicitly in the second order optical potential. Considerably more work is still required to put these ideas together in a fully quantitative theory, but the theory in its present form reproduces systematics of the elastic scattering and charge exchange sufficiently well to encourage us that lots of data and ideas will be successfully brough together in this forumulation.

The double delta terms constitute an important physical ingredient of our theory and have become controversial in the interpretation of recent double charge exchange data. It has been suggested by C. Morris and T. Fortune in a series of papers[20] that their very interesting measurements of double charge exchange in $T = 0 \rightarrow T = 2$, $0^+ \rightarrow 0^+$ transitions and $T = 1 \rightarrow T = 1$, $0^+ \rightarrow 0^+$ double isobaric analog transitions can be interpreted in terms of the amount of Δ_{33}^{++} in the nuclear wave-function of these nuclei. The evidence that they put forth is shown in Figs. 10 and 11. All cross sections of the former variety[21] exhibit a peaking in the resonance region, suggesting delta dominance. In the latter[22] one sees evidence for an interference between the sequential process (solid curve) and another term that is asserted to be proportional to the amplitude in the nonanalog transition. A phenomenological "two amplitude" model is studied in support of this idea.

We[23] have made an estimate of the double Δ terms shown in Fig. 12 based on the following assumptions. The two neutrons in ^{18}O are in the $d_{5/2}$ level. The Δ_{33} is allowed to couple to both the π and ρ fields. SU(4) arguments are used to relate the coupling of the π to nucleon and Δ_{33}, and the "strong ρ" coupling is taken. Monopole form factors of cutoff mass $\Lambda = 1.5$ GeV/c were used.

One immediate conclusion that one can draw is that the double Δ process in Fig. 12a dominates the two processes in Figs. 12b and c in the resonance region. This is a simple consequence of the fact that there are two on-shell Δ_{33} in the former but only one in the latter two figures. The more interesting conclusion is that the double delta terms are a factor of two to three too large in amplitude, for both analog and nonanalog transitions.

We see that the theory fails to reproduce the charge exchange data, and the question of immediate concern is why this has happened. We prefer not to speculate about this until our investigation is completed. However, it appears likely that double charge exchange is making a very strong statement about Δ-N dynamics in this result, and that our understanding of the way the Δ_{33} interacts in the nucleus will be refined as a result of the double charge exchange measurements.

To summarize the talk, let me merely reiterate that we are attempting to extend the optical model to heavy nuclei in order to develop a microscopic theory for pion elastic, single and double charge exchange in a unified framework. The theory works well for elastic scattering at low energy and in the resonance region if we employ an energy shift. For charge exchange we find:

(1) The relative (N,Z,A) dependence of experimental single and double charge exchange cross sections is reproduced to a factor of two, and we expect the discrepancies to reflect inadequacies in the nuclear structure models used.

(2) The overall scale of the cross sections reflects the importance of Δ_{33}-nucleus interactions. We find large second order terms in the theory, and discrepancies of roughly a factor of two with data in the absence of second order terms.

The main question for the future is whether these discrepancies can be reconciled to learn new and detailed information about nuclear structure and Δ-nucleus dynamics.

REFERENCES

1. See for example D. J. Ernst and M. B. Johnson, Phys. Rev. C17 (1978) 247; C22 (1980) 651 and references contained therein.

2. (a) M. Hirata, J. M. Koch, F. Lenz, and E. J. Moniz, Ann. Phys. (NY) 120, 205 (1979); (b) E. Oset and W. Weise, Phys. Lett. 77B, 159 (1978); (c) K. Klingerbeck, M. Dillig, and M. G. Huber, Phys. Rev. Lett.4

3. Y. Horikawa, M. Thies, and F. Lenz, Nucl. Phys. A345, 386 (1980).

4. G. E. Brown and M. Rho, Phys. Lett. 82B (1979) 177; S. Thieberge, A. W. Thomas, and G. A. Miller, Phys. Rev. D22 (1980) 2838; G. E. Brown, J. W. Durso, and M. B. Johnson, Nuc. Phys. A397 (1983) 447.

5. See, for example, J. W. Negele and D. Vautherin, Phys. Rev. C 5, 1472 (1972).

6. K. Stricker, H. McManus, and J. A. Carr, Phys. Rev. C19, 929 (1979); K. Stricker, J. A. Carr, and H. McManus, Phys. Rev. C22, 2043 (1980).

7. M. B. Johnson and E. R. Siciliano, Phys. Rev. C27, 730 (1983).

8. M. B. Johnson and E. R. Siciliano, Phys. Rev. C27, 1647 (1983).

9. M. B. Johnson and D. J. Ernst, Phys. REv. C27, 709 (1983).

10. M. B. Johnson and H. A. Bethe, Nucl. Phys. A305, 418 (1978); M. B. Johnson and B. D. Keister, Nucl. Phys. A305, 461 (1978).

11. M. B. Johnson, Proceedings of the International School of Physics, "Enrico Fermi" Course LXXIX, edited by A. Molinari (North-Holland, Amsterdam, 1981) p. 412.

12. W. B. Cottingame and D. B. Holtkamp, Phys. Rev. Lett. 45, 1828 (1980).

13. H. Baer, J. D. Bowman, M. D. Cooper, F. H. Cverna, C. M. Hoffman, M. B. Johnson, N. S. P. King, J. Piffaretti, E. R. Siciliano, J. Alster, A. Doren, S. Gilad, M. Moinster, P. R. Bevington, and E. Windkelmann, Phys. Rev. Lett. 45, 982 (1980); E. Piasetzky, private communication.

14. A summary of this data is given in P. A. Seidl, R. R. Kiziah, M. K. Brown, C. F. Moore, C. L. Morris, H. Baer, S. J. Greene, G. R. Burleson, W. B. Cottingame, L. C. Bland, R. Gilman, and H. T. Fortune, Phys. Rev. Lett. 50, 1106 (1983). The point for ^{48}Ca comes from K. Seth, private communication.

15. M. B. Johnson, Phys. Rev. C22, 192 (1980).

16. D. J. Ernst and M. B. Johnson, to be published.

17. D. J. Ernst, G. Miller, and D. Weiss, to be published in Phys. Rev. C.

18. J. P. Albanese, J. Arvieux, J. Bolger, E. Boshitz, and C. H. Q. Ingraham, Nucl. Phys. A350, 301 (1980).

19. See the invited talk by E. Moniz at this symposium for further discussion of the sign of the imaginary part of $U^{(2)}$.

