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## GEOMETRICAL SCALING, FURRY BRANCHING AND MINIJETS

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### ABSTRACT

Scaling properties and their violations in hadronic collisions are discussed in the framework of the geometrical branching model. Geometrical scaling supplemented by Furry branching characterizes the soft component, while the production of jets specifies the hard component. Many features of multiparticle production processes are well described by this model.

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## I. INTRODUCTION

Another possible title of this talk, though a less explicit one, would be: "An Amalgamation of the Conventional Wisdom in High-Energy Collisions". What is to be presented is a framework to describe a broad range of empirical features in hadronic collisions. They include: (a)  $\sigma_{\text{el}}$ , (b)  $\sigma_{\text{tot}}$ , (c)  $d\sigma/dt$ , (d)  $s$  dependence of  $\sigma_{\text{el}}/\sigma_{\text{tot}}$ , (e) geometrical structure of hadrons, (f) soft and hard components in  $\sigma_{\text{inel}}$ , (g) KNO scaling and nonscaling, (h) increase in  $\langle p_T \rangle$ , (i) rapidity-interval dependence of  $P_N$ , (j) forward-backward correlation, (k) jets, etc. All these properties are interrelated, so a framework that can accommodate all of them in a natural way is likely to have captured the essence of the physics of hadronic collisions at high energy.

The broad range of features mentioned above are manifestations of different properties of a hadron that are revealed under different conditions. A hadron has a geometrical size and consists of point-like constituents. The geometrical aspect results in some dimensionful quantities such as cross sections and  $\langle p_T \rangle$ , that have nontrivial  $s$  dependences. On the other hand, the masslessness of the gluon results in the dominance of soft interaction in the central region; the statistical nature of the many-body problem is revealed in the scaling properties of certain observables due to the absence of any scale in that aspect of the problem. But the scaling is broken by hard scattering which contains a basic QCD scale.

The above features of strong interaction, when considered separately, have been described by various theoretical constructs. In connection with the geometrical aspect of hadrons, the constructs are matter density, form factors, bag model, lattice gauge theory, etc. To emphasize the constituent aspect of the hadrons, we have the parton model, structure functions, valon model, and perturbative QCD. For the  $s$ -dependence of measurable quantities with dimension, we use the eikonal formalism dispersion relations, Pomeron theory, diffractive excitation, and we investigate the notion of geometrical scaling and its violation. Finally, for the multiplicity distribution of particles produced, stochastic processes and quantum statistics have been considered as possible means to describe the many-body system, for which a detailed dynamical analysis would be unfeasible. This last point is rather controversial and represents only one of the possible views<sup>1)</sup>, whereas there are far less controversies on the other three points. A successful description of hadronic interaction should incorporate all those theoretical constructs and fit them together in a cohesive way. Thus instead of a deductive analysis from a fundamental theory, the approach here is an inductive synthesis that is guided by phenomenology.

## 2. SCALING AND ITS VIOLATION

To construct a general framework it is essential to recognize first the important characteristics of the observables as energy is increased. Up to the top of the CERN-ISR energies, the following quantities have been found to be approximately constant. (a)  $\langle p_T \rangle_\pi \approx 350$  MeV, (b)  $\sigma_{el}/\sigma_{tot} \approx 0.17$ ,<sup>2)</sup> (c)  $B(0)/\sigma_{tot} \approx 0.3$  GeV<sup>-2</sup>/mb,<sup>3)</sup> where  $B(0)$  is the slope of the elastic peak at  $t = 0$ , (d)  $\langle n \rangle P_n$  vs.  $n/\langle n \rangle$ , i.e. KNO scaling, and (e) forward-backward multiplicity correlation,  $\langle n_B \rangle = A + B n_F$ ,  $B \approx 0.14$  for  $|\eta| > 1$ .<sup>4)</sup> In the CERN-S<sup>pp</sup>S collider energy region all of the above five scaling properties are broken. At  $\sqrt{s} = 540$  GeV, in particular, it is found: (a)  $\langle p_T \rangle_\pi \approx 430$  MeV,<sup>5)</sup> (b)  $\sigma_{el}/\sigma_{tot} \approx 0.21$ ,<sup>6)</sup> (c)  $B(0)/\sigma_{tot} \approx 0.25$ ,<sup>2)</sup> (d) violation of KNO scaling,<sup>7)</sup> and (e)  $B \approx 0.41$ .<sup>8)</sup>