20. C. L. Morris, H. T. Fortune, L. C. Bland, R. Gilman, S. J. Greene, W. B. Cottingame, D. B. Holtkamp, G. R. Burleson, C. F. Moore, Phys. Rev. C25, 3218 (1982); S. J. Greene, D. B. Holtkamp, W. B. Cottingame, C. F. Moore, G. R. Burleson, C. L. Morris, H. A. Thiessen, H. T. Fortune, Phys. Rev. C25, 924 (1982); R. A. Gilman, et al., Phys. Rev. C, to be published.

21. L. C. Bland, H. T. Fortune, M. A. Carlini, K. S. Dhuga, C. L. Morris, S. J. Greene, P. A. Seidl, and C. F. Morris, abstract submitted to this symposium.

22. S. J. Greene, W. J. Braithwaite, D. B. Holtkamp, W. B. Cottingame, C. F. Moore, G. R. Burelson, G. S. Blanpied, A. J. Viescar, G. H. Daw, C. L. Morris, and H. A. Thiessen, Phys. Rev. C25, 927 (1982).

23. M. B. Johnson, E. R. Siciliano, H. Toki, and A. Wirzba, to be published.

FIGURE CAPTIONS

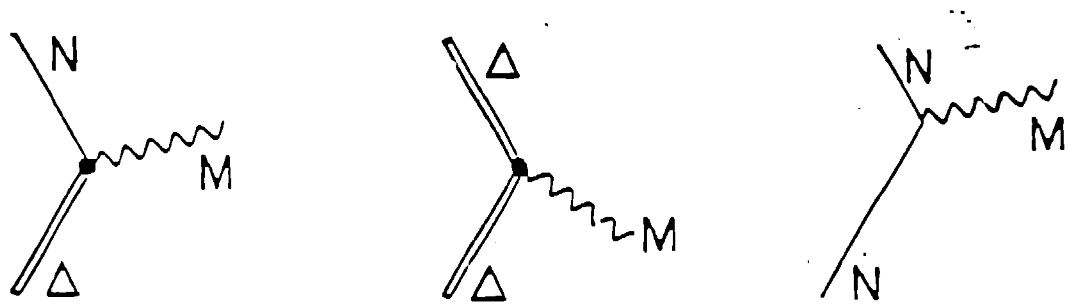
1. A few of the basic effective couplings which underly dynamical models in nuclear physics. The couplings (a) and (c) are rather well understood, but the "double Δ" process, (b), is subject to much uncertainty.
2. Two-nucleon processes contributing to the pion-nucleus optical potential. These terms are second order in the pion-nucleon scattering amplitude and are referred to as sequential scattering processes.
3. Additional two-nucleon processes contributing to the pion-nucleus optical potential. (a) is third order in the pion-nucleon scattering amplitude and is referred to as a reflection process, whereas (b)-(d) involve various isobar-medium effects. Each process has a corresponding exchange and crossed piece.
4. Comparison of the theoretical spreading potential of Ref. 10 (solid line) to the phenomenological result of Ref. 3. The triangles come from an analysis of ^{4}He, the squares ^{16}O, and the circles ^{12}C.
5. Forward cross section for SCX divided by scaling cross section of Eq. (6) as a function of A for 165 MeV pions. The data are from Ref. 13 and this compilation is due to M. Cooper.
6. Forward cross section for DCX divided by (N − Z)(N − Z − 1), as a function of A. The solid curve is $A^{-10/3}$, as expected from Eq. (7). The data are from Ref. 14.
7. Representation of the lowest order optical potential as a piece of a Feynman diagram. (a) Direct and crossed amplitudes are evaluated in terms of nucleon single paticle energies and wave functions obtained from a Hartree-Fock theory. (b) The amplitudes in pion-nucleon P-waves consist of the nucleon pole and the isobar, and in addition numerous small terms not shown.
8. The average potential $\tilde{U}_\Delta$ defined in Eq. (11) for ^{16}O. The pion wave-functions were evaluated in our coordinate space optical model and incorporate the Lorentz-Lorenz effect in the second order U. The solid curve corresponds to $U_\Delta = U_N$, and the dashed line to the full U_Δ in Eq. (10).

9. Calculations of π^+ elastic scattering from ^{16}O at 163 MeV with a series of approximations in the isobar propagator. The solid curve includes $\bar{U}_\Delta$ = -24 MeV and the dashed curve omits $\bar{U}_\Delta$. The dot-dashed curve corresponds to omitting both $\bar{U}_\Delta$ and the nucleon binding. Data are from Ref. 19.

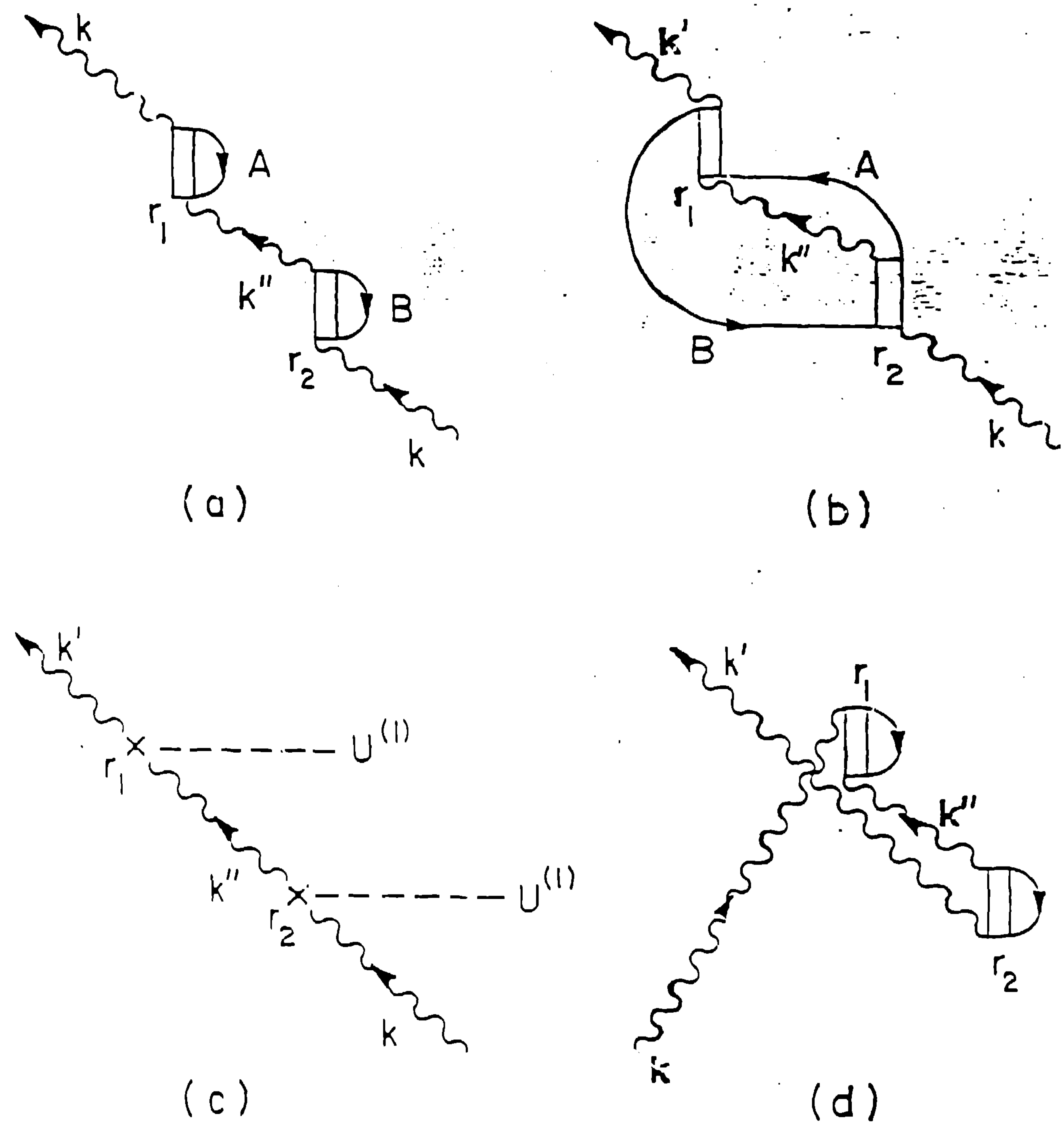
10. Zero degree cross sections for nonanalog $(T = 0, J^P = 0^+) \to (T = 2, J^P = 0^+)$ transitions. The peaking of excitation function near 160-180 MeV suggests a delta-dominated reaction mechanism. Data are taken from Ref. 21.

11. Zero degree cross section for the $^{18}O(\pi^+,\pi^-)^{18}Ne$ double analog transition. The solid curve is the sequential double charge exchange calculated from our coordinate space theory by R. Gilman and P. Seidl. Data are taken from Ref. 22.

12. Double delta terms which are conjectured to contribute significantly to double charge exchange.



(a) (b) (c)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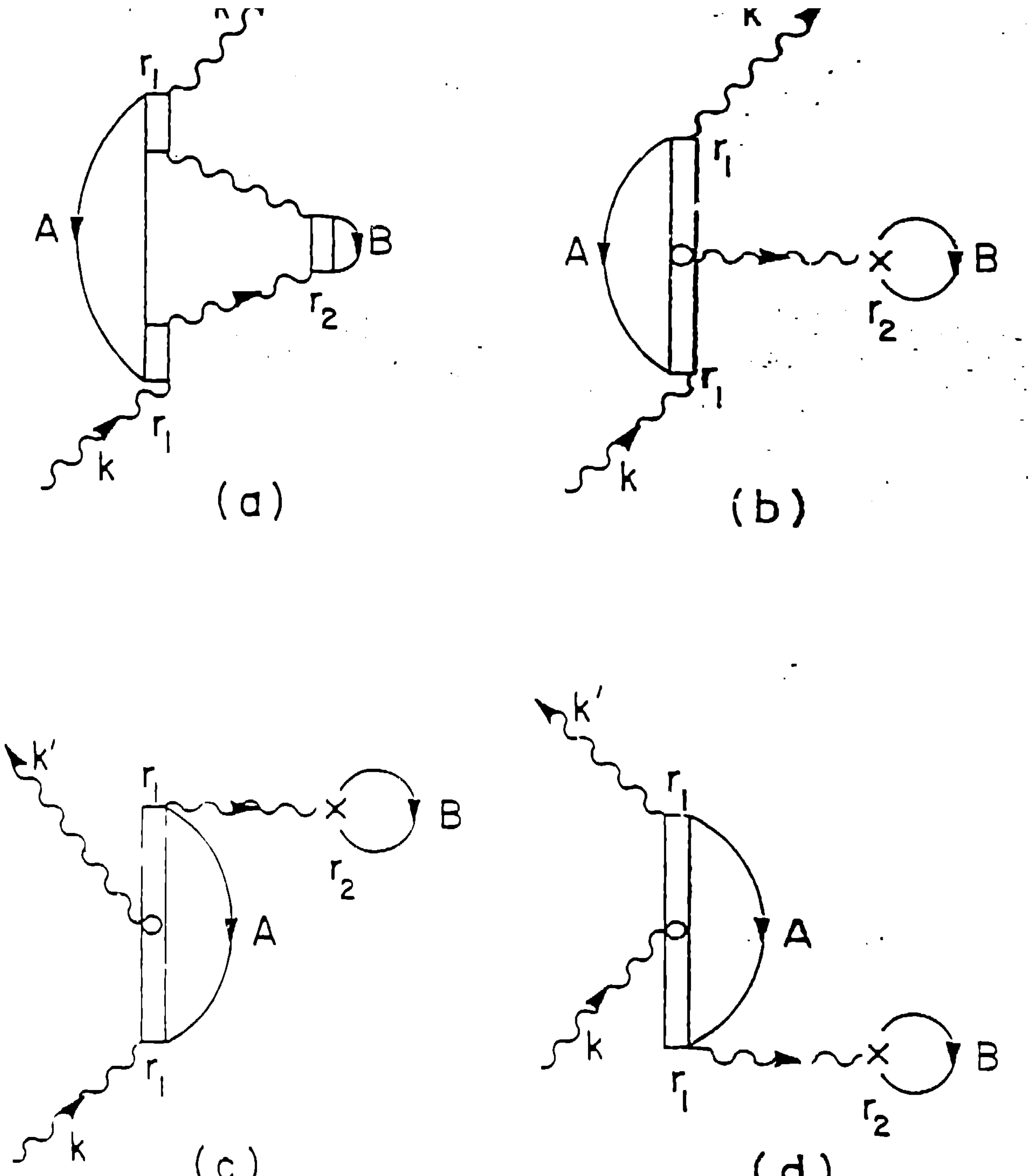