The qualitative explanations that one can give to these non-scaling behaviors are, respectively: (a) more large-angle scattering, (b) more absorptive, (c) same, (d) non-stochastic process becoming important, and (e) enhanced effect of impact-parameter smearing. The question is then: what accounts for all these changes occurring at about the same energy? The answer in my view is the production of jets. When hard scattering of partons takes place, one would naturally expect: (a) increase in  $\langle p_T \rangle$ , (b) increase in absorption, (c) absorptive point-like interaction that does not affect geometrical size, (d) increase in  $\langle n \rangle$  due to jet fragmentation, and (e) virtuality smearing in addition to impact-parameter smearing. Thus, qualitatively the production of jets provides a good explanation for the violation of scaling of all the phenomena mentioned. The problem is then the quantitative calculation of the scaling properties and their demise due to jets.

The effect of jets is a subject that has already been investigated extensively.<sup>9-14)</sup> What is different about this consideration? The answer is centered around the issue of what a jet is. If it is defined by an arbitrarily chosen  $p_T$  cut-off, then one person's jet may be another person's low- $p_T$  background. If  $p_T^{\text{cut}}$  is defined by fitting  $\sigma_{tot}$ , then it depends on what one assumes for the no-jet component of  $\sigma_{inel}$ . My view is that we should focus on the low- $p_T$  scaling part first before considering the non-scaling jet contribution.<sup>15)</sup>

Since the consideration of jets necessitates the introduction of a scale parameter  $p_T^{\text{cut}}$  (in addition to the QCD  $\Lambda$  parameter), one should expect the breaking of scaling of some quantity which is energy independent before jets become important. We identify that quantity to be the dimensionless ratio  $\sigma_{el}/\sigma_{tot}$  whose constancy is usually referred to as geometrical scaling.<sup>2,3)</sup> The constancy of  $B(0)/\sigma_{tot}$  is related. The range of energy through which geometrical scaling is found to be valid is not very wide, roughly  $10 \leq \sqrt{s} \leq 65$  GeV. At lower energies there are resonances and other complications. At the next available higher energy, 200 GeV, the scaling is already broken. Nevertheless, we should regard it as a significant foothold that plays a role similar to that of an unperturbed Hamiltonian. Our working principle is then:

geometrical scaling defines the soft component, and it is broken by hard processes. This is how we define our two components. Our task is not only to formulate this model, but also to show that the breaking of the scaling of the other quantities mentioned above follows naturally and their  $s$  dependences are quantitatively correct.

### 3. GEOMETRICAL BRANCHING MODEL (GBM)

Before discussing scaling violation, we must first consider scaling: geometrical scaling, KNO scaling, and  $\langle p_T \rangle$  scaling. At the present stage of our ignorance about strong interaction in hadronic collisions, we have no first-principle explanation for those three scaling properties. What we can do is to build them into a single framework. Our focus here will be on the first two scaling properties, since the last one on  $p_T$  can easily be incorporated. It should be noted that geometrical scaling involves cross sections, while KNO scaling involves multiplicity distributions. For the former we use the eikonal formalism, and for the latter we treat multiparticle production as a stochastic branching process.<sup>16)</sup> In Ref. 16 I have attempted to advance the point of view that Furry branching may be the unifying description of all production processes, both hard and soft. The amalgamation of the eikonal scattering and Furry branching results in the geometrical branching model (GBM).<sup>17)</sup>

In the eikonal formalism the inelastic cross section is expressed in terms of the eikonal function,  $\Omega(s, b)$ , as

$$\sigma_{in}(s) = \pi \int_0^{\infty} db^2 [1 - e^{-2\Omega(s, b)}] \quad (3.1)$$