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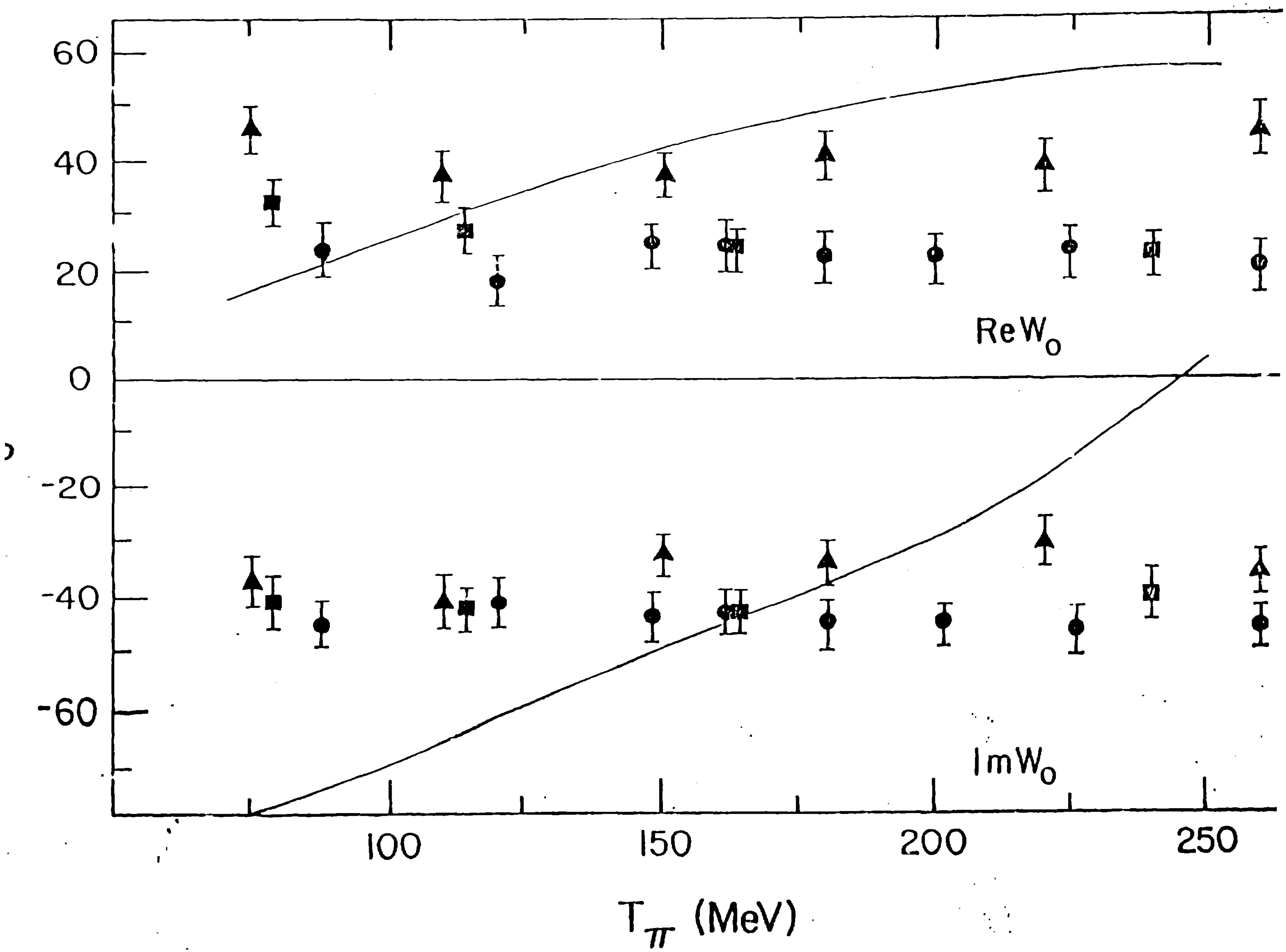

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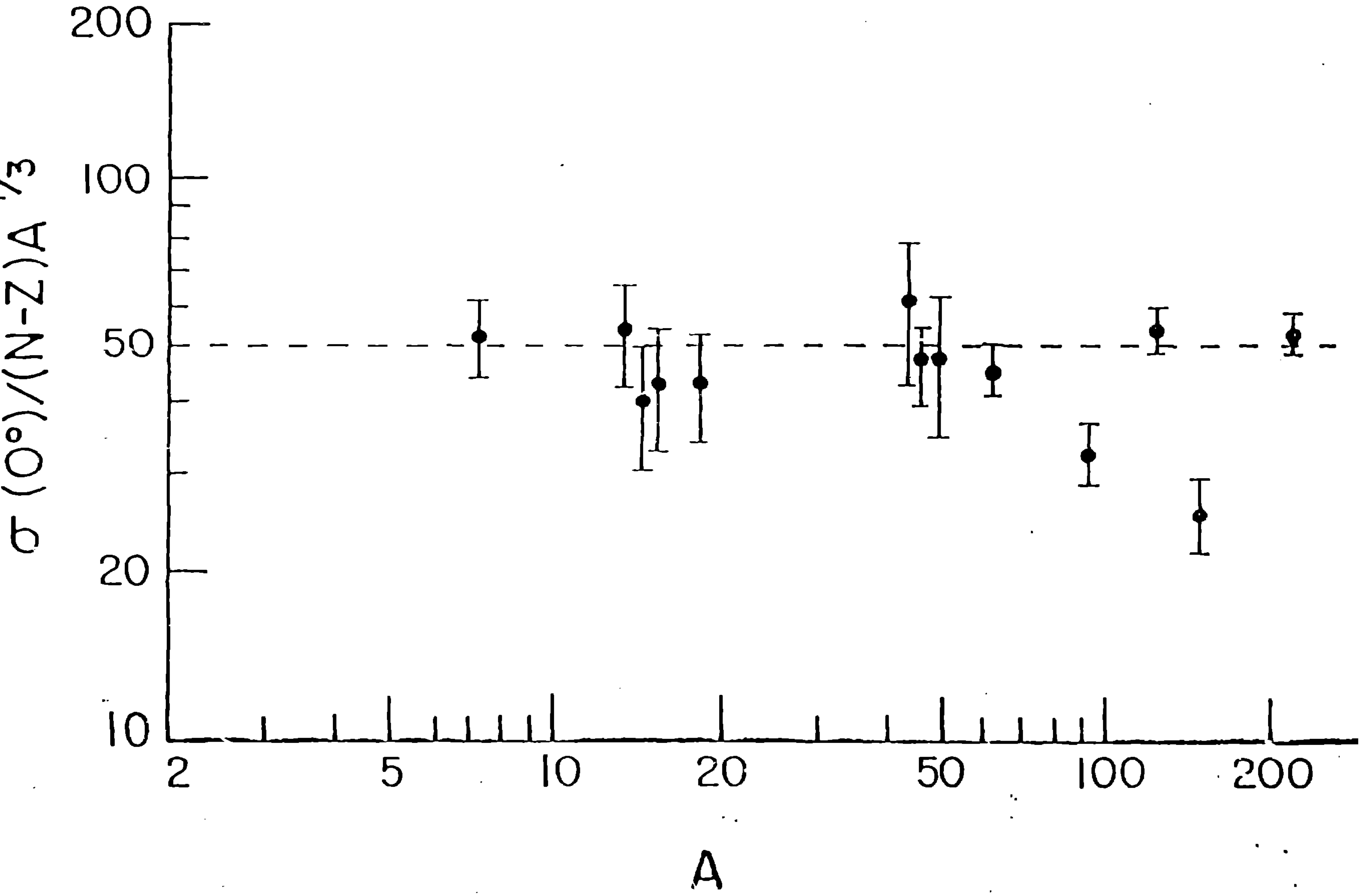
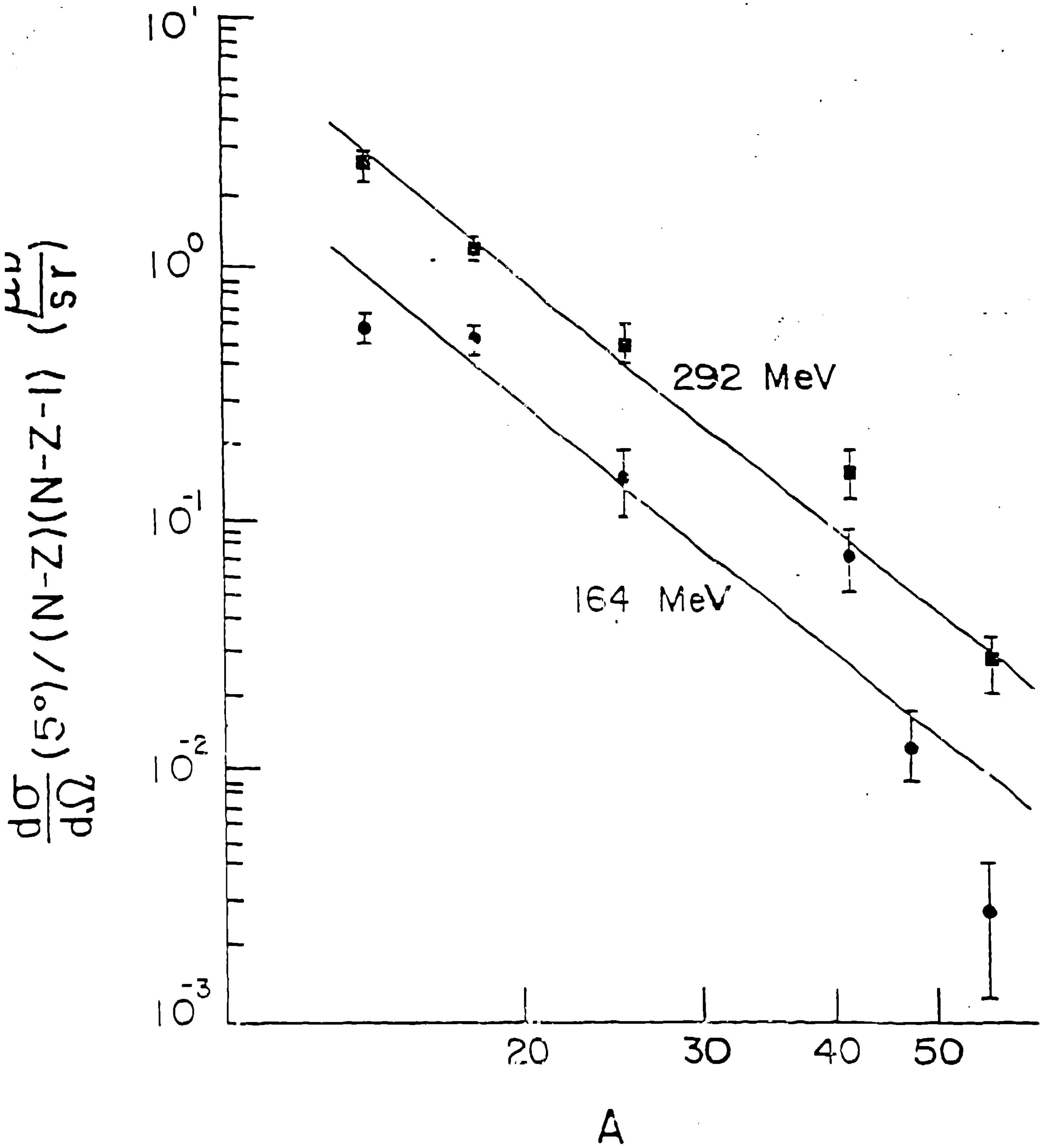
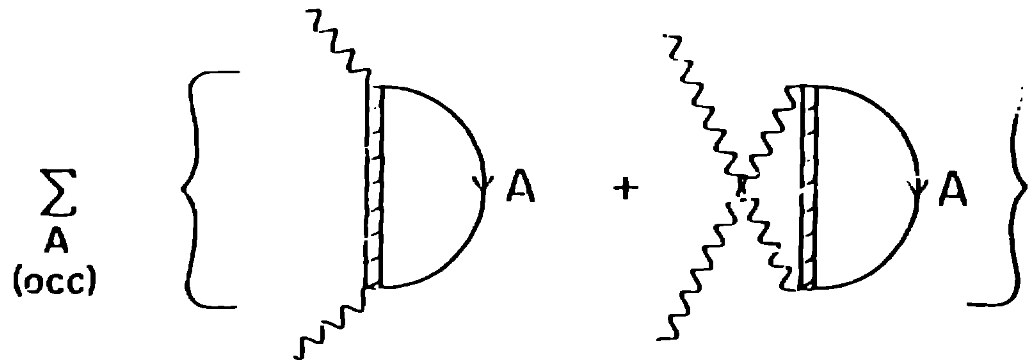
150
200
250
T_π (MeV)