For geometrical scaling the impact parameter  $b$  is written as  $b_0(s)R$ , where  $R$  is the scaled parameter, on which  $\Omega$  depends exclusively. Thus (3.1) becomes

$$\sigma_{in}(s) = \sigma_0(s) \int_0^{\infty} dR^2 [1 - e^{-2\Omega(R)}] \quad (3.2)$$

where  $\sigma_0 = \pi b_0^2$ . Since  $\sigma_{el}$  and  $\sigma_{tot}$  are proportional to the same  $\sigma_0(s)$ , their ratio is therefore  $s$  independent. The multiplicity distribution  $P_n(s)$  is an impact-parameter smearing of the Furry distribution

$$P_n(s) = \int_0^{\infty} dR^2 [1 - e^{-2\Omega(R)}] F_n^{k(R)}(s) \quad (3.3)$$

where<sup>16,18)</sup>

$$F_n^k = \frac{\Gamma(n)}{\Gamma(k)\Gamma(n-k+1)} \left(\frac{1}{w}\right)^k \left(1-\frac{1}{w}\right)^{n-k} \quad (3.4)$$

The parameter  $k$  denotes the number of initial clusters before branching, and  $w = \bar{n}/k$  is an evolution parameter that depends mildly on  $s$ .

Space does not permit an adequate description here of how KNO scaling is achieved for  $P_n$ . Suffice it to mention that the dispersion in fluctuation for (3.4) is

$$\sigma^2 = \bar{n}^2 - \bar{n}^2 = (w-1)/\bar{n} \quad (3.5)$$

which can possibly be  $s$  independent, if  $w$  increases with  $\bar{n}$ . What the actual variation should be depends on the  $R^2$ -smearing of  $k(R)$  in (3.3). Therein lies one of the central issues in the geometrical approach: what is the productivity of particles at each  $R$ ? This question has been investigated in Ref. 17. At the expense of using a free parameter we have been able to obtain an  $s$ -independent  $\langle n \rangle P_n$  versus  $n/\langle n \rangle$ , which happens also to have the correct KNO shape. Although this result is achieved by data-fitting, it is by no means obvious beforehand that such a fit is necessarily possible. But, more importantly, without any further adjustments it is possible to show that the forward-backward multiplicity correlation predicted in the GBM is exactly correct.<sup>15)</sup>

What we have learned in this model, besides being able to fit all the scaling behaviors in the ISR energy range with one parameter, is that

$$k(R) \propto [\Omega(R)]^\gamma \quad (3.6)$$

where  $\gamma$  is the adjustable parameter that has the value 0.27. The eikonal function,  $\Omega(R)$ , connects the GBM to the extensive work that has already been done on elastic scattering and its diffraction peak.<sup>19)</sup> The value of  $k(R)$  decreases with  $R$  in such a way that at large  $R$  [say around  $R^2 = 2.5$  where the inelasticity factor in the square bracket in (3.1) is roughly 0.1]  $k(R)$  becomes about 2, but at small  $R$ ,  $k(R)$  increases to around 6 to 7 near  $R = 0$ . Because of branching the multiplicity distribution at each  $R$  is quite wide; in this respect it is quite different from that for  $e^+e^-$  annihilation processes.

Thus the GBM is a framework that relates the description of the scattering cross sections in the geometrical picture on the one hand to the description of particle production as stochastic branching processes on the other. In addition to possessing all the scaling properties and the F/B correlation behavior, it also yields the correct rapidity-window dependence of the multiplicity distribution.<sup>20)</sup>

#### 4. THE GBM WITH JETS

When  $s$  is increased into the range of the CERN-S $\bar{p}$ pS collider, the production of jets breaks all scaling behaviors. In the eikonal formalism  $\sigma_{el}$  and  $\sigma_{tot}$  have the expressions

$$\sigma_{el}(s) = \sigma_0(s) \int_0^\infty dR^2 [1 - e^{-\Omega(s, R)}]^2 \quad (4.1)$$

$$\sigma_{tot}(s) = 2\sigma_0(s) \int_0^\infty dR^2 [1 - e^{-\Omega(s, R)}] \quad (4.2)$$

where  $\Omega(s, R)$  now has two components

$$\Omega(s, R) = \Omega_0(R) + \Omega_1(s, R) \quad (4.3)$$

It is a major point in our approach to assume that the soft part,  $\Omega_0(R)$ , is the geometrical-scaling component even when  $s$  is above the "threshold" for substantial jet production. This does not mean that the soft-components of any of the cross sections satisfy geometrical scaling, since unitarity mixes up the soft and hard contributions to the inelastic cross section. The hard component,  $\Omega_1(s, R)$ , is directly proportional to the jet cross section, for whatever criteria one may adopt for the definition of a jet. On the basis of probability argument,<sup>12)</sup> one has<sup>15)</sup>

$$\Omega_1(s, R) = \frac{\sigma_{jet}(s) \Omega_0(R)}{2\sigma_0(s) \int dR^2 \Omega_0(R)} \quad (4.4)$$