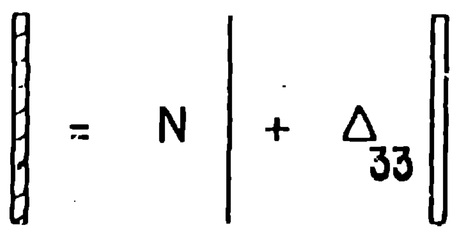

200
100
50
20
10
σ (0°)/(N-Z)A^-4/3
2
5
10
20
50
100
200
A



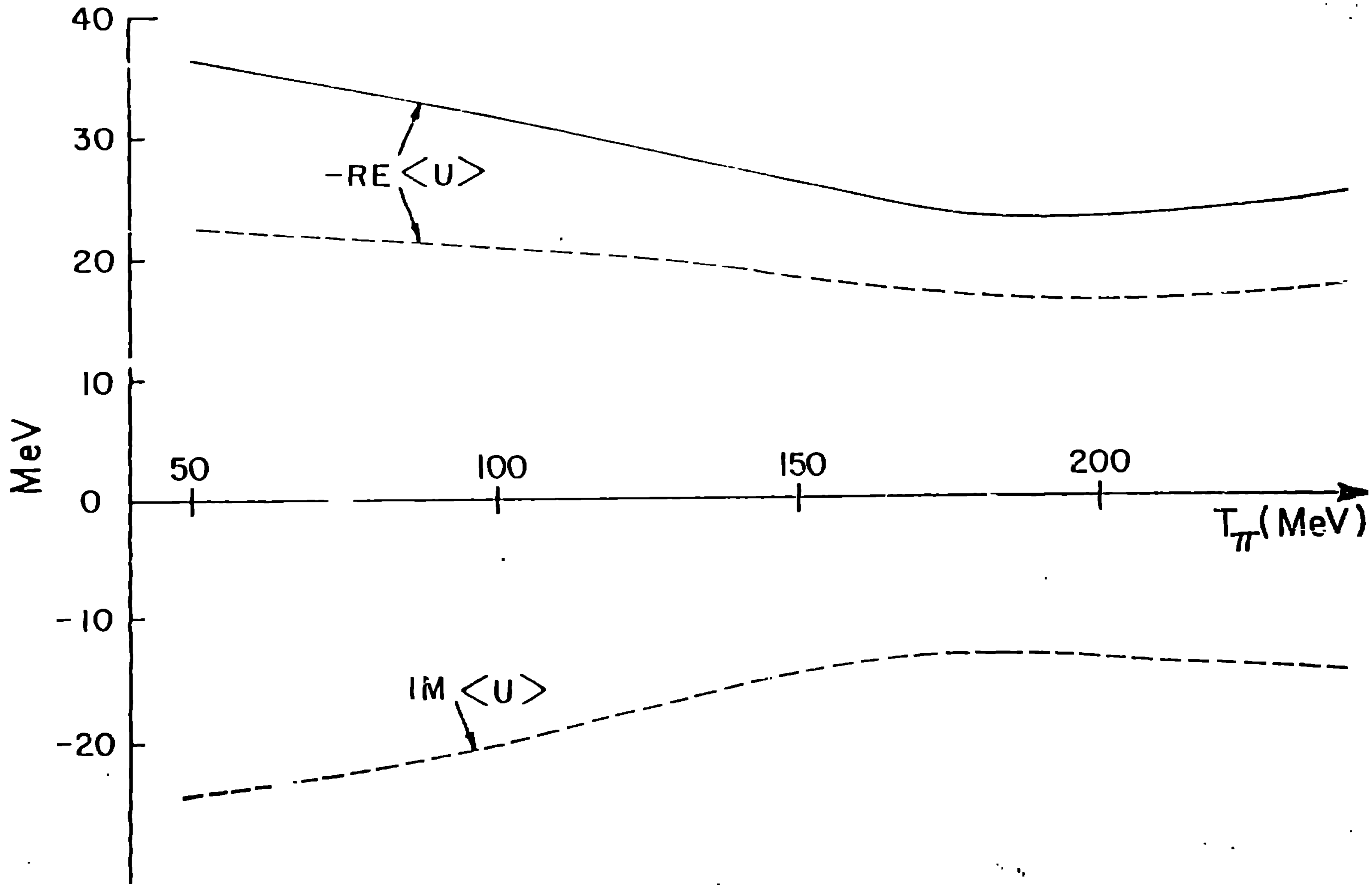
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
$\frac{d\sigma}{d\Omega}(5^\circ)/(N-Z)(N-Z-1)$ $\left(\frac{\mu b}{sr}\right)$
292 MeV
164 MeV
20
30
40
50
A

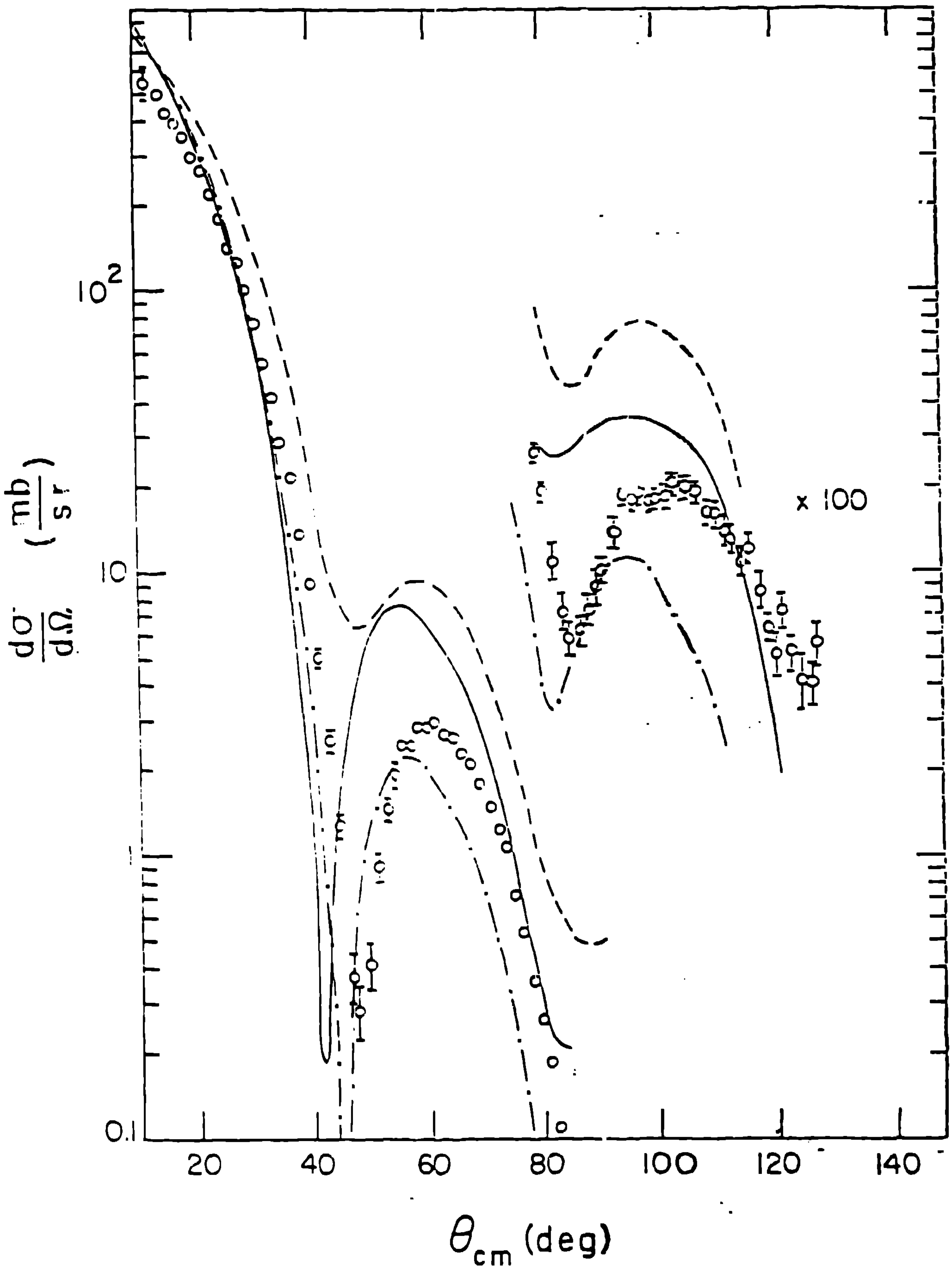


(a)

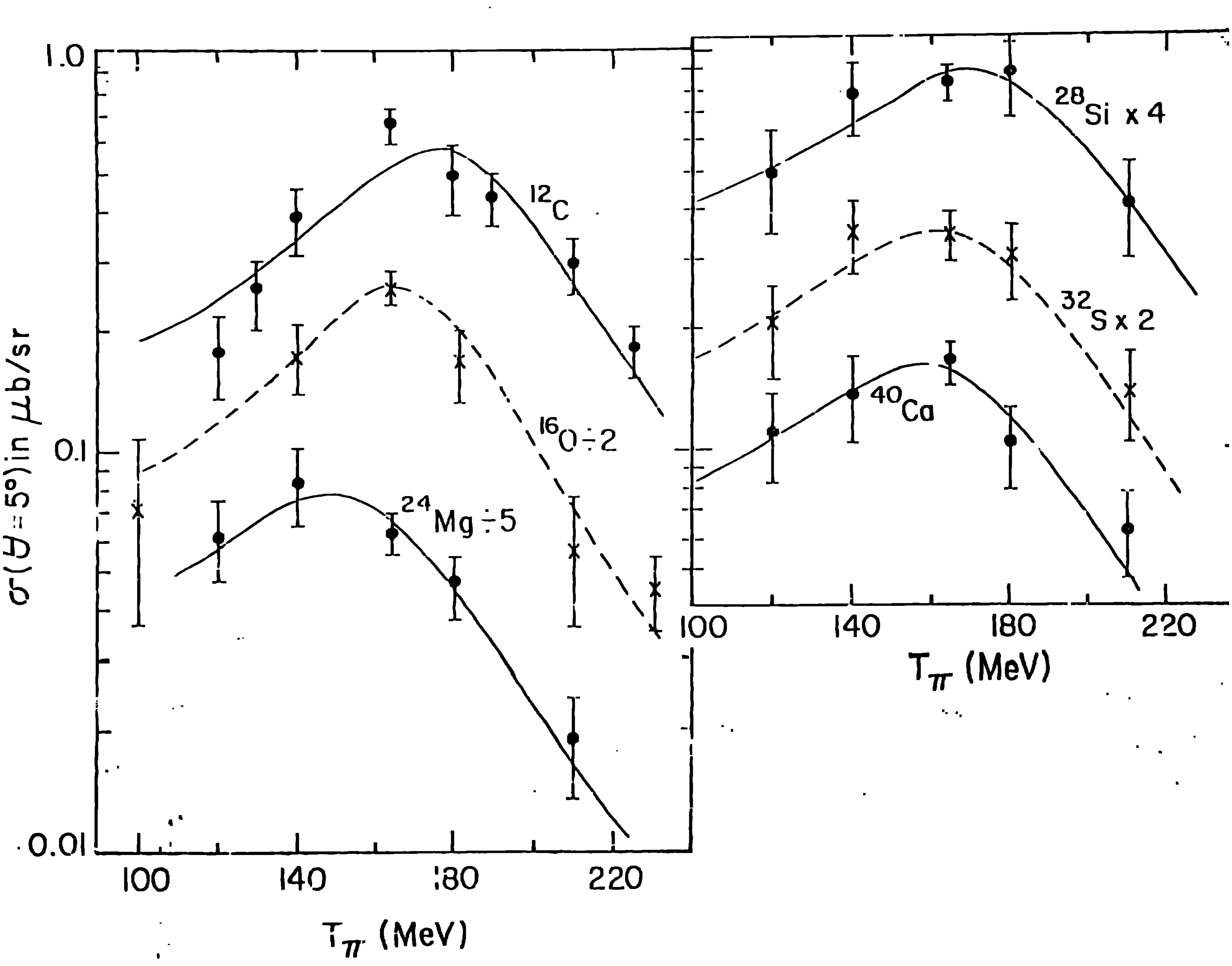


(b)