Using (4.1) to (4.4) one can calculate  $\sigma_{el}$  and  $\sigma_{tot}$  as functions of  $\sigma_0$  and  $\sigma_{jet}$ , with reliance on some given  $\Omega_0(R)$  determined from the ISR data. From the known data on  $\sigma_{el}$  and  $\sigma_{tot}$ ,<sup>6)</sup> the values of  $\sigma_0$  and  $\sigma_{jet}$  can then be fixed. The latter in turn determines the minimum parton transverse momentum  $k_T^{\min}$  that defines a jet in perturbative QCD.<sup>21)</sup> Note that in this scheme the jet is not defined by an arbitrary choice of  $p_T^{\text{cut}}$ , as done in an experimental procedure<sup>5)</sup>. It is the concept of the violation of geometrical scaling by jets that enables us to infer the scale parameter  $k_T^{\min}$ . In Ref. 2 we have found that  $\sigma_{jet}$  at 540 GeV is 24 mb, and that the corresponding value of  $k_T^{\min}$  is 2.7 GeV in lowest order perturbative QCD. What turns out is that for constant  $\sigma_0$  at 40 mb and the same fixed value of  $k_T^{\min}$ , the calculated cross sections in the GBM with jets agree with the data not only throughout the S $\bar{p}$ pS energy range but also into the cosmic ray region above  $10^4$  GeV.

The inelastic cross section breaks up into the soft,  $\sigma^s$ , and hard,  $\sigma^h$ , components as follows:<sup>15)</sup>

$$\sigma^s(s) = \sigma_0(s) \int_0^\infty dR^2 [1 - e^{-2\Omega_0(R)}] e^{-2\Omega_1(s, R)} \quad (4.5)$$

$$\sigma^h(s) = \sigma_0(s) \int_0^\infty dR^2 [1 - e^{-2\Omega_1(R)}] \quad (4.6)$$

The  $n$ -particle cross sections are<sup>15)</sup>

$$\sigma_n = \sigma_{in} P_n = \sigma_n^s + \sigma_n^h \quad (4.7)$$

$$\sigma_n^s = \sigma^s P_n^s = \sigma_0 \int_0^\infty dR^2 [1 - e^{-2\Omega_0}] e^{-2\Omega_1} F_n^k \quad (4.8)$$

$$\sigma_n^h = \sigma^h P_n^h = \sigma_0 \int_0^\infty dR^2 [1 - e^{-2\Omega_1}] H_n \quad (4.9)$$

In (4.8) it is the Furry branching distribution that describes the multiplicity distribution of the soft process at each  $R$ , while in (4.9) the hard distribution  $H_n$  involves jet fragmentation and initial-state bremsstrahlung, as well as particle production at low  $p_T$  associated with jets. Together they form a two-component description of how the KNO scaling is violated.<sup>21)</sup>

## 5. COMMENTS

We have presented a framework to describe the strong interaction of multiparticle production. The many attributes of this framework are:

- (a) Hadrons have sizes.
- (b) Low- $p_T$  production is described by Furry branching.
- (c) For  $\sqrt{s} < 100$  GeV, it possesses scaling properties: (i) geometrical scaling, (ii) KNO scaling, and (iii) constant  $\langle p_T \rangle$ .
- (d) Eikonal formalism for  $d\sigma/dt$  and various cross sections.
- (e) KNO distribution  $\psi(z)$  when scaling is valid.
- (f) A relationship between opacity and particle productivity at each  $R$ .
- (g) Forward-backward multiplicity correlation.
- (h) Hard scattering of partons as described by perturbative QCD.
- (i) Two-component description of hard and soft processes with geometrical scaling

characterizing the soft component.

- (j) Scaling violation of (c), (e) and (g) for  $\sqrt{s} > 100$  GeV.
- (k)  $\sigma_{\text{el}}$  and  $\sigma_{\text{tot}}$  increase due to enhanced absorption.
- (l) Correct rapidity-window dependence.

An ultimate description of strong interaction must address all the issues mentioned here, which evidently are intricately interrelated. Any understanding of elastic and diffractive scattering must therefore at some level also confront these issues. We have already seen how  $\sigma_{\text{el}}$  enters into our framework. The multiparticle production process that is described in this framework characterizes the nature of the Pomeron, which in turn specifies the properties of diffractive scattering, the topic of this conference. In view of the complex structure of our framework and the multitude of physics issues involved in the multiparticle intermediate state when the Pomeron is cut, it is very difficult to state precisely what a Pomeron is. It is therefore far from clear what one means by the structure function of a Pomeron when used in connection with the description of a diffractive hard process. Nevertheless, the notion of a factorizable Pomeron is not inconsistent with data and is operationally convenient. The relationship between such a description of diffractive scattering and the geometrical description of hadronic collisions as advanced here is one of the foremost problems in the theory of strong interaction that is urgently in need of a deeper understanding.

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#### REFERENCES

1. For an overview, see Multiparticle Production, Proc. of the Shandong Workshop, Jinan, China, 1987, edited by R.C. Hwa and Q.-B. Xie (World Scientific, Singapore, 1988).
2. A.J. Buras and J. Dias de Deus, Nucl. Phys. B71, 481 (1974).  
U. Amaldi and K.R. Schubert, Nucl. Phys. B166, 301 (1980).
3. P. Kroll, in Elastic and Diffractive Scattering, edited by B. Nicolescu and J. Tran Thanh Van (Editions Frontieres, France, 1985), p. 165.

4. S. Uhlig et al., *Nucl. Phys. B132*, 15 (1978).
5. B. Arnison et al. (to be published); G. Piano-Mortari, in Proc. of the Oregon Meeting, Annual Meeting of the DPF, Eugene, 1985, edited by R. C. Hwa (World Scientific, Singapore, 1986), p. 615.
6. M. Bozzo et al., *Phys. Lett. 147B*, 392 (1984); M. Haguenauer, in Elastic and Diffractive Scattering, loc. cit.
7. G.J. Alner et al., *Phys. Lett. 160B*, 193 (1985); *167B*, 476 (1986).
8. K. Alpgard et al., *Phys. Lett. 123B*, 361 (1983).
9. T. K. Gaisser and F. Halzen, *Phys. Rev. Lett. 54*, 1754 (1985); T.K. Gaisser, F. Halzen, A.D. Martin and C.J. Maxwell, *Phys. Lett. 166B*, 219 (1986).
10. G. Pancheri and C. Rubbia, *Nucl. Phys. A418*, 117 (1984); G. Pancheri and Y. Srivastava, *Phys. Lett. 159 B*, 69 (1985); 5th Topical Workshop on  $p\bar{p}$  Collider Physics, edited by M. Greco (World Scientific, Singapore, 1985), p. 505.
11. A. Capella, J. Tran Thanh Van, and J. Kwiecinski, *Phys. Rev. Lett. 58*, 2015 (1987).
12. L. Durand and H. Pi, *Phys. Rev. Lett. 58*, 303 (1987).
13. P. l'Heureux, B. Margolis and P. Valin, *Phys. Rev. D32*, 1681 (1985).
14. J. Dias de Deus and J. Kwiecinski, CERN-TH. 4759 (1987); J. Dias de Deus, CERN-TH 4815 (1987).
15. R.C. Hwa, OITS-354, *Phys. Rev.* (to be published).
16. R.C. Hwa, OITS-374, to be published in Hadronic Multiparticle Production, edited by P. Carruthers (World Scientific, Singapore, 1988).
17. W.R. Chen and R. C. Hwa, *Phys. Rev. D36*, 760 (1987).
18. W.H. Furry, *Phys. Rev. 52*, 569 (1937); N. Arley, Stochastic Processes and Cosmic Radiation, (J. Wiley, New York, 1948).
19. T.T. Chou and C.N. Yang, *Phys. Rev. 170*, 1591 (1968); *Phys. Rev. Lett. 20*, 1213 (1968); A.W. Chao and C.N. Yang, *Phys. Rev. D8*, 2063 (1973); T.T. Chou and C.N. Yang, *Phys. Lett. 116 B*, 301 (1982); *ibid. 128B*, 457 (1983).
20. R.C. Hwa, OITS-369, *Phys. Rev.* (to be published).
21. W.R. Chen and R.C. Hwa, OITS-372.

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