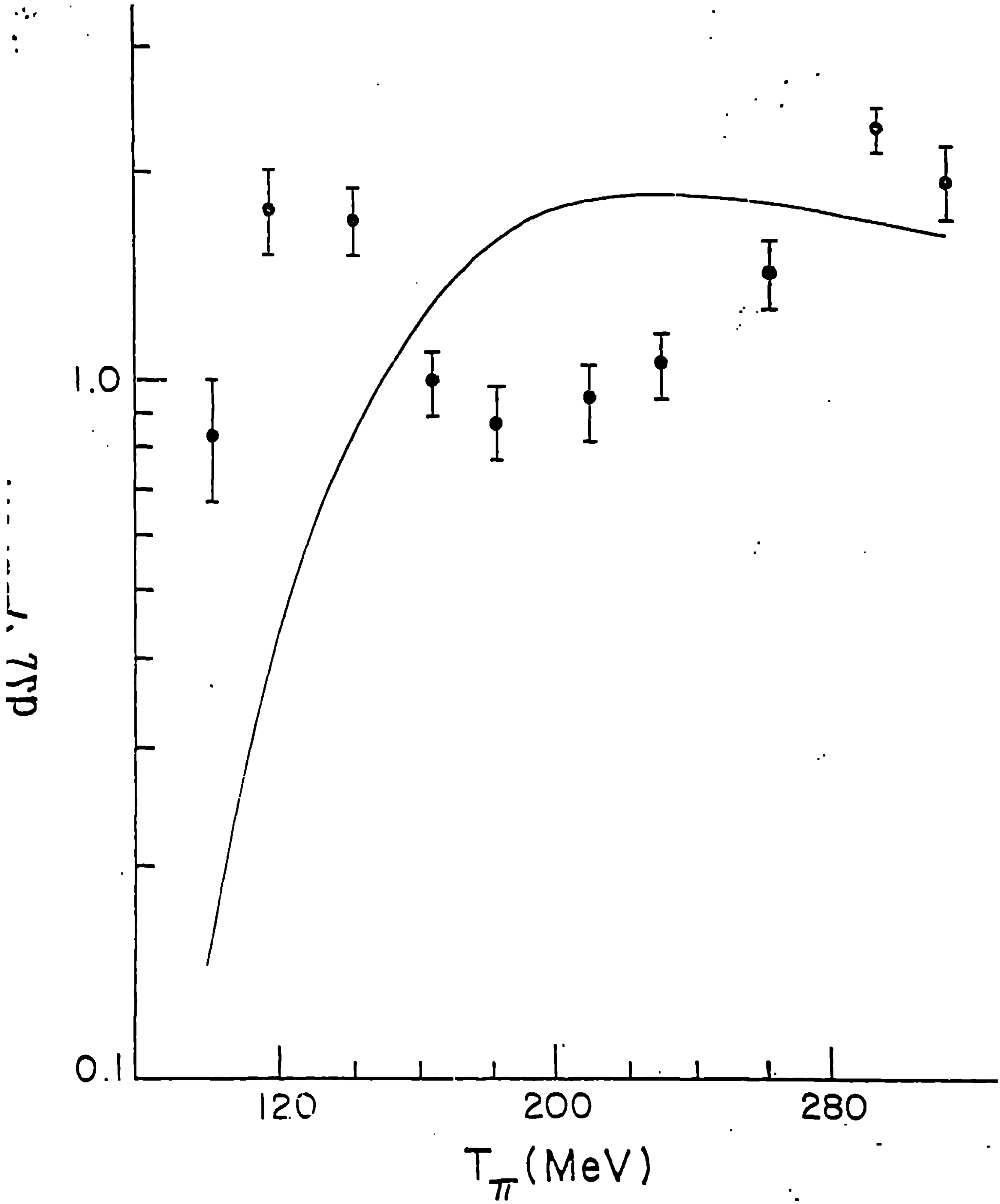

40
30
20
10
0
-10
-20
MeV
-RE <U>
IM <U>
50
100
150
200
T_π(MeV)



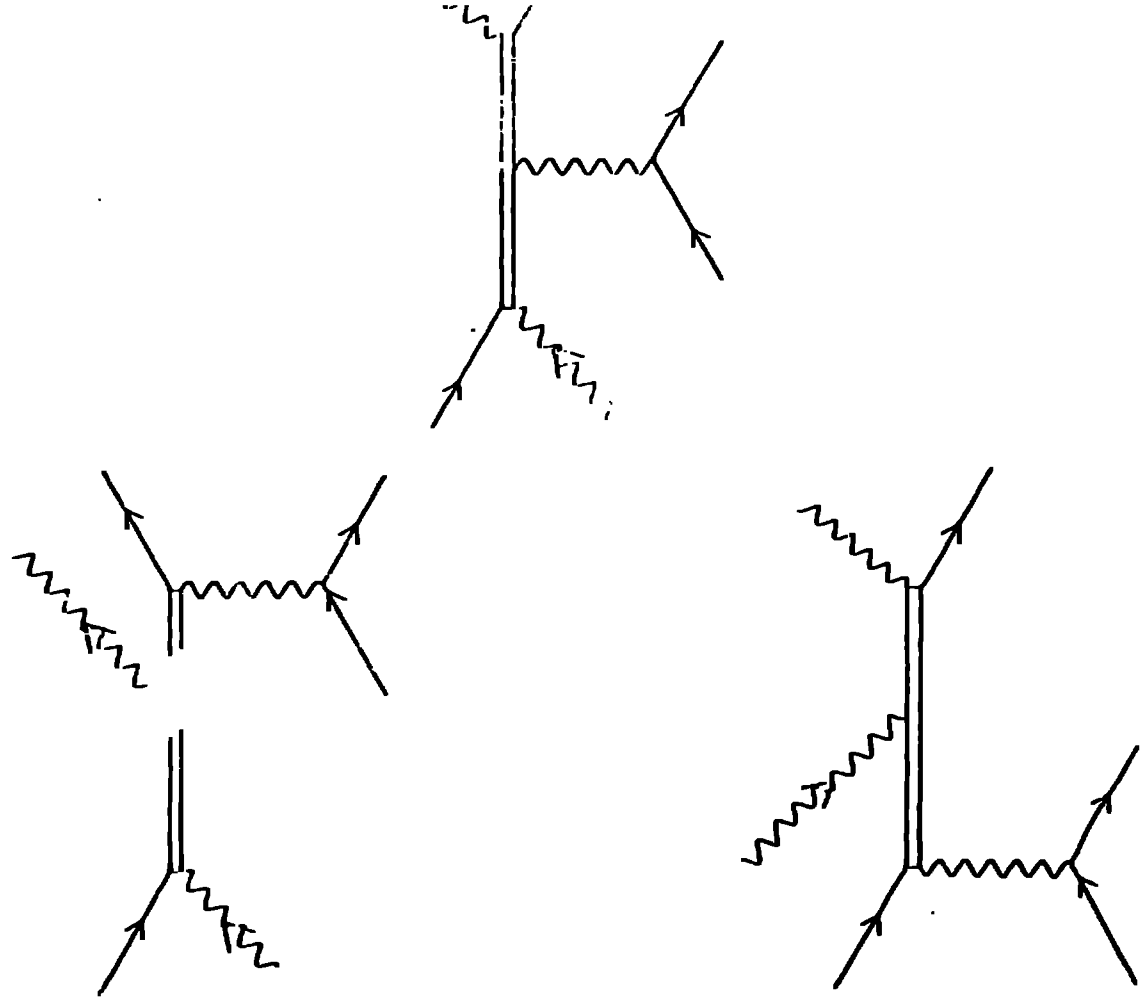
$\frac{d\sigma}{d\Omega}$ $\left(\frac{mb}{sr}\right)$
10²
10
1
0.1
x 100
20
40
60
80
100
120
140
θ_{cm} (deg)



1.0
0.1
0.01
σ(θ = 5°) in μb/sr
^{12}C
^{16}O ÷ 2
^{24}Mg ÷ 5
100
140
180
220
T_π (MeV)
^{28}Si x 4
^{32}S x 2
^{40}Ca
100
140
180
220
T_π (MeV)



1.0
0.1
120
200
280
T_π (MeV)



+ CROSSED + EXCHANGE